A NEW PARTON SHOWER ALGORITHM

Parton Evolution, Matching at LO and NLO level

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INTRODUCTION

- The parton shower is a tool to model high multiplicity event in particle collisions
  - QCD inspired model
  - Not a predictive model. The scale and unphysical parameters are rather uncontrolled.
  - Important tool for detector simulation
  - Very crude approximation but with the tuning the performance usually is very good.
  - Very popular

INTRODUCTION

- The shower is strictly pQCD object
  - We need to define a nice formalism
  - We have to rethink and improve the shower algorithm and the matching schemes
    - Kinematics, soft gluon, Lorentz invariance/covariance, ... T. Sjöstrand & P. Skands
    - Matching to Born matrix elements CKKW method
    - Matching to NLO calculation MC@NLO
    - Adding higher order correction → NNLO

CONFIGURATION SPACE

An *m*-parton configuration is

$$\{p,f,c\}_{\mathrm{a},\mathrm{b},m} \equiv \{\eta_{\mathrm{a}} p_A, a, c_A, \eta_{\mathrm{b}} p_B, b, c_B, p_1, f_1, c_1, ..., p_m, f_m, c_m\}$$

Basis vector in the configuration space $\big|\{p,f,c\}_{\mathrm{a},\mathrm{b},m}\big)$

Normalization:

$$\big(\{p',f',c'\}_{\mathrm{a},\mathrm{b},m'}\big|\{p,f,c\}_{\mathrm{a},\mathrm{b},m}\big) = \delta_{mm'}\,\delta_{a,a'}\,\delta_{c_A c'_A}\,\delta(\eta_{\mathrm{a}} - \eta'_{\mathrm{a}})$$
$$\times\,\delta_{b,b'}\,\delta_{c_B c'_B}\,\delta(\eta_{\mathrm{b}} - \eta'_{\mathrm{b}}) \prod_{i=1}^{m} \delta_{f,f'_i}\,\delta_{c_i c'_i}\,\delta^{(4)}(p_i - p'_i)$$

Completeness relation:

$$1 = \sum_m \int \big[d\{p,f,c\}_{\mathrm{a},\mathrm{b},m}\big]\,\big|\{p,f,c\}_{\mathrm{a},\mathrm{b},m}\big)\big(\{p,f,c\}_{\mathrm{a},\mathrm{b},m}\big|$$

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Basis vector in the configuration space $\big|\{p,f,c\}_{\mathrm{a,b},m}\big)$

Normalization:

$$\int \big[d\{p,f,c\}_{\mathrm{a,b},m}\big] \equiv \sum_a \int_0^1 d\eta_{\mathrm{a}} \sum_b \int_0^1 d\eta_{\mathrm{b}} \prod_{i=1}^{m} \Big\{ \sum_{f_i} \int d^4 p_i \Big\} \sum_{\{c\}_{\mathrm{a,b},m}}$$

Completeness relation:

$$1 = \sum_m \int \big[d\{p,f,c\}_{\mathrm{a,b},m}\big]\, \big|\{p,f,c\}_{\mathrm{a,b},m}\big)\big(\{p,f,c\}_{\mathrm{a,b},m}\big|$$

CONFIGURATION SPACE

An *m*-parton configuration is

A general state (e.g. jet function) is

$$(F| = \sum_m \int \big[d\{p,f,c\}_{\mathrm{a,b},m}\big]\, F(\{p,f,c\}_{\mathrm{a,b},m})\big(\{p,f,c\}_{\mathrm{a,b},m}\big| \ .$$

The unit vector is

$$(1| = \sum_m \int \big[d\{p,f,c\}_{\mathrm{a,b},m}\big]\, \big(\{p,f,c\}_{\mathrm{a,b},m}\big| \ .$$

Completeness relation:

$$1 = \sum_m \int \big[d\{p,f,c\}_{\mathrm{a,b},m}\big]\, \big|\{p,f,c\}_{\mathrm{a,b},m}\big)\big(\{p,f,c\}_{\mathrm{a,b},m}\big|$$

PHASE SPACE INTEGRAL

To define the phase space integral we have an operator

$$\begin{aligned}\Gamma = \sum_m \int \big[d\{p,f,c\}_{\mathrm{a,b},m}\big]\, \big|\{p,f,c\}_{\mathrm{a,b},m}\big)\big(\{p,f,c\}_{\mathrm{a,b},m}\big| \\ \times f_{a/A}(\eta_{\mathrm{a}},\mu_F^2)\, f_{b/B}(\eta_{\mathrm{b}},\mu_F^2)\, \frac{1}{2\eta_{\mathrm{a}}\eta_{\mathrm{b}} p_A\!\cdot\! p_B}\, \frac{1}{m!} \\ \times \prod_{i=1}^{m} \left\{ \frac{1}{(2\pi)^3}\delta_+(p_i^2) \right\} (2\pi)^4 \delta\Big(\eta_{\mathrm{a}} p_A + \eta_{\mathrm{b}} p_B - K - \sum_{i=1}^{m} p_i\Big)\end{aligned}$$

Cross section in the configuration space

$$\big|\sigma_m\big) = \Gamma\big|\mathcal{M}_m\big) \quad \Rightarrow \quad \sigma_m[F_{m-\mathrm{jet}}] = \big(F_{m-\mathrm{jet}}\big|\Gamma\big|\mathcal{M}_m\big)$$

