Back to basics: EDMs(θ) and comments on the strong CP problem

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Flambaum, MP, Ritz, Stadnik, 1912.13129 (PRD2020)
R. Bedi, T. Gherghetta, MP 2205.07948, (PRD2022)
Y. Ema, T. Gao, MP, A. Ritz, to appear

Congratulations to Pierre
on a long illustrious impactful career!

Plan

1. Re-assessment of the strong CP problem (still there!)
2. QCD axion vs discrete symmetry as solution to strong CP: some comments.
3. EDM observables induced by theta term:

- New result for electron EDM observables induced by theta term.
- Explanation why lattice QCD has difficulty in predicting $d_n(\theta)$.
- "Correct" choices of current for the lattice or QCD sum rule calculation of $d_n(\theta)$.
- Revisiting QCD sum rule calculations: generalizing earlier calculations to Ioffe current, $\beta = -1$. Consistent results.

Strong CP problem

Energy of QCD vacuum depends on θ-angle:

$$E(\bar{\theta}) = -\frac{1}{2}\bar{\theta}^2 m_* \langle \overline{q} q \rangle + \mathcal{O}(\bar{\theta}^4, m_*^2)$$

where $\langle \overline{q} q \rangle$ is the quark vacuum condensate and m_* is the reduced quark mass, $m_* = \frac{m_u m_d}{m_u + m_d}$. In CP-odd channel,

$$d_n \sim e \frac{\bar{\theta} m_*}{\Lambda_{\rm had}^2} \sim \bar{\theta} \cdot (6 \times 10^{-17})\, e\, {\rm cm}$$

Strong CP problem = naturalness problem = Why $|\bar{\theta}| < 10^{-9}$ when it could have been $\bar{\theta} \sim O(1)$? $\bar{\theta}$ can keep "memory" of CP violation at Planck scale and beyond. Suggested solutions

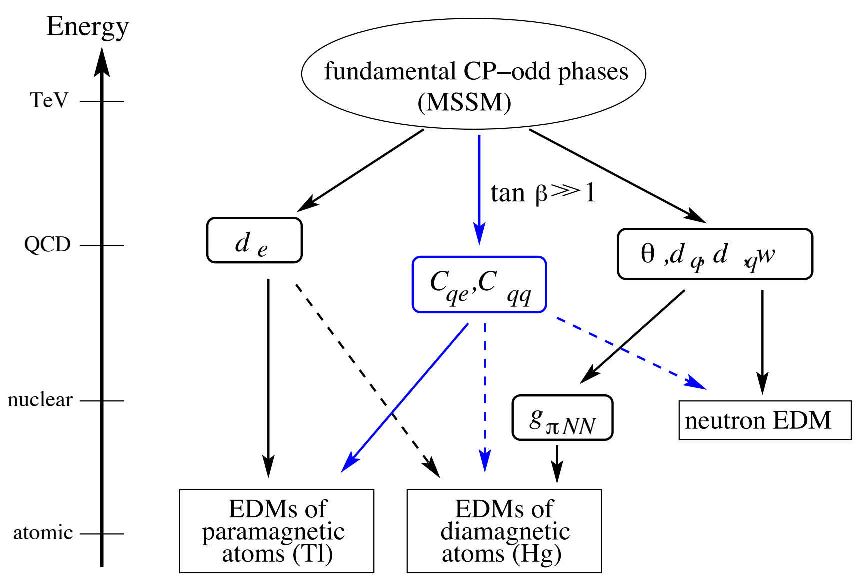
- Minimal solution $m_u = 0 \leftarrow$ apparently can be ruled out by the chiral theory analysis of other hadronic (CP-even) observables.
- $\bar{\theta} = 0$ by construction, requiring either exact P or CP at high energies + their spontaneous breaking. Tightly constrained scenario.
- Axion, $\bar{\theta} \equiv a(x)/f_a$, relaxes to $E = 0$, eliminating theta term. $a(x)$ is a very light field. Not found so far.

