Exotics and Glueballs on the lattice.

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Hybrids, glueballs, and multi-quark states are bound states of QCD that are not predicted by the naive QUARK model.

Motivation

QCD is a hard theory to solve outside perturbation theory.

Hadron masses depenend nonperturbatively on the coupling $M \sim Ce^{-1/g^2}$hence can't be computed using perturbation theory.

From a CLEO-c talk, Shipsey hep-ex/0203033.

Moreover, as we look to the future beyond the Standard Model, we anticipate that the larger theory in which the Standard Model is embedded will likely be strongly coupled, or have strongly coupled sectors. Both Technicolor, which is modeled on QCD and is ab initio strongly coupled, and Supersymmetry, which employs strongly coupled sectors to break the supersymmetry, are prime examples of candidates for physics beyond the Standard Model. Strong coupling is a phenomenon to be expected, weak coupling is the exception in field theory not the norm. However our ability to compute reliably in a strongly coupled theory is not well developed. Techniques such as lattice gauge theory that deal squarely with strongly coupled theories will eventually determine our progress on all fronts in particle physics. At the present time the absence of adequate theoretical tools significantly limits the physics we can obtain from the study heavy quarks.

Solving QCD is important?

If (like a thought experiment) we could solve QCD, then using one experimental number and the quark masses we could predict all of hadronic physics. To look for physics beyond the standard model, it is useful to extract the quark masses from data.

The current particle data table gives the mass of the strange quark mass as

$$m_s(2\text{ GeV}) = 80 \text{ to } 155 \text{ MeV}$$

Compare to the Higgs mass bounds in the PDG from precision electroweak data:

$$M_H = 88^{+53}_{-35}\text{ GeV}$$

(90 % confidence interval). A lot of money is being spent to "reduce" the error on M_H at the LHC at CERN.

The experimental mass of the kaon is 493.677 ± 0.013 MeV. The large error on the strange quark mass comes from THEORY!

Introduction to lattice QCD.

To solve QCD four bound state properties, "all" that is required is to compute

$$c_{ij}(t) = \frac{1}{Z}\int du \int d\psi \overline{d\psi} O(t)_i O(0)_j^\dagger e^{-S_F - S_G}$$

The path integral is regulated by the introduction of a space-time lattice.

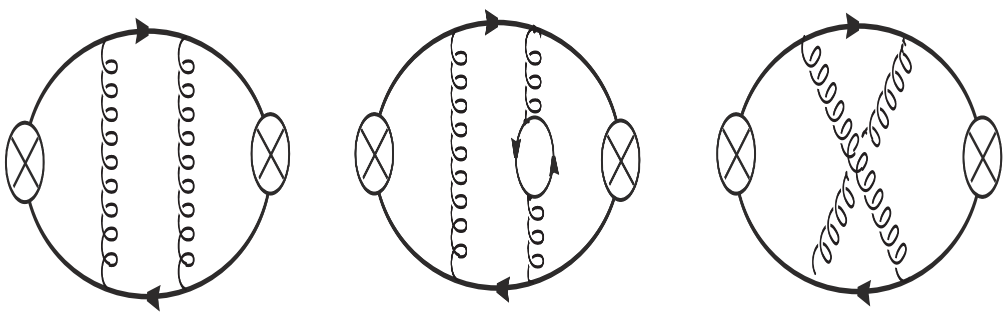
The integral is computed in Euclidean space using Monte Carlo techniques on the computer.

$$c_{ij}(t) = c_{ij}^0 e^{-m_0 t} + c_{ij}^1 e^{-m_1 t} \dots$$

The "art" is in the construction of the interpolating operators $O_i(t)$

Quenched QCD.

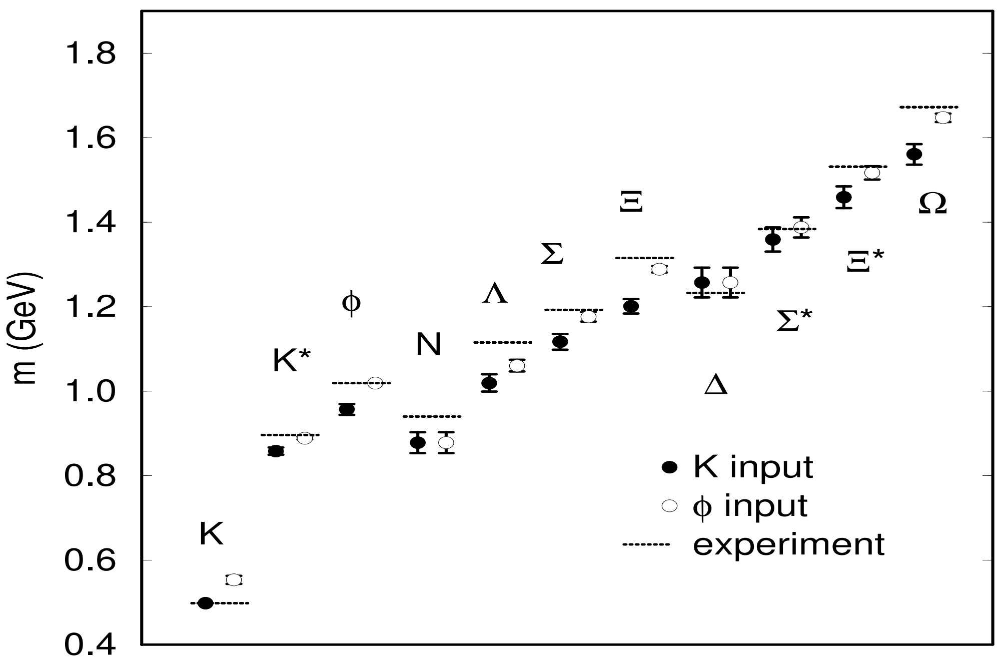
In the past may lattice calculations were done in quenched QCD. This is where the dynamics of the sea quarks is ignored. From Jiunn-Wei Chen, hep-lat/0205014



(a) (b) (c)

Successes of quenched QCD

Surprisingly, quenched QCD agrees with experiment for many quantaties quite well. For example, the largest spectroscopy study of quenched QCD from CP-PACS (hep-lat/9904012) used: lattice spacings between 0.1 and 0.05 fm, box sizes of 3 fm, and m_π/m_ρbetween 0.75 and 0.4.



The deviation of the non-singlet masses from experiment was < 11 %.

Failures of quenched QCD.

The CP-PACS collaboration (hep-lat/0004010) have computed the mass of the strange quark from full QCD.

n_f	input	$m_s(2GeV)$ MeV
2	ϕ	90^{+5}_{-11}
2	K	88^{+4}_{-6}
0	ϕ	132^{+4}_{-6}
0	K	110^{+3}_{-4}

Although this is not a definitive calculation of the strange quark mass, but it does illustrate that it is more important to work on unquenched QCD.

Typical parameters in dynamical lattice QCD calculations.

Collaboration	n_f	a fm	L fm	$\frac{M_{PS}}{M_V}$
MILC	2+1	0.09	2.5	0.4
CP-PACS	2	0.11	2.5	0.6
UKQCD	2	0.1	1.6	0.58
SESAM	2	0.08	2.0	0.56

Lattice QCD is being solved at these parameters, unfortunately they are not the physical parameters. The view at the moment is to use effective field theories such as chiral perturbation theory to extrapolate the results from lattice QCD calculations to the physical parameters (see Kronfeld hep-lat/0205021 for a review).

