Recent calculations in chiral perturbation theory

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1. Chiral perturbation theory
2. Photon induced reactions
3. Strong interactions
4. Summary

1. Chiral perturbation theory

$$\mathcal{L}_{QCD} \quad \to \quad \mathcal{L}_{eff}$$

quarks,gluons $\qquad \pi, K, \eta, p, n, \ldots$

$$\mathcal{L}_{eff}=\mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots$$

- chiral symmetry determines structure of $\mathcal{L}_n$
- coupling constants (LEC's) in $\mathcal{L}_n$ must be determined from experiment/lattice calculations
- the calculation with $\mathcal{L}_{eff}$ is a **systematic** procedure: Expansion in powers of momenta and of quark masses
- for appropriately chosen LEC's, the effective theory reproduces the S-matrix elements of QCD at low energies [1]

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[1] Weinberg 79, Leutwyler 94

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The procedure to calculate matrix elements with $\mathcal{L}_{eff}$ is called

Chiral perturbation theory [ChPT]

2 examples:

- $I = 0$ S-wave scattering length in $\pi\pi \to \pi\pi$

$$a_0^0 = \frac{7x}{32\pi}\left[1 + c_1 x + c_2 x^2 + O(x^3)\right]$$

where

$$x = M_\pi^2 / F_\pi^2$$

$$c_{1,2} = \text{numbers } +\chi\text{logs } + \text{ LEC's} \qquad [1]$$

$$\Rightarrow \qquad \boxed{a_0^0 - a_0^2 = 0.265 \pm 0.004} \qquad [2]$$

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[1] $7/32\pi$: Weinberg 66
c_1: Gasser, Leutwyler 83
c_2: Bijnens, Colangelo, Ecker, Gasser, Sainio 96

[2] Colangelo, Gasser, Leutwyler 00

- Lifetime of $\pi^+\pi^-$ bound state (Pionium) in ground state:

$$a_0^0 - a_0^2$$

$$\downarrow$$

$$\boxed{\tau = (2.9 \pm 0.1) \times 10^{-15}\text{sec}} \; [1]$$

$$\updownarrow$$

DIRAC - experiment at CERN [2]

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[1] Gall, Gasser, Lyubovitskij, Rusetsky 01

[2] PS212, DIRAC collaboration, CERN

Reactions considered in the following

$$\begin{aligned} \pi X &\to \pi\gamma X' \\ KX &\to K\gamma X' \\ \pi X &\to \pi\pi X', K\bar{K}X' \\ KX &\to K\pi X' \end{aligned}$$

Primakoff; one-pion exchange

No consideration of kinematical conditions performed

I do not discuss

$$K_{l3}, K_{l4}, K \to 2\pi, K \to 3\pi, \ldots$$

I understand that these are (will be) investigated e.g. at NA48

2. Photon induced reactions

1. Pion polarizabilities

Consider Compton scattering

$$\gamma\pi^{\pm} \to \gamma\pi^{\pm}$$

and expand the amplitude in powers of photon momenta. Quadratic terms contain the polarizabilities α, β

a) amplitude known to two loops [1]

b) α, β can a be expanded in powers of the pion mass - result is similar to the expansion of the $\pi\pi$ scattering lengths

c) knowledge of LEC's has been improved, structure of $\mathcal{L}_6$ is now known [2]. Bürgi's result should be re-investigated in this respect

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[1] Bürgi 96

[2] Bijnens, Colangelo, Ecker 99

2. Kaon polarizabilities

I am not aware of a calculation of kaon polarizabilities at the two-loop level

Is expansion in m_s, prediction will be less precise

Motivation for measuring polarizabilities:

i) polarizabilities start being nonzero at one-loop order - one explores loop effects rather directly

ii) at leading order, LEC's that occur in π, K polarizabilities are related to radiative pion decay $\pi \to e\nu\gamma$

iii) measurement of polarizabilities and comparison with the chiral expansion is of similar interest as the $\pi\pi$ scattering lengths