BSM physics and EDMs

$$\mathcal{L}_{eff}^{1\mathrm{GeV}} = \frac{g_s^2}{32\pi^2}\,\theta_{QCD} G^a_{\mu\nu}\widetilde{G}^{\mu\nu,a}$$

$$-\frac{i}{2}\sum_{i=e,u,d,s} d_i\,\overline{\psi}_i(F\sigma)\gamma_5\psi_i - \frac{i}{2}\sum_{i=u,d,s}\widetilde{d}_i\,\overline{\psi}_i g_s(G\sigma)\gamma_5\psi_i$$

$$+\frac{1}{3}w\; f^{abc}G^a_{\mu\nu}\widetilde{G}^{\nu\beta,b}G_\beta{}^{\mu,c} + \sum_{i,j=e,d,s,b} C_{ij}\,(\bar{\psi}_i\psi_i)(\bar{\psi}_j i\gamma_5\psi_j) + \cdots$$



ThO, HfF+

- One needs hadronic, nuclear, atomic matrix elements to connect Wilson coefficients to observables

- Extremely high scales [10-100 TeV] can be probed if new physics generating EDMs violates CP maximally.

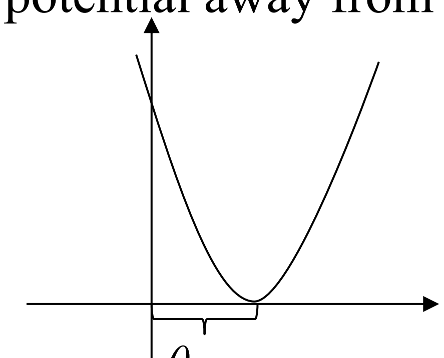
Axion mechanism in the presence of extra CP violation – proper UV decoupling

Imagine that at some scale Λ_{CP} there is some new CP-violating physics with phases δ_{CP}. Integrating it out, we end up with a series of effective CP-odd operators of various dimensions, O_{CP}.

$$\mathcal{L}_{eff} = \frac{\Delta\theta(\delta_{CP})g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} + \sum_q \overbrace{\tilde{d}_q(-i/2)\overline{q}(G\sigma)\gamma_5 q}^{O_5} + \overbrace{c_W(GG\tilde{G})}^{O_6} + \ldots$$

$\Delta\theta$ is an additive renormalization of theta-term and is unobservable in the axion model.

Higher dimensional operators induce axion tadpoles, leading to the minimum of the potential away from $\theta = 0$.



$$\theta_{ind} \propto \frac{\int d^4x T\langle 0|G\tilde{G}(0), O_5(x)|0\rangle}{\int d^4x T\langle 0|G\tilde{G}(0), G\tilde{G}(x)|0\rangle}$$

θ_{ind} will have the decoupling properties, i.e. $\theta_{ind} \rightarrow 0$ as $\Lambda_{UV} \rightarrow$ infinity.

Models based on quasi-exact discrete symmetries have to be "engineered"

- Models where θ is close to zero "by construction" (Parity, CP, mirror symmetry) have to be constructed rather carefully not to be in conflict with the neutron EDM bounds.

- Same refers to models with the so-called "heavy axions" where some BSM physics in the UV enhances topological susceptibility,

$$\chi(0) = -i \lim_{k\to 0} \int d^4x\, e^{ikx} \left\langle 0 \left| T\left\{ \frac{1}{32\pi^2} G\widetilde{G}(x), \frac{1}{32\pi^2} G\widetilde{G}(0) \right\} \right| 0 \right\rangle$$

If there is some scale of e.g. "small instantons", Λ_{SI}, that enhances χ, *and* there is extra amount of CP-violation, e.g. Λ_W^{-2}GGGdual, the induced theta term scales as

$\theta_{ind} \sim (\Lambda_{SI} / \Lambda_W)^2$, which is extremely UV sensitive. (Bedi, MP, Gherghetta, 2022). *No decoupling makes these models less appealing*

Is strong CP problem on solid foundation?

- *Confinement*? After all, all pheno consequences are for hadrons, but the original formulation in terms of [nonperturbative] gluons.

  Addressed by the famous Shifman-Vainshtein-Zakharov paper – at a level of a theorem.

