Multi-Jet Predictions in the High Energy Limit of QCD

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Outline

The High Energy Limit of Fixed Order Matrix Elements

Process	Diagrams	$\overline{\sum}|\mathcal{M}|^2/g^4$
$qq' \rightarrow qq'$		$\dfrac{4}{9}\dfrac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$
$q\bar{q} \rightarrow q'\bar{q}'$		$\dfrac{4}{9}\dfrac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$q\bar{q} \rightarrow gg$		$\dfrac{32}{27}\dfrac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \dfrac{8}{3}\dfrac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$

High Energy Limit: $|\hat{t}|$ fixed, $\hat{s} \rightarrow \infty$

t–channel dominance

Example: W+n-jet production at the LHC
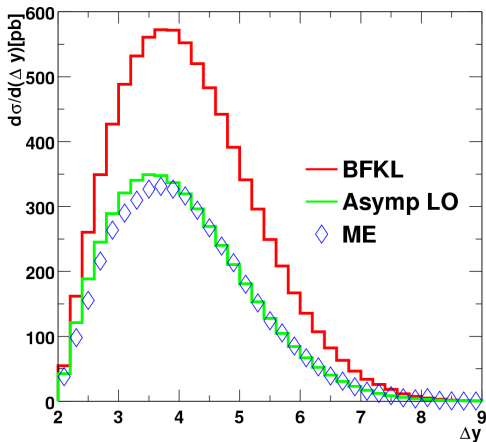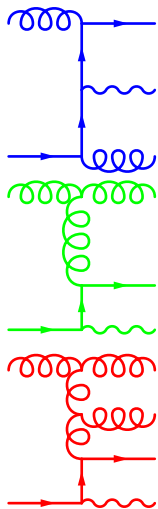
$$\Delta y = y_{j_2} - y_{j_1}, y_W, y_{j_2} \geq 1, y_{j_1} \leq -1$$

Observations

- In the limit of large rapidity spans, the fixed order matrix elements are dominated by contributions from diagrams with a t-channel gluon exchange

- This limit will be called The High Energy Limit and is generally characterised by the following phase space configuration of the final state particles

$$y_0 > y_1 > \cdots > y_n > y_{n+1}, \quad |k_0| \sim |k_i| \sim |k_{n+1}|$$

  i.e. multiple, isolated, hard parton production (multiple jets)

- Good agreement ($\sim 10\%$) with the full, fixed order result in the relevant limit

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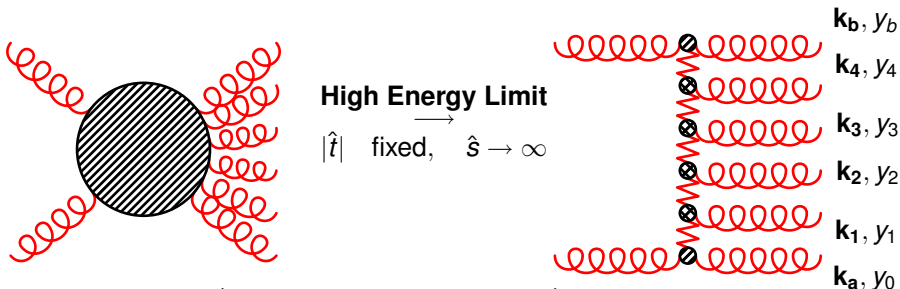
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The Possibility for Prediction of n-jet Rates
The Power of Reggeisation

High Energy Limit
$$|\hat{t}| \quad \text{fixed}, \quad \hat{s} \to \infty$$

$$\mathcal{A}^R_{2 \to 2+n} = \Gamma_{A'A} \left(\prod_{i=1}^{n} \frac{e^{\omega(q_i)(y_{i-1}-y_i)}}{q_i^2} V^{J_i}(q_i, q_{i+1}) \right) \frac{e^{\omega(q_{n+1})(y_n-y_{n+1})}}{q_{n+1}^2} \Gamma_{B'B}$$

$$\mathbf{q}_i = \mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l$$

At LL only gluon production; at NLL also quark–anti-quark pairs produced.

Prediction of any-jet rate possible.

Reggeisation and the BFKL Equation

The **evolution of the reggeised gluon** is described by the BFKL equation

$$\omega f_\omega \left(\mathbf{k}_a, \mathbf{k}_b \right) = \delta^{(2+2\epsilon)} \left(\mathbf{k}_a - \mathbf{k}_b \right) + \int d^{2+2\epsilon} \mathbf{k}' \mathcal{K}_\epsilon \left(\mathbf{k}_a, \mathbf{k}' \right) f_\omega \left(\mathbf{k}', \mathbf{k}_b \right)$$

ω: Mellin conjugated variable to the rapidity y along the evolution.