Currently a major goal of the lattice QCD community (hep-lat/0209086) is the reduction of the masses of the sea quarks. The target is to compute at $M_{PS}/M_V \sim 0.3$ (Sharpe hep-lat/9811006).

Going beyond the quark model.

The quark model (as described in the review section of the PDG) claims that light quark spectroscopy is "explained" by

✔ $\overline{\psi}\Gamma\psi$ Mesons

✔ $\psi\psi\psi$ Baryons

In QCD there are additional possible colour singlet operators that may form bound states

- Glueballs: lumps of glue.
- Hybrids mesons: quark and ante-quark with excited glue.
- Multiquark states:

States with exotic quantum numbers $J^{PC} = 1^{-+}$, 0^{+-}, 2^{+-}, $0^{--}$are defintely not described by the quark model.

Operators for exotics on the lattice.

Construct a meson operator with the correct J^{PC}quantum numbers out of quark fields and the field strength tensor F (e.g. MILC, Phys.Rev. D56 (1997) 7039)

Name	J^{PC}	Operator
π	0^{-+}	$\overline{\psi}\gamma_5\psi$
ρ	1^{--}	$\overline{\psi}\gamma_i\psi$
ρ(hybrid)	1^{--}	$\epsilon_{ijk}\overline{\psi}\gamma_5 F^{jk}\psi$
a_0	0^{++}	$\overline{\psi}\psi$
1^{-+}exotic	1^{-+}	$\overline{\psi}\gamma_j F_{ij}\psi$

Quenched 1^{-+} (light quarks)

Operator	hep-lat	Mass GeV
Glueball(Morningstar/Peardon)	9901004	Not seen (too heavy??)
$\overline{\mathrm{q}}qg$ (hybrid) MILC	9809087	2.1(1)$\pm sys$
$\overline{\mathrm{q}}qg$ (hybrid) MILC	9707008	2.0(1)$\pm sys$
$\overline{\mathrm{q}}qg$ (hybrid) UKQCD	9611011]	2.0(2)$\pm sys$
$\overline{\mathrm{q}}qg$ (hybrid) Mei and Luo	0206012	2.01(7)$\pm sys$

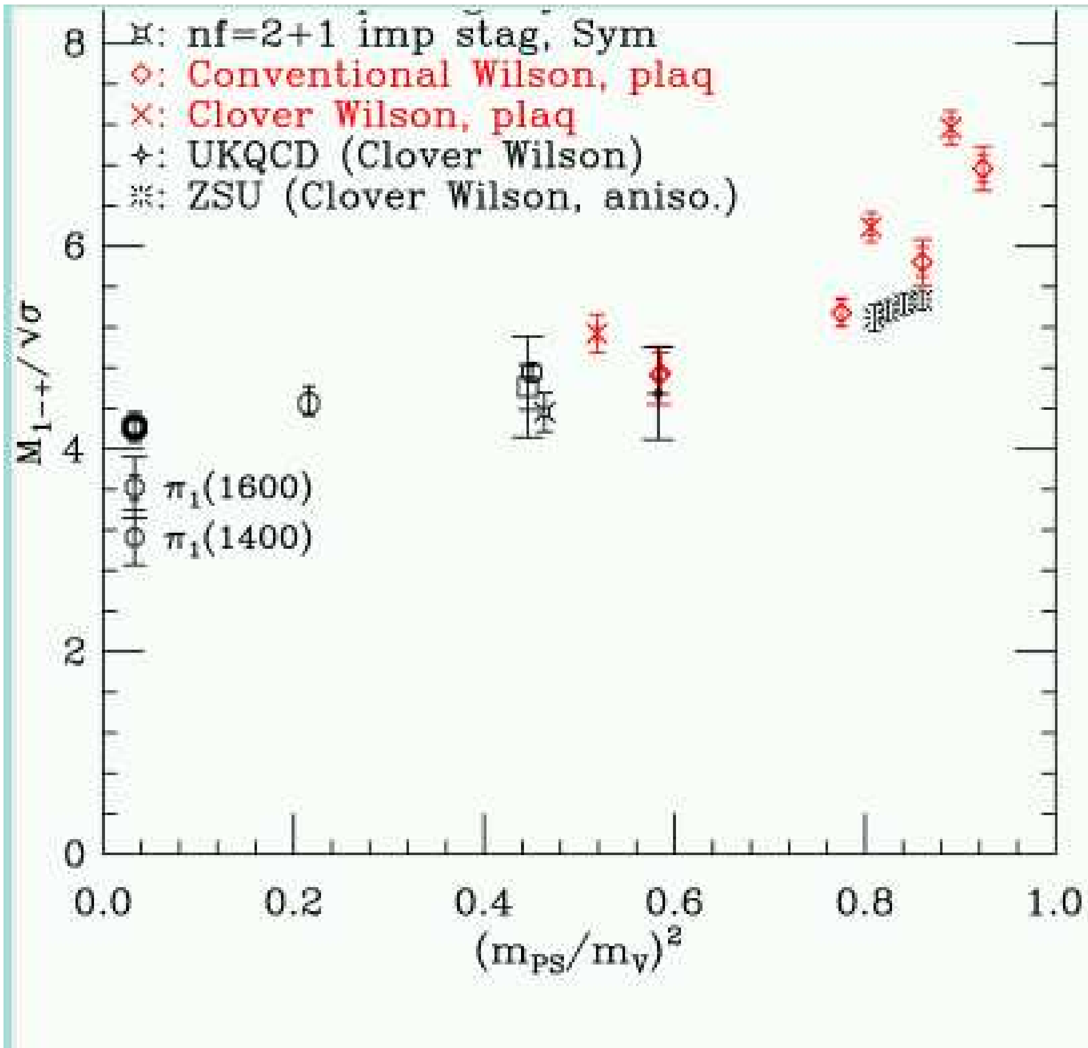
Although the calculations are consistent, no attempt has been made to do a continuum extrapolation.

There are experimental signals at 1400 and 1600 MeV.

Exotic masses from unquenched QCD.

An old calculation by SESAM (hep-lat/9809022) found $m_{1^{-+}} = 1.9(2)$ GeV.

New results from the MILC collaboration (hep-lat/0209097)



nf=2+1 imp stag, Sym
Conventional Wilson, plaq
Clover Wilson, plaq
UKQCD (Clover Wilson)
ZSU (Clover Wilson, aniso.)
$\pi_1(1600)$
$\pi_1(1400)$
$M_{1-+}/\sqrt{\sigma}$
$(m_{PS}/m_V)^2$
0
2
4
6
8
0.0
0.2
0.4
0.6
0.8
1.0

Chiral extrapolation of the 1^{-+}

A. W. Thomas, Adam P. Szczepaniak, CHIRAL EXTRAPOLATIONS AND EXOTIC MESON SPECTRUM, Phys.Lett.B526:72-78,2002

The terms that produce the most rapid chiral changes are for the decays $E \longrightarrow \pi H$ with E nearly degenerate with H. The dominant terms would be a transition between an isovector exotic and an isoscalar exotic an a pion.

The conclusion is that a chiral extrapolation at lower quark masses with their chiral extrapolation model could reduce the mass of the $1^{-+}$by 100 MeV.

