To the theory of forward elastic hadron scattering at LHC

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1. Introduction

Elastic nucleon collisions at high energies:

- due to hadron interactions at *all* t & Coulomb interactions at *small* $|t|$
- influence of both interactions (spins neglected) … Bethe (1953)

$$F^{C+N}(s,t) = F^{C}(s,t)e^{i\alpha\Phi} + F^{N}(s,t)$$

$F^{C}(s,t)$ … Coulomb (QED), $F^{N}(s,t)$ … hadronic amplitude

$\alpha\Phi(s,t)$ … relative phase ; $\alpha = 1/137.36$ …fine structure constant

- West–Yennie (1968) (for large s)

$$\alpha\Phi(s,t) = \mp\alpha\left[\ln\left(\frac{-t}{s}\right) - \int_{-4p^2}^{0}\frac{dt'}{|t-t'|}\left(1 - \frac{F^{N}(s,t')}{F^{N}(s,t)}\right)\right]$$

- some assumptions and adding dipole form factors $f_j(t)$ $\rightarrow$
  simplified West-Yennie formula

$$F^{C+N}(s,t) = \pm\frac{\alpha s}{t} f_1(t) f_2(t) e^{i\alpha\Phi} + \frac{\sigma_{tot}}{4\pi} p\sqrt{s}(\rho + i) e^{Bt/2}$$

relative phase $\alpha\Phi = \mp\alpha(\ln(-Bt/2) + \gamma)$ $\gamma = 0.577215$

- fitting data $\frac{d\sigma(s,t)}{dt} = \frac{\pi}{sp^2}|F^{C+N}(s,t)|^2$ $|t| \leq (\sim 0.01 \ \mathrm{GeV}^2)$

$\rightarrow$ constant and averaged values of $\sigma_{tot}, B, \rho = \frac{\Re F^N(s,t=0)}{\Im F^N(s,t=0)}$

- is WY integral formula correct?
- what approximations used for simplified WY?
- t independence of B, ρ ?

2. Limitations involved in West-Yennie approach

- general belief: no limitation of relative phase

$$\alpha\Phi(s,t) = \mp\alpha\left[\ln\left(\frac{-t}{s}\right) - \int_{-4p^2}^{0} \frac{dt'}{|t-t'|}\left(1 - \frac{F^N(s,t')}{F^N(s,t)}\right)\right]$$

phase real → imaginary part of integrand should be zero for *all* t ;

phase and modulus (s depressed): $F^N(s,t) = iF(t)e^{-i\zeta(t)}$

$$I(t) = I_1(t) - I_2(t) \equiv 0$$

$$I_1(t) = \int_{-4p^2}^{t} d\tau f(t,\tau)$$

$$I_2(t) = \int_{t}^{0} d\tau f(t,\tau)$$

$$f(t,\tau) = \begin{cases} \frac{\sin[\zeta(t)-\zeta(\tau)]}{t-\tau} F(\tau) & \text{for} \quad \tau \neq t \\ [\zeta(\tau)]' F(\tau) & \text{for} \quad \tau = t \end{cases}$$

$$t \in (-4p^2, 0)$$

- both $I_1(t), I_2(t)$ proper integrals if $\zeta(t)$ has bounded derivatives

$$[I_1(t)]' = \int_{-4p^2}^{t} d\tau \, \frac{\partial}{\partial t} f(t,\tau) + f(t,t) = \int_{-4p^2}^{t} d\tau \, g(t,\tau) + f(t,t)$$

$$[I_2(t)]' = \int_{t}^{0} d\tau \, \frac{\partial}{\partial t} f(t,\tau) - f(t,t) = \int_{t}^{0} d\tau \, g(t,\tau) - f(t,t)$$

$$g(t,\tau) = \frac{\partial}{\partial t} f(t,\tau) =$$

$$= \begin{cases} \frac{\cos[\zeta(t)-\zeta(\tau)][\zeta(t)]'(t-\tau)-\sin[\zeta(t)-\zeta(\tau)]}{(t-\tau)^2} F(\tau) & \text{for} \quad t \neq \tau \\ \frac{1}{2}[\zeta(\tau)]'' F(\tau) & \text{for} \quad t = \tau \end{cases}$$

- higher derivatives $\quad I_1^{(n)}(t) - I_2^{(n)}(t) \equiv 0$

- key question: what $\zeta(t)$ solves $\boxed{I_1(t) - I_2(t) = 0}$?