3. The anomaly

The word **anomaly** refers to anomalous Ward identities valid for the Green functions

a) $\pi^0 \to \gamma\gamma$

amplitude known at next-to-leading order [1]

prediction: $\Gamma_{\pi^0 \to \gamma\gamma} = 8.10 \pm 0.08$ eV [1]

PDG: $\Gamma_{\pi^0 \to \gamma\gamma} = 7.74 \pm 0.55$ eV [1,2]

measurement in progress at JLAB (PRIMEX)

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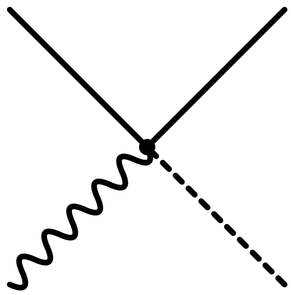
[1] Bernstein, Goity, Holstein 02

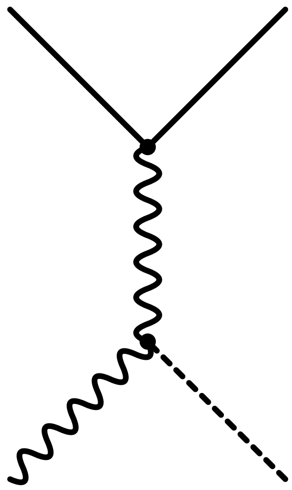
[2] PDG 00

b) $\pi\gamma \to \pi\pi$

Recent calculation at one-loop order [1], including terms of order e^2. Process is interesting, because electromagnetic effects are large. Two-loop calculations at $e^2 = 0$ also available [2]

At leading order, two diagrams contribute:





Solid (dashed) lines: charged (neutral) pion

wiggle line:photon

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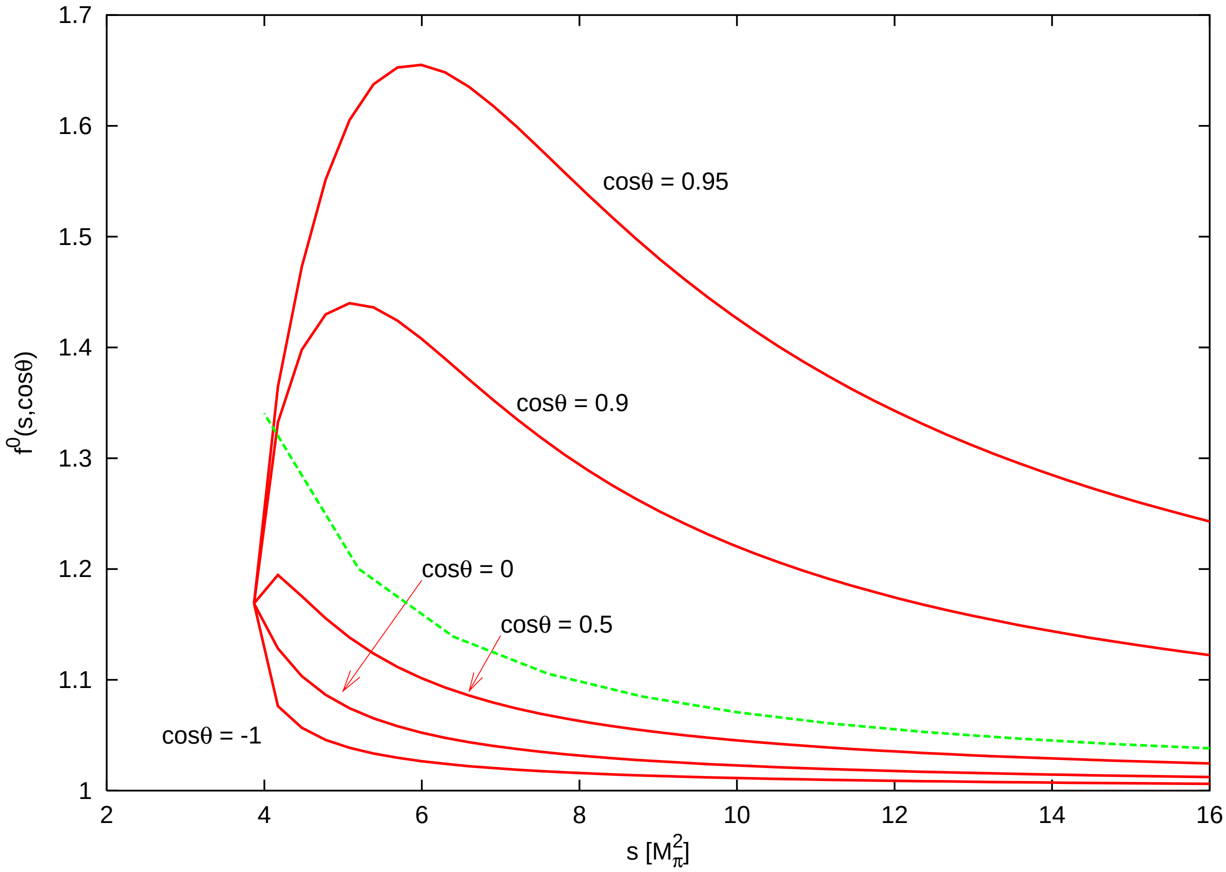
[1] Ametller, Knecht, Talavera 01

