Likhoded A.

IHEP, Protvino

Some problems in heavy quarkonium production

Introduction

- Bound $Q\bar{Q}$ states are similar to QED positronium
- Simple strong-interactive system
- Non-relativistic for $b\bar{b}$

  less for $c\bar{c}$
- small component velocity v

  and small a_s

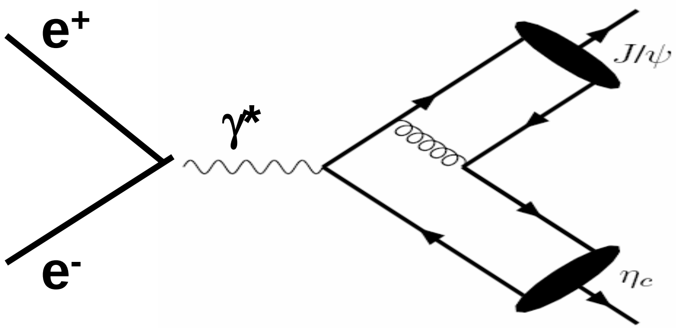
  are the expansion parameters in NRQCD
- The properties of these bound states, their decay and production channels are a good laboratory for QCD both in perturbative and non- perturbative modes
- The simplest system clarifying the transitions $Q\bar{Q}$ ® particles $(J/y, c, g)$

Double charmonium production

One of the most challenging open problems in heavy quarkonium is:

the large discrepancy of the double charmonium production cross sections measured in e^+e^- annihilation at B factories and the theoretical calculations from NRQCD.

Theoretical calculations (LO):



one of 4 diagrams

exclusive double charmonium production via e^+e^- annihilation to a γ^*. two charmonium states with opposite C parities

E. Braaten, J. Lee: **(PRD 67 (2003) 054007)**

$$\sigma(e^+e^- \to J/\psi\,\eta_c) = (2.31 \pm 1.09)\ \text{fb}$$
$$= 5.5^{+10.6}_{-3.5}\ \text{fb (inclu. relativistic correction.)}$$

K.-Y. Liu, Z.-G. He and K.-T Chao:

NRQCD factorization formalism CS dominant

$$\sigma(e^+e^- \to J/\psi\,\eta_c) = 5.5\ \text{fb}$$ **PLB557 (2003) 45**

K. Hagiwara, E. Kou and C.-F.Qiao,

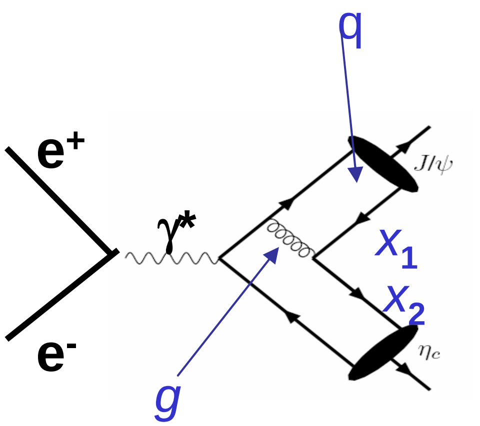
$$\sigma(e^+e^- \to J/\psi\,\eta_c) = 2.3\ \text{fb}$$ **PLB570 (2003) 39**

A. Berezhnoy, A. Likhoded:

$s(e^+e^-$ ® $J/y\ h_c) = 2.6$ fb

- The calculated exclusive cross sections are about an order of magnitude smaller than exp. results!
- The calculated inclusive cross section of J/ψ is about a factor of 5 smaller than exp. results!

- δ-approximation (Bethe-Heitler)



$$M = \int A^{\text{hard}}(p_i, q)\Psi(q)d^3q \approx$$

$$A^{\text{hard}}(p_i, 0)\int \Psi(q)d^3q \approx$$

$$A^{\text{hard}}(p_i, 0)\,\Psi(x)\big|_{x=0}$$

gluon virtuality

$$q^2 = Q^2 x_1 x_2$$

- In δ-approximation

$$q^2 \approx \frac{1}{4}Q^2, \quad x_1 = x_2 = 1/2$$

- In $p_\psi \to \infty$ frame the c-quark distribution over the momentum fraction *x* is