- $d\,I_1(t) - d\,I_2(t) = 0$
  $d\,I_1(t) + p_t\,d\,I_2(t) = 0, \ \ p_t > 0 \quad \rightarrow \quad \boxed{d\,I_1(t) \equiv 0, \ \ d\,I_2(t) \equiv 0}$

- boundary condition: $I_j(0) = I_j(-4p^2) = 0, \ j=1,2 \quad \rightarrow$

$$\boxed{I_1(t) = \int_{-4p^2}^{t} d\tau\, f(t,\tau) \equiv 0 \qquad I_2(t) = \int_{t}^{0} d\tau\, f(t,\tau) \equiv 0}$$

- $[I_j(0)]^{(n)} \equiv 0, \ j=1,2; \quad [I_2(t)]' = \int_{t}^{0} d\tau\, \frac{\partial}{\partial t} f(t,\tau) - f(t,t) \quad \rightarrow$

$f(0,0) = 0 \quad \rightarrow \quad \zeta'(0) = 0;$ similarly also $\zeta^{(n)}(0) = 0$

Taylor series expension at $t=0 \quad \rightarrow \quad \boxed{\zeta(t) = const} \quad \rightarrow$
$\rho(t) = \tan\zeta(t) = const$ ... independent of t !!!

3. West-Yennie approach and data

- t independence of ρ ? $\rightarrow$ first derivative of ρ is zero $\rightarrow$

$$\frac{d}{dt}\Re F^N(s,t) \quad \Im F^N(s,t) = \Re F^N(s,t) \quad \frac{d}{dt}\Im F^N(s,t)$$ valid for all t

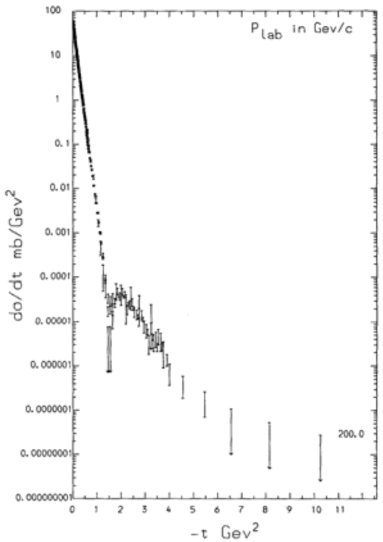
data: *diffractive minimum at* t_D $\rightarrow$ $\frac{d}{dt} \quad \frac{d\sigma(s,t)}{dt} = 0$

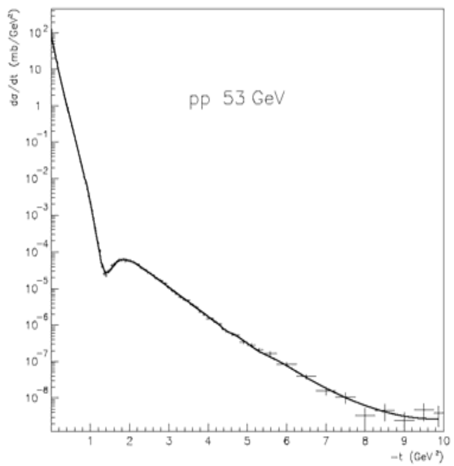
$$\rightarrow \quad \Re F^N(s,t_D) \, \frac{d}{dt}\Re F^N(s,t_D) = -\Im F^N(s,t_D) \, \frac{d}{dt}\Im F^N(s,t_D)$$