PARTON SHOWER EVOLUTION

We use an evolution variable e.g.:

$$\boxed{\log \frac{Q^2}{\hat{p}_1 \cdot \hat{p}_2} = t \ \in \ [0, \infty]}$$

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$$\log \frac{Q^2}{\hat{p}_1 \cdot \hat{p}_2} = t \ \in \ [0, \infty]$$

$$U(t_3, t_1) = \underbrace{N(t_3, t_1)}_{\text{No-splitting part}} + \overbrace{\int_{t_1}^{t_3} dt_2 \, U(t_3, t_2) \, \mathcal{H}(t_2) \, N(t_2, t_1)}^{\text{Splitting part}}$$

Preserves the
normalization

$$(1|A(t_0)) = 1 \quad \Rightarrow \quad (1|U(t, t_0)|A(t_0)) = 1$$

NO-SPLITTING OPERATOR

The operator $N(t',t)$ leaves the basis states $|\{p,f,c\}_{\mathrm{a,b},m})$ unchanged

$$N(t',t)|\{p,f,c\}_{\mathrm{a,b},m}) = \underbrace{\Delta(\{p,f,c\}_{\mathrm{a,b},m};t',t)}_{\text{Sudakov factor}}|\{p,f,c\}_{\mathrm{a,b},m})$$

From the normalization $(1|U(t,t')|\{p,f,c\}_{\mathrm{a,b},m}) = 1$

$$\begin{aligned} 1 = {} & \Delta(\{p,f,c\}_{\mathrm{a,b},m};t_3,t_1) \\ & + \int_{t_1}^{t_3} dt_2\ (1|\mathcal{H}(t_2)|\{p,f,c\}_{\mathrm{a,b},m})\,\Delta(\{p,f,c\}_{\mathrm{a,b},m};t_2,t_1) \end{aligned}$$

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From the normalization $(1|U(t, t')|\{p, f, c\}_{\mathrm{a,b},m}) = 1$

$$\Delta(\{p, f, c\}_{\mathrm{a,b},m}; t_2, t_1) = \exp\left(-\int_{t_1}^{t_2} dt\,(1|\mathcal{H}(t)|\{p, f, c\}_{\mathrm{a,b},m})\right)$$

SPLITTING OPERATOR

The splitting operator describes all the possible transitions that $\big|\{p, f, c\}_{\mathrm{a},\mathrm{b},m}\big) \rightarrow \big|\{\hat{p}, \hat{f}, \hat{c}\}_{\mathrm{a},\mathrm{b},m+n}\big)$

Since we are interested only at LL and NLL level we have only $1 \rightarrow 2$ splittings

SPLITTING OPERATOR

$$
\begin{aligned}
&\big(\{\hat{p},\hat{f},\hat{c}\}_{\mathrm{a},\mathrm{b},m+1}\big|\mathcal{H}(t)\big|\{p,f,c\}_{\mathrm{a},\mathrm{b},m}\big) \\
&\quad = \sum_{l=\mathrm{a},\mathrm{b},1,\dots,m}\ \sum_{\substack{k=\mathrm{a},\mathrm{b},1,\dots,m \\ k\neq l}} \int_0^1 \frac{dy}{y} \int_0^1 dz \int_0^{2\pi} \frac{d\phi}{2\pi}\, \delta\big(t + \log(T_{l,k}(p_l,p_k,z,y)/Q^2)\big) \\
&\qquad \times C_{l,k}\, \frac{\alpha_s\big(Q^2 e^{-t}\big)}{2\pi}\, S_{l,k}(z,y,\hat{f}_{l,1},\hat{f}_{l,2})\, \frac{\hat{\eta}_{\mathrm{a}}}{\eta_{\mathrm{a}}} \frac{f_{\hat{a}/A}(\hat{\eta}_{\mathrm{a}},\mu_F^2)}{f_{a/A}(\eta_{\mathrm{a}},\mu_F^2)}\, \frac{\hat{\eta}_{\mathrm{b}}}{\eta_{\mathrm{b}}} \frac{f_{\hat{b}/B}(\hat{\eta}_{\mathrm{b}},\mu_F^2)}{f_{b/B}(\eta_{\mathrm{b}},\mu_F^2)} \\
&\qquad \times \big(\{\hat{p},\hat{f},\hat{c}\}_{\mathrm{a},\mathrm{b},m+1}\big|\mathcal{R}_{l,k}(z,y,\kappa_\perp)\big|\{p,f,c\}_{\mathrm{a},\mathrm{b},m}\big)
\end{aligned}
$$

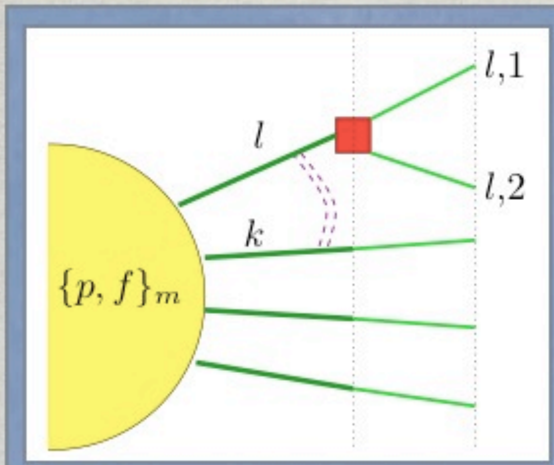
SPLITTING OPERATOR

$$\Big(\{\hat{p},\hat{f},\hat{c}\}_{\mathrm{a,b},m+1}\Big|\mathcal{H}(t)\Big|\{p,f,c\}_{\mathrm{a,b},m}\Big)$$