- *Recent papers doubting the existence of strong CP problem*. In the last few years, there were multiple claims that strong CP might be not a real problem: Ai, Garbrecht, Tamarit, 2020; G. Schierholz, 2023,2024; N. Yamanaka, 2022 etc. Contradiction with the SVZ paper is not given a satisfactory explanation.

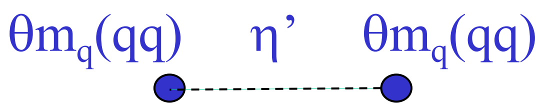
- *Lattice QCD having hard time deriving* $d_n(\theta)$. Starting from Aoki et al. (1990) – onward, Lattice QCD has a difficulty of handling $d_n(\theta)$ calculation

Axion mass and connection to $U(1)_A$

- There are multiple derivations of the the axion mass (aka topological susceptibility) result. The simplest one is using chiral transformation to “move” theta term in front of the quark mass.

$$\mathcal{L}_{QCD} = -\frac{1}{4}(G^a_{\mu\nu})^2 + \sum_{u,d,} \bar{q}(iD_\mu\gamma_\mu - m_q)q + \frac{\theta g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

$$\to m_*(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d)\theta + m_*(\bar{u}u + \bar{d}d)\theta^2/2 + \ldots$$



m_* is the reduced quark mass, $m_u m_d (m_u+m_d)$. The expectation value of the second term over the vacuum here is the vacuum energy dependence on the theta angle (and upon the rescaling the axion mass squared.)

We assume that U(1) problem is solved somehow, and the mass of the singlet is lifted. Otherwise, pole diagram with the singlet will cancel theta dependence.

Linear in $G\tilde{G}$ matrix elements

- Consider a matrix element of $\langle H_1\ \pi\ |\ \mathrm{GGdual}\ | H_2 \rangle$ operator, for the states of a soft pion, where H are arbitrary in- and out- states.

- Chiral PT / current algebra / soft pion theorem allow to "reduce" the pion so that

$\langle H_1\ \pi\ |\ \mathrm{GGdual}\ |\ H_2 \rangle \rightarrow i\,(F_\pi)^{-1} \langle H_1\ \ |\ m_q(uu\text{-}dd)\ |\ H_2 \rangle$. If H_1, H_2 are nucleons, we get a scalar-isovector matrix element, part of the n-p mass splitting.

- In *our world* with light quarks $m_\pi^2 = B\, m_q$ while $m_{\eta'}^2 = B\, m_q + m_0^2$, and heavy mass of η' requires m_0^2 to be large and m_q independent in the limit of large m_q. In an *imaginary world*, where eta-prime is light and $m_0^2 = 0$, there is a second diagram that cancel the first one (SVZ 1980, MP, Ritz 1999)

EDMs induced by θ_{QCD}

- Neutron EDM.

$$d_n \simeq e \times g_A \times (15 \times 10^{-3}\theta)\frac{\log(m_N^2/m_\pi^2)}{8\pi^2 F_\pi},$$

Crewther et al showed logarithmic sensitivity to m_π, and numerically this is ~ few 10^{-16} e cm. $\Theta < 10^{-10}$.

- ^{199}Hg EDM. This is the tightest constraint on atomic EDM, the sensitivity to theta is reduced because one has to use Schiff moment of the nucleus. Similar sensitivity to θ, with different systematics.

- Paramagnetic EDMs (aka electron EDM) – coupling of electric field to an unpaired electron spin. *What is the sensitivity to theta?*

Progress in paramagentic EDMs

$|d_e| < 1.6 \times 10^{-27}$ e cm → $|d_e| < 4.1 \times 10^{-30}$ e cm (HfF+), 1.1×10^{-29}(ThO)

- In the last ~ 10 years, improved by a factor of ~ 400.
- Sensitivity is usually quoted as d_e. Relativistically enhanced as $d_{Atom} \sim Z^3 \alpha^2 d_e$. In reality, d_{Atom} is a linear combination of d_e and a semileptonic operator. Using most sensitive results from ThO and HfF+ molecules, one can limit both sources. Diatomic molecules have strong internal field and can effectively "enhance" modest external E field.
- More progress is real (e.g. ACME III). Most daring proposals want to go down to $d_e \sim 10^{-34}$ e cm.
- Theoretically is the cleanest. Atomic theory is under control at ~ 10% accuracy. In many models - minimum of QCD/nuclear input. SM contributions (θ_{QCD} and δ_{CKM}) were calculated in the last three years. Benchmark CKM value $d_e^{eq} = 1.0 * 10^{-35}$ e cm.

