QED ⊗ QCD Exponentiation and Shower/ME Matching at the LHC

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- QED $\otimes$ QCD Threshold Corrections
- Shower / ME Matching at the LHC
- Conclusions

Motivation

- **FNAL / RHIC:** $t\bar{t}$, $b\bar{b}$, J/Ψ production, polarized pp processes
  - Soft n(g) effects already needed: $\Delta m_t = 5.1$ GeV with soft n(g) uncertainty ~ 2 – 3 GeV.
- **LHC / ILC:** Desire $O(\alpha_s^2)L$ event generator incorporating soft n(g) exponentiation in the presence of showers with exact phase space and no double-counting.

QED Effects

How important are QED corrections when QCD is controlled at ~1%?

- Estimates (Spiesberger, Sterling, Roth & Weinzierl) suggest QED corrects structure function evolution by a few per mille.
- QED effects are enhanced at thresholds, especially in resonance production.
- How big are these effects at the LHC?
- We will treat QCD and QED together in the YFS exponentiation to answer such questions.

Vector Boson Production

We are primarily interested in the processes for vector boson production:

$$pp \longrightarrow V + n(\gamma) + m(g) + X$$

$$\longrightarrow \bar{\ell}\,\ell' + n'(\gamma) + m(g) + X$$

where V = $W^{\pm}$, Z and the leptons are as in the table:

V	ℓ	ℓ'
Z	e, μ	e, μ
W+	e, μ	ν_e, ν_μ
W^-	ν_e, ν_μ	e, μ

Review of YFS Theory

- The MC structure will be based on **YFS exponentiation** – which was developed by S. Jadach et al. into a successful framework for handling the soft and collinear singularities in electroweak physics.
- This led to a series of YFS-exponentiated MC programs: YFS2, YFS3, BHLUMI, BHWIDE, KORALZ, KKMC, YFSWW3, YFSZZ, KoralW

Review of YFS Theory

For fermion pair production,

$e^+(p_1)\, e^-(q_1) \longrightarrow f(p_2)\, \bar{f}(q_2) + n(\gamma)(k_1, \ldots, k_n)$

renormalization-group improved YFS theory gives an exponentiated cross-section

$$d\sigma_{exp} = e^{2\alpha\, Re\, B + 2\alpha\, \bar{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j^0} \int \frac{d^4 y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum_j k_j)+D}$$

$$\bar{\beta}_n(k_1, \ldots, k_n) \frac{d^3 p_2\, d^3 q_2}{p_2^0 q_2^0}$$

Virtual YFS form function

Real YFS function

YFS residuals (IR-finite)

Review of YFS Theory

$$d\sigma_{exp} = e^{2\alpha\, Re\, B + 2\alpha\, \bar{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j^0} \int \frac{d^4 y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum_j k_j)+D}$$

$$\bar{\beta}_n(k_1,\ldots,k_n) \frac{d^3 p_2 d^3 q_2}{p_2^0 q_2^0}$$

with…

real IR function

$$2\alpha\,\tilde{B} = \int_{k\le K_{max}} \frac{d^3 k}{k_0} \tilde{S}(k)$$

All dependence on the cutoff K_{max} cancels.

$$D = \int d^3 k \frac{\tilde{S}(k)}{k^0} \left(e^{-iy\cdot k} - \theta(K_{max} - k)\right)$$

soft photon factor

$$\tilde{S}(k) = \frac{\alpha}{4\pi^2} \left[Q_f Q_{(\bar{f})'} \left(\frac{p_1}{p_1 \cdot k} - \frac{q_1}{q_1 \cdot k} \right)^2 + (\ldots) \right]$$

Extension of YFS Theory to QCD

QED:

$$d\sigma_{exp} = e^{2\alpha\, Re\, B + 2\alpha\, \tilde{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j^0} \int \frac{d^4 y}{(2\pi)^4} e^{iy(p_1 + q_1 - p_2 - q_2 - \sum_j k_j) + D}$$

$$\bar{\beta}_n(k_1, \ldots, k_n) \frac{d^3 p_2 d^3 q_2}{p_2^0 q_2^0}$$

QCD: parton level cross section

$$d\hat{\sigma}_{\rm exp} = e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j^0} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(P_1 + Q_1 - P_2 - Q_2 - \sum k_j) + D_{\rm QCD}}$$

$$\tilde{\bar{\beta}}_n(k_1, \ldots, k_n) \frac{d^3 P_2}{P_2^0} \frac{d^3 Q_2}{Q_2^0}$$

