LECTURE 2
SUPERSYMMETRY

MOTIVATION

HIERARCHY PROBLEM : NO SYMMETRY WHEN $m_h \to 0$ $\to$ LARGE CORRECTIONS...

WHAT IF SCALAR & FERMION WOULD SOMEHOW BE IN SAME MULTIPLET?

THE EXTRA CHIRAL SYMMETRY OF FERMION WOULD PROTECT THE SCALAR AS WELL.

- HOW COULD WE POSSIBLY HAVE DIFFERENT SPINS IN A SINGLE MULTIPLET?
- ENLARGE SPACE-TIME SYMMETRIES TO INCLUDE GENERATORS THAT CONVERT FERMIONS AND BOSONS INTO EACH OTHER.
- NEED TO HAVE SOME FERMIONIC ($\equiv$ SUPERSYMMETRY) GENERATORS.

- 1960'S : TRIED TO EXTEND POINCARÈ GROUP (= LORENTZ GROUP + + TRANSLATIONS) TO INCLUDE INTERNAL SYMMETRIES NO SUCCESS

COLEMAN- MANDULA : NOT POSSIBLE, NON-TRIVIAL GENERATORS ARE ONLY P_μ, $M_{\mu\nu}$.

- IF ALSO SUSY (FERMIONIC) GEN'S:

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\, \sigma^\mu_{\alpha\dot\beta} P_\mu$$

Q_α : SUSY GENERATOR (FERMIONIC)

Weyl spinor (2 comp) $\to$ 4 real supercharges.

$Q_\alpha |\text{fermion}\rangle = |\text{boson}\rangle$

$Q_\alpha |\text{boson}\rangle = |\text{fermion}\rangle$

1 Q_α GEN. $\to$ N=1 SUSY

- <u>HOW TO BUILD A SUSY THEORY?</u>

- POINCARÈ GROUP : FOR A QFT NEED TO USE IRREDUCIBLE REPRESENTATIONS OF POINCARÈ (≡ PARTICLES WITH GIVEN MASS & SPIN)

- SUSY : PARTICLES NEED TO FALL INTO SUSY REPRESENTATIONS (= BUILDING BLOCKS OF SUSY THEORY)

- EVERY PARTICLE WITHIN SUSY REP. HAS EQUAL MASS

$$[Q_\alpha, P_\mu] = 0 \rightarrow$$
$$[Q_\alpha, P^2] = 0$$

$P^2 = m^2$ COMMUTES WITH ALL GENERATORS $\rightarrow$ COMMON MASS

- EQUAL # OF FERMIONS & BOSONS WITHIN SUSY REP.

FERMION # OP $\rightarrow N_F$

$(-1)^{N_F}$ $+1$ BOSON, -1 FERMION

$$(-1)^F Q_\alpha = - Q_\alpha (-1)^{N_F}$$

$$\mathrm{Tr}\left[(-1)^{N_F}\{Q_\alpha^A, \bar{Q}_{\dot{\beta}B}\}\right] =$$
$$= \mathrm{Tr}\left[(-1)^{N_F}(\underbrace{Q_\alpha \bar{Q}_{\dot{\beta}} + \bar{Q}_{\dot{\beta}} Q_\alpha}_{P^\mu \sigma_{\mu\alpha\dot{\beta}}})\right] = 0$$

$$\mathrm{Tr}\left((-1)^{N_F} P^\mu\right) = 0 \qquad \boxed{\mathrm{Tr}(-1)^{N_F} = 0}$$

$\hookrightarrow$ fixed in a given rep.

SIMPLEST (AND MOST USEFUL) REPRESENTATIONS OF N=1 SUSY

- **CHIRAL SUPERFIELD**

COULD BE ANY REP OF GAUGE GROUP
{ ONE COMPLEX SCALAR φ 2 BOSONS
ONE (2 COMPONENT) WEYL FERMION ψ 2 FERMIONS }

- **VECTOR SUPERFIELD**

MUST BE IN ADJOINT
{ ONE MASSLESS GAUGE BOSON A_μ 2 BOSONS
ONE (2 COMPONENT) FERMION λ 2 FERMIONS }

- CAN SYSTEMATICALLY BUILD UP REPRESENTATIONS : ALL OTHERS CONTAIN SPIN > 1 → DON'T WANT
- SUSY HIGGS MECHANISM: VSF EATS CHIRAL SF