"Paramagnetic" EDMs:

- Paramagnetic EDM (EDM carried by electron spin) can be induced not only by a purely leptonic operator

$$d_e \times \frac{-i}{2}\, \overline{\psi}\sigma_{\mu\nu}\gamma_5 F_{\mu\nu}\psi$$

but by semileptonic operators as well:

$$C_S \times \frac{G_F}{\sqrt{2}}\, \overline{N}N\, \overline{\psi}i\gamma_5\psi$$

- Only a linear combination is limited in any single experiment. ThO 2018 ACME result is:

$|d_e| < 1.1\times10^{-29}$ e cm at $C_S = 0$

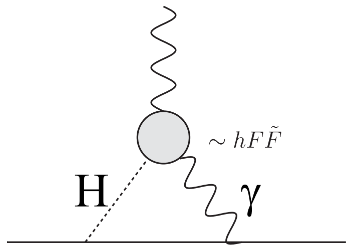
$|C_S^{singlet}| < 7.3\times10^{-10}$ at $d_e = 0$

$d_e^{\rm equiv} = d_e + C_S \times 1.5 \times 10^{-20}\, e\,{\rm cm}$ ← Specific for ThO

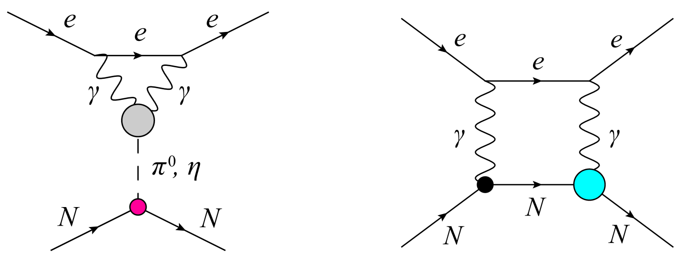
$d_e^{equiv} = d_e + C_S{*}0.9{*}10^{-20}\, e\, cm$ ← Specific for Hf F+

Hadronic CP violation → paramagnetic EDMs

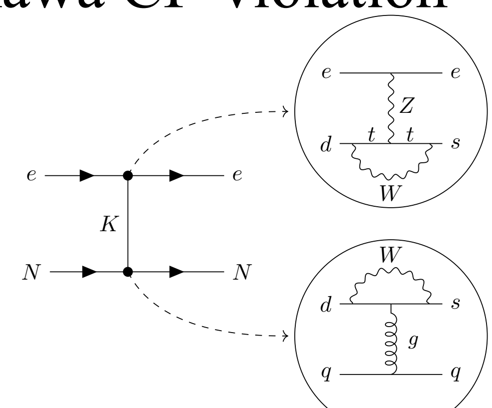
- CP violation in top-Higgs sector – Barr Zee diagrams, h-γ mediation



- Theta term, light quark, mu EDMs -- γ–γ mediation

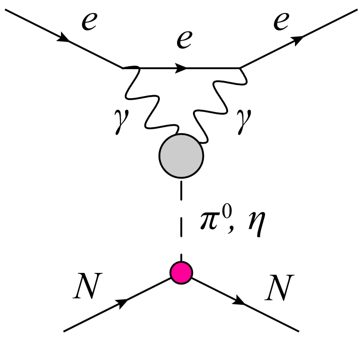


- Kobayashi-Maskawa CP-violation – Z (and WW) mediation



LO chiral contribution:

- T-channel pion exchange gives



$$\mathcal{L} = \theta \times \frac{1}{m_\pi^2} \times 0.017 \times 3.5 \times 10^{-7} (\bar{e} i \gamma_5 e)(\bar{n}n - \bar{p}p)$$
$$= (\bar{e} i \gamma_5 e)(\bar{n}n - \bar{p}p) \times \frac{3.2 \times 10^{-13}\theta}{\text{MeV}^2}.$$

implying $|\theta| < 8.4 \times 10^{-8}$ sensitivity. However, adding exchange of η_8,

$$1 \to 1 - \frac{1}{3}\frac{f_\pi^2 m_\pi^2}{f_\eta^2 m_\eta^2} \times \frac{m_d - m_u}{m_d + m_u} \times \frac{A \times \sigma_N}{\frac{m_d - m_u}{2} \langle p|\bar{u}u - \bar{d}d|p\rangle \times (N - Z)}$$

$$1 \to 1 - 0.88 \simeq 0.12.$$

The effect can completely cancel within error bars on nucleon sigma term σ_N.

Photon box diagrams:

- Diagrams are IR divergent but regularized by Fermi momentum in the Fermi gas picture of a nucleus (intermediate N is above Fermi surface).

$$\mathcal{L} = \bar{e} i \gamma_5 e \bar{N} N \times \frac{2m_e \times 4\alpha \times \overline{d\mu} \times 6.2}{\pi p_F} = \bar{e} i \gamma_5 e \bar{N} N \times 2.4 \times 10^{-4} \times \overline{d\mu}$$

$$\overline{d\mu} = \frac{Z}{A} \mu_p d_p + \frac{A-Z}{A} \mu_n d_n = \frac{e}{2m_p} \times (1.08 d_p - 1.16 d_n)$$

- Nucleon EDM (theta) is very much a triplet, $d_p \simeq -d_n \simeq 1.6 \times 10^{-3} e\mathrm{fm}\theta$

Full answer including chiral NLO. (accidental cancellation of π^0 and η)

$$\boxed{C_{SP}(\bar{\theta}) \approx \left[0.1_{\mathrm{LO}} + 1.0_{\mathrm{NLO}} + 1.7_{(\mu d)}\right] \times 10^{-2} \bar{\theta} \approx 0.03\, \bar{\theta}}$$

Limit on theta term from ThO (electron EDM) experiment:

$$\boxed{|\bar{\theta}|_{\mathrm{ThO}} \lesssim 3 \times 10^{-8}}$$

* Improved by a factor of ~ 2 in Dec 2022, $\theta < 1.5 * 10^{-8}$

Revisiting nonperturbative calculations of d_n

- Use chiral PT, rely on IR enhanced contributions, use some pheno input (or lattice input) to infer π-NN CP-odd couplings. (Crewther, DiVecchia, Veneziano, Witten, ++, 1980++)

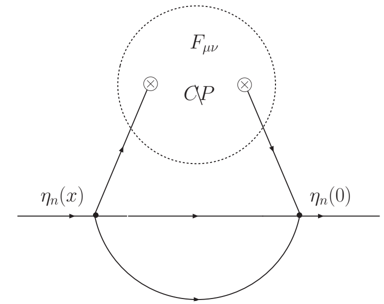
- MP, A. Ritz: 1999-2002: apply QCD sum rules to estimate the OPE coefficients in the external CP-violating and EM backgrounds, including the theta term.

- Preferable direction: set up proper lattice QCD calculations. Various nucleon matrix elements are calculated, but observables that are very sensitive to the quark mass, such as *$d_n(\theta)$ prove to be difficult.*

- *Ema, Gao, MP, Ritz* – to appear. Investigate chiral properties of the correlator of nucleon interpolating currents, re-derive $d_n(\theta)$.

Nonperturbative calculations of nucleon (hadronic) observables

$$\Pi(Q^2) = i\int d^4x\, e^{ip\cdot x}\langle 0|T\{\eta_N(x)\overline{\eta}_N(0)\}|0\rangle_{\not{C}\!\not{P},F,\pi},$$



- Interpolating η currents can be formulated in terms of 3 quarks with appropriate quantum numbers.

- $\Pi(x)$ can be calculated at short distances, using perturbative QCD + nonperturbative condensates. On the other hand, due to quark-hadron duality, we expect that $\Pi(Q^2)$ has also representation in terms of the hadronic resonances and their matrix elements. QCD sum rules *hopes* to match the two at some intermediate/borderline scale, $Q^2 \sim \mathrm{GeV}^2$.