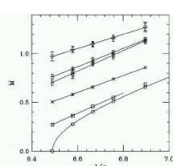


FIG. 4. Particle masses at $6/g^2 = 6.15$ and extrapolations to κ_c^{-1}. The symbols are the same as in Fig. 3, and the fancy pluses are a 0^{+-} exotic. Here the pion mass is fit to a quadratic in $1/\kappa$. Again we show two fits of the 1^{-+} meson for each κ, the lower one fitting both source operators simultaneously and the upper one fitting only the $1^{-+} \to 1^{-+}$ propagator. The $\kappa = 0.1350$ or $1/\kappa = 7.407$ charmonium point is not in the range shown here.

Heavy hybrids.

There has been a lot of work on the spectroscopy of heavy exotics $\overline{b}bg$ and $\overline{c}cg$

Heavy quarks are non-trivial to deal with in lattice QCD because the lattice spacing is of the same magnitude as the mass. Standard techniques (mostly) work for charm quarks.

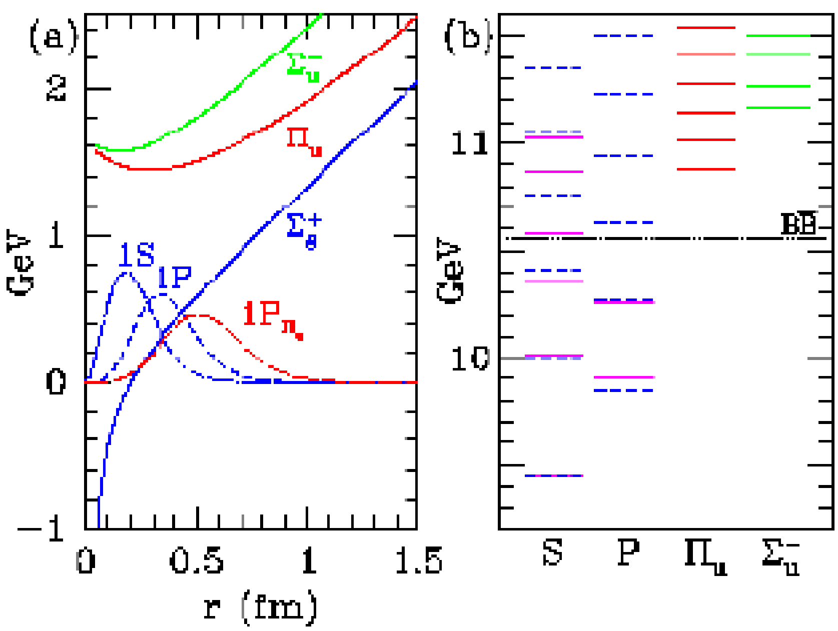
In the limit of infinitely heavy quarks potentials are constructed from Wilson lines.

The lowest order NRQCD Lagrangian (Lepage and Thacker, Phys.Rev.D43:196-208,1991) is

$$L_0 = \overline{Q}(D_t - \frac{D^2}{2M_Q a} - c\frac{\sigma.B}{2M_Q a} + ...)Q$$

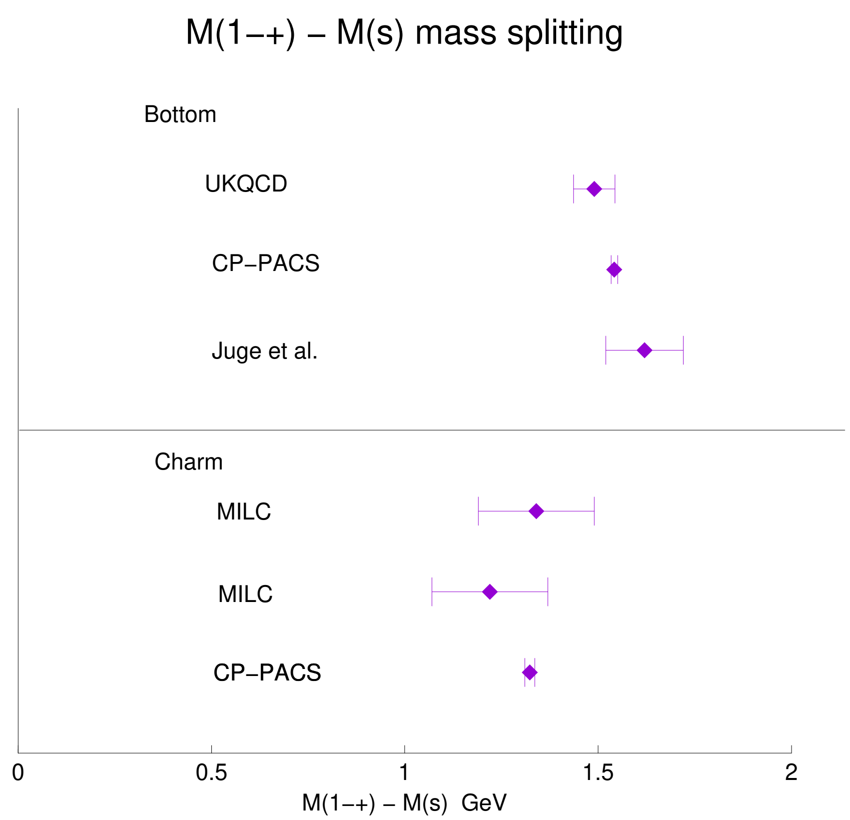
Results for static quarks.

From hep-ph/9902336, Juge, Kuti and Morningstar



Results for heavy ($\bar{c}cg$ and $\bar{b}bg$) 1^{-+}

Results from MILC (hep-lat/9707008,hep-lat/9809022), CP-PACS (hep-lat/9812017), and UKQCD(hep-lat/9809022)