$$= \boxed{\sum_{l=\mathrm{a,b},1,\dots,m}\ \sum_{\substack{k=\mathrm{a,b},1,\dots,m\\ k\neq l}}} \int_0^1 \frac{dy}{y} \int_0^1 dz \int_0^{2\pi} \frac{d\phi}{2\pi}\, \delta\big(t+\log(T_{l,k}(p_l,p_k,z,y)/Q^2)\big)$$

$$\times \boxed{C_{l,k}} \frac{\alpha_s\big(Q^2e^{-t}\big)}{2\pi}\, S_{l,k}(z,y,\hat{f}_{l,1},\hat{f}_{l,2})\, \frac{\hat{\eta}_{\mathrm{a}}}{\eta_{\mathrm{a}}} \frac{f_{\hat{a}/A}(\hat{\eta}_{\mathrm{a}},\mu_F^2)}{f_{a/A}(\eta_{\mathrm{a}},\mu_F^2)} \frac{\hat{\eta}_{\mathrm{b}}}{\eta_{\mathrm{b}}} \frac{f_{\hat{b}/B}(\hat{\eta}_{\mathrm{b}},\mu_F^2)}{f_{b/B}(\eta_{\mathrm{b}},\mu_F^2)}$$

$$\times \Big(\{\hat{p},\hat{f},\hat{c}\}_{\mathrm{a,b},m+1}\Big|\mathcal{R}_{l,k}(z,y,\kappa_\perp)\Big|\{p,f,c\}_{\mathrm{a,b},m}\Big)$$



$$C_{l,k} = \begin{cases} 1 & \text{if } l \text{ and } k \text{ are color connected} \\ 0 & \text{otherwise} \end{cases}$$

SPLITTING OPERATOR

$$\left(\{\hat{p},\hat{f},\hat{c}\}_{\mathrm{a,b},m+1}\middle|\mathcal{H}(t)\middle|\{p,f,c\}_{\mathrm{a,b},m}\right)$$

$$= \sum_{l=\mathrm{a,b},1,\dots,m}\ \sum_{\substack{k=\mathrm{a,b},1,\dots,m\\ k\neq l}} \boxed{\int_0^1 \frac{dy}{y}\int_0^1 dz \int_0^{2\pi}\frac{d\phi}{2\pi}\,\delta\big(t+\log(T_{l,k}(p_l,p_k,z,y)/Q^2)\big)}$$

$$\times\, C_{l,k}\,\frac{\alpha_s\big(Q^2e^{-t}\big)}{2\pi}\,S_{l,k}(z,y,\hat{f}_{l,1},\hat{f}_{l,2})\,\frac{\hat{\eta}_{\mathrm{a}}}{\eta_{\mathrm{a}}}\frac{f_{\hat{a}/A}(\hat{\eta}_{\mathrm{a}},\mu_F^2)}{f_{a/A}(\eta_{\mathrm{a}},\mu_F^2)}\,\frac{\hat{\eta}_{\mathrm{b}}}{\eta_{\mathrm{b}}}\frac{f_{\hat{b}/B}(\hat{\eta}_{\mathrm{b}},\mu_F^2)}{f_{b/B}(\eta_{\mathrm{b}},\mu_F^2)}$$

$$\times\left(\{\hat{p},\hat{f},\hat{c}\}_{\mathrm{a,b},m+1}\middle|\mathcal{R}_{l,k}(z,y,\kappa_\perp)\middle|\{p,f,c\}_{\mathrm{a,b},m}\right)$$

Sudakov parametrization of the new momenta:

$$\hat{p}_{l,1} = z p_l + y(1-z)p_k + k_\perp \qquad p_l + p_k = \hat{p}_{l,1} + \hat{p}_{l,2} + \hat{p}_k$$

$$\hat{p}_{l,2} = (1-z)p_l + yzp_k - k_\perp \qquad \hat{p}_{l,1}^2 = \hat{p}_{l,2}^2 = 0$$

$$\hat{p}_k = (1-y)p_k \qquad -k_\perp^2 = 2p_l\cdot p_k yz(1-z) = T_{l,k}$$

SPLITTING OPERATOR

$$\begin{aligned}
&\big(\{\hat{p},\hat{f},\hat{c}\}_{\mathrm{a,b},m+1}\big|\mathcal{H}(t)\big|\{p,f,c\}_{\mathrm{a,b},m}\big) \\
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&\qquad \times C_{l,k}\,\frac{\alpha_s\big(Q^2e^{-t}\big)}{2\pi}\,S_{l,k}(z,y,\hat{f}_{l,1},\hat{f}_{l,2})\,\frac{\hat{\eta}_\mathrm{a}}{\eta_\mathrm{a}}\frac{f_{\hat{a}/A}(\hat{\eta}_\mathrm{a},\mu_F^2)}{f_{a/A}(\eta_\mathrm{a},\mu_F^2)}\,\frac{\hat{\eta}_\mathrm{b}}{\eta_\mathrm{b}}\frac{f_{\hat{b}/B}(\hat{\eta}_\mathrm{b},\mu_F^2)}{f_{b/B}(\eta_\mathrm{b},\mu_F^2)} \\
&\qquad \times \big(\{\hat{p},\hat{f},\hat{c}\}_{\mathrm{a,b},m+1}\big|\mathcal{R}_{l,k}(z,y,\kappa_\perp)\big|\{p,f,c\}_{\mathrm{a,b},m}\big)
\end{aligned}$$