- The kernel $\mathcal{K}_\epsilon$ consists of the **virtual** corrections of the trajectory and the **real** corrections from the Lipatov vertices.

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Energy and Momentum Conservation in an Inclusive Framework

One of the benefits of BFKL : **Fully inclusive** any-jet cross
sections **can be calculated analytically**
$(p'_a, p'_b \rightarrow p_a, \{p_i\}, p_b)$

$$\mathrm{d}\hat{\sigma}(p_a, p_b) = \Gamma_a(\mathbf{p}_a) \, f(\mathbf{p}_a, -\mathbf{p}_b, \Delta) \, \Gamma_b(\mathbf{p}_b)$$
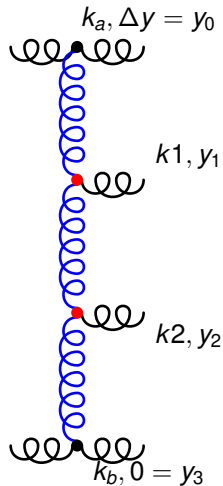
Inclusive partonic cross section depending on two **final state**
particles.
In order to reconstruct the initial state (impose energy and
momentum conservation, correct parton momentum etc.) and
calculate the convolution with pdfs, we need the full final state
information[1]!

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[1] Not resummation of soft, colinear radiation: large contribution to energy

$\Omega \, Q \, \curvearrowright$

Iteration at (Next to) Leading Logarithmic Accuracy

$$f(\mathbf{k}_a, \mathbf{k}_b, \Delta) = \exp\left(\omega_0\left(\mathbf{k}_a^2, \lambda^2, \mu\right)\Delta\right)\delta^{(2)}(\mathbf{k}_a - \mathbf{k}_b)$$

$$+ \sum_{n=1}^{\infty}\prod_{i=1}^{n}\int d^2\mathbf{k}_i\int_0^{y_{i-1}}dy_i\left[V\left(\mathbf{k}_i, \mathbf{k}_a + \sum_{l=0}^{i-1}\mathbf{k}_l, \mu\right)\right]$$

$$\times \exp\left[\omega_0\left(\left(\mathbf{k}_a + \sum_{l=1}^{i-1}\mathbf{k}_l\right)^2, \lambda^2, \mu\right)(y_{i-1} - y_i)\right]$$

$$\times \exp\left[\omega_0\left(\left(\mathbf{k}_a + \sum_{l=1}^{n}\mathbf{k}_l\right)^2, \lambda^2, \mu\right)(y_n - 0)\right]$$

$$\times \delta^{(2)}\left(\sum_{l=1}^{n}\mathbf{k}_l + \mathbf{k}_a - \mathbf{k}_b\right)$$

$k_a, \Delta y = y_0$

$k1, y_1$

$k2, y_2$

$k_b, 0 = y_3$

Direct BFKL Evolution @ LL&NLL

Solution to the BFKL equation at fixed Δ at both LL and NLL:

$$f(\mathbf{k}_a, \mathbf{k}_b, \Delta) = \sum_{n=0}^{\infty} \int d\mathcal{P}_n \, \mathcal{F}_n,$$

$$\int d\mathcal{P}_n = \left(\int \prod_{i=1}^{n} d\mathbf{k}_i \int_0^{y_0} dy_1 \int_0^{y_1} dy_2 \cdots \int_0^{y_{n-1}} dy_n \right) \delta^{(2)} \left(\mathbf{k}_a + \sum_{l=1}^{n} \mathbf{k}_l - \mathbf{k}_b \right)$$

$$\mathcal{F}_n = \left(\prod_{i=1}^{n} e^{\omega(\mathbf{q}_i)(y_{i-1}-y_i)} \, V(\mathbf{q}_i, \mathbf{q}_{i+1}) \right) e^{\omega(\mathbf{q}_{n+1})(y_n-y_{n+1})}$$

$$\int_0^{y_0} dy_1 \int_0^{dy_1} dy_2 \cdots \int_0^{dy_{n-1}} dy_n \left(\prod_{i=1}^{n} e^{\omega(\mathbf{q}_i)(y_{i-1}-y_i)} \right) e^{\omega(\mathbf{q}_{n+1})(y_n-y_{n+1})}$$

$$= \int_0^{\Delta} d\delta y_n \int_0^{\Delta-y_n} d\delta y_{n-1} \cdots \int_0^{\Delta-y_n-\cdots-y_2} d\delta y_1 \left(\prod_{i=1}^{n} e^{\omega(\mathbf{q}_i)\delta y_i} \right) e^{\omega(\mathbf{q}_{n+1})\delta y_{n+1}}$$

$$= \int_0^{\infty} d\delta y_{n+1} \int_0^{\infty} d\delta y_n \cdots \int_0^{\infty} d\delta y_1 \, \delta(\Delta - \sum_{i=1}^{n+1} \delta y_i) \prod_{i=1}^{n+1} e^{\omega(\mathbf{q}_i)\delta y_i}$$

