HEAVY QUARKONIA SECTOR IN PYTHIA 6.324: TEST AND VALIDATION

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OUTLINE

- Motivations for the inclusion of Heavy Quarkonium contribution in PYTHIA;
- Current status: new channels and new NRQCD matrix elements: values and tuning;
- Experimental settings chosen for tests and validation;
- Comparison with Tevatron data and perspectives for LHC.

MOTIVATIONS FOR THE INCLUSION OF NRQCD IN PYTHIA

- **Production of charm and beauty hidden flavor states in PYTHIA was incomplete:**
  - Only color singlet processes (Color Singlet Model), no NRQCD implementation;
  - CSM largely fails in shape and normalization;
- **Not too flexible**
  - Cannot allow simultaneous production of ψ's and Υ's, nor Υ(1S) and Υ(2S), etc.

→ **Following the discussion started at a LCG/GENSER meeting in March 2005, T. Sjostrand introduced NRQCD for heavy quarkonia production in PYTHIA 6.324.**

→ Work done in the framework of LHCb and GENSER

- For the GENSER side, precious collaboration with P. Bartalini
- For the LHCb side, work done in collaboration with V. Vagnoni
- Fundamental help from T. Sjostrand

CURRENT STATUS

- Integration of the original code (by Stefan Wolf) made by T. Sjostrand in PYTHIA 6.324.
  - This PYTHIA implementation for NRQCD already existed since a few years, but it was not validated and never included in official releases.
  - PYTHIA 6.324 now relays **both to charmonia and bottomonia sector**
  - The code is now under validation;
  - Realistic parameter values (e.g. NRQCD MEs) have to be fixed.

→ OTHER VISIBLE IMPLICATIONS:

- **Possibility to produce simultaneously J/ψ and Υ (introduced as different processes)**
- **is still not possible to generate Υ’ and ψ’ simultaneously, but can be implemented ‘in locum’**

IMPLEMENTATION DETAILS: NEW CHANNELS (1)

- Originally only the Color Singlet Model (CSM) contributions to the quarkonia production were available in PYTHIA 6.2
- ….BUT Non-Relativitic Quantum Chromodinamics (NRQCD) predicts large contributions via the color octet mechanism

→ Introduction of new processes:

ISUB	$g+g \to c\bar{c}[n]+g$	ISUB	$q+g \to q+c\bar{c}[n]$	ISUB	$q+\bar{q} \to g+c\bar{c}[n]$
421	$g+g \to c\bar{c}[{}^3S_1^{(1)}]+g$				
422	$g+g \to c\bar{c}[{}^3S_1^{(8)}]+g$	425	$q+g \to q+c\bar{c}[{}^3S_1^{(8)}]$	428	$q+\bar{q} \to g+c\bar{c}[{}^3S_1^{(8)}]$
423	$g+g \to c\bar{c}[{}^1S_0^{(8)}]+g$	426	$q+g \to q+c\bar{c}[{}^1S_0^{(8)}]$	429	$q+\bar{q} \to g+c\bar{c}[{}^1S_0^{(8)}]$
424	$g+g \to c\bar{c}[{}^3P_J^{(8)}]+g$	427	$q+g \to q+c\bar{c}[{}^3P_J^{(8)}]$	430	$q+\bar{q} \to g+c\bar{c}[{}^3P_J^{(8)}]$

IMPLEMENTATION DETAILS: NEW CHANNELS (2)

…where ISUB = 421 is almost completly equivalent to ISUB =86 except from the fact that the CSM factors out the wave function $|R(0)|^2$ at the origin, while NRQCD parametrizes the non-perturbative part with the so-called '*NRQCD matrix elements*'.

- For χ_c: were implemented only the gluon-gluon fusion mode: again new modes implemented (from ISUB = 87-89 to ISUB =431- 433) with rearrenged constant as before
- Some photoproduction channels have been implemented in PYTHIA 6.2, even if they have not been tested
  - For PYTHIA 6.3 these channels have not been introduced yet!
- These new processes can be switched ON through 3 parameters MSEL:
  - 61: switch ON all charmonium processes, ISUB = 421 – 439;
  - 62: switch ON all bottomonium processes, ISUB = 461 – 479;
  - 63: switch ON both of above, ISUB = 421 - 439, 461 -479.

χ_c implementations in PYTHIA 6.3: g-g, q-g, q-q channels

ISUB	$g+g\to c\bar{c}[^3P_J^{(1)}]+g$	ISUB	$q+g\to q+c\bar{c}[^3P_J^{(1)}]$	ISUB	$q+\bar{q}\to g+c\bar{c}[^3P_J^{(1)}$
431	$g+g\to c\bar{c}[^3P_0^{(1)}]+g$	434	$q+g\to q+c\bar{c}[^3P_0^{(1)}]$	437	$q+\bar{q}\to g+c\bar{c}[^3P_0^{(1)}]$
432	$g+g\to c\bar{c}[^3P_1^{(1)}]+g$	435	$q+g\to q+c\bar{c}[^3P_1^{(1)}]$	438	$q+\bar{q}\to g+c\bar{c}[^3P_1^{(1)}]$
433	$g+g\to c\bar{c}[^3P_2^{(1)}]+g$	436	$q+g\to q+c\bar{c}[^3P_2^{(1)}]$	439	$q+\bar{q}\to g+c\bar{c}[^3P_2^{(1)}]$

Bottomonia implementation in PYTHIA 6.3

ISUB	$g+g\to b\bar{b}[n]+g$	ISUB	$q+g\to q+b\bar{b}[n]$	ISUB	$q+\bar{q}\to g+b\bar{b}[n]$
461	$g+g\to b\bar{b}[^3S_1^{(1)}]+g$				
462	$g+g\to b\bar{b}[^3S_1^{(8)}]+g$	465	$q+g\to q+b\bar{b}[^3S_1^{(8)}]$	468	$q+\bar{q}\to g+b\bar{b}[^3S_1^{(8)}]$
463	$g+g\to b\bar{b}[^1S_0^{(8)}]+g$	466	$q+g\to q+b\bar{b}[^1S_0^{(8)}]$	469	$q+\bar{q}\to g+b\bar{b}[^1S_0^{(8)}]$
464	$g+g\to b\bar{b}[^3P_J^{(8)}]+g$	467	$q+g\to q+b\bar{b}[^3P_J^{(8)}]$	470	$q+\bar{q}\to g+b\bar{b}[^3P_J^{(8)}]$

χ_b implementations in PYTHIA 6.3: g-g, q-g, q-q channels

ISUB	$g+g \rightarrow b\bar{b}[^3P_J^{(1)}]+g$	ISUB	$q+g \rightarrow q+b\bar{b}[^3P_J^{(1)}]$	ISUB	$q+\bar{q} \rightarrow g+b\bar{b}[^3P_J^{(1)}$
471	$g+g \rightarrow b\bar{b}[^3P_0^{(1)}]+g$	474	$q+g \rightarrow q+b\bar{b}[^3P_0^{(1)}]$	477	$q+\bar{q} \rightarrow g+b\bar{b}[^3P_0^{(1)}]$
472	$g+g \rightarrow b\bar{b}[^3P_1^{(1)}]+g$	475	$q+g \rightarrow q+b\bar{b}[^3P_1^{(1)}]$	478	$q+\bar{q} \rightarrow g+b\bar{b}[^3P_1^{(1)}]$
473	$g+g \rightarrow b\bar{b}[^3P_2^{(1)}]+g$	476	$q+g \rightarrow q+b\bar{b}[^3P_2^{(1)}]$	479	$q+\bar{q} \rightarrow g+b\bar{b}[^3P_2^{(1)}]$

NEW PARAMETERS: THE NRQCD MATRIX ELEMENTS (1)

- As CSM, NRQCD parametrises the non-perturbative fragmentation of the $Q\overline{Q}$ pair into the quarkonium state…..**BUT**:
  - while CSM requires only two parameters (|R(0)|^2 and |R'(0)|^2 = wave function at the origin, and first derivative squared: PARP(38) and PARP(39)):

$$\left\langle O^{J/\psi}[{}^3S_1^{(1)}]\right\rangle = \frac{3N_C}{2\pi}|R(0)|^2 ,$$

$$\left\langle O^{\chi_c}[{}^3P_0^{(1)}]\right\rangle = \frac{3N_C}{2\pi}|R'(0)|^2 .$$

→ NRQCD requires **INDIPENDENT** matrix elements:

$$\left\langle O^{H}[{}^{2S+1}L_J^{(C)}]\right\rangle$$

to denote the probability that a $Q\overline{Q}$ pair in a state $^{2S+1}L_J^{(C)}$ build up the bound state H.

These matrix elements fullfils the relation due to heavy quark spin symmetry:

$$\left\langle O^{\chi_{cJ}}[{}^3P_J^{(8)}]\right\rangle = (2J+1)\left\langle O^{J/\psi}[{}^3P_0^{(8)}]\right\rangle,$$

$$\left\langle O^{\chi_{cJ}}[{}^3P_J^{(1)}]\right\rangle = (2J+1)\left\langle O^{\chi_{c0}}[{}^3P_0^{(1)}]\right\rangle.$$

NEW PARAMETERS: THE NRQCD MATRIX ELEMENTS (2)

→ The rates for these new processes are regulated by 10 NEW NRQCD matrix elements values (their default values are set to one in the current release, and need tuning):

PARP(141)	$\langle O^{J/\psi}[{}^3S_1^{(1)}]\rangle$
PARP(142)	$\langle O^{J/\psi}[{}^3S_1^{(8)}]\rangle$
PARP(143)	$\langle O^{J/\psi}[{}^1S_0^{(8)}]\rangle$
PARP(144)	$\langle O^{J/\psi}[{}^3P_0^{(8)}]\rangle / m_c^2$
PARP(145)	$\langle O^{\chi_{c0}}[{}^3P_0^{(1)}]\rangle / m_c^2$
PARP(146)	$\langle O^{\Upsilon}[{}^3S_1^{(1)}]\rangle$
PARP(147)	$\langle O^{\Upsilon}[{}^3S_1^{(8)}]\rangle$
PARP(148)	$\langle O^{\Upsilon}[{}^1S_0^{(8)}]\rangle$
PARP(149)	$\langle O^{\Upsilon}[{}^3P_0^{(8)}]\rangle / m_b^2$
PARP(150)	$\langle O^{\chi_{b0}}[{}^3P_0^{(1)}]\rangle / m_b^2$