Appropriately modified functions

IR Functions for QCD

How do we construct the YFS functions for QCD? We can start with the same structure that worked in the abelian case, with minimal modifications needed to adapt it to QCD…

virtual YFS function:

$$B_{QCD} = \frac{i}{8\pi^2}\int \frac{d^4k}{k^2 - m_\gamma^2 + i\epsilon}\left[C_F\left(\frac{2P_1+k}{k^2+2P_1\cdot k+i\epsilon} + \frac{2Q_1-k}{k^2-2Q_1\cdot k+i\epsilon}\right)^2 + \Delta C_S \frac{2(2P_1+k)\cdot(2Q_1-k)}{(k^2+2P_1\cdot k+i\epsilon)(k^2-2Q_1\cdot k-i\epsilon)} + \ldots\right]$$

soft gluon factor:

$$\tilde{S}_{QCD}(k) = \frac{\alpha_s}{4\pi^2}\left\{C_F\left(\frac{P_1}{P_1\cdot k} - \frac{Q_1}{Q_1\cdot k}\right)^2 - \Delta C_s \frac{2P_1\cdot Q_1}{P_1\cdot k Q_1\cdot k} + \ldots\right\}$$

(There is some freedom here.)

$$C_F = 4/3, \qquad \Delta C_s = \begin{cases} -1 & \text{for } qq' \\ -1/6 & \phantom{\text{for }} q\bar{q}' \end{cases} \text{incoming}, \ \ldots$$

real YFS function:

$$2\alpha_s \tilde{B}_{QCD} = \int_{k<K_{max}} \frac{d^3k}{k^0}\tilde{S}_{QCD}(k)$$

DGLAP Synthesis

Using these equations in

$$d\sigma_{exp} = \sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{exp}(x_i x_j s)$$

would lead to double counting, since the DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) equation for the structure function evolution already includes all leading logs from initial state radiation of gluons, and all related collinear IR effects.

DGLAP Synthesis

The double-counting can be avoided by removing part of the soft gluon factor corresponding to the YFS limit of the DGLAP kernel: Define ***nls* functions** (non-leading in the *s* channel) via…

$$\tilde{S}^{nls}_{QCD} = \tilde{S}_{QCD} - \tilde{S}_{DGLAP}$$

$$\tilde{S}_{DGLAP}(k) = \frac{C_F \alpha_s}{4\pi^2}\left[\frac{\theta(z)4(1-zv)^2}{k_\perp^2 + z^2v^2m_q^2} + \frac{\theta(-z)4(1+zv)^2}{k_\perp^2 + z^2v^2m_q^2}\right]\left(1 - \frac{m_q^2v^2z^2}{k_\perp^2 + z^2v^2m_q^2}\right)$$

$$z = k_z/k, \qquad v = 2k^0/\sqrt{s}$$

$$2\alpha_s\tilde{B}_{QCD} \to 2\alpha_s\tilde{B}^{nls}_{QCD} = \int_{k<K_max}\frac{d^3k}{k^0}\tilde{S}^{nls}_{QCD}(k)$$

Similarly, remove all leading logs from the virtual YFS function.

YFS Residuals for QCD

The YFS residuals are defined by rearranging the cross section so that…

$$d\hat{\sigma}^n = \frac{1}{n!} e^{2\alpha_s \operatorname{Re} B_{QCD}} \int \prod_{m=1}^{n} \frac{d^3 k_m}{k_m^0} \delta(P_1 + Q_1 - P_2 - Q_2 - \sum_{i=1}^{n} k_i)$$
$$\times \bar{\rho}^{(n)}(P_1, Q_1, P_2, Q_2, k_1, \ldots, k_n) \frac{d^3 P_2 d^3 Q_2}{P_2^0 Q_2^0}$$

with

$$\bar{\rho}^{(n)} = \tilde{S}_{QCD}(k_1) \cdots \tilde{S}_{QCD}(k_n) \bar{\beta}_0 + \sum_{i=1}^{n} \prod_{j \neq i} \tilde{S}_{QCD}(k_j) \bar{\beta}_1(k_i) + \cdots + \bar{\beta}_n(k_1, \ldots, k_n)$$

In an abelian gauge theory, the $\bar{\beta}_i$ would now be IR finite. In QCD, there are additional non-abelian IR divergences which remain in the $\bar{\beta}_i$.