$$c(x) = x^{-\alpha_\psi}(1-x)^{g^2-\alpha_\psi}$$

α_ψ - J/ψ trajectory intercept

$\alpha_\psi = -2.2 \div -3$

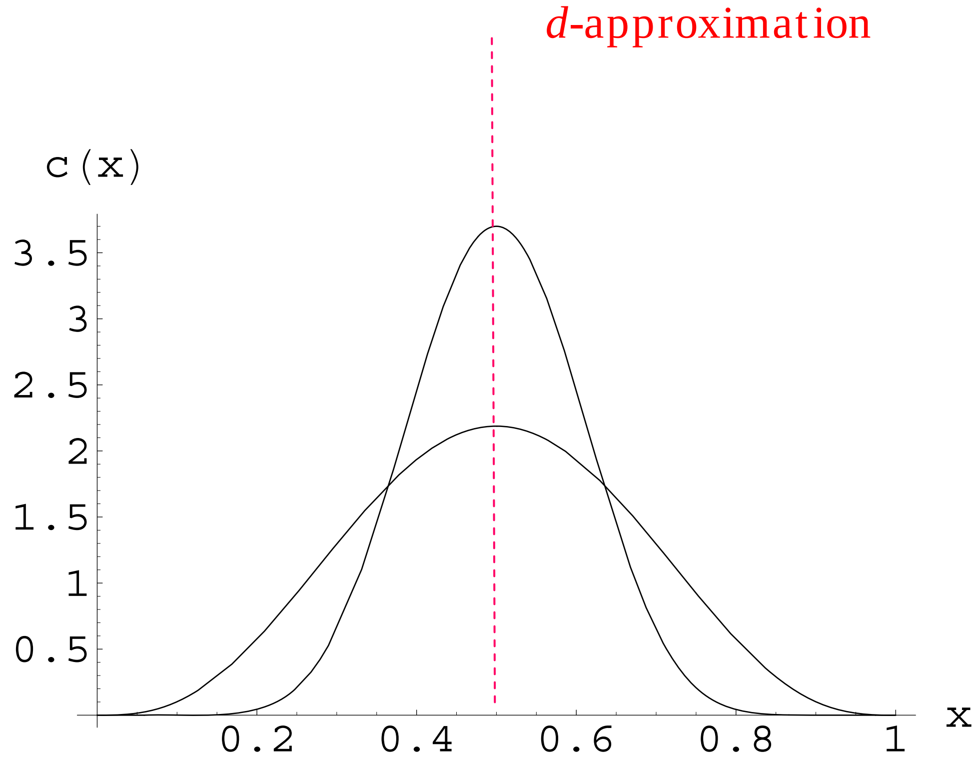
$\langle x_c \rangle = 0.488$

$\langle x_g \rangle = 0.023$

J.P. Ma, Z.G. Si

A.E. Bondar, V.L. Chernyak,

V. Braguta, A. Luchinsky, A. Likhoded



d-approximation
c(x)
3.5
3
2.5
2
1.5
1
0.5
0.2
0.4
0.6
0.8
1
x

Light-cone formalism

processes $e^+e^- \to \psi\,\eta_c,\ \psi'\,\eta_c,\ \psi\,\eta_c'$

$$\sigma\left(e^+e^- \to VP\right) = \frac{\pi\alpha^2}{6}\left|\frac{2|\mathrm{p}|}{\sqrt{s}}\right|^3 |F_{VP}|^2$$

where F_{VP} is defined from

$$\left\langle V(p_1,\lambda),P(p_2)\,|\,J_\mu\,|\,0\right\rangle = \epsilon_{\mu\nu\alpha\beta}\,p_1^\alpha p_2^\beta \epsilon^\nu F_{VP}$$

Leading asymptotic behavior

$$\left\langle M_1(p_1,\lambda_1)M_2(p_2,\lambda_2)\,|\,J_\mu\,|\,0\right\rangle \sim \left(\frac{1}{\sqrt{s}}\right)^{|\lambda_1-\lambda_2|+1}$$

$M_1(p_1,\lambda_1) = V(p_1,\lambda_1)$ $\qquad M_2(p_2,\lambda_2) = P(p_2) \Rightarrow \lambda_2 = 0$

