VOLUME-BASED REPRESENTATION OF THE MAGNETIC FIELD

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Abstract

The access to the magnetic field has a large impact on both CPU performance and accuracy of simulation, reconstruction and analysis software.

An approach to the magnetic field access based on a volume geometry is described. The volumes are constructed in such a way that their boundaries correspond to field discontinuities, which are due to changes in magnetic permeability of the materials. The field in each volume is continuous.

The value of the field at a given point of a volume is obtained by interpolation from a regular grid of values resulting from a TOSCA calculation or, when it is available, from a parameterization.

To allow global access to the magnetic field, a volume finding algorithm that exploits explicitly the layout and the symmetries of the detector is used.

The main clients of the magnetic field, which are the simulation (GEANT) and the propagation of track parameters and errors in the reconstruction, can be made aware of the magnetic field volumes by connecting the per-volume magnetic field providers to the corresponding volume in the respective geometries. In this way the global volume search is by-passed and the access to the field is sped up significantly.

INTRODUCTION

The Compact Muon Solenoid (CMS) [1] is based on a large superconducting solenoid, allowing the measurement of the momentum of charged particles based on the bending of their trajectories in the central region and within the iron return yoke. The layout of the CMS detector is illustrated in Fig. 1.

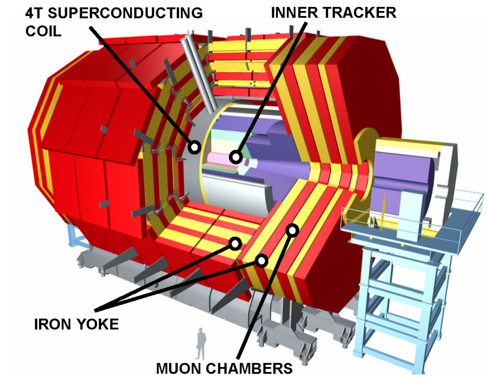

Figure 1: Layout of the CMS Detector.

A precise knowledge of the magnetic field over the full detector volume is essential for the proper simulation and reconstruction of events. In a complex detector, the access to the magnetic field is performed hundreds, even thousands, of times during the simulation or the reconstruction of a charged particle track; It can take a significant fraction of the total execution time. Optimizing the magnetic field access is therefore a worthwhile task.

The magnetic field is accessed dominantly from the code solving the equations of motion of a charged particle, which involves numerical solution of differential equations. The numerical methods for this problem rely on the continuity, and, for higher order methods, on the smoothness of the magnetic field.

In a detector with a central solenoid and iron return yoke like CMS the magnetic field has a physical discontinuity at every iron/air border (more precisely, at every magnetic/non-magnetic material border). In the case of CMS, the iron of the return yoke is instrumented with muon chambers; muon tracking, both for simulation and reconstruction, requires “transport”, or “propagation”, of the track parameters and errors traversing many volumes of iron and non-magnetic materials.

A single global grid for parameterizing the CMS magnetic field with the accuracy required for simulation and reconstruction would be prohibitively large. For example, a single grid implementation in which the retrieved field values are guaranteed not to be wrong by more than 100% over distances of more than 1 mm would require of the order of 10^{11} nodes.

MAGNETIC GEOMETRY

To address these problems a description of the magnetic field based on a description of the magnetic and non-magnetic volumes has been chosen. Volumes of identical magnetic properties are grouped in a single volume if the resulting shape can be described in a simple way. For example, all volumes inside the solenoid are grouped in a single cylinder: the materials used inside the solenoid are required to be non-magnetic. The “magnetic geometry” thus contains the minimal number of volumes required to accurately describe all iron/air boundaries, and the coil of the magnet. In the case of CMS this amounts to 271 simple (non-boolean) volumes for a 30 degrees slice of the detector.

In order to serve as a basis for a field map the magnetic geometry must cover the entire volume of interest (the entire detector) without any holes, and preferably without overlaps.

FIELD CALCULATIONS

The actual computation of the magnetic field of CMS is performed using TOSCA [2]. The TOSCA geometry corresponds exactly to the volume geometry used for field access. In every volume a regular 3D grid is generated corresponding to the shape of the volume [3] (e.g. regular in Cartesian coordinates for a box, and in cylindrical coordinates for a cylinder). Near the iron-air interface the boundary grid points of each volume are placed just inside the volume, with a small tolerance (100 microns). The number of points and the grid step size are optimized for each volume to provide sufficient level of accuracy with a minimal number of points. The current model has 822492 grid nodes. The magnetic field values (the Cartesian components of the field vector) are computed in the post-processor OPERA-3d (an OPerating environment for Electromagnetic Research and Analysis) [2] at each grid node, and stored in a file. The file is converted to a compact binary format optimized for fast reading. The current model uses about 9 MB of data.

A color plot of the magnetic flux density in a section of the CMS detector is shown in Fig. 2.

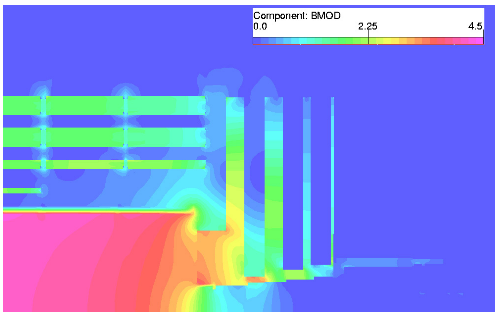

Figure 2: Magnetic flux density (Tesla) in the CMS horizontal plane. The plots covers one quarter of the detector.