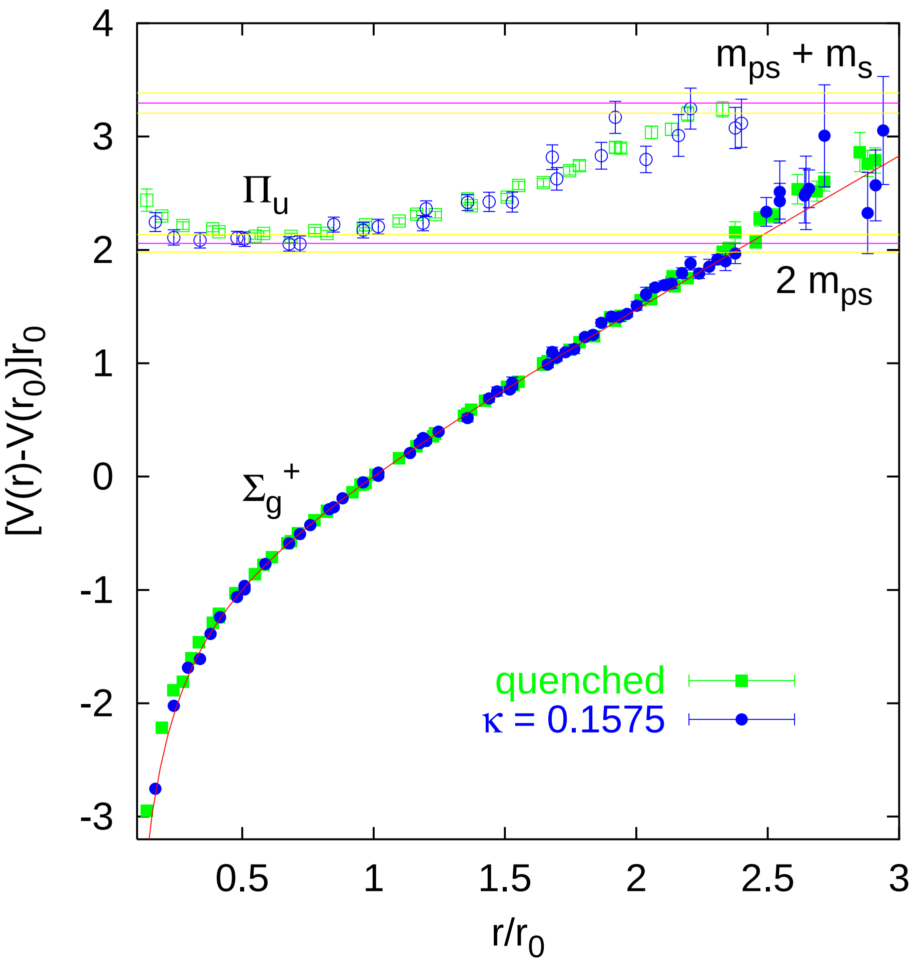


Results for heavy ($\overline{c}cg$ and $\overline{b}bg$) 1^{-+}

Results from MILC (hep-lat/9707008,hep-lat/9809022), CP-PACS (hep-lat/9812017), and UKQCD(hep-lat/9809022)

Effect of sea quarks on hybrid mesons.

The few calculations that have included the dynamics of sea quarks on hybrid mesons have found only a small effect. For example, from SESAM, hep-lat/0003012.



Non-exotic hybrid mesons

Creation operators for non-exotic mesons (e.g. $J^{PC} = 1^{--}$) that contain "excited glue" can be written down, but do they effect the dynamics?

Burch et al. (hep-lat/0103025)studied this in the bottonium system using NRQCD.

$$H = \frac{-\overrightarrow{D}^2}{2M_Q} + c_B \frac{-g}{2M_Q} \overrightarrow{\sigma}.\overrightarrow{B}$$

For the 1^{--} state (Y)they found (at $a \sim 0.13 fm$, L$\sim 2.6 fm$)

$$\mid Y \rangle = 0.99826(6) \mid Q\overline{Q} \rangle - 0.059(1) \mid Q\overline{Q}g \rangle$$

where $\mid Q\overline{Q} \rangle \equiv \overline{Q} \sigma_i Q$ and $\mid Q\overline{Q}g \rangle \equiv \overline{Q} B_i Q$ (transverse electric gluon mode in the bag model)

Why is hadronic spectroscopy so confusing?

Computing hadronic decay widths is hard (this is the holy grail of spectroscopy)!

To identify or understand a hadron its decay widths must be computed.

Computing decay widths is hard for models as it is not clear what interaction should be used to model a strong decay (Barnes, hep-ph/0202157, discusses recent tests of the $^3P_0$ decay model).

On the lattice computing decay widths is hard because:

- The calculations are done in Euclidean space, so matrix elements are real.

- The finite box size quantities momentum in chunks of order $\frac{2\pi}{aL}$(e.g. 800 MeV)

Strong decay formalism on the lattice.

The pioneering study was by Weingarten et al. (hep-lat/) for decay widths of the scalar glueball. (See also Michael and McNeile hep-lat/0010019, Michael et al. hep-lat/0201006) .

The transition amplitude ($\langle H \mid AB \rangle$) is computed for the decay $H \longrightarrow A + B$

$\langle H_E^{\dagger}(t) A_E(0) B_E(0) \rangle = \sum_{t_1=0}^{T} \langle H_E \mid H \rangle \langle H \mid AB \rangle \langle AB \mid A_E B_E \rangle e^{-E_H t} 1^{-(E_A+E_B)(t-t_1)} + \ldots$

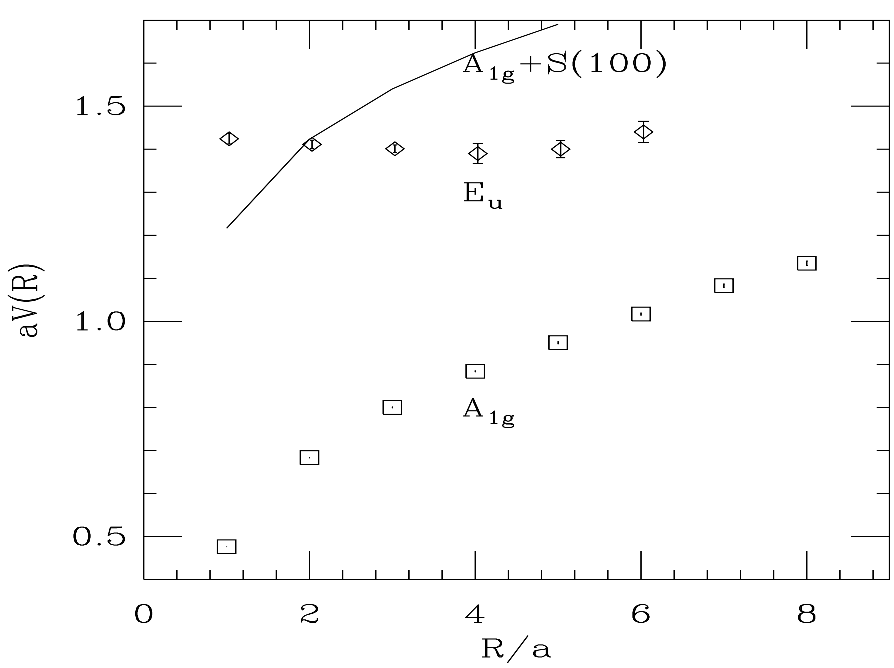
In the above sum the decay occurs at time t_1. The extraction of $\langle H \mid AB \rangle$ is complicated by excited hadrons also contributing to above fit formula. In practice the parameters have to be tuned for $E_H \sim E_A + E_B$ so the signal grows with t ,but the excited state contaminations are suppressed exponentially.

The decay width (Γ) is then computed from Fermi's golden rule.

De-excitation of an excited string

McNeile, Michael, and Pennanen, hep-lat/0201006.

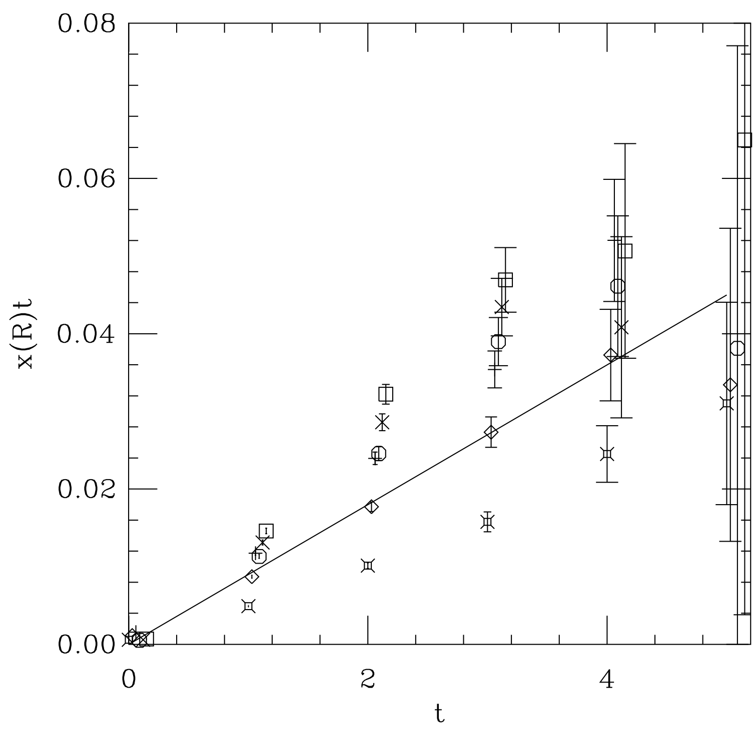
Consider a potential V(R) between quark anti-quark pair separated by a distance R.