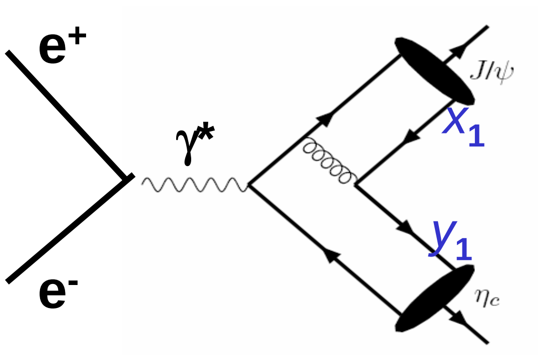
$$\left\langle V(p_1,\lambda_1)P(p_2)\,|\,J_\mu\,|\,0\right\rangle \sim \frac{1}{s}$$

$$V(p,l) = \left\langle V(p,l) \mid \bar{Q}_b(z) Q_a(-z) \mid 0 \right\rangle$$

$$P(p) = \left\langle P(p) \mid \bar{Q}_b(z) Q_a(-z) \mid 0 \right\rangle$$

Light-cone wave functions, that are expressed as the expansion over poorly known wave functions. For example, from the asymptotical Regge behavior

$$x_1^{-a_y} x_2^{-a_y} d(1 - x_1 - x_2)$$



$$d(x_1, y_1) = \left(x_1 + \frac{d}{y_1}\right)\left(y_1 + \frac{d}{x_1}\right)$$

$$s(x_1) = x_1 + \frac{Z_s M_Q}{y_1(1 - y_1)s}$$

$$s(y_1) = y_1 + \frac{Z_s M_Q}{x_1(1 - x_1)s}$$

$$d = \frac{\left(Z_m(k)\bar{M}_Q\right)^2}{s}$$

In d-approximation we have $\frac{1}{4}\frac{1}{s}$

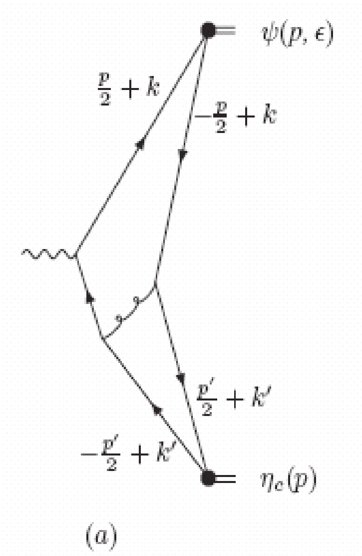
Light-cone wave functions

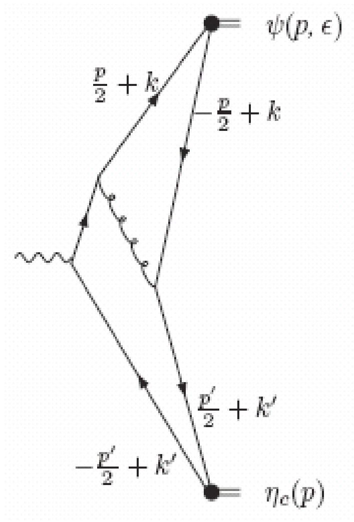
$$P_A, P_P; V_T, V_\perp, V_A, \ldots$$

are unknown.

This is a drawback of the light-cone formalism!

Can be estimated from the wave-functions obtained in the framework of potential models.





Light cone wave functions.

$$\mathbf{J/\Psi, \Psi'}: \quad \langle V_\lambda(p)|\bar{Q}_\beta(z)\, Q_\alpha(-z)|0\rangle = \frac{f_V M}{4}\int_0^1 dx_1\, e^{i(pz)(x_1-x_2)}\Big\{\widehat{e}_\lambda\, V_\perp(x)+$$

$$+\widehat{p}\,\frac{(e_\lambda z)}{(pz)}\,\widetilde{V}(x) + f_v^t(\sigma_{\mu\nu}e_\lambda^\mu\, p^\nu)\, V_T(x) + f_v^a(\epsilon_{\mu\nu\alpha\beta}\gamma_\mu\gamma_5\, e_\lambda^\nu\, p^\alpha z^\beta)\, V_A(x)\Big\}_{\alpha\beta}.$$

$$\eta_c, \eta_c': \quad \langle P(p)|\bar{Q}_\beta(z)\, Q_\alpha(-z)|0\rangle = i\frac{f_P M}{4}\int_0^1 dx_1 e^{i(pz)(y_1-y_2)}\Big\{\frac{\widehat{p}\,\gamma_5}{M}\, P_A(y) -$$

$$- f_p^p\,\gamma_5\, P_P(y) + f_p^t\,(\sigma_{\mu\nu}\, p^\mu\, z^\nu)\, P_T(y)\Big\}_{\alpha\beta}.$$

$$\chi_{c0}: \quad \langle \chi_{c0}(p)|\bar{Q}_\beta(z)\, Q_\alpha(-z)|0\rangle = f_V^{(1)}\frac{M_\chi}{4}\int_0^1 dx_1\, e^{i(pz)(x_1-x_2)}\Big\{\frac{\hat{p}}{M_\chi} S_V(x) + f S_S(x)\Big\}_{\alpha\beta}.$$