SIMULATION SETTINGS

- Several data samples produced under the following Tevatron settings:
  - $p\text{-}\bar{p}$ collisions;
  - 980.0 GeV Beam Momentum;
  - Energy reference for Tevatron: 1960 GeV;
  - processes on:
    - **all new numbered processes: both for CSM and for COM**
    - **only J/ψ processes considered, both direct or produced from χc, excluding all B decays.**
    - **Fragmentation processes on;**
  - Rapidity region between -0.6 ÷ 0.6 ;
  - CTEQ6L used as PDF set
  - Different min. p_T cuts applied: standard (1 GeV), 2 GeV and 2.5 GeV

CURRENT STATUS FOR COM MATRIX ELEMENTS

- 10 new values for NRQCD matrix elements inserted based on values extracted from: hep-ph/0003142
  - **CSM values extracted from Buchmuller-Tye (Eichten-Quigg) potential model (hep-ph/9503356)**
- Renormalization and factorization scale $\boxed{\mu=\sqrt{p_t^2+4m_c^2}}$
- Charm quark mass: **m_c= 1.5 GeV**
- Different p_T cuts methods applied:
  - **CKIN(3) min. p_T cut**
  - **Reweighting function PYEVWT (activated with MSTP(142)=2)**

Current Status (values)

- New Corresponding Matrix elements inserted:

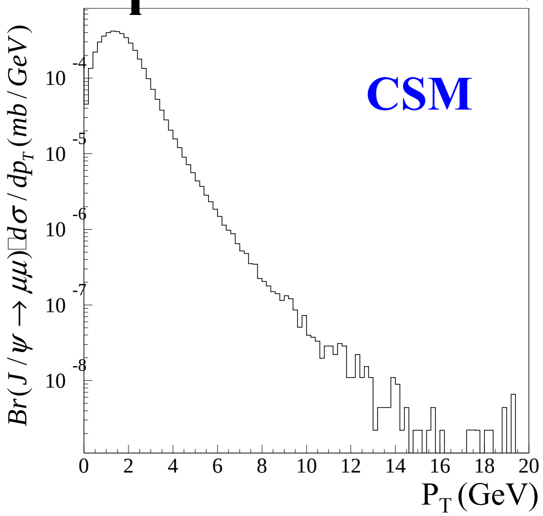
PARP(141)	$\langle O^{J/\psi}[^3S_1^{(1)}]\rangle$	1.16
PARP(142)	$\langle O^{J/\psi}[^3S_1^{(8)}]\rangle$	0.0119
PARP(143)	$\langle O^{J/\psi}[^1S_0^{(8)}]\rangle$	0.01
PARP(144)	$\langle O^{J/\psi}[^3P_0^{(8)}]\rangle / m_c^2$	0.01
PARP(145)	$\langle O^{\chi_{c0}}[^3P_0^{(1)}]\rangle / m_c^2$	0.05
PARP(146)	$\langle O^{\Upsilon}[^3S_1^{(1)}]\rangle$	9.28
PARP(147)	$\langle O^{\Upsilon}[^3S_1^{(8)}]\rangle$	0.15
PARP(148)	$\langle O^{\Upsilon}[^1S_0^{(8)}]\rangle$	0.02
PARP(149)	$\langle O^{\Upsilon}[^3P_0^{(8)}]\rangle / m_b^2$	0.48
PARP(150)	$\langle O^{\chi_{b0}}[^3P_0^{(1)}]\rangle / m_b^2$	0.09

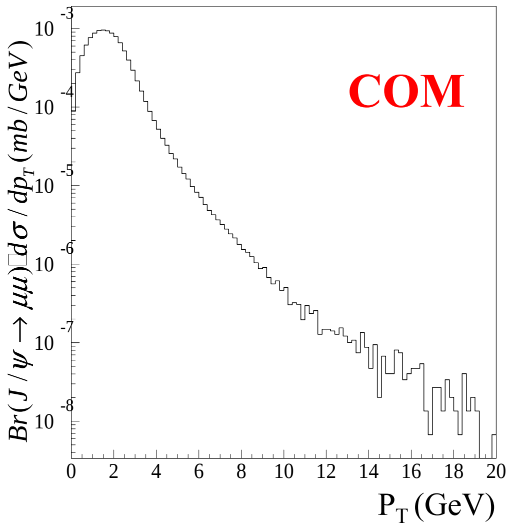
STATUS WITH CSM/COM ONLY (1GEV P_T MIN CUT)

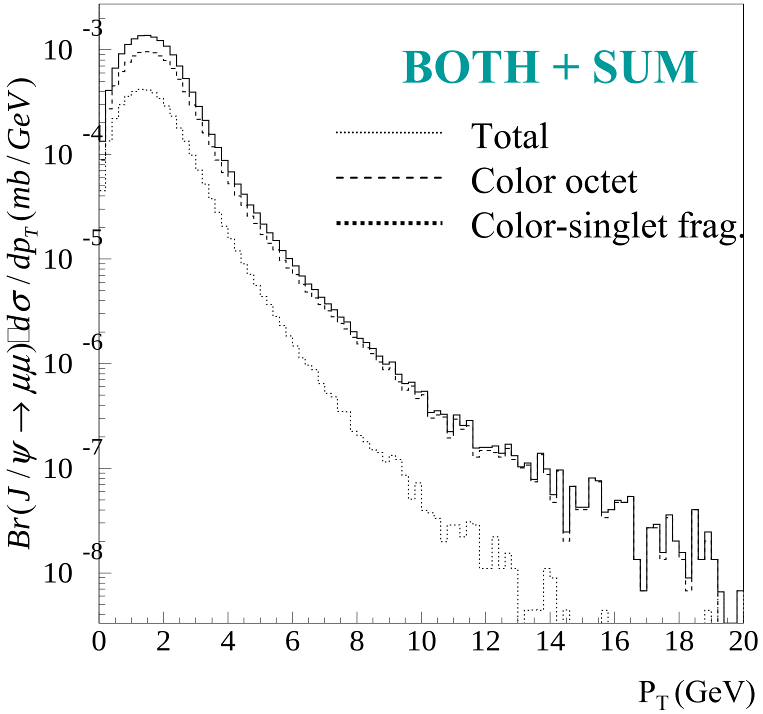
- CSM:
  - 10.0 million events produced with CSM model processes:
    - msub 421 active (same as 86): (S Wave):
      - $g+g \rightarrow c\bar{c}[{}^3S_1^{(1)}]+g$
    - msub 431, 432, 433 (same as 87, 88, 89): (P Wave)
      - $g+g \rightarrow c\bar{c}[{}^3P_0^{(1)}]+g$
      - $g+g \rightarrow c\bar{c}[{}^3P_1^{(1)}]+g$
      - $g+g \rightarrow c\bar{c}[{}^3P_2^{(1)}]+g$
    - all COM inactive

- COM:
  - 10.0 million events produced with COM model processes:
    - msub 422-430 active
    - all CSM inactive

x: p_T distribution, in y: $d\sigma/dp_T$*Br (in mb)).