Computing the decay amplitude.

Compute the amplitude x for the decay $H \to A + B$

$$x = \lim_{t\to\infty} \frac{1}{t} \frac{C_{H\to A+B}(t)}{\sqrt{C_H(t) C_{AB\to AB}(t)}}$$



Results for heavy hybrid decays.

Consider the decay of a static hybrid: $H \to X + Y$

There are selection rules (P. Page, Phys.Lett. B402 (1997)183)that forbid decays to X and Y with the same non-relativistic structure and L=0.

Possible decays are $\overline{b}bg \to \overline{b}q_{L=1} + \overline{q}b_{L=0}$, however the lattice results for $\overline{b}bg$ masses are below this threshold.

McNeile, Michael, and Pennanen, hep-lat/0201006, at $\beta = 5.2$, NP clover, $\kappa = 0.1355$,$16^3 32$:

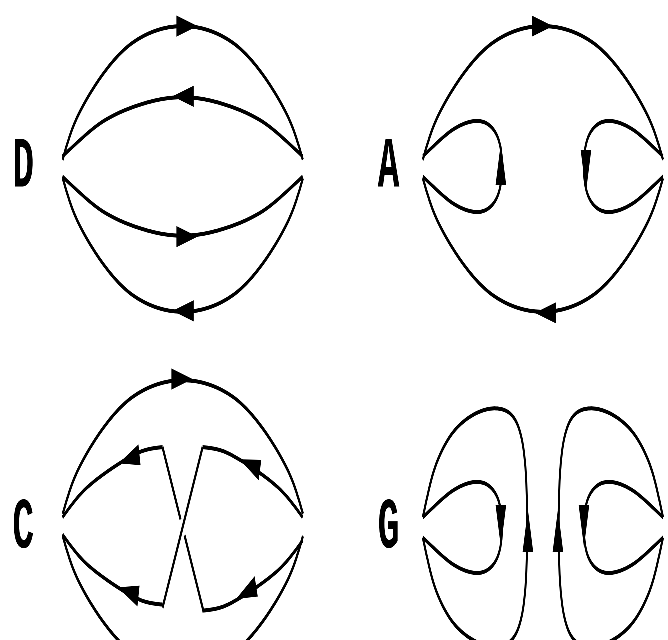
$$\Gamma(H \to \chi_b \eta) < 1 \text{ MeV}$$

$$\Gamma(H \to \chi_b S) \sim 80 \text{ MeV}$$

Is the 1.4 GeV 1^{-+}signal state a $\overline{qq}qq$?

One possibility is that the experimental candidate at 1.4 MeV might be a $\overline{qq}qq$ state. This would not couple to operators such of type $\overline{q}qg$ (see for example Klempt, hep-ex/0101031).

There has been a study of $\overline{qq}qq$ states on the lattice by Alford and Jaffe (hep-lat/0001023). These states are hard (but not impossible) to calculate from lattice QCD.



$\overline{qq}qq$ states for the 0^{++}

The motivation for the calculation (hep-lat/0001023) of Alford and Jaffe) was to investigate the a_0(980) and f_0(980) that some people think are $\overline{qq}qq$ states ($\overline{K}K$ molecules for example).

Only the direct and exchange diagrams were computed, I believe it would be consistent to compute one of the annihilation diagrams in the quenched approximation. Also, since that paper has been written, work by Isgur et al. (hep-lat/0106008) found a severe problem with the 0^{++}isovector channel, due to the quenched approximation.

$$\delta E_b = E_b - 2m_P = \frac{T_b}{L^3}(1 + 2.8373\frac{m_P T_b}{4\pi L} + \ldots)$$

where T_b is the scattering amplitude at threshold for channel a.

Conclusions of hep-lat/0106008: I=0 ($\overline{qq}qq$) is bound, I=2 $\overline{qq}qq$ is unbound.

Maybe something similar could be done for the 1^{-+}state.

0^{++} from experiment (PDG)

Particle	M MeV	Best guess at content
a_0(980)	980	$\overline{q}\overline{q}qq$/molecule?????
a_0(1450)	1450	$\overline{q}q$
f_0(400-1200)	400-1200	?????????????
f_0(980)	980(10)	$\overline{q}\overline{q}qq$/molecule ?????
f_0(1370)	1200-1500	glueball + $\overline{q}q$??
f_0(1500)	1500(10)	glueball + $\overline{q}q$??
f_0(1710)	1715(7)	glueball + $\overline{q}q$??

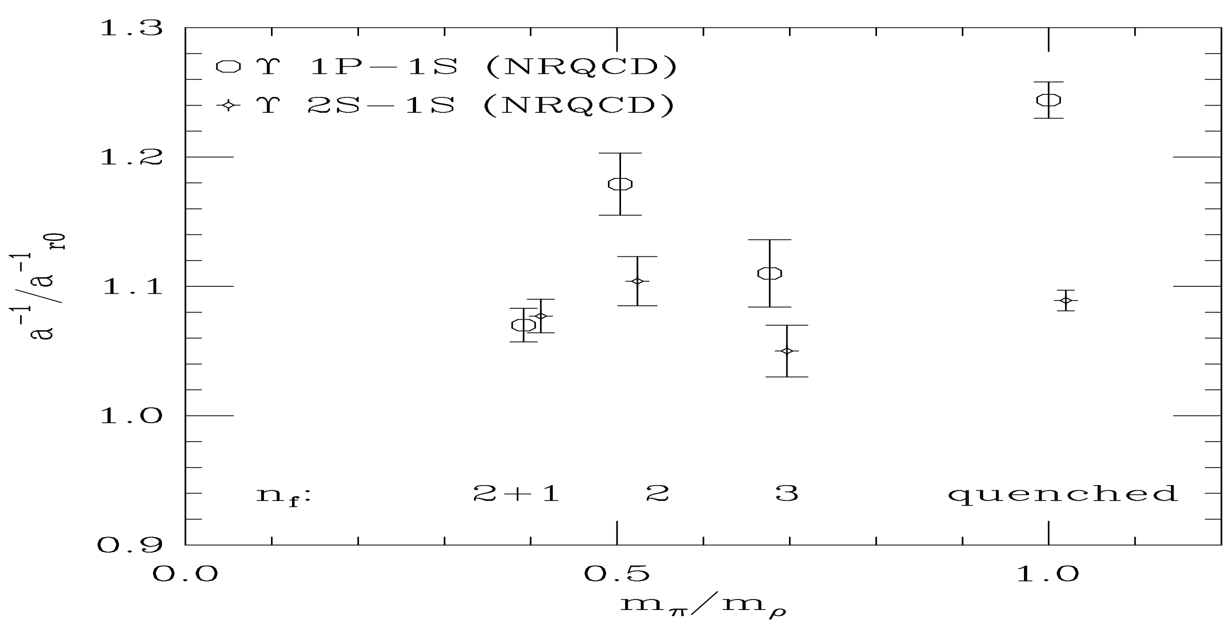
If only $\overline{q}q$ were the only important operators then the f_0 states could all be all classified in SU3 nonets. Experimentally there seems to be too many $I = 0, J^{PC} = 0^{++}$mesons to put in SU3 nonets.