YFS Residuals for QCD

Thus, we define a modified residual via

$$\bar{\beta}_n^{(m)} = \tilde{\bar{\beta}}_n^{(m)} + D\bar{\beta}_n^{(m)}$$

IR finite — IR divergent part

where the IR divergent part is defined by a minimal subtraction scheme. In terms of the modified residuals, the complete parton cross section becomes

$$d\hat{\sigma}_{\rm exp} = e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_j^0} \int \frac{d^4y}{(2\pi)^4} e^{iy\cdot(P_1+Q_1-P_2-Q_2-\sum k_j)+D_{\rm QCD}}$$

$$\times \quad \tilde{\bar{\beta}}_n(k_1,\ldots,k_n) \frac{d^3P_2}{P_2^0} \frac{d^3Q_2}{Q_2^0}$$

YFS Residuals for QCD

- Earlier papers (Delaney et al, PRD 52 (1995) 108 and PLB 342 (1995) 239) were not sufficient to derive this result, since they did not account for the effect of genuine non-abelian IR divergences in the original YFS residuals.

- The essentially non-abelian virtual IR singularities in $\int d\mathrm{PS}\ \bar{\beta}_n$ are now known to cancel the non-abelian real IR singularities in $\int d\mathrm{PS}\ \bar{\beta}_{n+1}$, and we can use this fact to isolate an IR finite residual, $\tilde{\bar{\beta}}$.

- There is some freedom in the choice of $\tilde{S}_{\mathrm{QCD}}$, since adding any term $\Delta\tilde{S}_{\mathrm{QCD}}$ with $|k|^2\ \Delta\tilde{S}_{\mathrm{QCD}} \to 0$ as $|k| \to 0$ gives an equally valid set of residuals. We could use this freedom to facilitate matching to a shower generator, for example.

Nonabelian Exponentiation: A Comparison

Another approach to QCD exponentiation is due to Gatheral (PLB 133 (1983) 90).

- Gatheral showed that the eikonal cross sections in nonabelian gauge theory exponentiate.
- Our exponentiation may be considered to be an exact treatment of the $N = 1$ term in Gatheral's result. The approaches agree in the eikonal limit.
- The $N = 1$ term should be adequate for a 1% MC, but higher N terms in the nonabelian exponentiation could be included if necessary, permitting systematic improvement.

Extension to QED⊗ QCD

Simultaneous exponentiation of QED and QCD effects gives

$$B^{nls}_{QCD} \to B^{nls}_{QCD} + B^{nls}_{QED} \equiv B^{nls}_{QCED},$$
$$\tilde{B}^{nls}_{QCD} \to \tilde{B}^{nls}_{QCD} + \tilde{B}^{nls}_{QED} \equiv \tilde{B}^{nls}_{QCED},$$
$$\tilde{S}^{nls}_{QCD} \to \tilde{S}^{nls}_{QCD} + \tilde{S}^{nls}_{QED} \equiv \tilde{S}^{nls}_{QCED}$$

leading to

$$d\hat{\sigma}_{\rm exp} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \frac{1}{m!n!} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}}$$

$$\times \prod_{j_2=1}^{m} \frac{d^3 k'_{j_2}}{k'_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum k'_{j_2})+D_{\rm QCED}}$$

$$\times \; \tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3 p_2}{p_2^0}\frac{d^3 q_2}{q_2^0},$$

n hard gluons,
m hard photons

All QCED definitions ($B_{\rm QCED}$, $\tilde{B}_{\rm QCED}$, …) are analogous to those given previously.

Infrared Algebra for QCED

The average values of x for QED and QCD effects are respectively,

$$x_{avg}(QED) \cong \gamma(QED)/(1+\gamma(QED))$$
$$x_{avg}(QCD) \cong \gamma(QCD)/(1+\gamma(QCD))$$
$$\gamma(A) = \frac{2\alpha_A C_A}{\pi}(L_s - 1),\ A = QED, QCD$$

where $\mathcal{C}_A = Q_f^2, C_F$ for A = QED, QCD.

The QCD-dominant corrections happen an order of magnitude earlier than for QED.

Threshold Corrections

For an example, shall apply the simultaneous QED⊗QCD exponentiation to single Z production with leptonic decay at the LHC to focus on the ISR alone.

Order α_s results are known due to Baur *et al.*, and order α_s^2 results are known due to van Neerven, and Matsuura and Anastasiou *et al*.

We will use

$$d\sigma_{exp} = \sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{exp}(x_i x_j s)$$

with semi-analytical methods and structure functions from Martin *et al*.