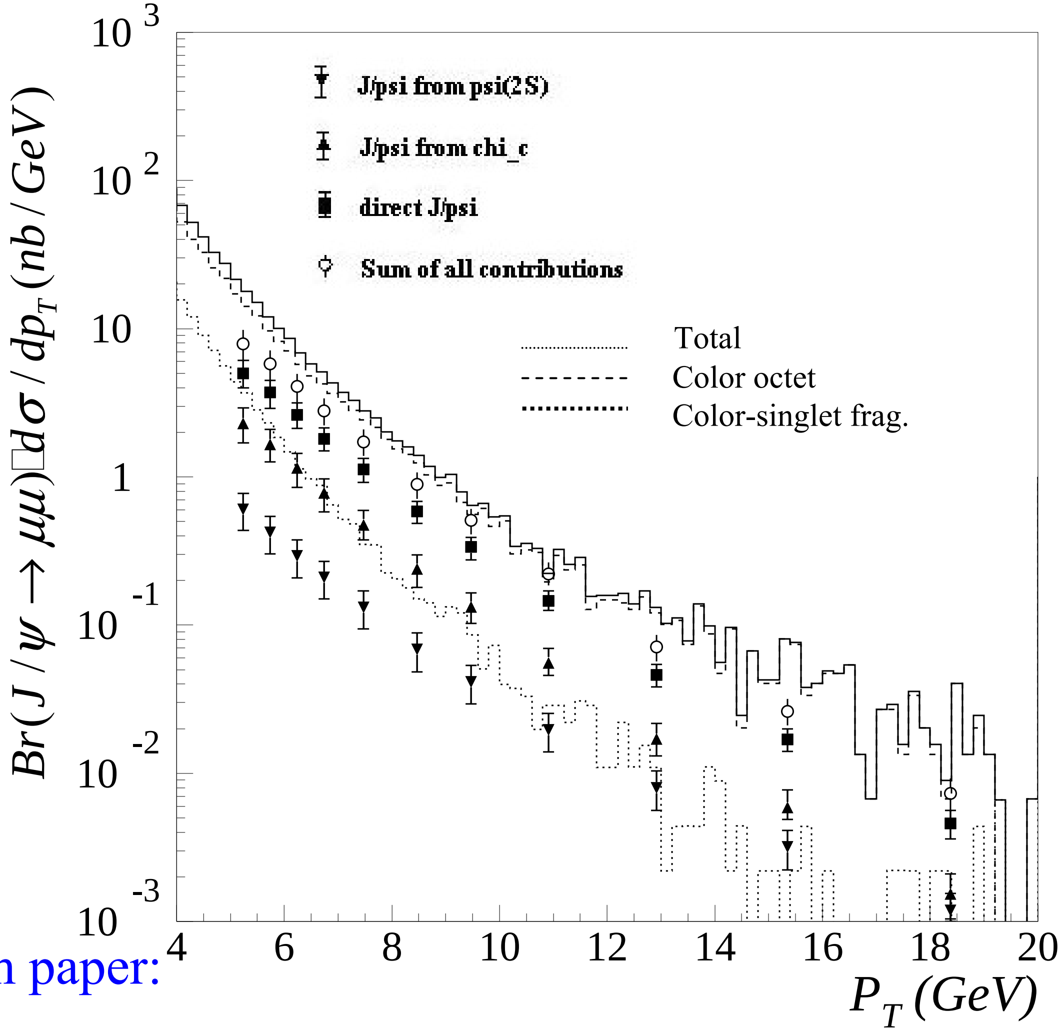




STATUS WITH CSM+COM

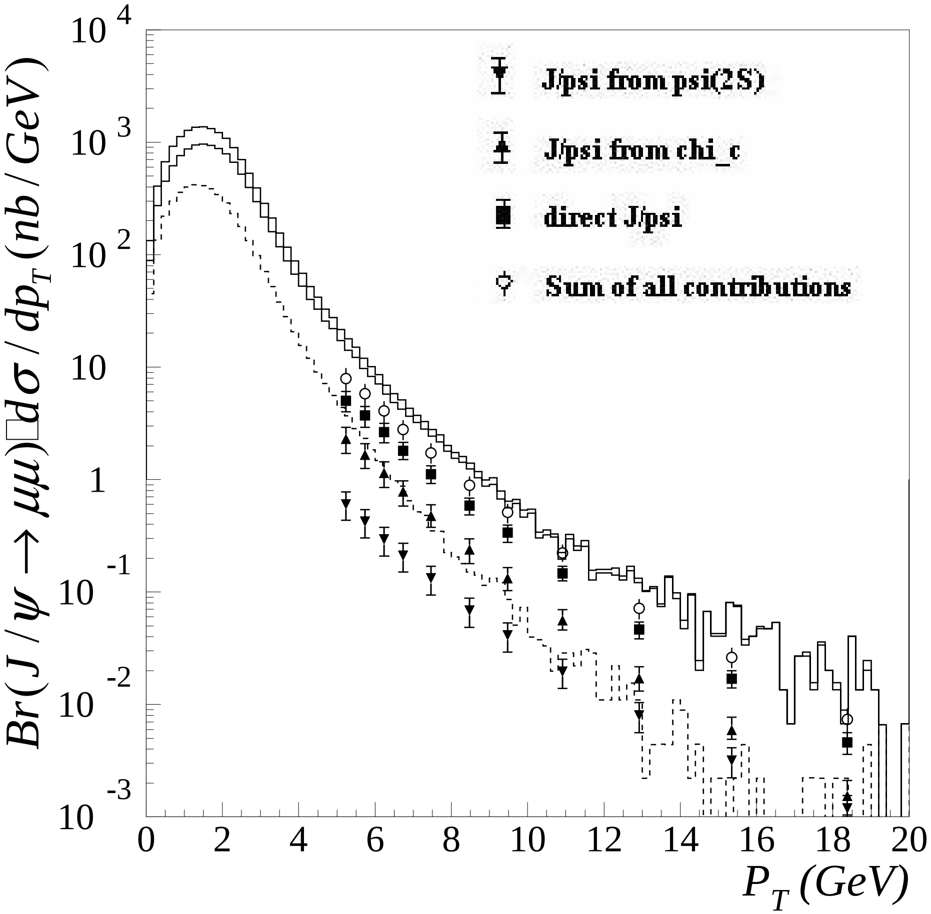
(1GEV P_T MIN CUT)

- msub :421, 422, 423, 424, 425, 426, 427, 428, 429, 430 active (all CSM and COM process for S wave implemented so far)
- msub 431, 432, 433 (same as 87, 88, 89) and more:
  - 434, 435, 436 active: are the qg contribution for P wave
  - 437, 438, 439 active: are the $q\bar{q}$ contribution for P wave

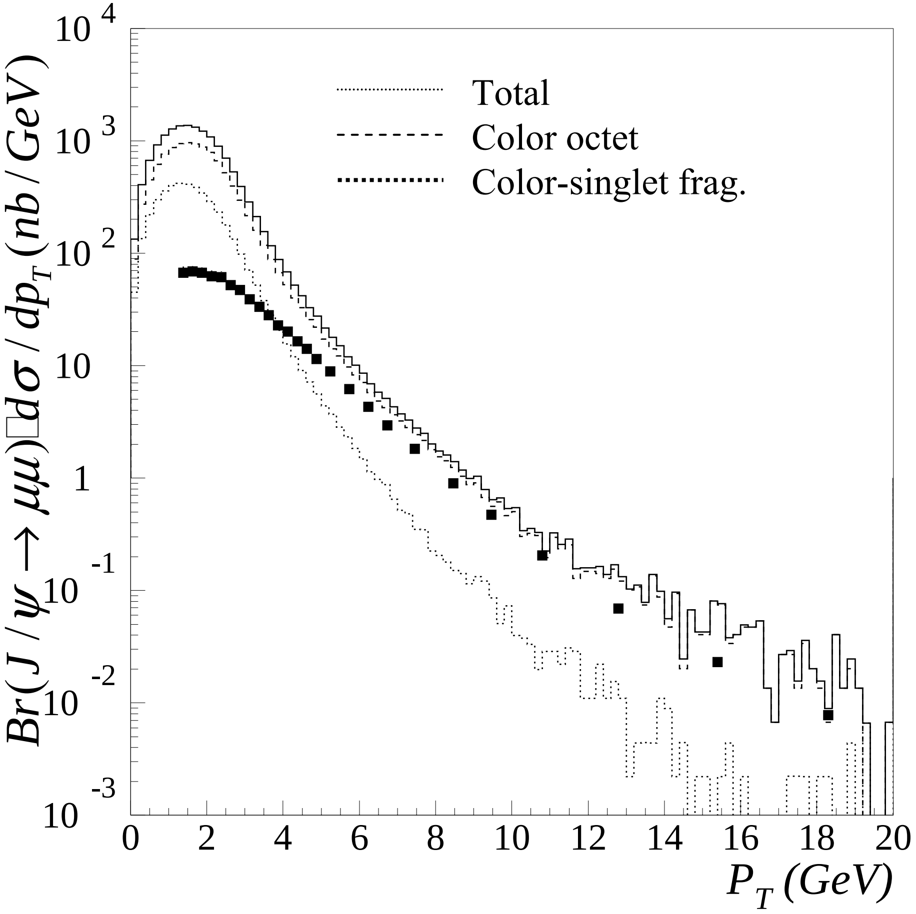


TEVATRON data as estracted from paper:
Phys. Rev.Lett.79:578-583, 1997

FULL SPECTRA @1 GEV P_T MIN CUT



On Full size scale

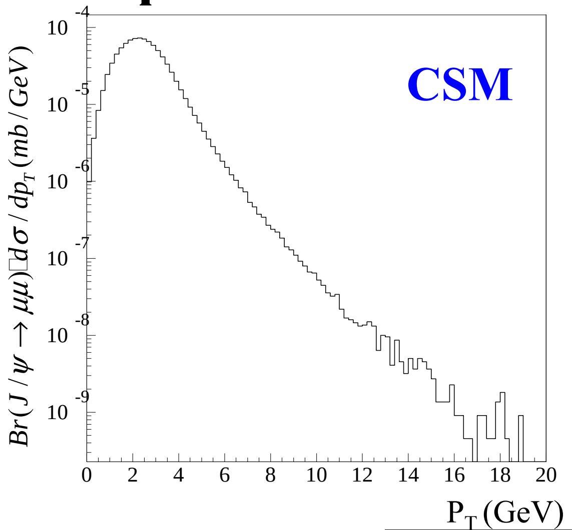


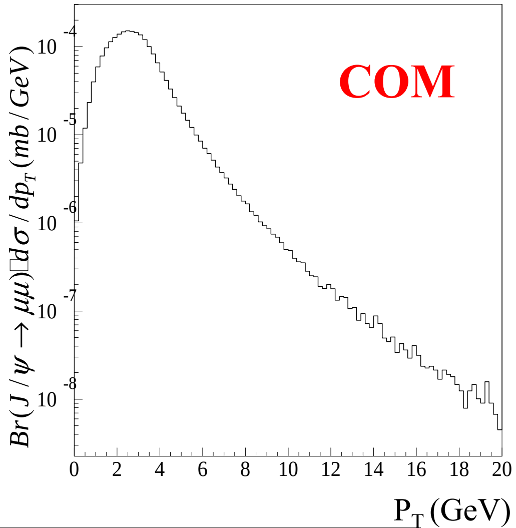
- ➢FERMILAB-PUB-04-440-E.

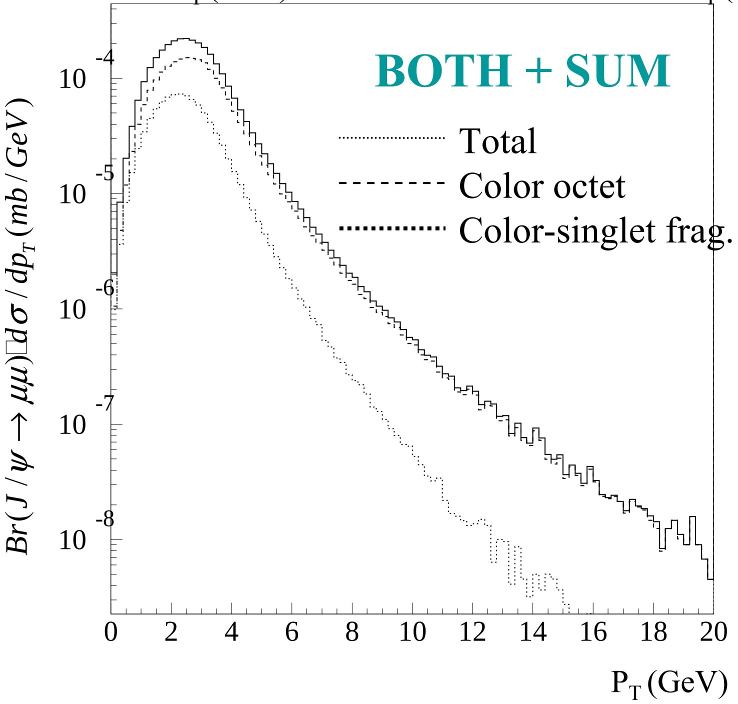
Status with CSM/COM only (2GeV p_T min cut)

- CSM:
  - 9.2 million events produced with CSM model processes:
  - msub 421 active (same as 86): (S Wave):
    - $g+g\rightarrow c\bar{c}[{}^3S_1^{(1)}]+g$
  - msub 431, 432, 433 (same as 87, 88, 89): (P Wave)
    - $g+g\rightarrow c\bar{c}[{}^3P_0^{(1)}]+g$
    - $g+g\rightarrow c\bar{c}[{}^3P_1^{(1)}]+g$
    - $g+g\rightarrow c\bar{c}[{}^3P_2^{(1)}]+g$
  - all COM inactive
- COM:
  - 9.8 million events produced with COM model processes:
  - msub 422-430 active
  - all CSM inactive

x: p_T distribution, in y: $d\sigma/dp_T$*Br (in mb)).