The phase space is exact after the splitting:

$$d\Gamma^{(m+1)}(\{\hat{p}\}_{m+1};Q)\,\frac{1}{2\hat{p}_{l,1}\cdot\hat{p}_{l,2}} = d\Gamma^{(m)}(\{p\}_m;Q)\,\frac{dy}{y}\,dz\,\frac{d\phi}{2\pi}\,\frac{1-y}{16\pi^2}$$

SPLITTING OPERATOR

$$\begin{aligned}
&\big(\{\hat{p},\hat{f},\hat{c}\}_{\mathrm{a,b},m+1}\big|\mathcal{H}(t)\big|\{p,f,c\}_{\mathrm{a,b},m}\big)\\
&\quad=\sum_{l=\mathrm{a,b},1,\dots,m}\;\sum_{\substack{k=\mathrm{a,b},1,\dots,m\\k\neq l}}\int_0^1\frac{dy}{y}\int_0^1 dz\int_0^{2\pi}\frac{d\phi}{2\pi}\,\delta\big(t+\log(T_{l,k}(p_l,p_k,z,y)/Q^2)\big)\\
&\qquad\times C_{l,k}\boxed{\frac{\alpha_s\big(Q^2e^{-t}\big)}{2\pi}\,S_{l,k}(z,y,\hat{f}_{l,1},\hat{f}_{l,2})}\,\frac{\hat{\eta}_{\mathrm{a}}}{\eta_{\mathrm{a}}}\frac{f_{\hat{a}/A}(\hat{\eta}_{\mathrm{a}},\mu_F^2)}{f_{a/A}(\eta_{\mathrm{a}},\mu_F^2)}\,\frac{\hat{\eta}_{\mathrm{b}}}{\eta_{\mathrm{b}}}\frac{f_{\hat{b}/B}(\hat{\eta}_{\mathrm{b}},\mu_F^2)}{f_{b/B}(\eta_{\mathrm{b}},\mu_F^2)}\\
&\qquad\times\big(\{\hat{p},\hat{f},\hat{c}\}_{\mathrm{a,b},m+1}\big|\mathcal{R}_{l,k}(z,y,\kappa_\perp)\big|\{p,f,c\}_{\mathrm{a,b},m}\big)
\end{aligned}$$

E.g.: Final state splitting with final state spectator, $q \to q + g$

$$S_{l,k}(z,y,q,g) = C_{\mathrm{F}}\left[\frac{2}{1-z(1-y)} - (1+z)\right]$$

SPLITTING OPERATOR

$$\left(\{\hat{p},\hat{f},\hat{c}\}_{\mathrm{a,b},m+1}\middle|\mathcal{H}(t)\middle|\{p,f,c\}_{\mathrm{a,b},m}\right)$$

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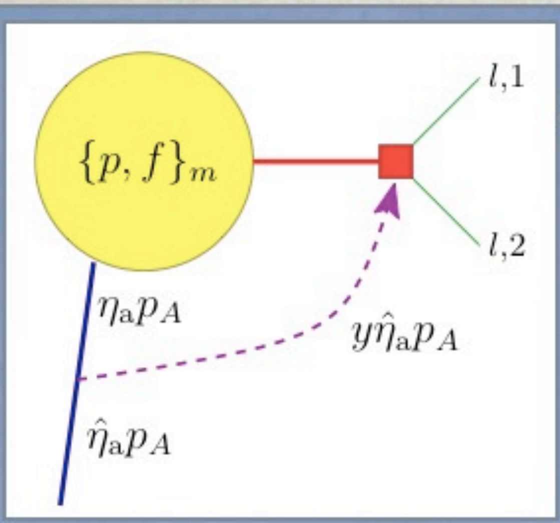
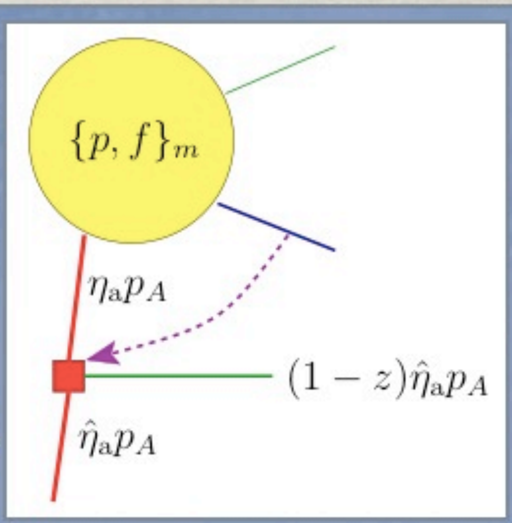
$$\times\, C_{l,k}\,\frac{\alpha_s\big(Q^2e^{-t}\big)}{2\pi}\,S_{l,k}(z,y,\hat{f}_{l,1},\hat{f}_{l,2})\boxed{\frac{\hat{\eta}_{\mathrm{a}}}{\eta_{\mathrm{a}}}\frac{f_{\hat{a}/A}(\hat{\eta}_{\mathrm{a}},\mu_F^2)}{f_{a/A}(\eta_{\mathrm{a}},\mu_F^2)}\,\frac{\hat{\eta}_{\mathrm{b}}}{\eta_{\mathrm{b}}}\frac{f_{\hat{b}/B}(\hat{\eta}_{\mathrm{b}},\mu_F^2)}{f_{b/B}(\eta_{\mathrm{b}},\mu_F^2)}}$$

$$\times\left(\{\hat{p},\hat{f},\hat{c}\}_{\mathrm{a,b},m+1}\middle|\mathcal{R}_{l,k}(z,y,\kappa_\perp)\middle|\{p,f,c\}_{\mathrm{a,b},m}\right)$$