Threshold Corrections

A semi-analytical result for the integral gives, at leading order ($\tilde{\tilde{\beta}}_{00}^{(0,0)}$)

$$\hat{\sigma}_{exp}(x_1 x_2 s) = \int_0^{v_{max}} dv \gamma v^{\gamma-1} F_{YFS}(\gamma) e^{\delta_{YFS}} \hat{\sigma}_{Born}((1-v)x_1 x_2 s)$$

where

$$\gamma = \left\{ 2Q_f^2 \frac{\alpha}{\pi} + 2C_F \frac{\alpha_s}{\pi} \right\} L_{nls} \qquad \textbf{with} \qquad L_{nls} = \ln(x_1 x_2 s/\mu^2)$$

$$F_{YFS}(\gamma) = \frac{e^{-c_E \gamma}}{\Gamma(1+\gamma)} \qquad \textbf{with} \qquad c_E = 0.5772\ldots$$

$$\delta_{YFS}(\gamma) = \frac{\gamma}{4} + \left(Q_f^2 \frac{\alpha}{\pi} + C_F \frac{\alpha_s}{\pi} \right) \left(\frac{\pi^2}{3} - \frac{1}{2} \right)$$

Threshold Corrections

Numerically, for parameters relevant to the LHC and Tevatron, with and without the QED contribution,

$$\frac{\sigma_{\text{exp}}}{\sigma_{\text{Born}}} = \begin{cases} 1.1901, & \text{QCED = QCD} \quad \text{QED, LHC} \\ 1.1872 & \text{QCD, LHC} \\ 1.1911 & \text{QCED = QCD} \quad \text{QED, Tevatron} \\ 1.1879 & \text{QCD, Tevatron} \end{cases}$$

- QED enters at 0.3% for both LHC & FNAL.
- This is stable under scale variations.
- We agree with Baur *et al*., Hamberg *et al*., van Neerven and Zilstra.
- The QED effect is similar in size to what is seen in the structure functions.

Shower / Matrix Element Matching

- We are not attempting to replace HERWIG or PYTHIA: We plan to combine $d\hat{\sigma}_{\text{exp}}(x_i x_j s)$ with HERWIG or PYTHIA. A recent parton evolution algorithm by Jadach & Skrzypek (Acta Phys. Pol. B35, 745 (2004) could also be used. ISAJET cannot (no color coherence.)
- HERWIG or PYTHIA can generate a shower starting from (x_1, x_2) at factorization scale m after this point has been provided by $\{F_i\}$.
- The combination can be systematically improved order-by-order in α_s and α to keep up with the state of the art.
- The multi-gluon / multi-photon phase space is represented exactly.

Shower / Matrix Element Matching

Two methods are under investigation…

- transverse momentum matching
- shower-subtracted residuals

Transverse momentum matching

- $P_\perp$ matching uses the fact that the factorization scale μ in DGLAP theory represents the largest $p_\perp$ of the gluon emission included in the structure function. The $p_\perp$ must be limited in the calculation of the residuals to avoid double counting.
- **In practice:** make an event which gives an initial beam state at (x_1, x_2) for two hard partons at scale μ together with a final state X. This event is showered employing backward evolution of the initial partons, with $p_\perp$ restricted as noted above.

Shower-subtracted residuals

Just as the residuals $\tilde{\bar{\bar{\beta}}}_{n,m}$ are computed order by order, the shower formula that generates the backward evolution can also be expanded in powers of α_s and α.

These two expansions can be combined with an expansion of the exponentials at each order in $\alpha^n \alpha_s^{\,m}$ to give the complete result at this order. New "shower-subtracted" residuals can then be constructed which may be used over the entire gluon phase space, with no cut on the transverse momentum. These could be called

$$\hat{\tilde{\bar{\bar{\beta}}}}_{n,m}(\, k_1,\ldots,\, k_n;\, k_1',\ldots,\, k_m') \;.$$

Since the shower includes all soft and collinear effects, this amounts to a redefinition of what is meant by the soft contribution which is subtracted to define the residual. This procedure could, in principle, give more accurate results than the previous proposal.

Conclusions

- YFS Theory extends to non-abelian gauge theory and allows the simultaneous exponentiation of QED and QCD with proper shower/ME matching built in.
- A full MC event generator realization is possible.
- Semi-analytical results for QED and QCD threshold effects in Z production agree with the literature.
- Since QED enters at the 0.3% level, it is essential for reaching 1% theory predictions for the LHC.
- A firm basis for constructing the complete order α_s^2, $\alpha\alpha_s$, α^2 MC results needed for physics at FNAL, LHC, RHIC, and the ILC has been described, and all of the required processes are being calculated in this framework.