FIELD ACCESS DECOMPOSITION

Using this geometry, the problem of magnetic field lookup at a given point in global coordinates is decomposed in:

- the problem of finding the volume (in the magnetic geometry) that contains the point;
- the computation of the field value in the volume;
- the transformation of coordinates between the global and the volume reference frames. This is done with a direct translation and rotation between the two frames.

VOLUME SEARCH

For a generic geometry, it is difficult to solve efficiently the problem of finding the volume that contains a given point. However, practical geometries have regularities that can be exploited to organize the volumes in a hierarchical structure. For example, the CMS barrel region can be organized in cylindrical layers, each one composed of sectors consisting in several rods of adjacent volumes.

Once volumes are organized in such a structure, volume finding is reduced to a simple binning problem for each level of the hierarchy.

Even after dedicated optimization the average CPU time required for volume finding is significant compared to the time spent in the evaluation of the field within the volume. However, the actual access patterns of simulation and reconstruction are very localized: the trajectory of a charged particle is followed for many consecutive steps, resulting in many field queries for the same volume. A simple caching mechanism for the last accessed volume provides about 98% hit rate for both the GEANT4 simulation and the reconstruction of CMS events, substantially reducing the CPU time spent in volume search.

FIELD INTERPOLATION

The simplest way to obtain a continuous magnetic field given a regular grid of known values is linear interpolation. In three dimensions the simplest tri-linear interpolation uses the values at 8 corners of a grid "cube" which contains the point at which the field is evaluated.

The 3D grid cell is really a cube only in the reference frame where the grid step is constant along all three dimensions; the transformations from/to this frame are non-linear in the general case. In principle one could take this non-linearity into account when computing the weight of the 8 values contributing to the interpolation to reflect correctly the true distance from the point to the grid nodes; however, the non-linearities are small at the scale of a single grid cell, and there is no clear gain in in field quality if these additional calculations are performing. Therefore the effect of the grid non-linearity on the interpolated value has been neglected.

Using higher order parameterizations in three dimensions is impractical, since the number of required field values increases rapidly (27 for quadratic interpolation compared to 8 for linear). To achieve a smoother field (including field derivatives) field parameterization is a more promising approach.

FIELD PARAMETERIZATION

The volume approach to the magnetic field access allows the use of a different algorithm for the field computation in different volumes instead of using a linear interpolation everywhere.

The approximation of the field, as defined on the grid nodes, by a parametric function has the advantages of:

- using much less memory (only a few parameters per volume);
- being potentially faster (depending on the parametric function);
- providing a higher degree of smoothness.

A high degree of smoothness is essential for the efficient application of advanced numerical methods for solving the equations of motion. For example, a 4^{th} order Runge-Kutta method loses it's 4^{th} order properties if applied to a field which is smooth only to zeroth order (like the field resulting from linear interpolation).

A full parameterization of the field in all volumes has not yet been studied in detail, but the special case of the central tracking volume, where most of the field accesses during reconstruction occur, has been parameterized successfully.

Parameterization in the Central Tracking Volume The applicability of the ideal solenoid formula in the CMS Tracker volume has been studied. The parameters of the formula are the field value B_0 at the origin, the length L and the radius a of the solenoid. These formulas for B_z and B_r are expressed as infinite series in terms of the derivatives $B_z^{(n)}(0,z)$ of $B_z(0,z)$. The two leading terms of the series expansions are [4]:

$$
\begin{aligned}
B_z(r,z) &= B_z(0,z) - B_z''(0,z)\left(\frac{r}{2}\right)^2 + \cdots \\
B_r(r,z) &= -B_z'(0,z)\left(\frac{r}{2}\right) + \frac{1}{2}B_z'''(0,z)\left(\frac{r}{2}\right)^3 + \cdots
\end{aligned}
$$

Notice that for a constant z the dependence in r is simply polynomial for the two components. The on-axis formula $B_z(0,z)$ can be expressed in closed form as follows:

$$
B_z(0,z) = \frac{1}{2}\, B_0 \sqrt{1+\bar{a}^2}\; [f[(1-\bar{z})/\bar{a}] + f[(1+\bar{z})/\bar{a})]
$$

where $f(x) = x/\sqrt{1+x^2}$. The quantities $\bar{z}$ and $\bar{a}$ are defined as: $\bar{z} = 2z/L$ and $\bar{a} = 2a/L$. The third component B_ϕ is small in the Tracker volume and is neglected.

For a non-ideal solenoid, like the CMS coil, the constants a and L are considered as formal parameters which are fitted so as to match the parameterized field as closely as possible with the finite-element-calculated field map .

The parameterization works well, resulting in per mill accuracy on all components of the field in the entire tracker volume.

RELATION TO OTHER GEOMETRIES

The CMS detector simulation program currently uses the magnetic field via the global reference frame: when a field value is needed at some point, the point is converted to global coordinates, the field is queried, and the resulting field vector is converted back to local coordinates. The magnetic geometry described above can be connected volume-by-volume to the simulation geometry in order to avoid the conversion from/to the global frame and to find directly the magnetic field volume that contains a given simulation volume.

The reconstruction geometry, which is much less detailed than the simulation one, can be connected to the magnetic field geometry in a similar way.

Connecting the application geometries with the magnetic geometry will allow to bypass the volume search and to go directly to the interpolation/parameterization within the magnetic volume.

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