Final state emitter with initial state spectator

Initial state emitter with final state spectator



SPLITTING OPERATOR

$$\big(\{\hat{p},\hat{f},\hat{c}\}_{\mathrm{a,b},m+1}\big|\mathcal{H}(t)\big|\{p,f,c\}_{\mathrm{a,b},m}\big)$$

$$= \sum_{l=\mathrm{a,b},1,\dots,m}\ \sum_{\substack{k=\mathrm{a,b},1,\dots,m\\ k\neq l}} \int_0^1 \frac{dy}{y}\int_0^1 dz \int_0^{2\pi}\frac{d\phi}{2\pi}\,\delta\big(t+\log(T_{l,k}(p_l,p_k,z,y)/Q^2)\big)$$

$$\times\, C_{l,k}\,\frac{\alpha_s\big(Q^2e^{-t}\big)}{2\pi}\,S_{l,k}(z,y,\hat{f}_{l,1},\hat{f}_{l,2})\,\frac{\hat{\eta}_{\mathrm{a}}}{\eta_{\mathrm{a}}}\,\frac{f_{\hat{a}/A}(\hat{\eta}_{\mathrm{a}},\mu_F^2)}{f_{a/A}(\eta_{\mathrm{a}},\mu_F^2)}\,\frac{\hat{\eta}_{\mathrm{b}}}{\eta_{\mathrm{b}}}\,\frac{f_{\hat{b}/B}(\hat{\eta}_{\mathrm{b}},\mu_F^2)}{f_{b/B}(\eta_{\mathrm{b}},\mu_F^2)}$$

$$\times\,\boxed{\big(\{\hat{p},\hat{f},\hat{c}\}_{\mathrm{a,b},m+1}\big|\mathcal{R}_{l,k}(z,y,\kappa_\perp)\big|\{p,f,c\}_{\mathrm{a,b},m}\big)}$$

$$\big(\{\hat{p},\hat{f}\}_{\mathrm{a,b},m+1}\big|\mathcal{R}_{l,k}(z,y,\kappa_\perp)\big|\{p,f\}_{\mathrm{a,b},m}\big)$$

$$= \frac{1}{2}(1-y)\,\delta^{f_l}_{\hat{f}_{l,1}+\hat{f}_{l,2}}\,\delta^a_{\hat{a}}\,\delta(\hat{\eta}_{\mathrm{a}}-\eta_{\mathrm{a}})\,\delta^b_{\hat{b}}\,\delta(\hat{\eta}_{\mathrm{b}}-\eta_{\mathrm{b}})\prod_{\substack{i=1\\ i\neq l}}^{m}\delta^{f_i}_{\hat{f}_i}$$

$$\times\,\delta^{(4)}(\hat{p}_k-(1-y)p_k)\prod_{\substack{i=1\\ i\neq l,k}}^{m}\delta^{(4)}(\hat{p}_i-p_i)$$

$$\times\,\delta^{(4)}\big(\hat{p}_{l,1}-zp_l-y(1-z)p_k-[2p_l\cdot p_k yz(1-z)]^{1/2}\kappa_\perp\big)$$

$$\times\,\delta^{(4)}\big(\hat{p}_{l,2}-(1-z)p_l-yzp_k+[2p_l\cdot p_k yz(1-z)]^{1/2}\kappa_\perp\big)$$

SHOWER CROSS SECTION

The evolution starts from the simplest configuration, e.g.: $p\bar{p} \to \text{jets}$, the simplest configurations are

$$p\bar{p} \longrightarrow 2 \text{ partons}$$

The shower cross section is

Starting hard scale

$$\sigma[F] = \big(F\big|D(t_\mathrm{f})U(t_\mathrm{f},t_2)\big|\sigma_2\big)$$

Hadronization

Infrared cutoff scale

SUMMARY: SHOWER EVOLUTION

- We defined a nice operator formalism for describing the parton shower.
- Improvements
  - Kinematics: Exact phase space in every steps.
  - Lorentz covariant and invariant.
  - Better soft gluon treatment.
  - No external parameters.
- What about the *freedom*?
  - Adding finite terms to the splitting kernels.
  - Freedom to improve this algorithm:
    - Better soft gluon treatment: Including subleading color contributions.
    - Adding higher order contributions: $1 \to 3, 2 \to 4, \ldots$

MATCHING BORN LEVEL MATRIX ELEMENTS TO PARTON SHOWER

Outlines:

- Definition of the scheme
- Connection to the slicing method (CKKW method)

ADJOINT SPLITTING OPERATOR

Let us define the operator $\mathcal{H}^\dagger(t)$ according to

$$\big(F\big|\mathcal{H}(t)\Gamma\big|\mathcal{M}_2\big) = \big(\mathcal{M}_2\big|\mathcal{H}^\dagger(t)\Gamma\big|F\big)$$

Since $\mathcal{H}(t)$ always increases the number of partons $\mathcal{H}^\dagger(t)$ always decreases it.