- Lattice QCD can perform these calculations "honestly", $x \rightarrow$ large

Nucleon Interpolating Currents

$$\eta_n = j_1^{(n)} + \beta j_2^{(n)}.$$

$$j_1^{(n)} = 2\epsilon_{ijk}\left(d_i^T \mathcal{C}\gamma_5 u_j\right) d_k, \quad j_2^{(n)} = 2\epsilon_{ijk}\left(d_i^T \mathcal{C} u_j\right)\gamma_5 d_k,$$

- $\beta = 0$, $\eta = j_1$, is the so-called QCD current i.e. the current used the most in the lattice QCD community. It takes its origin in the naïve quark model, because it is j_1 that has a nonrelativistic limit.

- $\beta = -1$ can be called “Ioffe current”, and it has been used the most in various QCD SR literature of 1980s-1990s.

- $\beta = +1$ found to be the most convenient choice (MP and Ritz) for the neutron EDM calculations created by external sources.

Recap of d_n results (QCD SR, $\beta = 1$)

- Use odd-number of γ-matrices for the SR, and spurious phases of the 2-point functions will never appear

- Simple estimate based on the leading term of the OPE has a strong correspondence with the NQM (according to "Ioffe formula", the coefficient outside the square brackets below = 1).

$$d_n^{\rm est} = \frac{8\pi^2|\langle\overline{q}q\rangle|}{m_n^3}\left[-\frac{2\chi m_*}{3}e(\bar{\theta}-\theta_{\rm ind}) + \frac{1}{3}(4d_d - d_u) + \frac{\chi m_0^2}{6}(4e_d\tilde{d}_d - e_u\tilde{d}_u)\right],$$

- Here, c stands for another vacuum condensate, EM susceptibility of the QCD vacuum, $\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle = F_{\mu\nu}\times eQ_q\langle 0|\bar{q}q|0\rangle \times \chi$

Numerically, $\chi \sim -6\ \text{GeV}^{-2}$.

Back to basics: QCD + theta term

$$\mathcal{L}_{QCD} = -\frac{1}{4}(G^a_{\mu\nu})^2 + \sum_{u,d,} \bar{q}(iD_\mu\gamma_\mu - m_q)q + \frac{\theta g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

Do a standard iso-singlet quark chiral rotation to eliminate θGGdual.

$$\rightarrow m_*(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d)\theta + m_*(\bar{u}u + \bar{d}d)\theta^2/2 + ...$$

m_* is the reduced quark mass, $m_u m_d (m_u + m_d)$. The expectation value of the second term over the vacuum here is the vacuum energy dependence on the theta angle (and upon the rescaling the axion mass squared.) Expectation value of the second term over nucleon, gives theta-dependence of nucleon mass.

All observables that depend on θ should also depend on m_* and vanish in the chiral limit! Also, observables do not depend on how you distribute θ, putting some parts to quark mass, and some to GGdual.

QCD + theta term + Nucleon Source

$$\mathcal{L} = -\frac{1}{4}(G^a_{\mu\nu})^2 + \sum_{u,d,} \bar{q}(iD_\mu\gamma_\mu - m_q)q + \frac{\theta g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} + \text{Source} \times (j_1 + \beta j_2) + h.c.$$

- This is the basis for studying nucleon properties. It is almost QCD, but not quite!

- Let us perform a chiral rotation, as on the previous slide. If this transformation would lead to

$$\text{Source} \times (j_1 + \beta j_2) \;\rightarrow\; \text{Source} \times e^{i\alpha\gamma_5} \times (j_1 + \beta j_2)$$

then it is an innocent transformation, and the new phase can be reabsorbed into the source. Otherwise, θ dependence will persist even in $m_q \rightarrow 0$ limit.

- This is true only for β=1 and β=-1 current choices. It is specifically not true for the lattice current β=0. It has unphysical L←→R quark transitions.