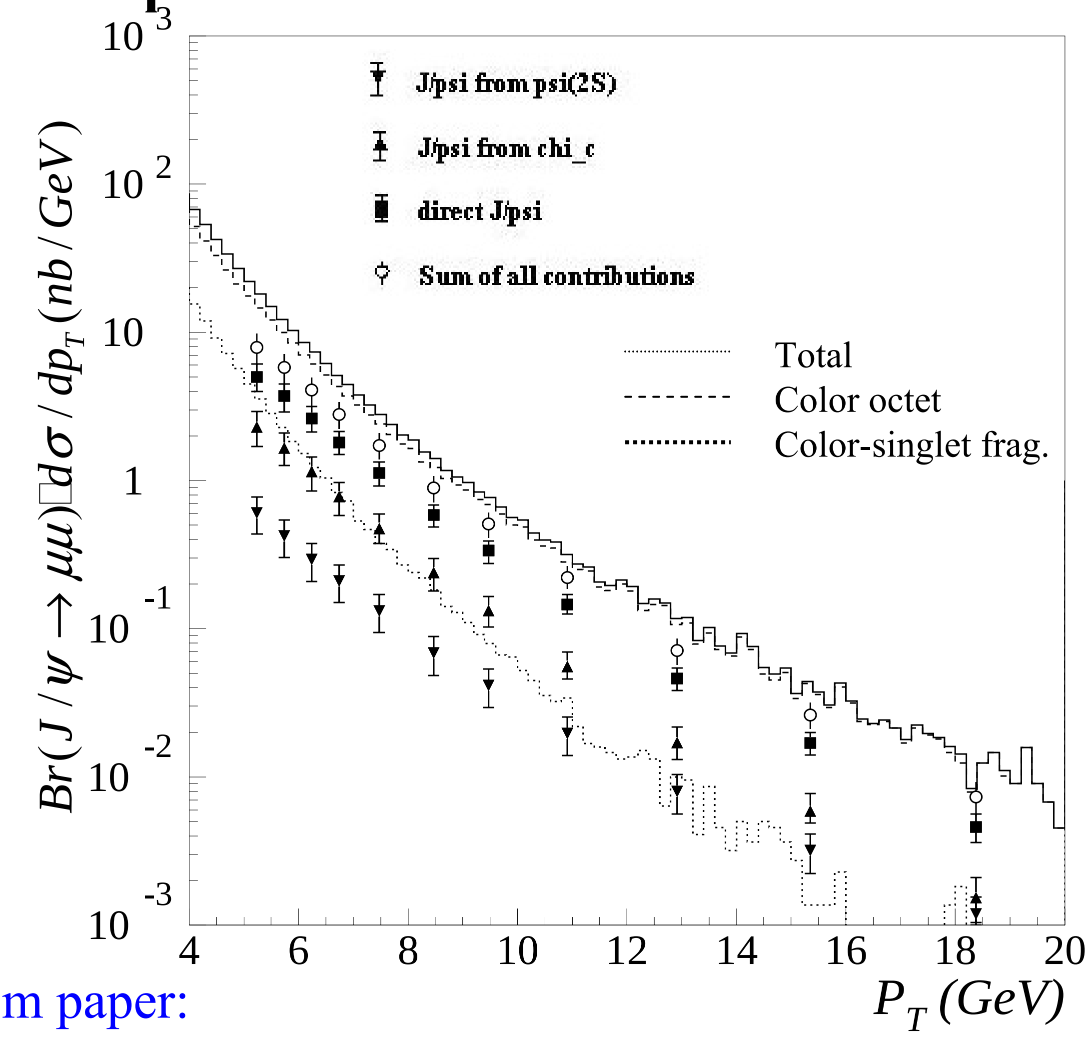




STATUS WITH CSM+COM

(2GEV P_T MIN CUT)

- msub :421, 422, 423, 424, 425, 426, 427, 428, 429, 430 active (all CSM and COM process for S wave implemented so far)
- msub 431, 432, 433 (same as 87, 88, 89) and more:
  - 434, 435, 436 active: are the qg contribution for P wave
  - 437, 438, 439 active: are the $q\bar{q}$ contribution for P wave

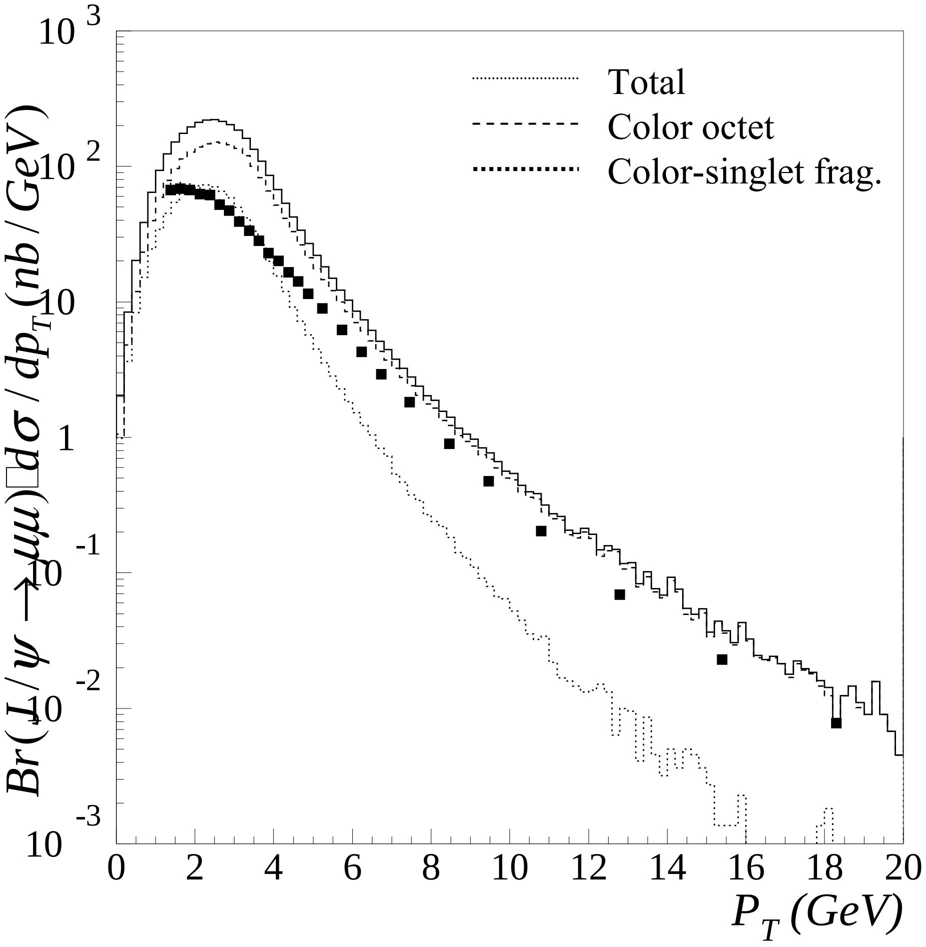


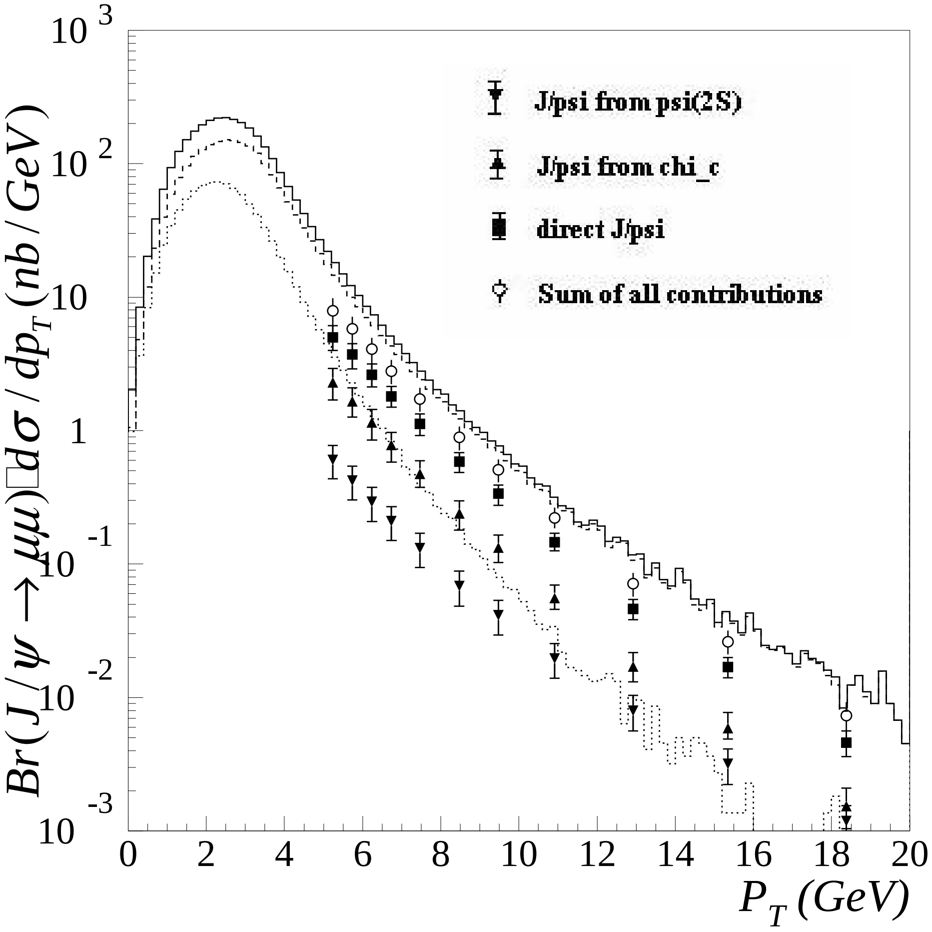
TEVATRON data as estracted from paper:

Phys. Rev.Lett.79:578-583, 1997

Full spectra @ 2 GeV p_T min cut

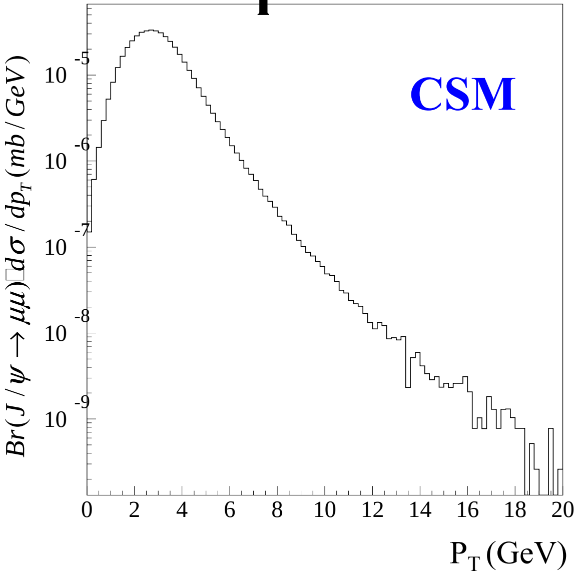
On Full size scale

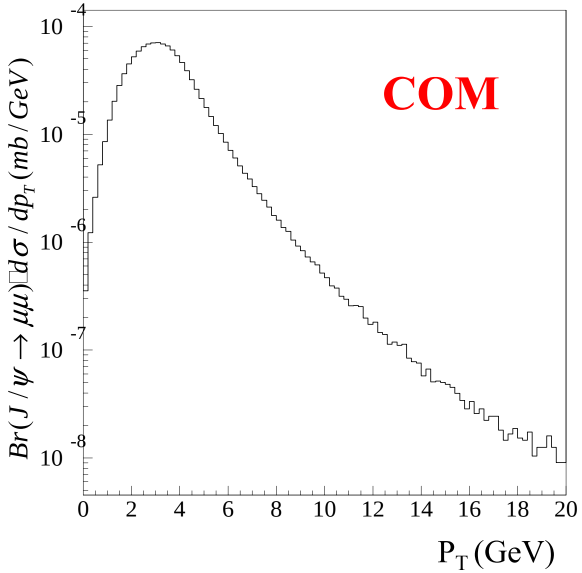


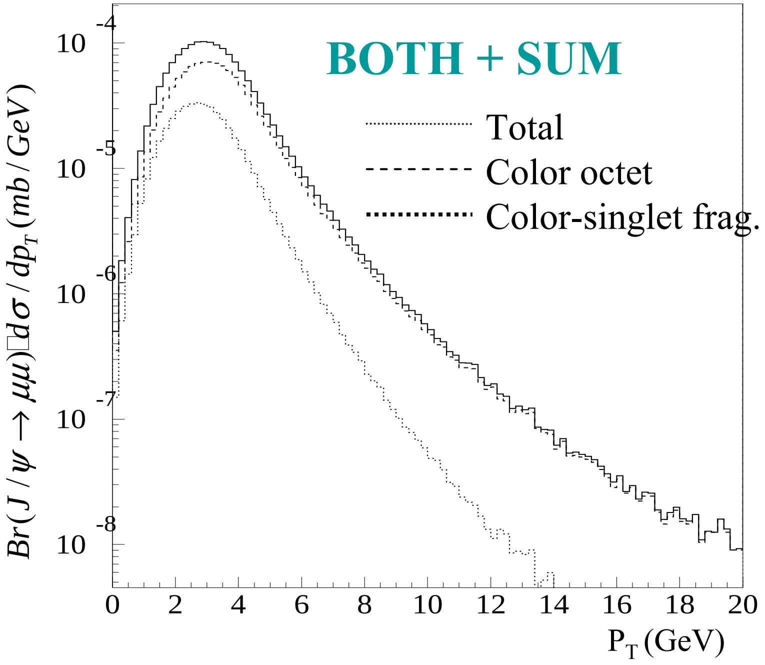


STATUS WITH CSM/COM ONLY (2.5 GEV p_T MIN CUT)

- CSM:
  - 9.9 million events produced with CSM model processes:
    - msub 421 active (same as 86): (S Wave):
      - $g+g \to c\bar{c}[^3S_1^{(1)}]+g$
    - msub 431, 432, 433 (same as 87, 88, 89): (P Wave)
      - $g+g \to c\bar{c}[^3P_0^{(1)}]+g$
      - $g+g \to c\bar{c}[^3P_1^{(1)}]+g$
      - $g+g \to c\bar{c}[^3P_2^{(1)}]+g$
    - all COM inactive
- COM:
  - 9.9 million events produced with COM model processes:
    - msub 422-430 active
    - all CSM inactive