For multiple emission:

$$\begin{aligned}&\big(F\big|\mathcal{H}(t_m)\mathcal{H}(t_{m-1})\cdots\mathcal{H}(t_3)\Gamma\big|\mathcal{M}_2\big)\\ &\qquad = \big(\mathcal{M}_2\big|\mathcal{H}^\dagger(t_3)\cdots\mathcal{H}^\dagger(t_{m-1})\mathcal{H}^\dagger(t_m)\Gamma\big|F\big)\end{aligned}$$

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$$\begin{aligned}
&\big(\{\tilde{p},\tilde{f},\tilde{c}\}_{\mathrm{a,b},m}\big|\mathcal{H}^\dagger(t)\big|\{p,f,c\}_{\mathrm{a,b},m+1}\big) \\
&\qquad = \sum_{\substack{i,j\\ \text{pairs}}}\sum_{k\neq i,j} \frac{1}{2p_i\!\cdot\!p_j}\,\frac{\eta_\mathrm{a}}{\tilde{\eta}_\mathrm{a}}\frac{\eta_\mathrm{b}}{\tilde{\eta}_\mathrm{b}}\,V_{ij,k}(p_i,p_j,p_k,f_i,f_j,c_i,c_j,c_k) \\
&\qquad\quad \times \delta\big(t+\log(T_{ij,k}(p_i,p_j,p_k)/Q^2)\big) \\
&\qquad\quad \times \big(\{\tilde{p},\tilde{f},\tilde{c}\}_{\mathrm{a,b},m}\big|\mathcal{Q}_{ij,k}\big|\{p,f,c\}_{\mathrm{a,b},m+1}\big)
\end{aligned}$$

APPROX. MATRIX ELEMENT

If $\big|\{p,f,c\}_{\mathrm{a,b},m}\big)$ was generated by a shower procedure then the following is a good approximation:

$$\big(\mathcal{M}_m\big|\{p,f,c\}_{\mathrm{a,b},m}\big) \approx \underbrace{\int_{t_2}^{t_\mathrm{f}} dt_3 \int_{t_3}^{t_\mathrm{f}} dt_4 \cdots \int_{t_{m-1}}^{t_\mathrm{f}} dt_m \prod_{k=3}^{m} \frac{\alpha_s(\mu_R^2)}{\alpha_s(Q^2 e^{-t_k})} \times\big(\mathcal{M}_2\big|\mathcal{H}^\dagger(t_3)\mathcal{H}^\dagger(t_4)\cdots\mathcal{H}^\dagger(t_m)\big|\{p,f,c\}_{\mathrm{a,b},m}\big)}_{\big(\mathcal{A}_m(t_\mathrm{f},t_2)\big|\{p,f,c\}_{\mathrm{a,b},m}\big)}$$

$$w_M(\{p,f,c\}_{\mathrm{a,b},m}, t_\mathrm{f}, t_2) = \begin{cases} \dfrac{\big(\mathcal{M}_m\big|\{p,f,c\}_{\mathrm{a,b},m}\big)}{\big(\mathcal{A}_m(t_\mathrm{f},t_2)\big|\{p,f,c\}_{\mathrm{a,b},m}\big)} & \text{if } \mathcal{M}_m \text{ is known} \\ 1 & \text{otherwise} \end{cases}$$

APPROX. MATRIX ELEMENT

If $\big|\{p,f,c\}_{\mathrm{a,b},m}\big)$ was generated by a shower procedure then the following is a good approximation:

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Matrix element reweighting operator:

$$W_M(t_\mathrm{f},t_2) = \sum_m \int \big[d\{p,f,c\}_{\mathrm{a,b},m}\big]\,\big|\{p,f,c\}_{\mathrm{a,b},m}\big)\big(\{p,f,c\}_{\mathrm{a,b},m}\big| \times w_M(\{p,f,c\}_{\mathrm{a,b},m},t_\mathrm{f},t_2)$$

MATCHING AT BORN LEVEL

Expanding the first step of the shower cross section:

$$\big|\sigma(t_{\mathrm{f}})\big) = N(t_{\mathrm{f}}, t_2)\big|\sigma_2\big) + \int_{t_2}^{t_{\mathrm{f}}} dt_3\, U(t_{\mathrm{f}}, t_3)\, \underbrace{\mathcal{H}(t_3) N(t_3, t_2)\big|\sigma_2\big)}_{\sim \big|\mathcal{A}_3(t_{\mathrm{f}}, t_2)\big)}$$

It is better to use the 3-parton matrix element in the second term. Assuming we know $\mathcal{M}_3$

$$\big|\sigma_M(t_{\mathrm{f}})\big) = N(t_{\mathrm{f}}, t_2)\big|\sigma_2\big) + \int_{t_2}^{t_{\mathrm{f}}} dt_3\, U(t_{\mathrm{f}}, t_3) W_M(t_{\mathrm{f}}, t_2) \mathcal{H}(t_3) N(t_3, t_2)\big|\sigma_2\big)$$