The model for the light cone wave functions:

$P_A, P_P, V_T, V_L, V_\perp, V_A$ for $1S$ meson:

$$\phi_i(x, v^2) = c_i(v^2)\, \phi_i^a(x) \left\{ \frac{x_1 x_2}{[1 - 4x_1 x_2 (1 - v^2)]} \right\}^{1-v^2}$$

$P_A, P_P, V_T, V_L, V_\perp, V_A$ for $2S$ meson:

$$\phi_i(x, v^2) = c_i(v^2)\, \phi_i^a(x) \left(1 - 8v^2\beta \frac{(1 - v^2)x_1 x_2}{[1 - 4x_1 x_2 (1 - v^2)]}\right) \left\{ \frac{x_1 x_2}{[1 - 4x_1 x_2 (1 - v^2)]} \right\}^{1-v^2}$$

S_V, S_S for $1P$ meson:

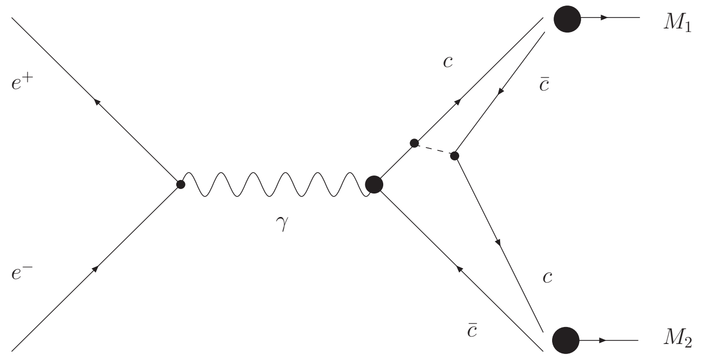
$$\phi_i(x, v^2) = c_i(v^2)\, \phi_i^a(x) \left\{ x_1 x_2 \frac{1 - 2x_1 x_2 (1 - v^2)}{[1 - 4x_1 x_2 (1 - v^2)]^2} \right\}^{1-v^2}$$

The property of the wave functions:

- $v \to 0 \;\Rightarrow\; \varphi(x) \to \delta(x - \frac{1}{2})$
- $v \to 1 \;\Rightarrow\; \varphi(x) \to \varphi^{as}(x)$

A.E. Bondar, V.L. Chernyak, Phys.Lett.B612:215, (2005)

$e^+e^- \to V(^3S_1)\ P(^1S_0)$



$$\langle V(p_1,\lambda), P(p_2)|J_\mu|0\rangle = \epsilon_{\mu\nu\rho\sigma}e^\nu p_1^\rho p_2^\sigma F_{vp}, \quad F_{vp} = \frac{32\pi}{9}\left|\frac{f_V f_P M_P M_V}{q_0^4}\right| I_0\,,$$

$$I_0 = \int_0^1 dx_1 \int_0^1 dy_1 \alpha_s(k^2)\left\{\frac{M_P}{M_V^2}\frac{Z_t Z_p V_T(x) P_P(y)}{d(x,y)\,s(x)} - \frac{1}{M_P}\frac{\overline{M}_Q^2}{{M_V}^2}\frac{Z_m Z_t V_T(x) P_A(y)}{d(x,y)\,s(x)} + \right.$$
$$\left.\frac{1}{2M_P}\frac{V_L(x)\,P_A(y)}{d(x,y)} + \frac{1}{2M_P}\frac{(1-2y_1)}{s(y)}\frac{V_\perp(x)\,P_A(y)}{d(x,y)} + \frac{1}{8}\left(1 - Z_t Z_m^k \frac{4\overline{M}_Q^2}{{M_V}^2}\right)\frac{1}{M_P}\frac{(1+y_1)V_A(x)P_A(y)}{d^2(x,y)}\right\}.$$