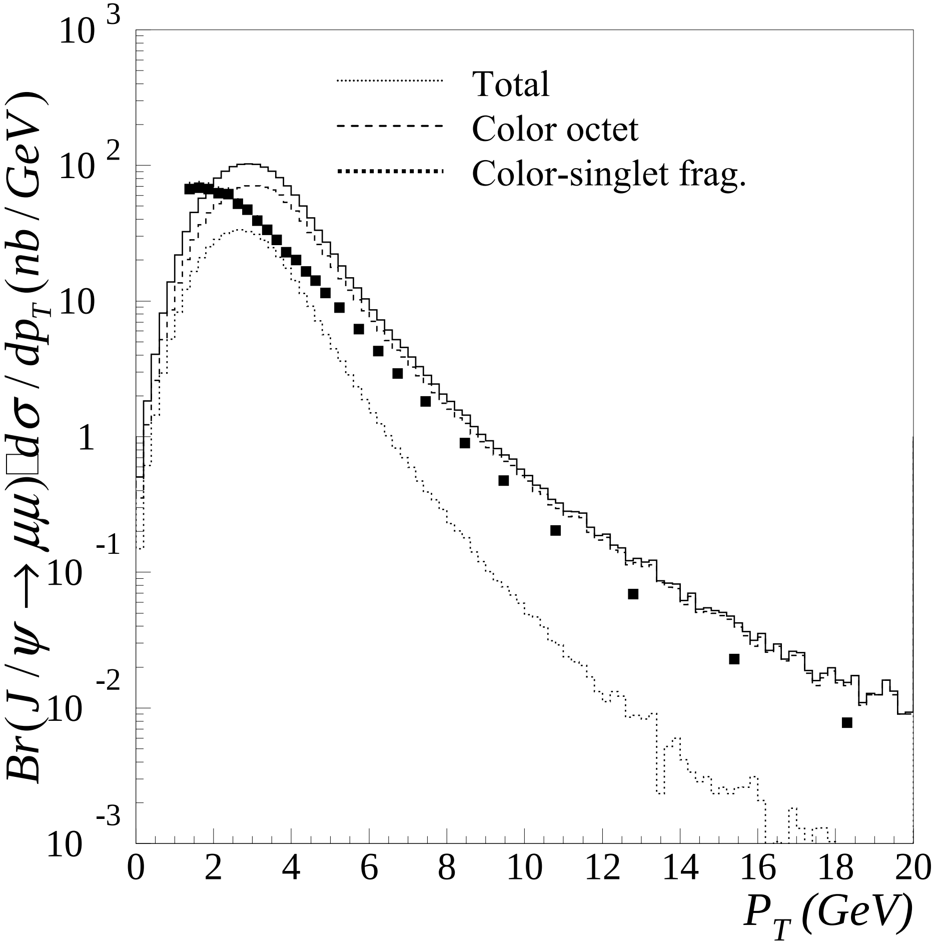


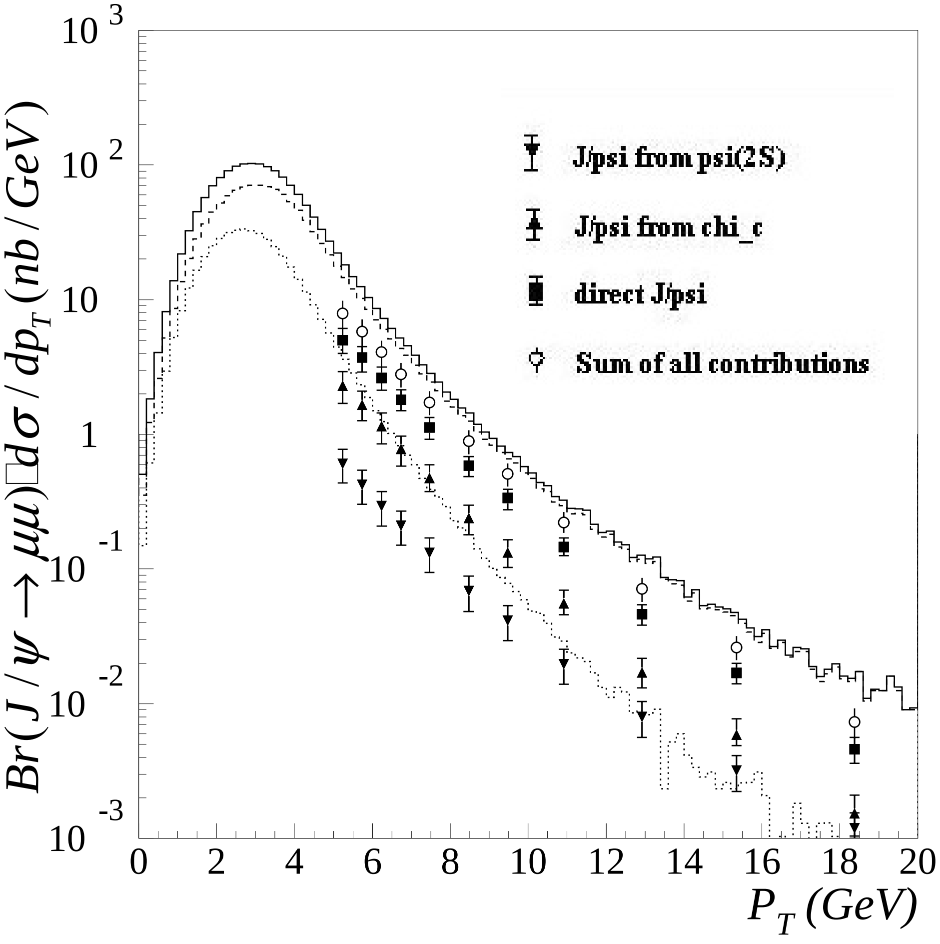




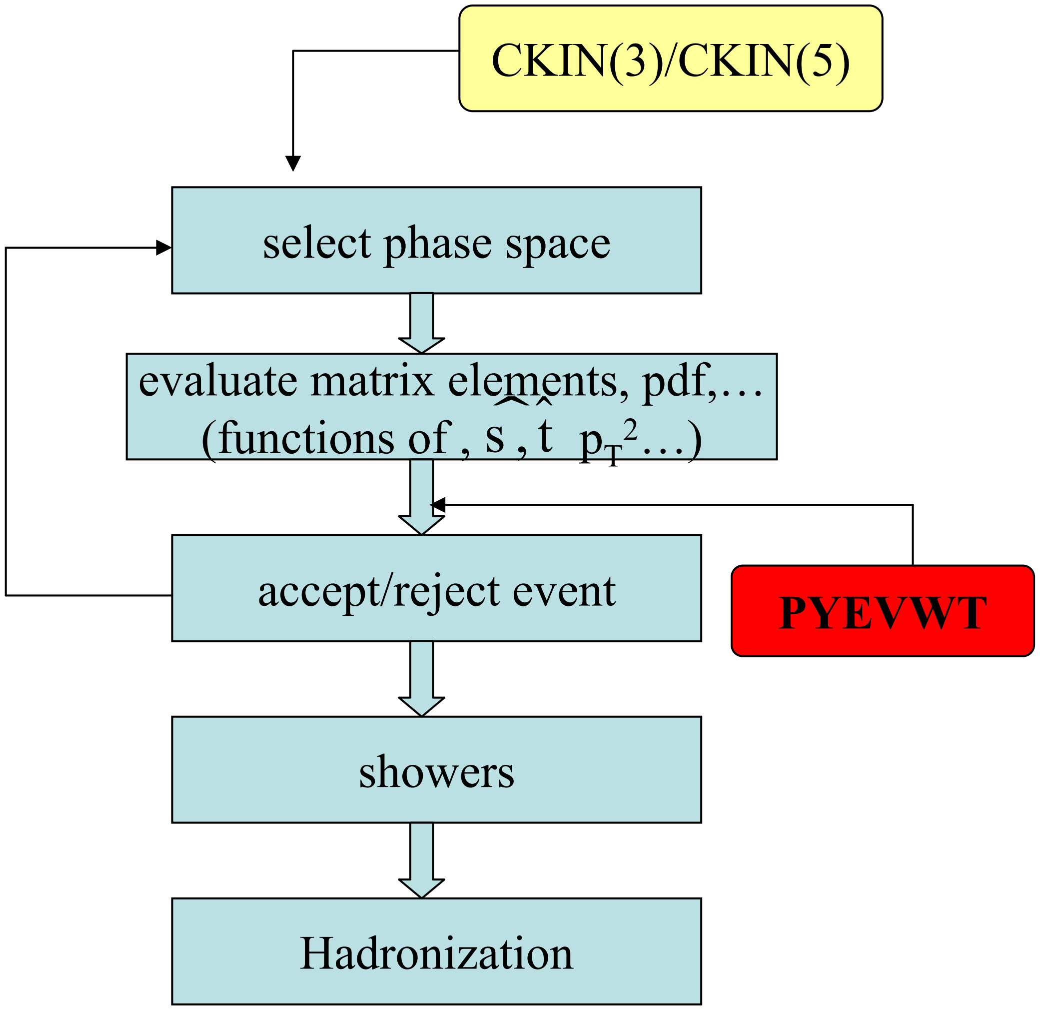
x: p_T distribution, in y: $d\sigma/dp_T$*Br (in mb)).

Full Spectra @ 2.5 GeV p_T min cut





A DIFFERENT APPROACH: PYEVWT



CKIN(3)/CKIN(5)

select phase space

evaluate matrix elements, pdf,…

(functions of , $\hat{s}$, $\hat{t}$ p_T^2…)

accept/reject event

PYEVWT

showers

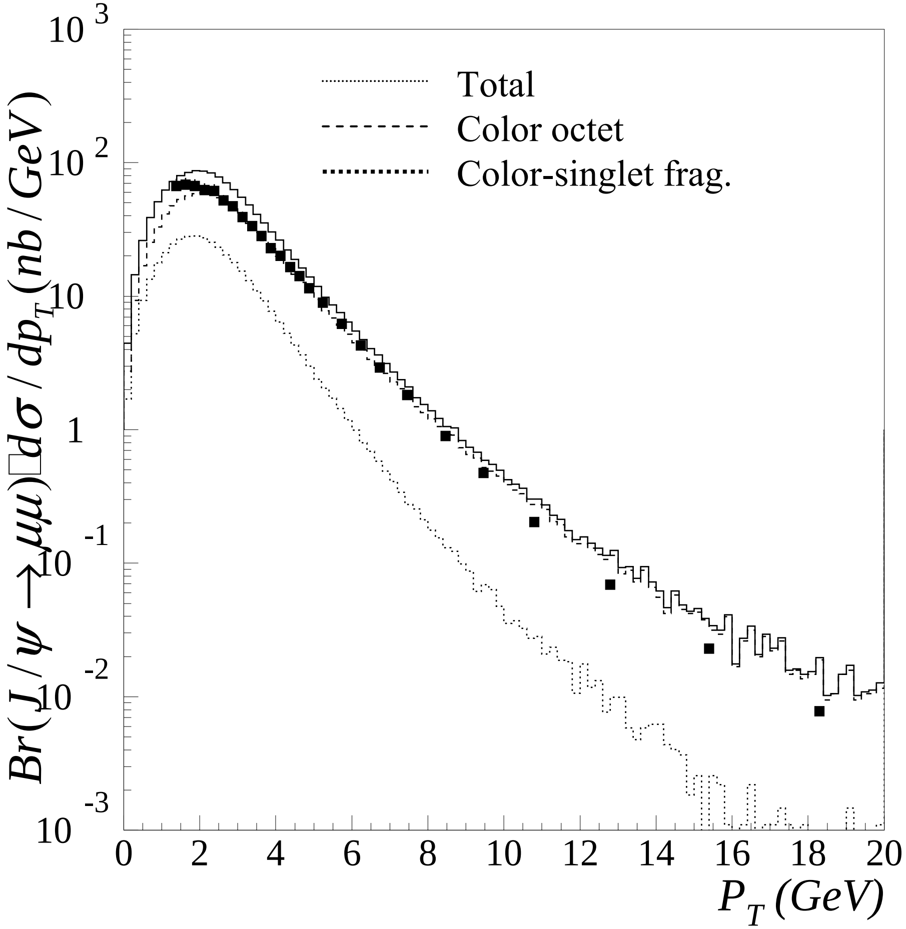
Hadronization

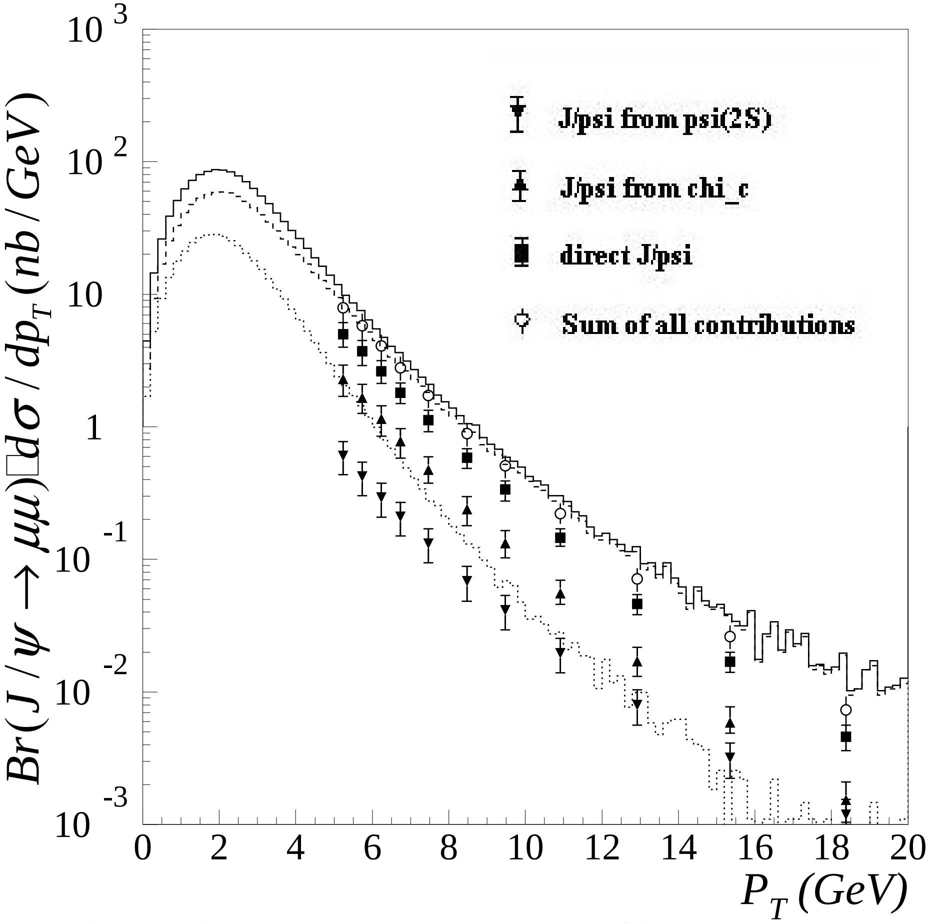
- Call PYEVWT with MSTP(142)=2 allows to reweight event cross section by process type and kinematics of the hard scattering.
  - In the present case, it's assumed that the true cross section have to be modified by a multiplicator factor WTXS set by us.