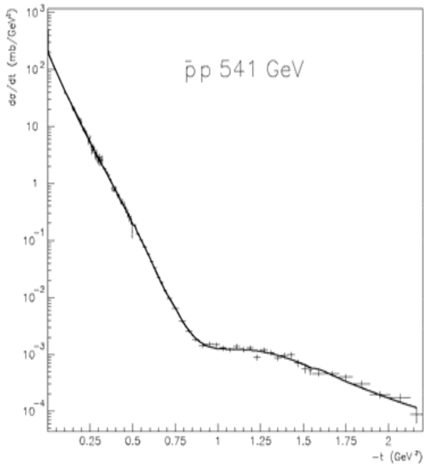
$\rightarrow$ $\rho^2 = -1$ …contradiction $\rightarrow$ diffr. minimum excludes $\rho = const$

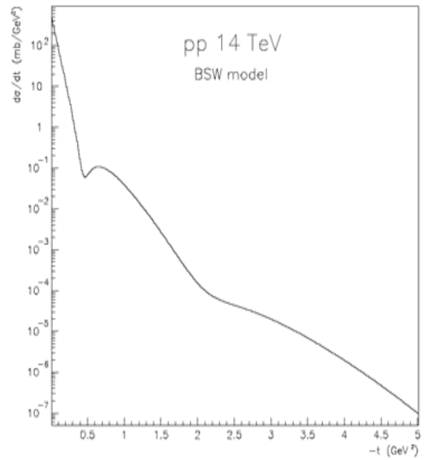
- assumptions needed for simplified WY formula:
  - spin neglected
  - $|F^N(s,t)| \sim e^{Bt}$ for *all* $t \in (-4p^2, 0)$
- are assumptions fulfilled by data?

V.K., M. Lokajíček, Phys. Lett. B 611 (2005) 102









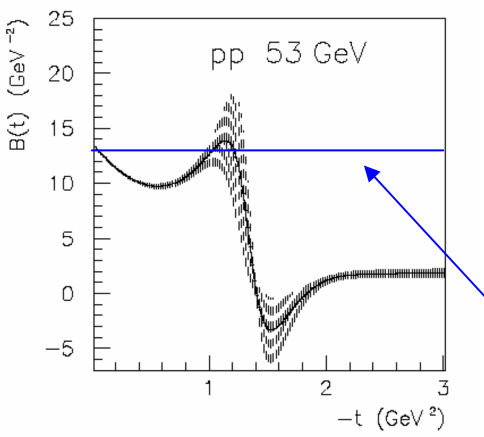
change of magnitude $\frac{d\sigma}{dt}$ from optical point to diffractive minimum:

	~ 8	~ 7÷8	~ 5÷6	~ 4
change of $\|F^N(s,t)\|$:	~ 4.	~ 3.5 ÷ 4.	~ 2.5 ÷ 3	~ 2
-t [GeV^2]	(0., 1.5)	(0., 1.35)	(0., 0.8)	(0., 0.4)

• $|F^N(s,t)|$ "approximately" exponential for small region of t *becomes narrower when energy increases with* $|t|$ *!!!*

• deviations from exponential behavior rise strongly with increasing s

• measure of deviations: diffractive slope



$$B(s,t) = \frac{d}{dt}\left[\ln \frac{d\sigma^N}{dt}\right] = \frac{2}{|F^N(s,t)|}\frac{d}{dt}|F^N(s,t)|$$

constant slope $B(s)$ corresponding to WY formula for $F^{C+N}(s,t)$

• ***integral and simplified WY formulas contradict experimental data***

• however: before ISR experiments nothing known about diffractive structure $\rightarrow$ WY amplitude might be used

4. Extrapolation of hadronic amplitude to $t \sim 0$

- strongly interacting hadronic amplitudes → conservation of isospin

(see Appendix A)

$$F_{ch.e.}(s,t) = F^N_{pp}(s,t) - F^N_{np}(s,t)$$

(valid at any s and t)