[2] Hannah 01

Exchange in t-channel rather sensitive to scattering angle [1]:



The lowest order amplitude $f^{(0)}(s, \cos\theta)$ including radiative corrections as a function of s and for various values of the center-of-mass scattering angle θ

[1] Ametller, Knecht, Talavera 01

At one loop, infrared divergences occur - are canceled by including the process

$$\gamma\pi \to \pi\pi\gamma$$

Result rather insensitive to the photon energy of the outgoing photon [1]

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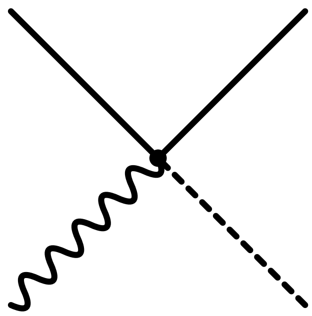
[1] Ametller, Knecht, Talavera 01

c) $\gamma K \to \pi K$

This process is again dominated by anomalous terms in the effective lagrangian. Is SU(3) version of anomaly. Calculations are yet to be performed. Again: expansion in m_s, is less precise than expansion in $m_{u,d}$

Interesting, because singularity in t-channel is absent in the neutral kaon case:



Solid (dashed) lines: neutral kaon (pion)

wiggle line: photon

$\Rightarrow$ Can disentangle pure anomaly contribution

N_c-dependence

$\pi^0 \to \gamma\gamma, \gamma\pi \to \pi\pi$ *independent* of N_c [1]

amplitude $\gamma K^\pm \to \pi^0 K^\pm \simeq (N_c + 1)$ [2]

amplitude $\gamma K^0 \to \pi^0 K^0 \simeq (N_c - 1)$ [2]

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[1] Rudaz 90; Abbas 90, 00; Bär, Wiese 01

[2] Bär, Wiese 01

3. Strong interactions

1. $\pi K \to \pi K$

recent calculations:

i) dispersive approach to determine $L_{1,2,3,4}$ using data on this reaction [1]

ii) $\pi^- K^+$atom $\leftrightarrow \pi K$ amplitude [2]

iii) scalar form factor of pion - need input from $\pi\pi \to K\bar{K}$ [3]

Motivation:

a) pin down LEC's

b) question of chiral symmetry breakdown: is the vacuum affected by heavy $\bar{s}s$ pairs? [4]

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[1] Ananthanarayan, Büttiker, Moussallam 01

[2] Schweizer, lavori in corso

[3] Donoghue, Gasser, Leutwyler 90

[4] Fuchs, Knecht, Moussallam, Stern 96

$\boxed{\text{Data } \pi K \to \pi K}$

$\Rightarrow$ Available data: I have not yet scanned the literature in detail

Needed [1]:

$\Rightarrow$ separation of isospin $1/2$ and $3/2$ amplitudes

$\Rightarrow$ $\pi\pi \to K\bar{K}$ in the region of $K\bar{K}$ threshold

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[1] Ananthanarayan, Büttiker, Moussallam, private communication

2. $\pi\pi \to \pi\pi$

Why interesting?