$$d(x,y) = \frac{k^2}{q_0^2} = \left(x_1 + \frac{\delta}{y_1}\right)\left(y_1 + \frac{\delta}{x_1}\right), \qquad \delta = \left(Z_m \frac{\overline{M}_Q}{q_0}\right)^2,$$
$$s(x) = \left(x_1 + \frac{(Z_m \overline{M}_Q)^2}{y_1 y_2\, q_o^2}\right), \quad s(y) = \left(y_1 + \frac{(Z_m \overline{M}_Q)^2}{x_1 x_2\, q_o^2}\right),$$

$$Z_p = \left[\frac{\alpha_s(k^2)}{\alpha_s(\overline{M}_Q^2)}\right]^{\frac{-3C_F}{b_o}}, \quad Z_t = \left[\frac{\alpha_s(k^2)}{\alpha_s(\overline{M}_Q^2)}\right]^{\frac{C_F}{b_o}}, \quad Z_m(\mu^2) = \left[\frac{\alpha_s(\mu^2)}{\alpha_s(\overline{M}_Q^2)}\right]^{\frac{3C_F}{b_o}},$$

$e^+e^- \to V(^3S_1)\ S(^3P_0)$

$$\langle V(p_1,e_1)\chi_{c0}(p_2)|J^\mu_{el}|0\rangle = \frac{\pi}{9}\frac{f_V^{(1)} f_V}{s^2} M_V M_S \Big\{ f_1(p_1^\mu - p_2^\mu)(e_1 p_2) + f_2((e_1p_2)p_1^\mu - e_1^\mu(p_1p_2)), \Big\}$$

$$f_1 = \int_0^1 dx_1 \int_0^1 dy_1 \alpha_s(k^2)\Big(-16 Z_1 \frac{S_V(y)V_L(x)}{M_S}\frac{1}{d(x,y)}\Big)$$

$$f_2 = \int_0^1 dx_1 \int_0^1 dy_1 \alpha_s(k^2)\Big(2Z_1\frac{S_V(y)V_A(x)}{M_S}(1 - Z_m Z_t \frac{4m_Q^2}{M_V^2})\frac{1+y_1}{d(x,y)^2} + 16 Z_t Z_1 Z_m \frac{S_V(y)V_T(x)}{s(x)d(x,y)}\frac{m_Q^2}{M_V^2 M_S}$$
$$-8Z_1\frac{S_V(y)}{M_S d(x,y)}(\tilde{V}(x) - V_\perp(x) - \frac{V_\perp(x)}{s(y)}) + 32 Z_p Z_t \frac{S_S(y)V_T(x)}{s(x)d(x,y)}\frac{m_Q}{M_V^2} f_2\Big).$$

$$Z_1 = \left[\frac{\alpha_s(k^2)}{\alpha_s(\overline{M}_Q^2)}\right]^{\frac{8C_F}{9b_0}}.$$

Numerical results.

$e^+e^- \to H_1H_2$	$\sigma_{BaBar} \times Br_{H_2 \to charged>2}(\text{fb})$	$\sigma_{Belle} \times Br_{H_2 \to charged>2}(\text{fb})$	$\sigma_{LO}(\text{fb})$	$\sigma_{NRQCD}(\text{fb})$
$\psi(1S)\eta_c(1S)$	$17.6 \pm 2.8^{+1.5}_{-2.1}$	$25.6 \pm 2.8 \pm 3.4$	26.7	2.3
$\psi(2S)\eta_c(1S)$	$-$	$16.3 \pm 4.6 \pm 3.9$	16.3	1.0
$\psi(1S)\eta_c(2S)$	$16.4 \pm 3.7^{+2.4}_{-3.0}$	$16.5 \pm 3.0 \pm 2.4$	26.6	1.0
$\psi(2S)\eta_c(2S)$	$-$	$16.0 \pm 5.1 \pm 3.8$	14.5	0.4

H_1H_2	$\sigma_{BaBar} \times Br_{H_2 \to charged>2}(\text{fb})$	$\sigma_{Belle} \times Br_{H_2 \to charged>2}(\text{fb})$	$\sigma_{LO}(\text{fb})$	$\sigma_{NRQCD}(\text{fb})$
$\psi(1S)\chi_{c0}$	$10.3 \pm 2.5^{+1.4}_{-1.8}$	$6.4 \pm 1.7 \pm 1.0$	$14.4\ ^{15.5}_{\ 13.3}$	2.3
$\psi(2S)\chi_{c0}$	$-$	$12.5 \pm 3.8 \pm 3.1$	$7.8\ ^{8.3}_{7.3}$	1.0