- data ($p_{lab} \sim 300\ GeV/c$)

pp [Burq et al: Nucl. Phys. B217 (1983) 285], np [Arefiev et al: Nucl. Phys. B232 (1984) 365)

np → pn [Barton et al: Phys. Rev. Lett. 37(1976) 1656, 1659 Landolt-Bornstein Vol. 9, Springer 1980]

	$(pp)_{el}$	$(np)_{el}$	np → pn
-t [GeV^2]	dσ/dt [mb/GeV^2]	dσ/dt [mb/GeV^2]	dσ/dt [μb/GeV^2]
.003	103.34 ± 4.1	77.09 ± .80	6.14 ± .006
.023	58.27 ± 1.1	61.80 ± .71	4.24 ± .004

→ $d\sigma/dt\ [np \to pn] \sim 10^{-5} * d\sigma/dt[np]$

$$\to \quad F^N_{pp}(s,t) \equiv F^N_{np}(s,t)$$

- np measured up to $t = 10^{-5}\ GeV^2 \to$ *compatible with* e^{Bt}

(Arefiev et al (1984))

5. Approaches based on impact param. representation

(Franco (1966,1973), Lapidus et al. (1978), Cahn (1982), …, V. K., M. Lokajíček (1994))

- used eikonal models based on approximate form of Fourier-Bessel transformation valid at asymptotic s and small $|t|$

$$F(s,q^2=-t)=\frac{s}{4\pi i}\int\limits_{\Omega_b} d^2b\, e^{i\vec{q}\vec{b}}\left[e^{2i\delta(s,b)}-1\right]$$

- mathematically rigorous formulation (valid at any s and t)

(Adachi et al., Islam (1965 – 1976))

additivity of pottentials $\rightarrow$ additivity of eikonals (Franco (1973))

$$\delta^{C+N}(s,b)=\delta^{C}(s,b)+\delta^{N}(s,b)$$

- total scattering amplitude

$$F^{C+N}(s,t=-q^2)=\frac{s}{4\pi i}\int\limits_{\Omega_b} d^2b\, e^{i\vec{q}\vec{b}}\left[e^{2i(\delta^{C}(s,b)+\delta^{N}(s,b))}-1\right] \quad \rightarrow$$

$$F^{C+N}(s,t) = F^{C}(s,t) + F^{N}(s,t)$$
$$+ \frac{s}{4\pi i} \int_{\Omega_b} d^2 b e^{i\vec{q}\vec{b}} \left[e^{2i\delta^{C}(s,b)} - 1 \right] \left[e^{2i\delta^{N}(s,b)} - 1 \right]$$

$$F^{C+N}(s,t) = F^{C}(s,t) + F^{N}(s,t) + \frac{i}{\pi s} \int_{\Omega_{q'}} d^2 q' F^{C}(s, q'^2) F^{N}(s, [\vec{q} - \vec{q}']^2)$$

convolution integral

- equation describes simultaneous actions of both Coulomb and hadronic interactions; to the sum of both amplitudes new complex function (convolution integral) is added
- at difference with WY amplitude (Coulomb amplitude multiplied by phase factor only)
- valid at any s and t

• general formula valid up to terms linear in α (V. K., M. Lokajíček, Z. Phys. C63 (1994) 619)

$$F^{C+N}(s,t) = \pm \frac{\alpha s}{t} f_1(t) f_2(t) + F^N(s,t)\left[1 \mp i\alpha G(s,t)\right]$$

$$G(s,t) = \int_{t_{min}}^{0} dt' \left\{ \ln\left(\frac{t'}{t}\right) \frac{d}{dt'}\left[f_1(t') f_2(t')\right] + \frac{1}{2\pi}\left[\frac{F^N(s,t')}{F^N(s,t)} - 1\right] I(t,t') \right\}$$