- $\pi\pi$ scattering is the perfect observable to access chiral structure of QCD vacuum

- Example: quark mass expansion of the pion mass:

$$M_\pi^2 = M^2 - M^4 \bar{l}_3/(32\pi^2 F^2) + O(M^6)$$

M^2 is proportional to the quark condensate and to the quark mass

Is the first term in this expansion leading? [1]

Can be answered from precision data on $\pi\pi$ scattering

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[1] Fuchs, Knecht, Moussallam, Stern 96

Recent work on $\pi\pi$ scattering:

Theory:

B. Ananthanarayan, G. Colangelo,
J. Gasser, H. Leutwyler:

Phys. Lett. **B 488** 261 (2000)

Nucl. Phys. **B 603** 125 (2001)

Phys. Rev. Lett. **86** 5008 (2001)

Phys. Rept. **353** 207 (2001)

Experiments:

E865 at Brookhaven, $\simeq$ 400 000 K_{e4} decays [1]

DIRAC, $|a_0^0 - a_0^2|$ [2]

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[1] BNL-E865 collaboration 01

[2] PS212 experiment at CERN, running

ChPT + Roy-equations: Precise prediction of phase shifts below 800 MeV. Precision decreases as the energy increases.

Example:

$$I = 0\ S\text{-wave}$$

ChPT + Roy-equations [1]:

$$a_0^0 = 0.220 \pm 0.005$$

E865 at BNL [2]:

$$a_0^0 = 0.216 \pm 0.013(\text{stat}) \pm 0.004(\text{syst}) \pm 0.005(\text{theor})$$

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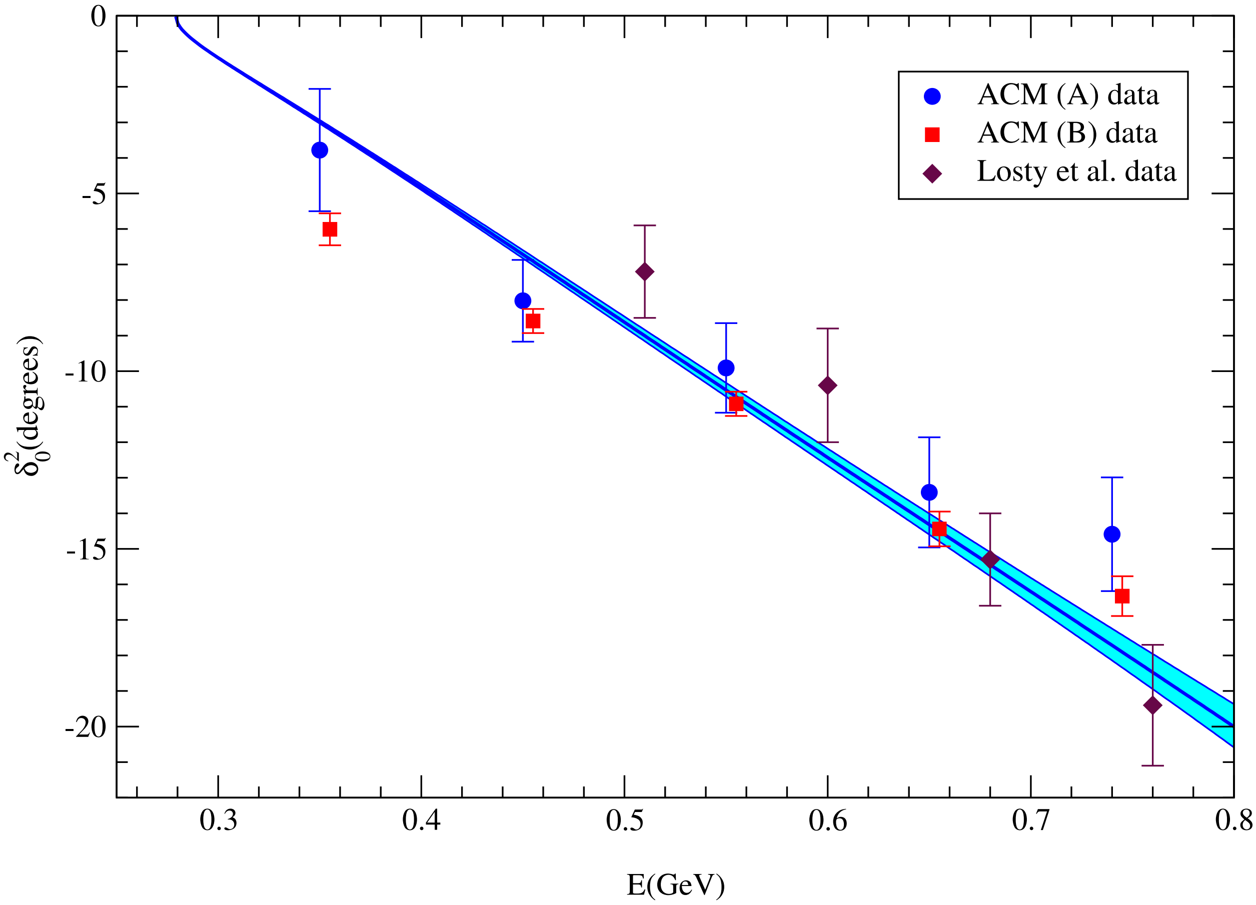
[1] Colangelo, Gasser, Leutwyler 00