Different views, Minkowski, Ochs, hep-ph/0209225.

0^{++} from quenched lattice QCD

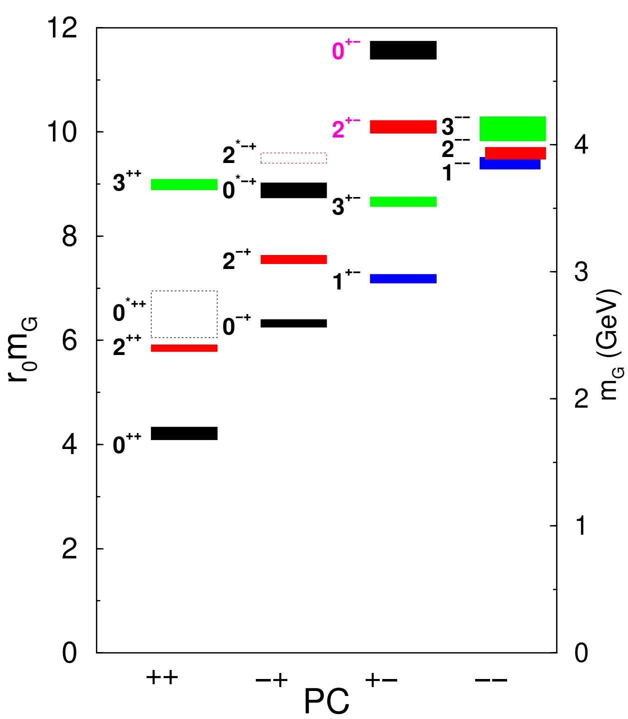
The glueball mass of $0^{++}$from the lattice QCD calculation of Morningstar and Peardon (hep-lat/9901004) was 1730 (50) (80)MeV. The second error is the uncertainty in $1/r_0$. This is consistent with the earlier results of the GF11 and UKQCD groups. Unfortunately, the second error (80 MeV) can not be reduced in quenched QCD.

From Gray et al., hep-lat/0209022.



Other highlights from glueball studies in quenched QCD.

- No spin exotic glueballs with masses under 4 GeV.
- Next lightest glueball 2^{++}, $m_{2^{++}}$= 2232 (220) (220) MeV



Concerns about the glueball spectrum from quenched QCD?

There is a sizable community of people who believe that the glueball spectrum from quenched QCD has **NOTHING** to do with nature! Computing the glueball spectrum is a well defined theoretical task in pure Yang-Mills theory (apart from the problem of setting the scale).

For example: hep-ph/9612457, S Narison, MASSES, DECAYS AND MIXINGS OF GLUONIA IN QCD. The properties of $\overline{q}q$mesons are computed from sum rules

$$\Gamma(\sigma_B \rightarrow \pi\pi) = 0.7 \text{ GeV} M_{\sigma_B} = 1 \text{ GeV}$$

$$\Gamma(\sigma'_B \rightarrow \pi\pi) = 1.3 \text{ GeV } M_{\sigma_{B'}} = 1.3 \text{ GeV}.$$

If the decay widths are large then unquenched glueball mass could be dramatically different to the quenched number.

Decay widths of the 0^{++} on the lattice

Sexton et al. (hep-lat/9510022) computed the decay width of the pure glue glueball $0^{++}$to two pseudoscalars. The result $\Gamma = 108(28)$ MeV at one (coarse) lattice spacing ($\beta = 5.7$). The decay widths increased with mass, which was inconsistent to the naive expectation, that the coupling of the 0^{++}coupled to pseudoscalar pairs in a flavour neutral way.

Reviewed by Klempt, hep-ex/0101031

A glueball is flavour singlet so can decay into two octet mesons $1 \rightarrow 8 \times 8$. SU(3) symmetry can predict the ratios:

(glueball)$\rightarrow$($K\overline{K}$, $\pi\pi$, $\eta_8\eta_8$, $\overline{K}K$) $=\frac{1}{\sqrt{8}}$(2 , 3, -1, -2)$^{1/2}$

Mixing analysis .

The glueball operators wil mix fermionic operators with the same quantum numbers. One way to model this is think in terms of a mixing matrix, written down by Weingarten and Lee (hep-lat/9910008)

$$\left(\begin{array}{c} \mid f_0(1710)\rangle \\ \mid f_0(1500)\rangle \\ \mid f_0(1310)\rangle \end{array} \right) = \left(\begin{array}{ccc} m_G & f & \sqrt{2}fr \\ f & m_S & 0 \\ \sqrt{2}fr & 0 & m_N \end{array} \right) \left(\begin{array}{c} \mid G\rangle \\ \mid \overline{s}s\rangle \\ \frac{1}{\sqrt{2}}(\overline{u}u + \overline{d}d) \end{array} \right)$$

.

The (main) assumptions are that:

- The mixing is between the mesons in the region 1300 to 1750 MeV.
- The unmixed states are the "quenched states".

Results of Weingarten and Lee

Weingarten and Lee studied this issue (hep-lat/9910008) in a large systematic quenched lattice QCD calculation.

1)The mass of the a_0 was computed in the quenched QCD.

2)They explicitly calculated the glueball-$\overline{q}q$ mixing for light and strange quarks.

The conclusion from the study was that the mixing is very weak. Weingarten and Lee have exact specific predictions, however the main content of the mesons are

$$f_0(1370) \sim \frac{1}{\sqrt{2}}(\mid \overline{u}u\rangle + \mid \overline{d}d)\rangle$$

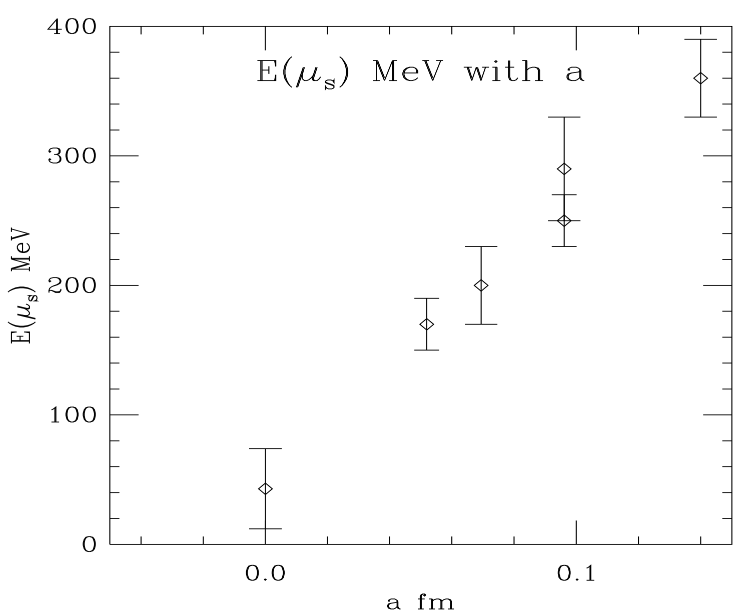
$$f_0(1500) \sim \mid \overline{s}s\rangle$$

$$f_0(1710) \sim \mid G\rangle$$

It is not clear whether this mixing pattern is consistent with the measured decays widths (hep-ph/0103173, Close and Kirk),