$$I(t,t') = \int_0^{2\pi} d\Phi'' \frac{f_1(t'') f_2(t'')}{t''} \qquad t'' = t + t' + 2\sqrt{tt'}\cos\Phi''$$

$$t_{min} = -s + 4m^2$$

• $[1 \pm i\alpha\, G(s,t)] \sim \exp(\pm i\alpha\, G(s,t))$

$$F^{N+C}(s,t) = F^C(s,t) + F^N(s,t) e^{\mp i\alpha G(s,t)}$$

→ complex *G(s,t)* cannot be interpreted as mere change of phase

→ *G(s,t) ... real* ↔ *ρ(s,t) ... constant in t*

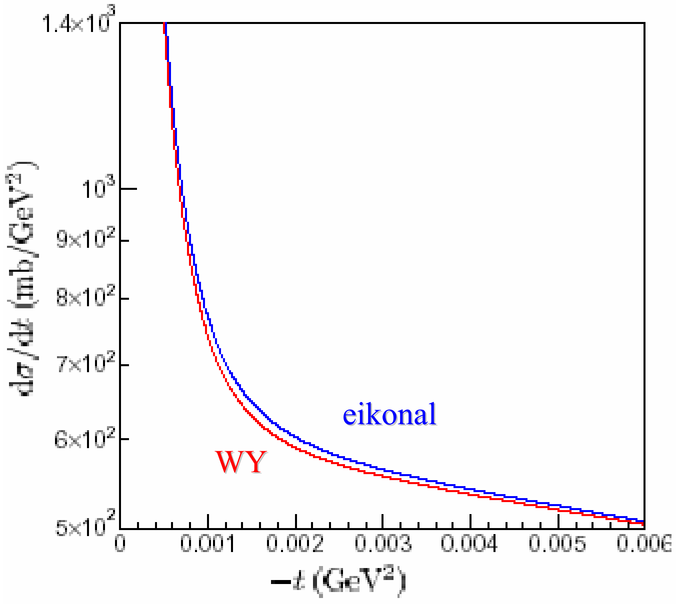
• use: data analysis (see Appendix B) or model predictions

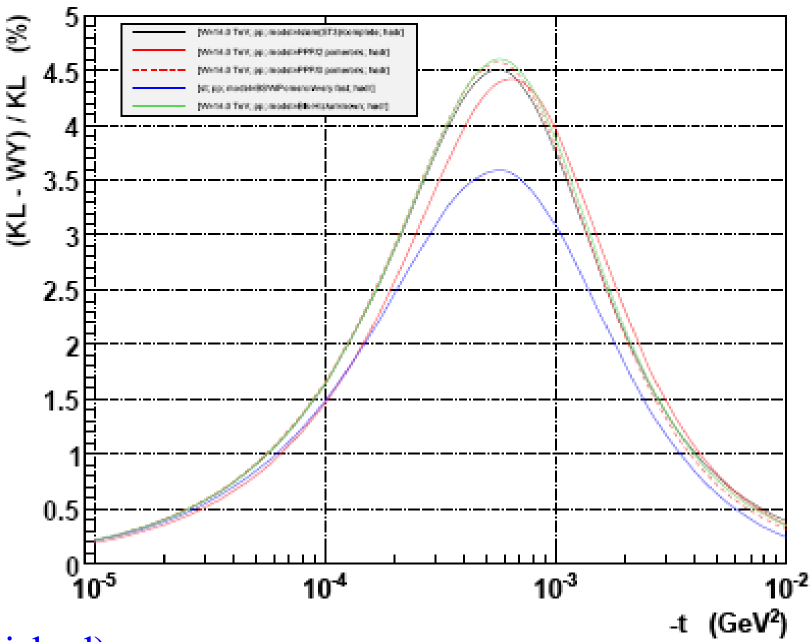
6. General formula and luminosity at LHC

$$\frac{1}{\mathcal{L}}\left[\frac{dN_{el}}{dt}\right]_{t\to 0} = \frac{\pi}{sp^2}\left[|F^{C+N}(s,t)|^2\right]_{t\to 0} \to \frac{4\pi\alpha^2}{|t|^2}$$