Direct BFKL Evolution, 2

$$f(\mathbf{k}_a, \mathbf{k}_b, \Delta) = \sum_{n=0}^{\infty} \int d\mathcal{P}_n \, \mathcal{F}_n,$$

$$\int d\mathcal{P}_n = \left(\prod_{i=1}^{n} \int d\mathbf{k}_i \int_0^{\infty} d\delta y_i \right) \int_0^{\infty} d\delta y_{n+1} \, \delta^{(2)} \left(\mathbf{k}_a + \sum_{l=1}^{n} \mathbf{k}_l - \mathbf{k}_b \right) \delta \left(\Delta - \sum_{i=1}^{n+1} \delta y_i \right)$$

$$\mathcal{F}_n = \left(\prod_{i=1}^{n} e^{\omega(\mathbf{q}_i)\delta y_i} \, V(\mathbf{q}_i, \mathbf{q}_{i+1}) \right) \, e^{\omega(\mathbf{q}_{n+1})\delta y_{n+1}}$$

$f(\mathbf{k}_a, \mathbf{k}_b, \Delta)$: the value at $\Delta \equiv \sum_{i=1}^{n+1} \delta y_i$ of the product of vertices $V(\mathbf{q}_i, \mathbf{q}_{i+1})$ at rapidity $y_i = \sum_{j=1}^{i} \delta y_j$ connected with Regge factors $e^{\omega(\mathbf{q}_i)\delta y_i}$ describing the probability of no (resolved) emission between two adjacent (in rapidity) vertices

Direct BFKL Evolution, 3

1. Choose a random number of vertices for the evolution, $n \geq 0$

2. Generate a set $\{\mathbf{k}_i\}_{i=1,\ldots,n}$ of transverse momenta (the outgoing momenta are $\{-\mathbf{k}_i\}_{i=1,\ldots,n}$)

3. Calculate the corresponding set of trajectories $\{\omega(\mathbf{q}_i)\}_{i=1,\ldots,n+1}$, and vertex factors $\{V(\mathbf{q}_i, \mathbf{q}_{i+1})\}_{i=1,\ldots,n}$, $\mathbf{q}_i = k_a + \sum_{l=1}^{i-1} \mathbf{k}_l$

4. Generate the inter-vertex rapidity separations $\{\delta y_i\}$ according to the distributions $e^{\omega(\mathbf{q}_i)\delta y_i}$

5. Calculate the corresponding $\Delta = \sum_{i=1}^{n+1} \delta y_i$ and return $\prod_{i=1}^{n} V(\mathbf{q}_i, \mathbf{q}_{i+1})$

Can construct full final state[2]! Trivial to impose energy and momentum conservation and do **proper jet studies**.

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[2]See later

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Observation

1. Imposing Energy and Momentum conservation (i.e. restricting phase space integral to that accessible at a given energy) is completely unrelated to the NLL corrections to the evolution.

2. To calculate an observable to full NLL accuracy, three ingredients are necessary:
   - NLL Impact Factors
   - NLL Evolution
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But I thought E&M-conservation *was* a NLL effect –
Why do you say it is not taken into account by the NLL
corrections to the kernel?!

$$\Delta y \approx_{\text{LL}} \ln s/s_0 = \ln s/s_1 + \ln s_1/s_0$$

Identifies **correctly** E&M-conservation as a **NLL effect**.
However, the very form of the BFKL equation means that effects from
**constraints in phase space cannot be taken into account by the BFKL
kernel**:

1. The kernel evolves between two transverse momenta; the BFKL
   equation is uniform in rapidity. Emission of (energetic) particles does not
   influence the phase space of emission later (or earlier) in the evolution

2. The NLL corrections to the kernel itself involves a fully inclusive phase
   space integral over two-particle production vertices.

Poses no problem in diffractive studies (where no particles are emitted from
the evolution) - however, very significant effects in jet studies.

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The Ingredients of the NLL Vertex

$$V(\mathbf{q}_1, \mathbf{q}_2) = \left| \text{} \right|^2 + \int d\mathcal{P} \left| \text{} \right|^2 + \int d\mathcal{P} \left| \text{} \right|^2$$

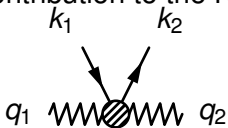Two methods for obtaining the vertices at NLL:

- Fadin & Lipatov:

- V. Del Duca:

Divergences and Strategy

Quark contribution to the NLL vertex:

$$k_1 \qquad k_2$$

$$q_1 \qquad \qquad q_2$$

Divergences separate into two categories:

$\Delta = \mathbf{q}_1 - \mathbf{q}_2 = 0$: Regulated by the **NLL Trajectory**

$\mathbf{k}_1 \to x\Delta$: Regulated by the quark contribution to the one-loop correction to the one-gluon production vertex (x is the light-cone momentum fraction of the anti-quark)

Strategy:

1. Implement Lipatov Vertices and perform integration, while having access to **full final state information**. Only possibility of **combining energy and momentum conservation** with **NLL evolution**.