→unlike the CKIN(3) factor that cuts from a certain p_T onward as a box function, the PYEVWT reweights the cross sections definig a p_{T0} bound to the center of mass energy, as used in multiple interactions. The WTXS is defined as:

WTXS = (PT2/(PT02+PT2))2**

Preliminary Results using PYEVWT for event-by-event reweighting



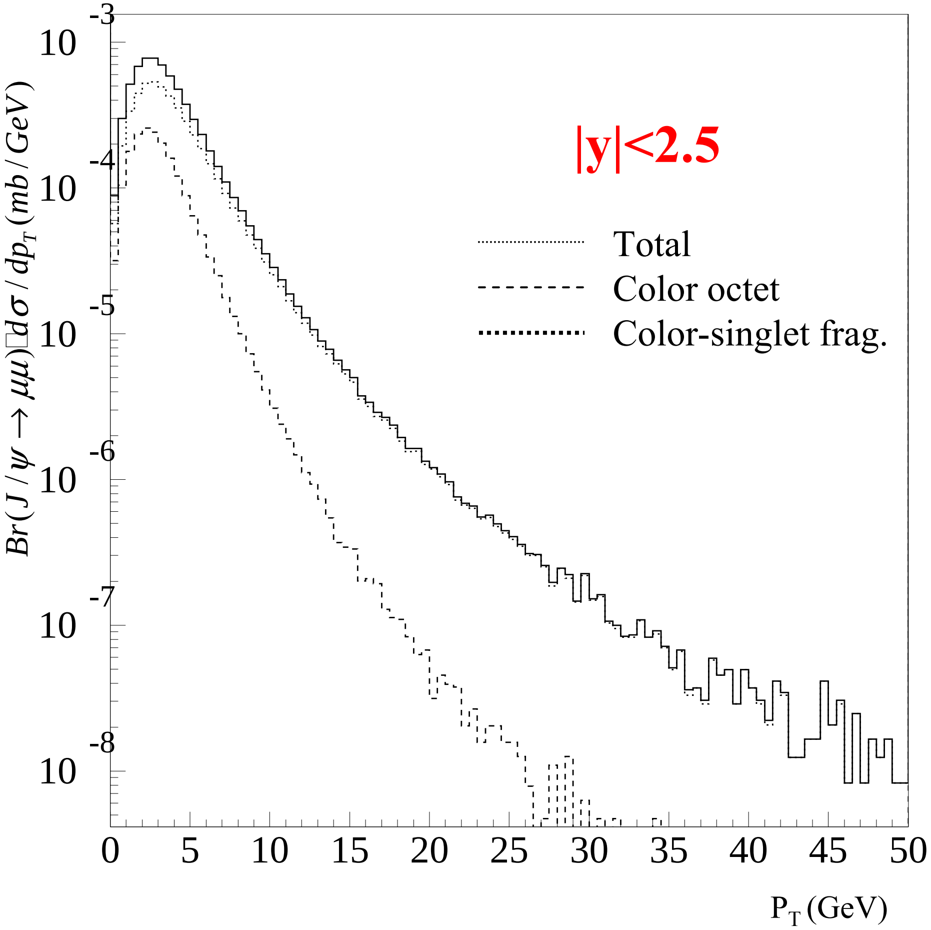


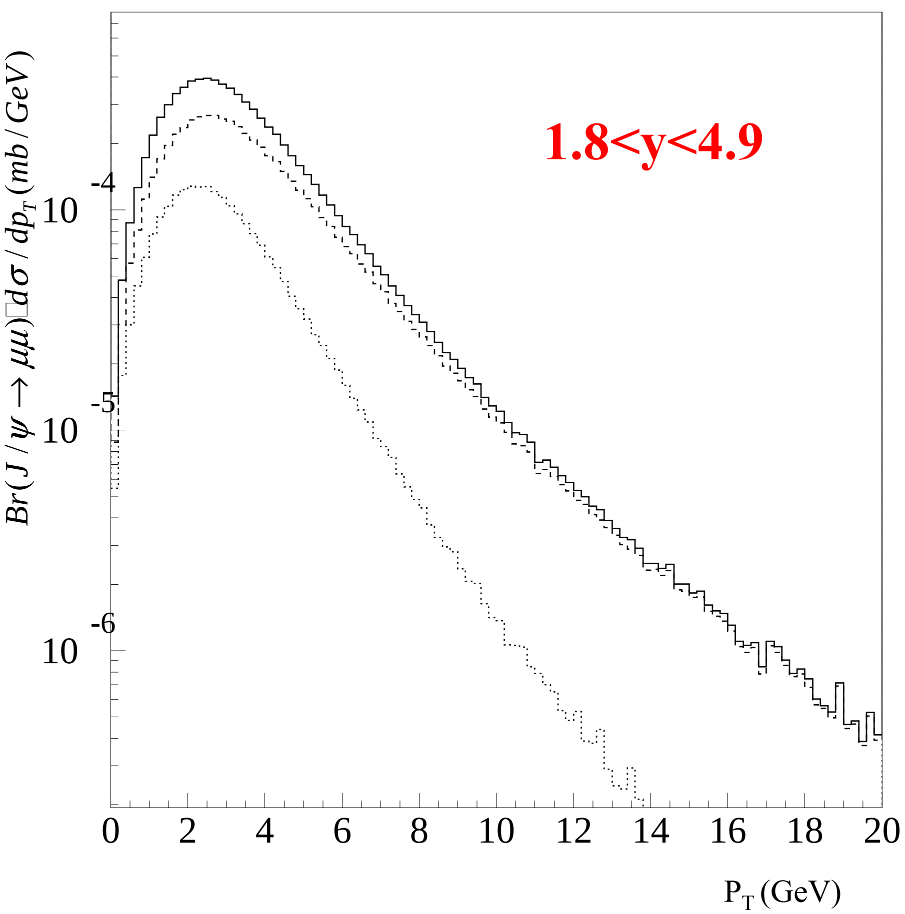
WEIGHT = (P_T**2/(P_{T0}**2+P_T**2))**2 P_{T0}=2.5 GeV

PERSPECTIVES FOR LHC (1)

- Using the reweightening approach:
  - P_{T0} extrapolated to 14 TeV by (see LHCb note 99-028):
    P_{T0} = 2.5 GeV*(14 TeV / 1.96 TeV)0.16 = 3.42 GeV**
  - Analogously as done for extrapolating the P_T min cut for multiple parton-parton interactions in Pythia
  - Parameters chosen according to LHCb tuning for multiple parton interactions;
  - 2 rapidity region: -2.5 – 2.5 (Atlas, CMS), 1.8 – 4.9 (LHCb)
    - **Total cross section*BR(μμ): 3.34 μb for |y|<2.5**
    - **Total cross section*BR(μμ) for LHCb : 1.58 μb for 1.8<y<4.9**
    - **Total cross section*BR(μμ) without acceptance cut: 6.48 μb**

PERSPECTIVES FOR LHC (2)





CONCLUSIONS

- Actual scenario:
  - Studies with fragmentation contributions at different low p_T cuts: unsatisfactory results with 1, 2 and 2.5 GeV with CKIN low p_T cut.
  - More promising results with PYEVWT re-weighting routine
  - Next step at LHC energies: wider production and tests.
- Future studies:
  - p_T cut not universal, need to check the extrapolation at LHC energies
    - Can use total cross section calculation available at NLO
  - Test to be performed also for Y (missing at the moment the possibility to produce $\psi(2S)$ and Y(2S) at the same time)

NRQCD QUICK THEORY SLIDES

Color Singlet Model (CSM)

Quarkonia inclusive decay rates and cross section were calculated at LO (*Leading Order*), with assumption of factorization:

→ short distance part, describing the annihilation (or creation) of the heavy quark pair in a COLOR SINGLET state;

→ non perturbative long distance factor, accounting for the soft part of the process.

The $c\bar{c}$ pair is created in a color neutral state with the same quantum numbers as the final charmonium state:

→CSM (Color Singlet Model)

✓For charmonia S-wave, NO infrared divergences of CSM for one-loop corrections;

✓ BUT in P-wave decays in light hadrons, appearance of infrared singularities in short distance coefficients → PROBLEM !

Experimental tests of CSM

In fact: during the last 10 years, found orders of magnitude of disagreement between CSM prediction and new measurements of J/ψ and ψ' production at several collider facilities.