Uncertainties:

- ❑ Poor knowledge of the light cone wave functions
- ❑ Radiative corrections
- ❑ 1/s corrections

Conclusion:

- ❑ Formfactor = NRQCD x Internal Motion
- ❑ WFs of charmonium are wide ⇨ NRQCD is not applicable

The results of papers: Phys.Rev.D72:094018,2005, Phys.Lett. B 635, 299, 2006

Quark-hadron duality

Let us consider the process

$$e^{+}e^{-} \text{ ® } c\bar{c}c\bar{c}$$

- LO - $\sigma_{tot}(4c) \sim O(\alpha_s)^2$ depends on α_s and m_c

  for m_c=1.25 GeV and α_s=0.24

$$\sigma_{tot}(4c)=372 \text{ fb}$$

In the duality interval of m_{cc} in color singlet

$$2m_c<m_{cc}<2m_D+\Delta, \qquad \Delta=0.5\text{GeV}$$

$$\sigma(e^+e^- \rightarrow (c\bar{c})_{sing}+c+\bar{c})=280 \text{ fb}$$

It should be compared with the sum of S- and P-wave resonances. In δ-approximation

σ(ψ+η+χ)=216 fb

B.K.L Phys.Lett.B 323 (1994) 411

Liu et al PR D69 (2004) 094027

In the same duality interval

$$\sigma(c\bar{c})_{\rm sing}^{S=1} = 204\,\text{fb},$$

$$\sigma(c\bar{c})_{\rm sing}^{S=0} = 76\,\text{fb}$$

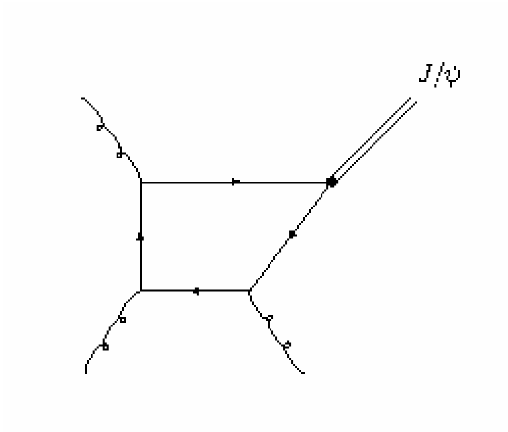
If we restrict region mass $c\bar{c}$ in the process $e^+e^- \to J/\psi + c\bar{c}$ by duality interval

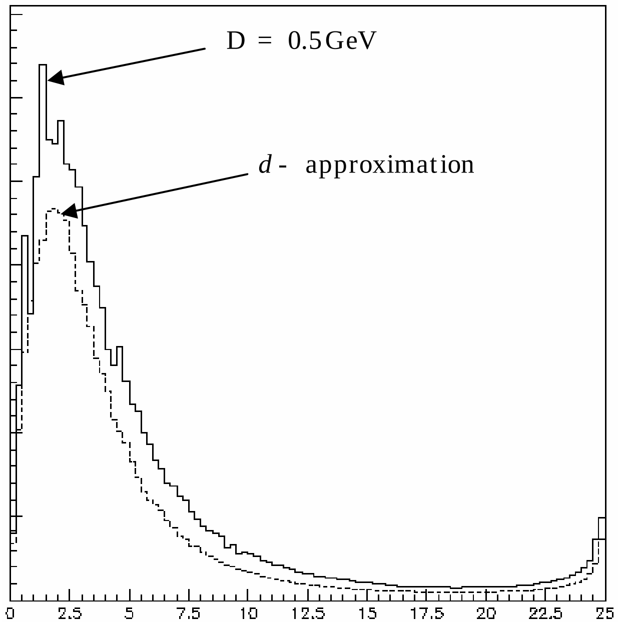
$$\sigma(J/\psi + \text{charmonium}) = 40\,\text{fb}$$