2. **Check** that the **numerical integration over full phase space** agrees with the result of Fadin & Lipatov (or Camici & Ciafaloni)

Structure of the Amplitude

$$K_r^{(2),q\bar{q}}(\mathbf{q}_1, \mathbf{q}_2) \propto \int d\kappa \; d\rho_f \; \delta^{(D)}(q_1 - q_2 - k_1 - k_2) \sum_{i_1, i_2, f} \left| \gamma_{i_1 i_2}^{q\bar{q}}(q_1, q_2, k_1, k_2) \right|^2$$

$$\sum_{i_1, i_2, f} \left| \gamma_{i_1 i_2}^{q\bar{q}}(q_1, q_2, k_1, k_2) \right|^2$$

$$\propto \left[(N_c^2 - 1) \left(\left| A_{+-}^{\bar{q}q}(k_1, k_2) \right|^2 + \left| A_{+-}^{\bar{q}q}(k_2, k_1) \right|^2 \right) + 2 A_{+-}^{\bar{q}q}(k_1, k_2) A_{+-}^{*\bar{q}q}(k_2, k_1) \right]$$

Symmetry properties of the divergent part of the amplitude ensures that the $1/N_c^2$ suppressed contribution is **finite**.

Phase space slice regularisation of the divergent pieces ensures cancellation between the divergences from the quark–antiquark production and the corresponding ones from the virtual corrections to the one-gluon production.
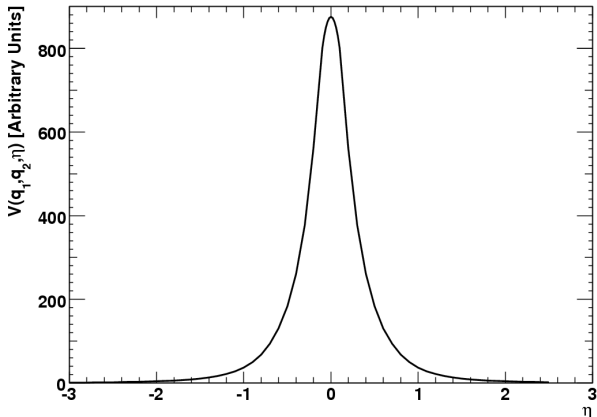
First Check. . .
Check of finite part
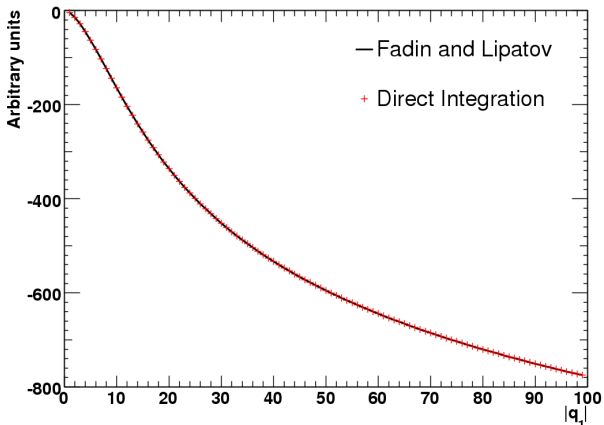
$1/N_c^2$ suppressed terms: 100% agreement.
Calculation under control.

Study of Rapidity Distribution within the $q\bar{q}$-Vertex

Antiquark: $(\mathbf{k_1}, \eta)$, quark: $(\mathbf{k_2}, -\eta)$
$q_1 = (20, 0)\mathrm{GeV}, q_2 = (15, 1)\mathrm{GeV}$

Summary and Conclustions

- Have constructed a **very efficient** method for obtaining the BFKL evolution as an approximation to multi-leg processes Also applicable to small-x studies etc.

- Have started the program to obtain **fully exclusive final state information** of the NLL BFKL Evolution **necessary for** energy and momentum conservation and thus **full NLL accuracy**

- Conclusion from the study of the exclusive NLL quark–anti-quark vertex:
  Exclusive information absolutely **crucial for realistic phenomenology**, since the $q\bar{q}$-vertex gets contributions from relatively large invariant masses of the $q\bar{q}$-pair. Cannot assign a single rapidity to the quark and the anti-quark.

- http://www.hep.phy.cam.ac.uk/~andersen/BFKL