[2] BNL-E865 collaboration 01

Point I wish to make for COMPASS:

• In my opinion, it would be very welcome to have more experimental information on $I = 2$ $S-$wave



<u>Comment added</u>: The above figure replaces the figure that I displayed in my talk. It is identical to Fig. 9 in Ref. [1]

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[1] Colangelo, Gasser, Leutwyler NPB 603 (2001) 125

4. Summary

	ref. theory	status of theory	status of experiment
photons			
$\gamma\pi \to \gamma\pi$	[1]	**(*)	*
$\gamma K \to \gamma K$	[2]	*	
anomaly			
$\pi^0 \to \gamma\gamma$	[3]	***	**
$\gamma\pi \to \pi\pi$	[4]	***	*
$\gamma K \to K\pi$	[5]		
strong			
$\pi K \to \pi K$	[6]	**	*
$\pi\pi \to \pi\pi$	[7]	****	***

- the distribution of *'s is subject to personal taste

- a complete list of references is relegated to the written version of this talk

References

[1] U. Burgi, Nucl. Phys. B **479** (1996) 392 [arXiv:hep-ph/9602429]; Phys. Lett. B **377** (1996) 147 [arXiv:hep-ph/9602421].

[2] None found

[3] J. L. Goity, A. M. Bernstein and B. R. Holstein, arXiv:hep-ph/0206007.

[4] J. Bijnens, A. Bramon and F. Cornet, Phys. Lett. B **237** (1990) 488;
T. Hannah, Nucl. Phys. B **593** (2001) 577 [arXiv:hep-ph/0102213];
L. Ametller, M. Knecht and P. Talavera, Phys. Rev. D **64** (2001) 094009 [arXiv:hep-ph/0107127].

[5] None found

[6] B. Ananthanarayan, P. Buttiker and B. Moussallam, Eur. Phys. J. C **22** (2001) 133 [arXiv:hep-ph/0106230].

[7] B. Ananthanarayan, G. Colangelo, J. Gasser and H. Leutwyler, Phys. Rept. **353** (2001) 207 [arXiv:hep-ph/0005297];
G. Colangelo, J. Gasser and H. Leutwyler, Phys. Lett. B **488** (2000) 261 [arXiv:hep-ph/0007112]; Phys. Rev. Lett. **86** (2001) 5008 [arXiv:hep-ph/0103063]; Nucl. Phys. B **603** (2001)

125 [arXiv:hep-ph/0103088];
S. Descotes, N. H. Fuchs, L. Girlanda and J. Stern, Eur. Phys. J. C **24** (2002) 469 [arXiv:hep-ph/0112088].