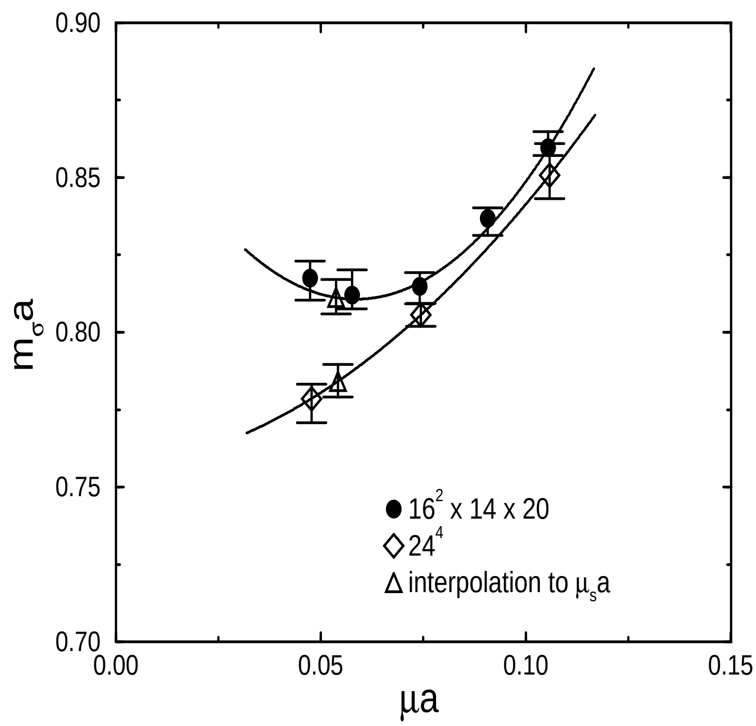
(Constructive) criticism of the Weingarten and Lee calculation I.

In their calculation Weingarten and Lee use four lattice spacings. The continuum result depends on the data with the smallest lattice spacings, however they may have underestimated the systematic errors in their fits (in time).



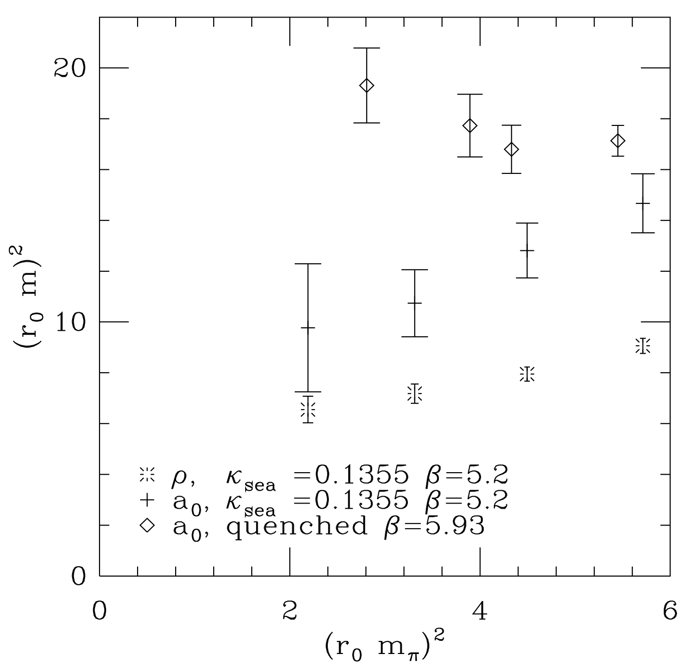
(Constructive) criticism of the Weingarten and Lee calculation II.

The Weingarten-Lee analysis depends on the mass of the a_0 being around 1300 MeV in the quenched approximation. Recently, Bardeen, Isgur et al. (hep-lat/0106008) found that the a_0 channel has a ghost state in the quenched theory. The $\eta' - \pi$ intermediate state is treated incorrectly in the quenched theory.



a_0 masses from dynamical QCD

Do a partially quenched analysis of the $a_0$at $\beta = \mathbf{5.2}$, $\mathbf{16^3 32}$, NP clover, unquenched gauge configurations, κ_{sea}=0.1355.

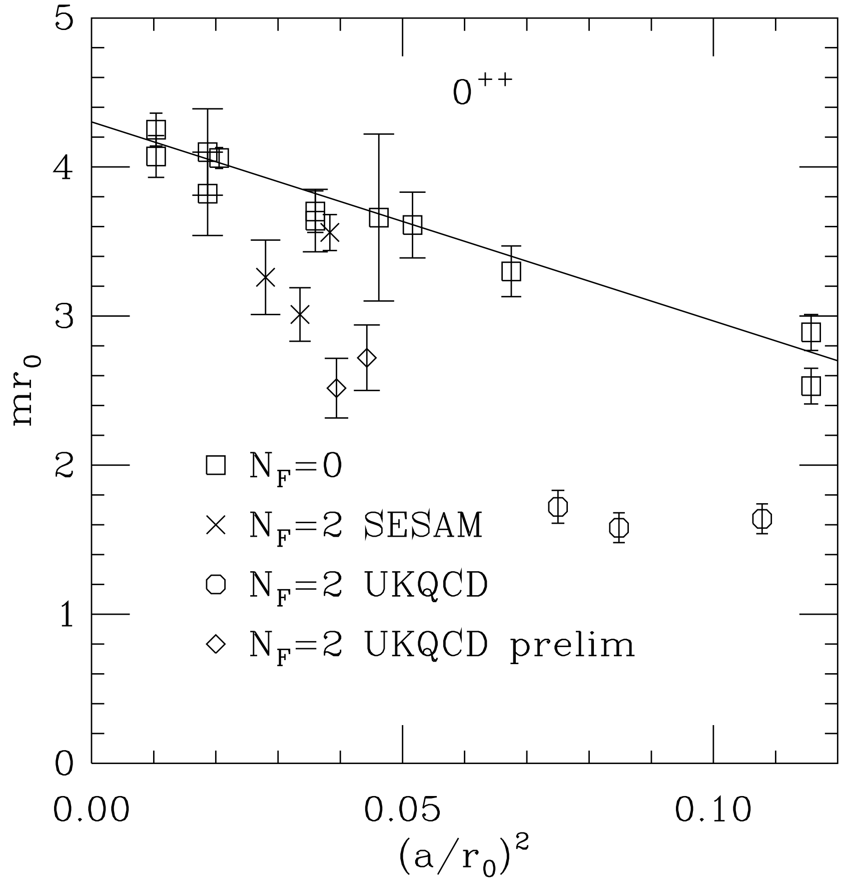


Singlet scalar mesons in full QCD.

If we were solving lattice QCD with the physical parameters, then the masses of the $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ states should be reproduced by 0^{++}operators. The quenched result does actually provide some insight into physics of scalar mesons.