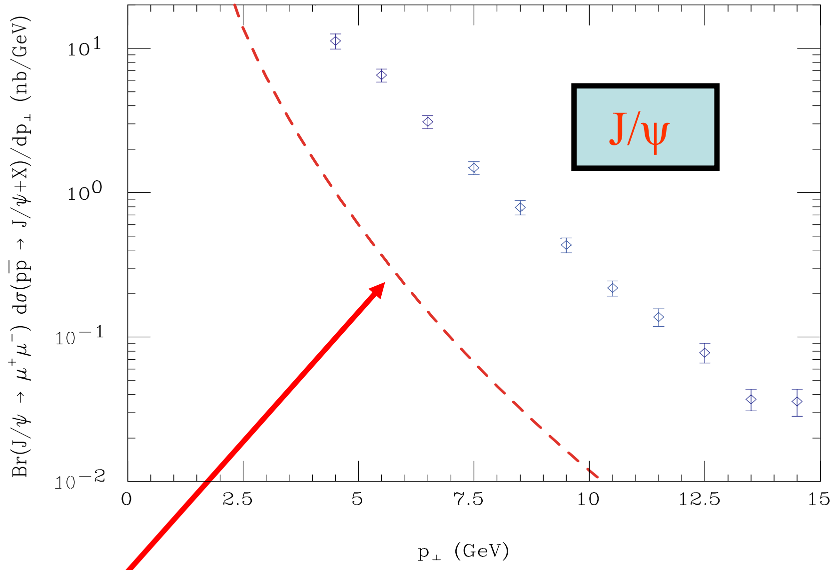
An example is the striking observation by CDF of large p_T J/ψ and ψ' states

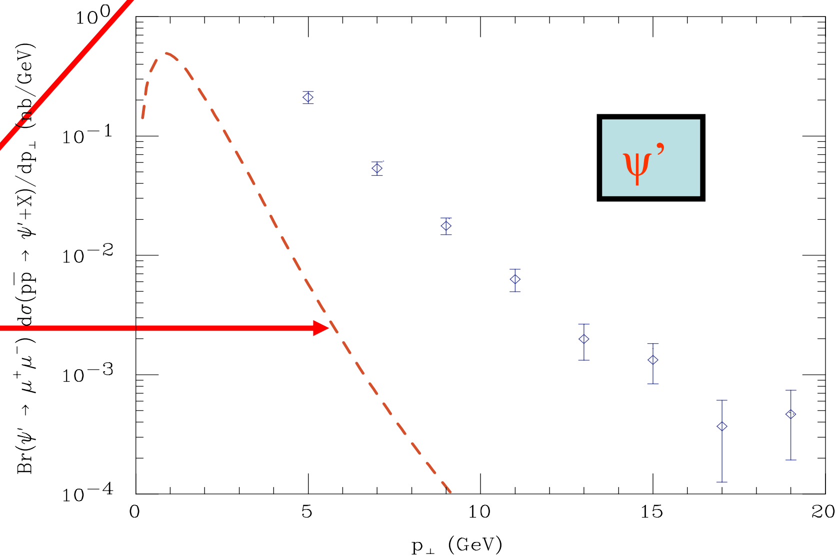
→ more than 1 order of magnitude larger than the theoretical predictions by CSM !

Tevatron transverse momentum differential cross sections:

Color Singlet predictions

both for J/ψ and ψ' production





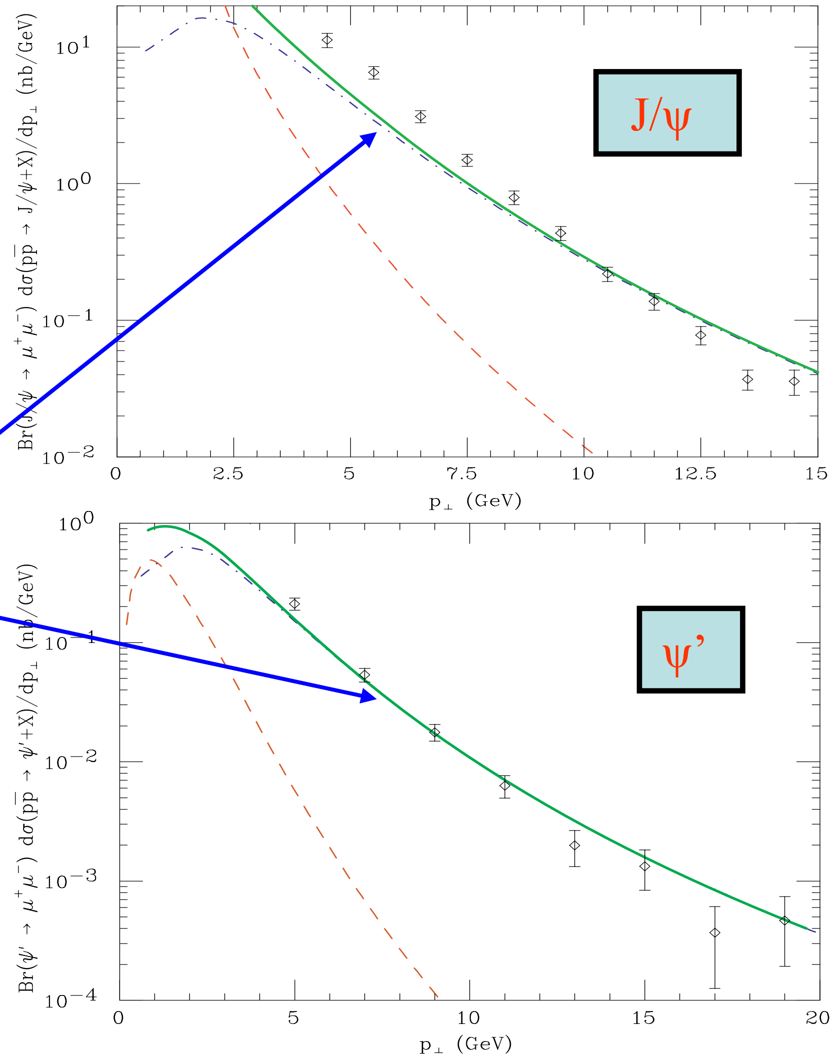
NRQCD

- Possible solution? → Effective field theory introduced → Non-Relativistic QCD (NRQCD).
  - quarkonium production and decay take place via intermediate $q\bar{q}$ states with different quantum numbers than the physical quarkonium state, that is producing or decaying.
  - a transition probability $\left\langle O^{H}_{1,8}(n) \right\rangle$ describes the transition of $c\bar{c}$ pair (color octet + color singlet) into the final $q\bar{q}$ state;
  - The NRQCD factorization formula for the production cross section of state H is:

$$\sigma^{H} = \sum_{n} \sigma^{c\bar{c}}_{1,8}(n) \left\langle O^{H}_{1,8}(n) \right\rangle$$

  - $\sigma^{c\bar{c}}_{1,8}(n)$: short-distance production of a $q\bar{q}$ pair in color, spin and angular momentum state *n* ($^{2S+1}L_J^{[1,8]}$);
  - $\left\langle O^{H}_{1,8}(n) \right\rangle$: describes the hadronization of the pair into the observable state H.

NRQCD predictions



→With the addiction of color octet contributions, the Tevatron transverse momentum cross sections AGREE well with the NRQCD predictions for both of charmonium states.

BACKUP

Photoproduction channels implemented in PYTHIA 6.2 only: the tests of the proper implementation of these channels only include the expression of partonic amplitude squared (PYSIGH). Not tested yet

ISUB	$g+\gamma\rightarrow c\bar{c}[^{(2S+1)}L_J^{(C)}]+g$	ISUB	$g+\gamma\rightarrow q+c\bar{c}[^{(2S+1)}L_J^{(C)}]$
440	$g+\gamma\rightarrow c\bar{c}[^3S_1^{(1)}]+g$		
441	$g+\gamma\rightarrow c\bar{c}[^3S_1^{(8)}]+g$	444	$g+\gamma\rightarrow q+c\bar{c}[^3S_1^{(8)}]$
442	$g+\gamma\rightarrow c\bar{c}[^1S_0^{(8)}]+g$	445	$g+\gamma\rightarrow q+c\bar{c}[^1S_0^{(8)}]$
443	$g+\gamma\rightarrow c\bar{c}[^3P_J^{(8)}]+g$	446	$g+\gamma\rightarrow q+c\bar{c}[^3P_J^{(8)}]$

ALTARELLI-PARISI EVOLUTION (1)

- Contributions from $Q\overline{Q}[^3S_1^{(8)}]$ partly come from the fragmentation of a gluon → since the gluon could have splitted into 2 gluons before fragmentation, this effect have to be included:

  •2 NEW switches: MSTP(148) to switch ON & OFF the splitting:

$$Q\overline{Q}[^3S_1^{(8)}] \rightarrow Q\overline{Q}[^3S_1^{(8)}] + g$$

  and MSTP(149) to choose if it's ensured that the QQ pair always takes the larger fraction of the four-momentum. This evolution obeys the Altarelli-Parisi evolution for g→g+g

- Handling of the Altarelli-Parisi evolution of $Q\overline{Q}[^3S_1^{(8)}]$, done with the parameter MSTP(148) (defalt value 0), allows the final- state shower evolution both for $c\overline{c}[^3S_1^{(8)}]$ and for $b\overline{b}[^3S_1^{(8)}]$.

ALTARELLI-PARISI EVOLUTION (2)

- ATTENTION! **switching MSTP(148) ON may exaggerate shower effects, since not all $QQ[^3S_1^{(8)}]$ comes from the fragmentation component where radiation is expected!!!! :** Since the fragmentation contribution of $QQ[^3S_1^{(8)}]$ to production processes is the most important contribution, the higher the transverse momentum of the QQ pair is….. →**highly advisable to switch ON the Altarelli-Parisi evolution for events with large transverse momentum**

- →If the $Q\bar{Q}[^3S_1^{(8)}]$ states are allowed to radiate [MSTP(148) = 1], the parameter MSTP(149) determines the kinematic of the $Q\bar{Q}[^3S_1^{(8)}] \rightarrow Q\bar{Q}[^3S_1^{(8)}] + g$ branching:
  - **MSTP(149) = 0**, daughter $Q\bar{Q}[^3S_1^{(8)}]$ picks always the larger momentum fraction (z > 0.5);
  - **MSTP(149) = 1**, daughter $Q\bar{Q}[^3S_1^{(8)}]$ picks momentum fraction equally z < 0.5 and z > 0.5

POLARIZATION

- Possibility to swich ON & OFF the polarized generation of quarkonia through the parameter MSTP(145) [0=unpolarized, 1=polarized, with selection of helicity states or density matrix elements]

  →**FOR EXPERTS ONLY:**

  - The selection of the different polarization reference is done through MSTP(146) whose possible states are:
    - **1: Recoil (recommended since it matches how PYTHIA defines particle directions);**
    - **2: Gottfried-Jackson;**
    - **3: Target;**
    - **4: Collins-Soper**
  - The selection of the different helicity states or density matrix is done through MSTP(147) (with MSTP(145)=1):

0: helicity 0;
1: helicity +-1;
2: helicity +-2;
3: density matrix element rho_{0,0};
4: density matrix element rho_{1,1};
5: density matrix element rho_{1,0};
6: density matrix element rho_{1,-1}.