Adding and subtracting the same terms we have

$$\big|\sigma_M(t_{\mathrm{f}})\big) = \underbrace{U(t_{\mathrm{f}}, t_2)\big|\sigma_2\big)}_{\text{Standard shower}} + \int_{t_2}^{t_{\mathrm{f}}} dt_3\, U(t_{\mathrm{f}}, t_3) \underbrace{\big[W_M(t_{\mathrm{f}}, t_2), \mathcal{H}(t_3)\big]}_{W_M(t_{\mathrm{f}}, t_2)\mathcal{H}(t_3) - \mathcal{H}(t_3) W_M(t_{\mathrm{f}}, t_2)} N(t_3, t_2)\big|\sigma_2\big)$$

MATCHING AT BORN LEVEL

Expanding the first step of the shower cross section:

$$\big|\sigma(t_\mathrm{f})\big) = N(t_\mathrm{f},t_2)\big|\sigma_2\big) + \int_{t_2}^{t_\mathrm{f}} dt_3\, U(t_\mathrm{f},t_3)\, \underbrace{\mathcal{H}(t_3) N(t_3,t_2)\big|\sigma_2\big)}_{\sim\,|A_2(t_\mathrm{f},\,t_2))}$$

It

se

$$\big[W_M(t_\mathrm{f},t_2),\mathcal{H}(t_3)\big]\big|\sigma_2\big) \sim \big|\sigma_3\big) - \mathcal{H}(t_3)\big|\sigma_2\big)$$

$$\big|\sigma_M(t_\mathrm{f})\big) = N(t_\mathrm{f},t_2)\big|\sigma_2\big) + \int_{t_2}^{t_\mathrm{f}} dt_3\, U(t_\mathrm{f},t_3) W_M(t_\mathrm{f},t_2)\mathcal{H}(t_3) N(t_3,t_2)\big|\sigma_2\big)$$

Adding and subtracting the same terms we have

$$\big|\sigma_M(t_\mathrm{f})\big) = \underbrace{U(t_\mathrm{f},t_2)\big|\sigma_2\big)}_{\text{Standard shower}} + \int_{t_2}^{t_\mathrm{f}} dt_3\, U(t_\mathrm{f},t_3)\, \underbrace{\big[W_M(t_\mathrm{f},t_2),\mathcal{H}(t_3)\big]}_{W_M(t_\mathrm{f},t_2)\mathcal{H}(t_3)-\mathcal{H}(t_3)W_M(t_\mathrm{f},t_2)} N(t_3,t_2)\big|\sigma_2\big)$$

MATCHING AT BORN LEVEL

Assuming we know $\mathcal{M}_3$, $\mathcal{M}_4$, ..., $\mathcal{M}_n$, the matched shower cross section is

$$
\begin{aligned}
&\big|\sigma_M(t_\mathrm{f})\big) = N(t_\mathrm{f}, t_2)\big|\sigma_2\big) \\
&\quad + \sum_{m=3}^{n-1} \int_{t_2}^{t_\mathrm{f}} dt_3 \int_{t_3}^{t_\mathrm{f}} dt_4 \cdots \int_{t_{m-1}}^{t_\mathrm{f}} dt_m \, N(t_\mathrm{f}, t_m) W_M(t_\mathrm{f}, t_2) \mathcal{H}(t_m) N(t_m, t_{m-1}) \\
&\qquad\qquad \times \mathcal{H}(t_{m-1}) N(t_{m-1}, t_{m-2}) \cdots \mathcal{H}(t_3) \, N(t_3, t_2)\big|\sigma_2\big) \\
&\quad + \int_{t_2}^{t_\mathrm{f}} dt_3 \int_{t_3}^{t_\mathrm{f}} dt_4 \cdots \int_{t_{n-1}}^{t_\mathrm{f}} dt_n \, U(t_\mathrm{f}, t_n) W_M(t_\mathrm{f}, t_2) \mathcal{H}(t_n) N(t_n, t_{n-1}) \\
&\qquad\qquad \times \mathcal{H}(t_{n-1}) N(t_{n-1}, t_{n-2}) \cdots \mathcal{H}(t_3) \, N(t_3, t_2)\big|\sigma_2\big)
\end{aligned}
$$

MATCHING AT BORN LEVEL

After some algebraic manipulation:

$$\big|\sigma_M(t_\mathrm{f})\big) = \big|\sigma_\Delta(t_\mathrm{f})\big) \equiv N(t_\mathrm{f},t_2)\big|\sigma_2\big) + \sum_{m=3}^{n-1}\int_{t_2}^{t_\mathrm{f}} dt_m\, N(t_\mathrm{f},t_m)W_\Delta(t_\mathrm{f},t_m,t_2)\big|\sigma_m\big) + \int_{t_2}^{t_\mathrm{f}} dt_n\, U(t_\mathrm{f},t_n)W_\Delta(t_\mathrm{f},t_n,t_2)\big|\sigma_n\big)$$

$$W_\Delta(t_\mathrm{f},t,t_2) = \sum_m \int \big[d\{p,f,c\}_{\mathrm{a,b},m}\big]\,\big|\{p,f,c\}_{\mathrm{a,b},m}\big)\big(\{p,f,c\}_{\mathrm{a,b},m}\big| \times \lim_{\delta\to 0}\int_{t_2}^{t} dt_{m-1}\int_{t_2}^{t_{m-1}} dt_{m-2}\cdots\int_{t_2}^{t_4} dt_3 \times \frac{\big(\mathcal{M}_2\big|N(t_3,t_2)\mathcal{H}^\dagger(t_3)\cdots N(t,t_{m-1})\mathcal{H}^\dagger(t)\big|\{p,f,c\}_{\mathrm{a,b},m}\big)}{\big(\mathcal{A}_m(t_\mathrm{f},t_2)\big|\{p,f,c\}_{\mathrm{a,b},m}\big)+\delta}$$