It have to be compared with the Belle and BaBar results

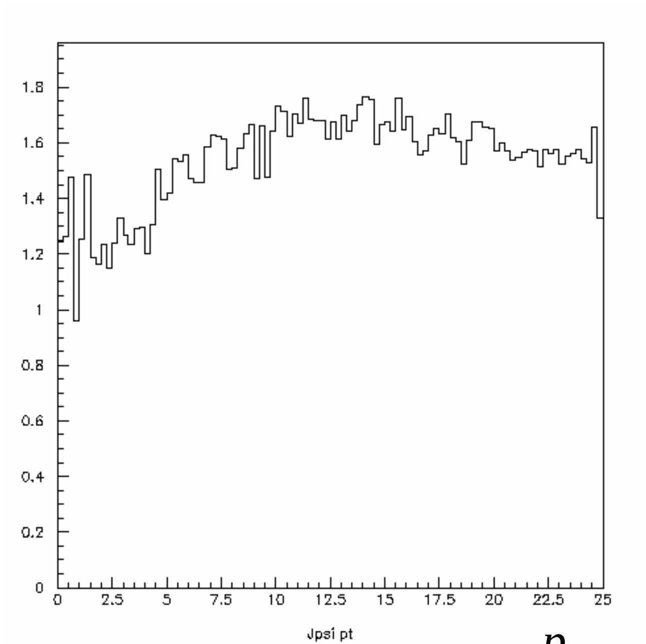
$$\sigma(J/\psi + \text{charmonium}) = \begin{cases} 55 \pm 10\,\text{fb (Belle)} \\ 44.3 \pm 9\,\text{fb (BaBar)} \end{cases}$$

$gg \to J/\psi\, g$





p_T



p_T

Work D. Kang et al (PR D71, 0904019 (2005)) used quark-hadron duality combined with the Color Evaporation Model.

Model cannot be applied to our process because the quantum numbers J/ψ and χ.

Conclusion

1) δ-approximation does not work
2) Light-cone approach gives reasonable agreement with experiment
3) Quark-Hadron Duality gives qualitative description of data
4) In hadronic production of J/ψ Quark-Hadron Duality gives an enhancement factor ~2 for Color Singlet Contribution

Doubly-charmed baryon production

hep-ex/0605076 BaBar

"Search for Doubly Charmed Baryons"

$\sigma^{+} B^{+} < 4.3\,\text{fb}$ $\qquad$ $\sigma^{++} B^{++} < 6.6\,\text{fb}$

$\Lambda_c^{+} K^{-} \pi^{+}\ :\ 2\%$ $\qquad$ $\Lambda_c^{+} K^{-} \pi^{+} \pi^{+}\ :\ 5\%$

$\sigma^{+} < 200\,\text{fb}$ $\qquad$ $\sigma^{++} < 130\,\text{fb}$

This have to be compared with

$$\sigma\left[e^{+}e^{-} \rightarrow (cc)_{\bar{3}} + (\bar{c}\,\bar{c})_{3}\right] = 32\,\text{fb}$$

In the duality interval $2m_c < m_{cc} < 2m_D + \Delta$

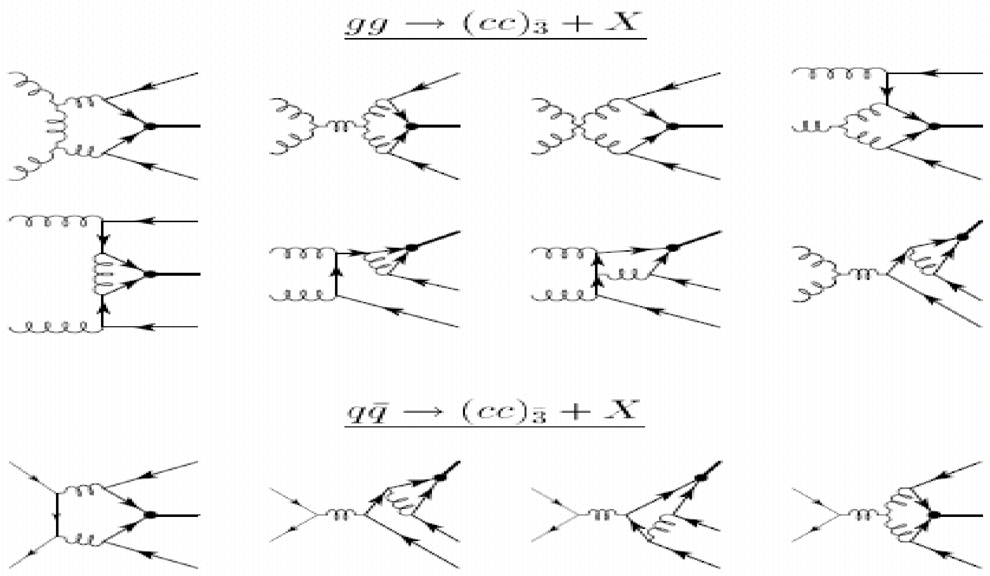
So there is a large gap between experiment and theoretical predictions