- different total (eikonal and WY) amplitudes → different luminosity determinations

$$R(t) = \frac{|F^{C+N}_{eik}(s,t)|^2 - |F^{C+N}_{WY}(s,t)|^2}{|F^{C+N}_{eik}(s,t)|^2} * 100.$$





(Kašpar & Smotlacha: to be published)

7. Conclusion

- WY integral formula: *hadronic amplitudes with constant quantity ρ*
- simplified WY amplitude: *in contradiction with data*
- WY approach leads to *false* results at high energies
- approach based on eikonal model ≡ suitable tool for analyzing high-energy elastic hadron scattering amplitude $\rightarrow$
t* dependence of its modulus and phase needed at all allowed *t
- dynamical characteristics of elastic hadronic amplitude determined by its t dependence, i.e., σ_{tot}, ρ, B are *model dependent quantities*
- influence of Coulomb scattering *cannot be neglected at higher $|t|$*

Appendix A: Conservation of isospin

- elastic hadronic amplitude $F^N(s,t)$ (strong interactios) → conservation of isospin

- NN scattering … isospin states

$$|NN> \equiv |j_1 j_2; m_1 m_2> = \sum_{J,M} |j_1 j_2; JM><j_1 j_2; JM|j_1 j_2; m_1 m_2>$$

$$|pp> = |½½; 11>, \quad |np> = \sqrt{½}\, |½½; 10> + \sqrt{½}\, |½½; 00>,$$

$$|nn> = |½½; 1\text{-}1>$$

- define (isospin conserved)

$$<I\, I_3|F^N(s,t)|I'\, I_3'> = F_{2I}(s,t)\ \delta_{II'}\, \delta_{I_3 I_3'}$$

$$<pp|F^N(s,t)|pp> = F_2(s,t), \quad <np|F^N(s,t)|np> = ½\, F_2(s,t) + ½ F_0(s,t)$$

$$<np|F_{ch.e.}(s,t)|pn> = ½\, [F_2(s,t) - F_0(s,t)] \qquad \rightarrow$$

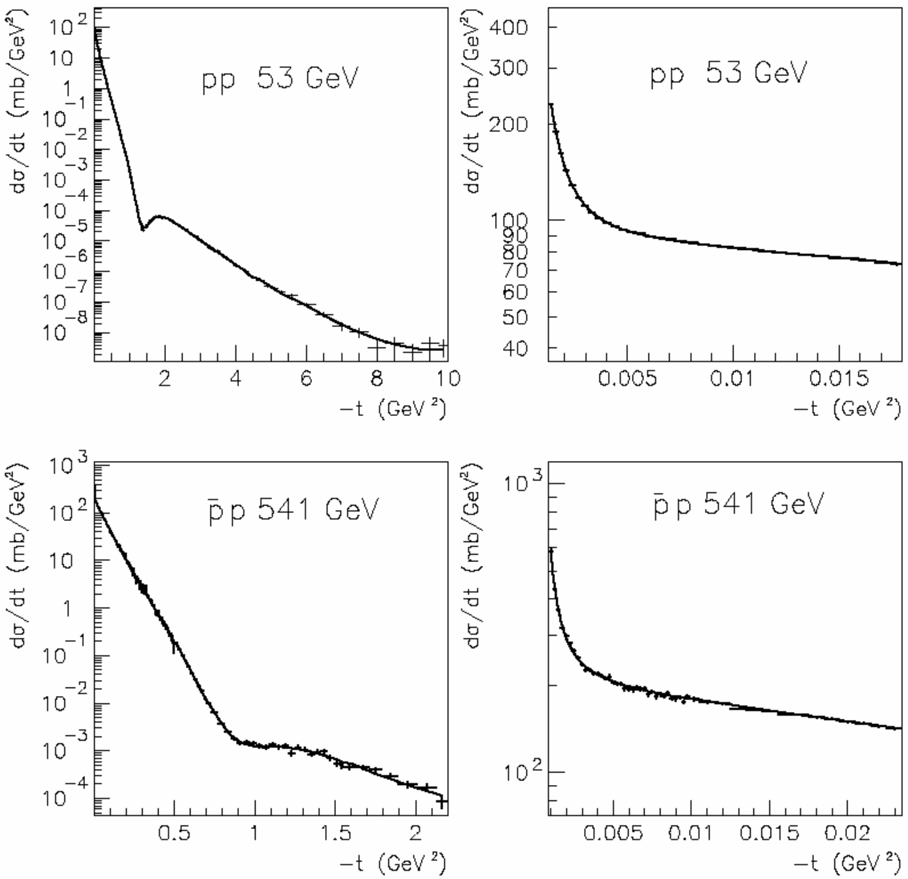
$$F_{ch.e.}(s,t) = F^N_{pp}(s,t) - F^N_{np}(s,t)$$