SLICING METHOD

(***C***atani-***K***rauss-***K***uhn-***W***ebber method)

Defining the matching scale $t_\mathrm{f} > t_\mathrm{ini} > t_2$ and using the group decomposation property:

$$\big|\sigma(t_\mathrm{f})\big) = U(t_\mathrm{f}, t_\mathrm{ini})U(t_\mathrm{ini}, t_2)\big|\sigma_2(t_2)\big) \approx U(t_\mathrm{f}, t_\mathrm{ini})\big|\sigma_\Delta(t_\mathrm{ini})\big)$$

The CKKW method use a simplified Sudakov rewighting operator based on the $k_\perp$ jet algorithm

$$\begin{aligned}\big|\sigma_\mathrm{CKKW}(t_\mathrm{f})\big) = {}& U(t_\mathrm{f}, t_\mathrm{ini})N(t_\mathrm{ini}, t_2)\big|\sigma_2\big) \\ &+ \sum_{m=3}^{n-1}\int_{t_2}^{t_\mathrm{ini}} dt_m\, U(t_\mathrm{f}, t_\mathrm{ini})N(t_\mathrm{ini}, t_m)W_\mathrm{CKKW}(t_\mathrm{ini}, t_m, t_2)\big|\sigma_m\big) \\ &+ \int_{t_2}^{t_\mathrm{ini}} dt_n\, U(t_\mathrm{ini}, t_n)W_\mathrm{CKKW}(t_\mathrm{ini}, t_n, t_2)\big|\sigma_n\big)\end{aligned}$$

Matching Parton Shower to NLO computation

PARTON SHOWER AT NLO

Let us calculate the N-jet cross section. The matrix element improved cross section is

$$\begin{aligned}(F_N|\sigma_\Delta(t_\mathrm{f})) = &\int_{t_2}^{t_\mathrm{f}} dt_N\,\big(F_N\big|N(t_\mathrm{f},t_N)W_\Delta(t_\mathrm{f},t_N,t_2)\big|\sigma_N\big)\\ &+\int_{t_2}^{t_\mathrm{f}} dt_{N+1}\,\big(F_N\big|U(t_\mathrm{f},t_{N+1})W_\Delta(t_\mathrm{f},t_{N+1},t_2)\big|\sigma_{N+1}\big)\end{aligned}$$

Expanding it in α_s then we have

"Error term" from $1/N_c^2$ approx. : $E = E^{(0)} + \frac{\alpha_s}{2\pi}E^{(1)} = \mathcal{O}\left(\frac{1}{N_c^2}\right)$

$$(F_N|\sigma_\Delta) = \int_N d\sigma^B\left(1 + E + \frac{\alpha_s}{2\pi}W_\Delta^{(1)}\right) + \underbrace{\int_{N+1}\left[d\sigma^R - d\sigma^A\right]}_{\text{Real - Dipoles}} + \mathcal{O}(\alpha_s^2)$$

Born term

"Quasi virtual"

PARTON SHOWER AT NLO

The NLO parton shower for an N-jet cross section is

$$\begin{aligned}\big(F_N\big|\sigma_{\mathrm{NLO}}(t_{\mathrm{f}})\big) = & \int_{t_2}^{t_{\mathrm{f}}} dt_N \,\big(F_N\big|N(t_{\mathrm{f}},t_N)W_\Delta(t_{\mathrm{f}},t_N,t_2)\big|\sigma_N\big) \\ & + \int_{t_2}^{t_{\mathrm{f}}} dt_{N+1}\,\big(F_N\big|U(t_{\mathrm{f}},t_{N+1})W_\Delta(t_{\mathrm{f}},t_{N+1},t_2)\big|\sigma_{N+1}\big) \\ & + \int_{t_2}^{t_{\mathrm{f}}} dt_N\,\big(F_N\big|U(t_{\mathrm{f}},t_N)W_\Delta(t_{\mathrm{f}},t_N,t_2)\big|\sigma_N^{(1)}\big)\end{aligned}$$

$$\sim |\mathcal{M}_N|^2_{1-\mathrm{loop}} + \boldsymbol{I}\otimes|\mathcal{M}_N|^2$$

$$\big|\sigma_N^{\mathrm{NLO}}\big) = -\frac{\alpha_s}{2\pi}W_\Delta^{(1)}\,\big|\sigma_N\big) + \big|\sigma_N^{I+V}\big) + \big|\sigma_N^{P+K}\big)$$

$$\sim (\boldsymbol{P}(\mu_F)+\boldsymbol{K})\otimes|\mathcal{M}_N|^2$$

SUMMARY

- We defined a new formalism for describing the parton shower
- Exact kinematics, Lorentz invariant and covariant formalism, improved soft gluon
- No phase space cut parametes at all, only the infrared cutoff parameter
- Clear way to add higher order to the shower
- It is possible to add massive fermions in the same way
- Matched to the LO matrix elements
- Matched to the “NLO matrix elements”