Doubly Heavy Baryon at LHC

- DHB is absolutely new type of objects
- DHB is similar to $\bar{Q}\,q$ -meson, where the role of $\bar{Q}$ -antiquark is played by QQ -diquark
- DHB have 2 types of excitations:
  i. diquark-light quark
  ii. excitations of diquark

In DHB production we have used the following approximations:

- QQ' -diquark is produced in $\bar{3}$ -color state
- We have considered 36 $O(a_s^4)$ diagrams:



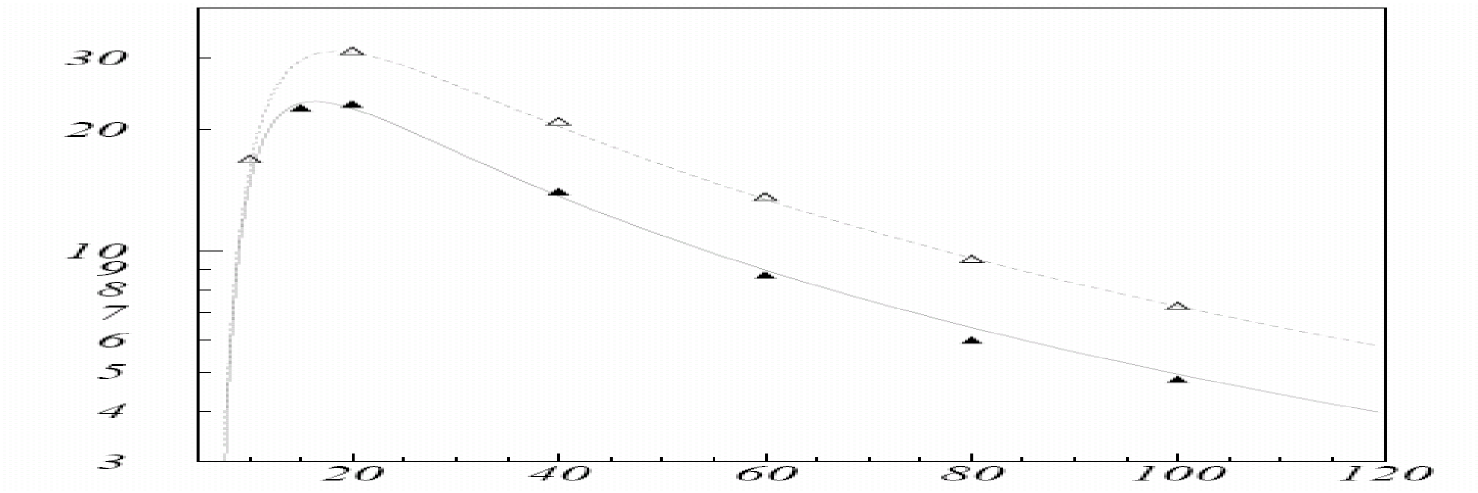
Three basis parameters were used:

- a_s
- m_c
- $R_{QQ'}(0)$ $\quad$ $R_{QQ'}(r)$ is the wave function of QQ' -diquark

$\sqrt{s} = 14\,\mathrm{TeV}$

Cross section at LHC

$$s_{\mathrm{tot}}\left(\langle cc \rangle_{\bar{3}} \right) = 240\,\mathrm{nb} \qquad |y| < 1$$



If the analogy with the case of charmed hadron is used

$$\mathrm{Br}(\mathrm{X}_{cc}^{++} \circledR K^{0(*)} \mathrm{S}_c^{++}) : \mathrm{Br}(\mathrm{X}^{+} \circledR K^{0*}(\mathrm{S}_c^{+} + l_c)) : 4 ´ 10^{-2}$$

One expect : 10^7 such decays at LHC

LHC is the only machine where we can observe DHB