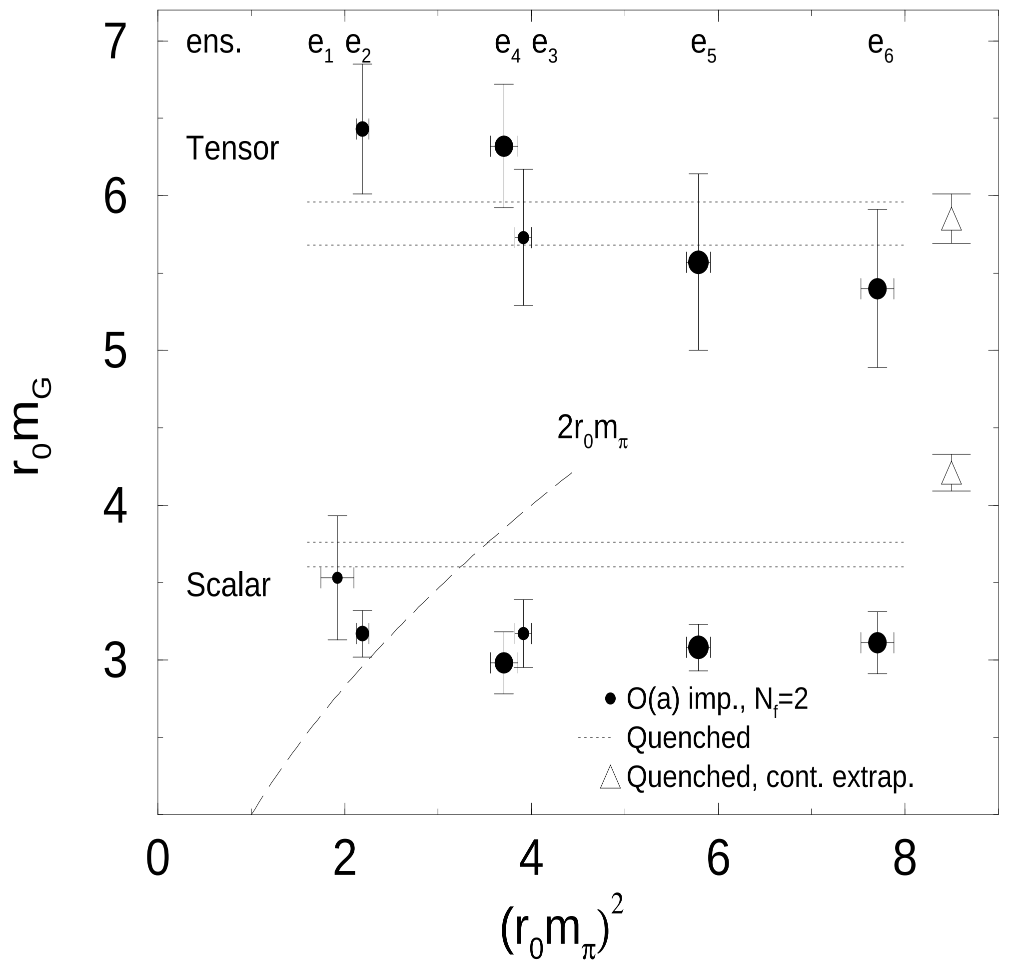
BUT, the mass differences are about 200 MeV. So at the very least the calculations have to be accurate to 50 MeV!

Result for the glueball mass



Opening of decay channels.

Hart and Tepper, hep-lat/0108022



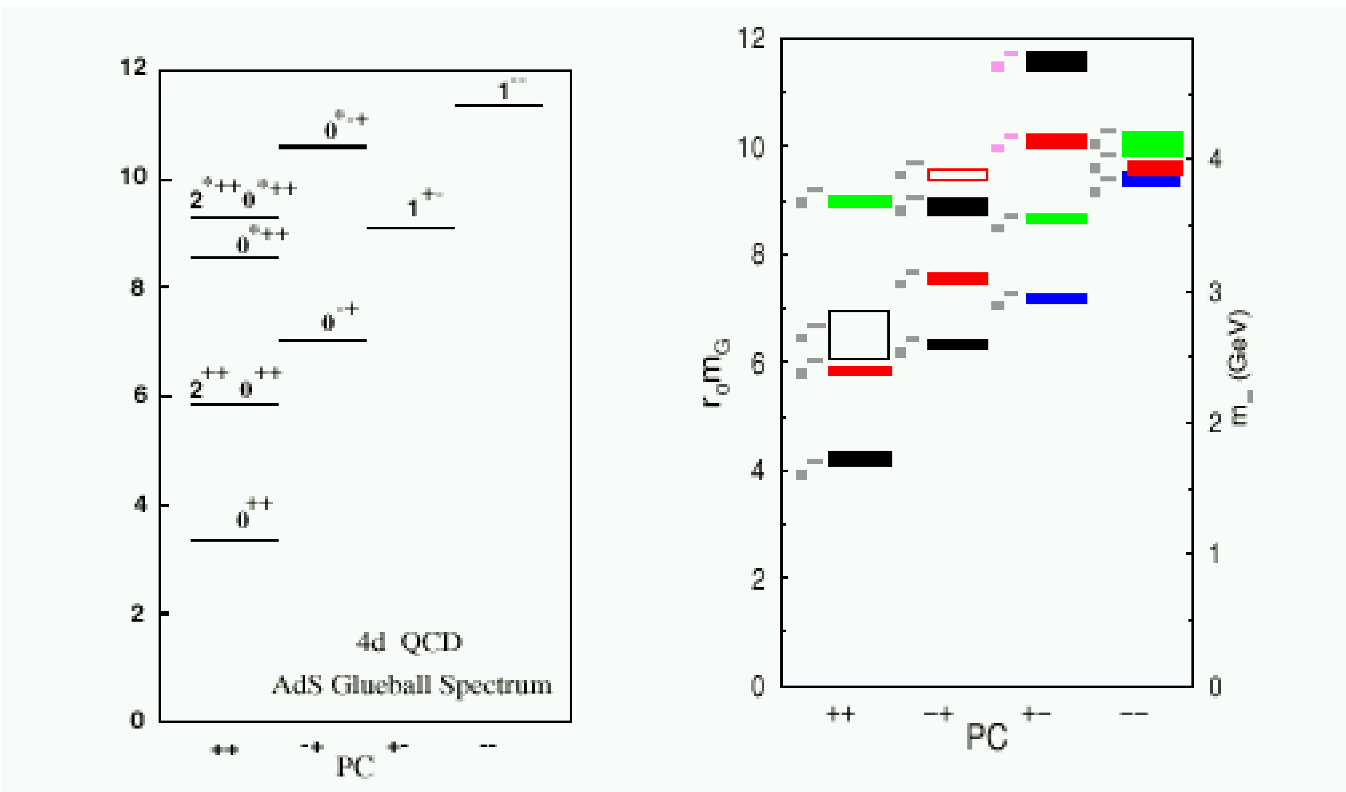
Large N limit of QCD

One hope for an analytical solution of QCD rests on large N (the number of colours) expansions.

For example in the paper: Glueball Spectrum for QCD from AdS Supergravity Duality, Brower, Mathur, Tan, (hep-th/0003115), the Maldacena duality conjecture is used to compute the glueball spectrum in the strong coupling limit ($g^2N \rightarrow \infty$) for pure gauge theory. The calculation consists of taking the normal modes for the bosonic components of the supergraviton multiplet in $AdS^7 \times S^4$. In the guts of the calculation, various wave equations are solved (numerically[1]).

[1] Thus removing them for the competition for proving a mass gap in QCD, that has prize money of $1,000,000.

Results at large N (hep-th/0003115)



Conclusions

- The first unquenched calculations of the mass of the 1^{-+}have been done by MILC.
- The 0^{++}sector is confusing! The lattice spacing and quark masses used in the lattice calculations should be reduced.
- The inclusion of $\overline{qq}qq$ type interpolating operators in lattice QCD calculations will be very interesting. This requires some algorithm work. The current data is (just) light enough for f_0 decay.
- Quenched QCD is a being superceeded by unquencehd calculations.
- Future progress depends on the quality of the unquenched calculations.