Appendix B: Analysis of data

(V.K., Lokajíček, Z. Phys. C 63 (1994) 619)

data	prof.	σ_{tot} [mb]	B [GeV^{-2}]	ρ	χ^2/df
pp	per.	42.89 ± 0.12	13.55 ± 0.05	0.06 ± 0.009	$253/201$
53	cent.	42.65 ± 0.23	13.25 ± 0.05	0.07 ± 0.009	$329/204$
GeV	WY	42.38 ± 0.15	12.87 ± 0.14	0.077 ± 0.009	1.43
$\bar{p}p$	per.	62.56 ± 1.16	16.67 ± 0.09	0.11 ± 0.022	$233/213$
541	cent.	62.70 ± 0.78	16.00 ± 0.05	0.096 ± 0.013	$354/217$
GeV	WY	62.17 ± 1.50	15.50 ± 0.10	0.135 ± 0.015	1.1

- different values of σ_{tot}, B, ρ



modulus

$$|F^N(s,t)| =$$

$$= (a_1 + a_2 t)e^{b_1 t + b_2 t^2 + b_3 t^3}$$

$$+ \ (c_1 + c_2 t)e^{d_1 t + d_2 t^2 + d_3 t^3}$$

phase (peripheral)

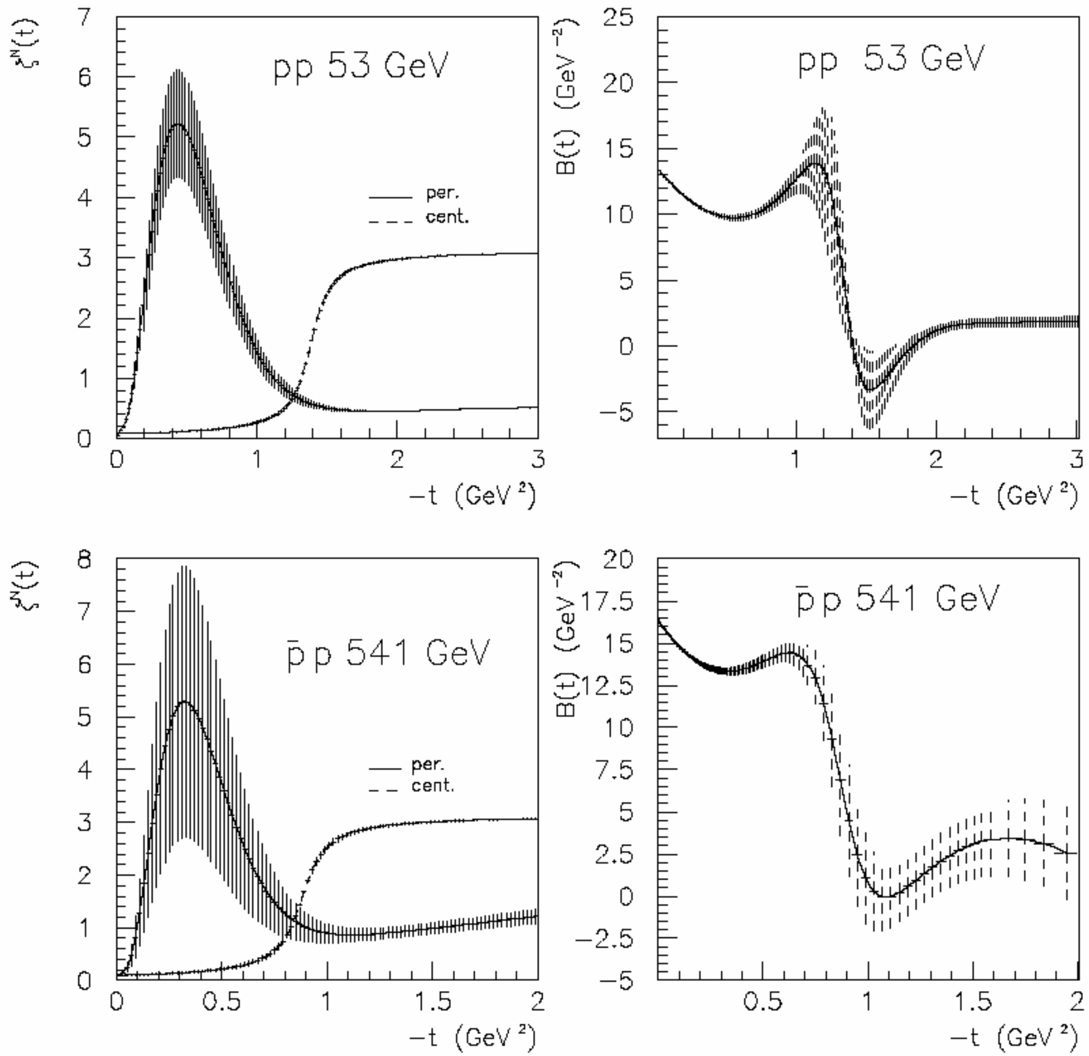
$$\zeta^N(s,t) = \zeta_0 + \zeta_1 \left|\frac{t}{t_0}\right|^{\kappa} e^{\nu t}$$