Unphysical θ dependence of some correlators

- Under the isosinglet chiral transformation, the lattice current changes

$$j_1 = 2\epsilon^{abc} d^a (d^{bT} C\gamma_5 u^c) \to 2\epsilon^{abc} e^{i\theta\gamma_5} d^a (d^{bT} C\gamma_5 e^{i2\theta\gamma_5} u^c)$$

- This results in a rephasing-invariant theta-dependent pieces in the OPE:

$$\frac{aM^4}{16} \times \left(6(1-\beta^2)e^{i2\theta\gamma_5} + (1-\beta)^2 e^{-i2\theta\gamma_5}\right) \dots$$
$$= |\lambda|^2 e^{i\alpha\gamma_5}(m_N + i\gamma_5 m_{N5}) e^{i\alpha\gamma_5} \times \exp(-m_N^2/M^2),$$

where a is $|\langle \bar{q}q \rangle| \times 4\pi^2$.

- If the correlator $\Pi(M^2)$ is matched to physical observables (e.g. hadron masses, they will acquire θ-dependence in the strict chiral limit.)
- Absolutely same problems will persist in the $d_n(\theta)$ calculation performed with the "lattice current". There will be dependences, in general, on unphysical phases, related to the chirality breaking built into the interpolating current itself.

Repeating $d_n(\theta)$ calculation for Ioffe current

- Original calculation by Ritz and MP was using $\beta = +1$, and a channel with odd number of gamma-matrices so that the EDM correlator is insensitive to the phase of the two-point function:

$$\lambda_I^2 \exp\{i\alpha\gamma_5\} \frac{i}{\not p - m_n - O} \exp\{i\alpha\gamma_5\} = i\lambda_I^2 \exp\{i\alpha\gamma_5\} \frac{(\not p + m_n)O(\not p + m_n)}{(p^2 - m_n)^2} \exp\{i\alpha\gamma_5\}$$

- For $\beta = -1$, one needs to use even number of gamma matrices, and evaluate both two- and three-point function.
- *Assuming* that ground state (i.e. the neutron) contributes dominantly to $\Pi(M^2)$, after some calculations, we derive the sum rule result for this channel:

$$d_n = -\theta\mu_n \left[\frac{3}{4\pi^2} \frac{m_*}{\chi\langle\bar{q}q\rangle} \left(\log(M^2/\Lambda_{IR}^2) - \frac{Q_d}{Q_u} \right) - \frac{2m_*}{m_n} \right].$$

- The results, $\beta = -1$ and $\beta = +1$, are A. having the same sign, B. consistent, C. predict EDM at $O(10^{-16}$ e cm $\theta)$ level, D. Same sign as the chiral estimate answer, a little smaller value.

Conclusions

- Strong CP problem is not challenged by recent works, in my opinion. Standard lore (decoupling of singlet meson creates non-derivative vertices of GGdual) stands.

- Among theoretical approaches to the strong CP problem, axion solution is the most natural/elegant, as it ensures smooth decoupling of heavy scales. Models based on discrete symmetries do not have these properties and are susceptible to extra amount of CP-violating → have to be carefully constructed all the way to the Planck scale.

- The paramagnetic EDMs (*experiments looking for d_e*) are also induced by the semi-leptonic operators of (electron pseudoscalar)*(nucleon scalar) type. C_S is induced by theta term via a two-photon exchange resulting in sensitivity $|\theta| < 1.5\times10^{-8}$. Further progress by O(100) for d_e type of experiments will bring the sensitivity to hadronic CP violation on par with current d_n limits.

Conclusions, continued

- Chiral properties of the nucleons interpolating currents, under $U(1)_A$ rotations, are crucial for obtaining observables such as those dependent on θ, and vanishing in $m_* \rightarrow 0$ limit.
- The "lattice currents" do not transform covariantly under $U(1)_A$ rotations, leading to spurious dependences of correlators on unphysical angles. I.E.: *all existing lattice QCD calculations of $d_n(\theta)$ have a wrong starting point.*
- The physical behavior of nucleon correlators is guaranteed with $\beta=1$ and $\beta=-1$ current choices. We have repeated the $d_n(\theta)$ with $\beta=-1$ current choice, achieving results consistent with Pospelov and Ritz, 1999 (magnitude and sign). We explicitly check that there is no dependence on unphysical phases. *Lattice should learn from this!*