$$+ \zeta_2 \left|\frac{t}{t_0}\right|^{\lambda}, t_0 = 1 \ \ \mathrm{GeV}^2$$

- analysis in the broadest interval of t; method enables natural normalization of data from measured t intervals in individual experiments
- pp at 53 GeV, $-t \in (0.00126, 9.75)$ GeV^2
  $\bar{p}p$ at 541 GeV, $-t \in (0.00075, 2.13)$ GeV^2

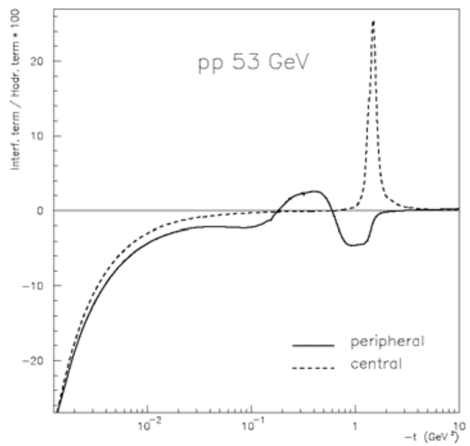
phase (central)

$$\zeta^N(s,t) = \arctan \frac{\rho_0}{1 - \left|\frac{t}{t_{diff}}\right|}$$



$$F^N(s,t) =$$
$$= i|F^N(s,t)|e^{-i\zeta^N(s,t)}$$

$$B(t) = \frac{d}{dt}\left(\ln \frac{d\sigma^N}{dt}\right)$$



Influence of Coulomb scattering cannot be neglected at higher |t| !!!

- exact values of $B(s,t), \rho(s,t)$ at any t
- statistical errors