MASSIVE GB	3
SCALAR	1
2 WEYL FERM. = DIRAC	4

THE SUSY LAGRANGIAN

- CHIRAL SUPERFIELDS

SHOULD CONTAIN KINETIC TERMS, MASS TERM WITH EQUAL MASS

$$\mathcal{L}_{kin+mass} = \partial_\mu \varphi^* \partial^\mu \varphi + \overbrace{i\, \partial_\mu \bar{\psi}\, \bar{\sigma}^\mu \psi}^{\text{KIN. FOR WEYL}} - m^2 \varphi^* \varphi \underbrace{- \frac{m}{2} \psi\psi - \frac{m}{2} \bar{\psi}\bar{\psi}}_{\text{MASSTERM FOR WEYL FERMION}}$$

↳ ANALOG OF DIRAC MATRICES

INTERACTION TERMS (CAN ALSO INCLUDE MASS TERM THERE):

MUST BE DERIVABLE FROM SUPERPOTENTIAL

$W(\varphi)$: A HOLOMORPHIC FUNCTION (= NO φ^*) OF SCALAR COMPONENT

- AND INTERACTION TERMS:

$$-\mathcal{L}_{int} = \underbrace{\frac{1}{2}\frac{\partial^2 W(\varphi)}{\partial\varphi_i\partial\varphi_j}\psi_i\psi_j + h.c.}_{\text{YUKAWA INTERACTIONS}} + \underbrace{\left|\frac{\partial W}{\partial\varphi_i}\right|^2}_{\text{SCALAR POTENTIAL}}$$

- FOR EXAMPLE

$$W(\varphi) = \frac{1}{2}m\varphi^2 + \frac{1}{6}\lambda\varphi^3$$

$$\Downarrow$$

$$-\mathcal{L}_{int} = \frac{1}{2}m\psi\psi + h.c. + \frac{\lambda}{2}\varphi\psi\psi + h.c.$$
$$+ \left|m\varphi + \frac{1}{2}\lambda\varphi^2\right|^2$$

ENSURES: $m_\varphi = m_\psi$

$$\lambda^2_{Yukawa} = \lambda_{\substack{quartic\\ scalar}}$$

- QUADRATIC DIVERGENCE

$m_F = m_B$

CANCEL

$= 0$ Leading piece

- VECTOR SUPERFIELDS

USUAL GAUGE KINETIC TERM + MASSLESS FERMION WITH COVARIANT DERIVATIVE

$$\mathcal{L} = -\tfrac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - i \lambda \sigma^\mu D_\mu \bar{\lambda}$$

INTERACTION TERMS ALREADY FORCED ON YOU BY GAUGE INVARIANCE

- INTERACTION BETWEEN CHIRAL AND VECTOR SUPERFIELDS:

CHIRAL SUPERFIELD (≡ BOTH SCALAR AND FERMION) TRANSFORM UNDER GAUGE SYMMETRY

- ADDITIONAL TERMS

$$\left.\begin{aligned} \partial_\mu \varphi^* \partial^\mu \varphi &\longrightarrow D_\mu \varphi^\dagger D^\mu \varphi \\ i\partial_\mu \bar{\psi} \bar{\sigma}^\mu \psi &\longrightarrow i D_\mu \bar{\psi} \bar{\sigma}^\mu \psi \end{aligned}\right\} \text{AS ALWAYS}$$

$$\sum_a \frac{1}{2} g^2 \left[\sum_i \varphi_i^\dagger T^a \varphi_i\right]^2$$ EXTRA TERM IN SCALAR POT

$$g \lambda^a \varphi_i^* T^{a\,i}{}_j \psi^j + h.c.$$ EXTRA YUKAWA COUPLING

- REPRESENTATIONS, GAUGE COUPLING + SUPERPOTENTIAL COMPLETELY DETERMINE INTERACTIONS

- SCALAR POTENTIAL

$$V(\varphi) = \sum_i \left|\frac{\partial W}{\partial \varphi_i}\right|^2 + \frac{1}{2} g^2 \sum_a \left[\sum_i \varphi_i^\dagger T^a \varphi_i\right]^2$$

UNBROKEN SUSY $\Longleftrightarrow$ $V(\varphi) = 0$

HOW TO MAKE THE SM SUPERSYMMETRIC?

VECTOR SF's

A_μ^a GAUGE FIELDS — ADJOINT OF SU(3), SU(2), U(1)

$\downarrow$

λ^a GAUGINO FIELD (GAUGE FERMION)

CHIRAL SF's

ψ_i LEPTONS, QUARKS — FUNDAMENTAL OF SU(3), SU(2), U(1)

$\downarrow$

φ_i SLEPTONS, SQUARKS

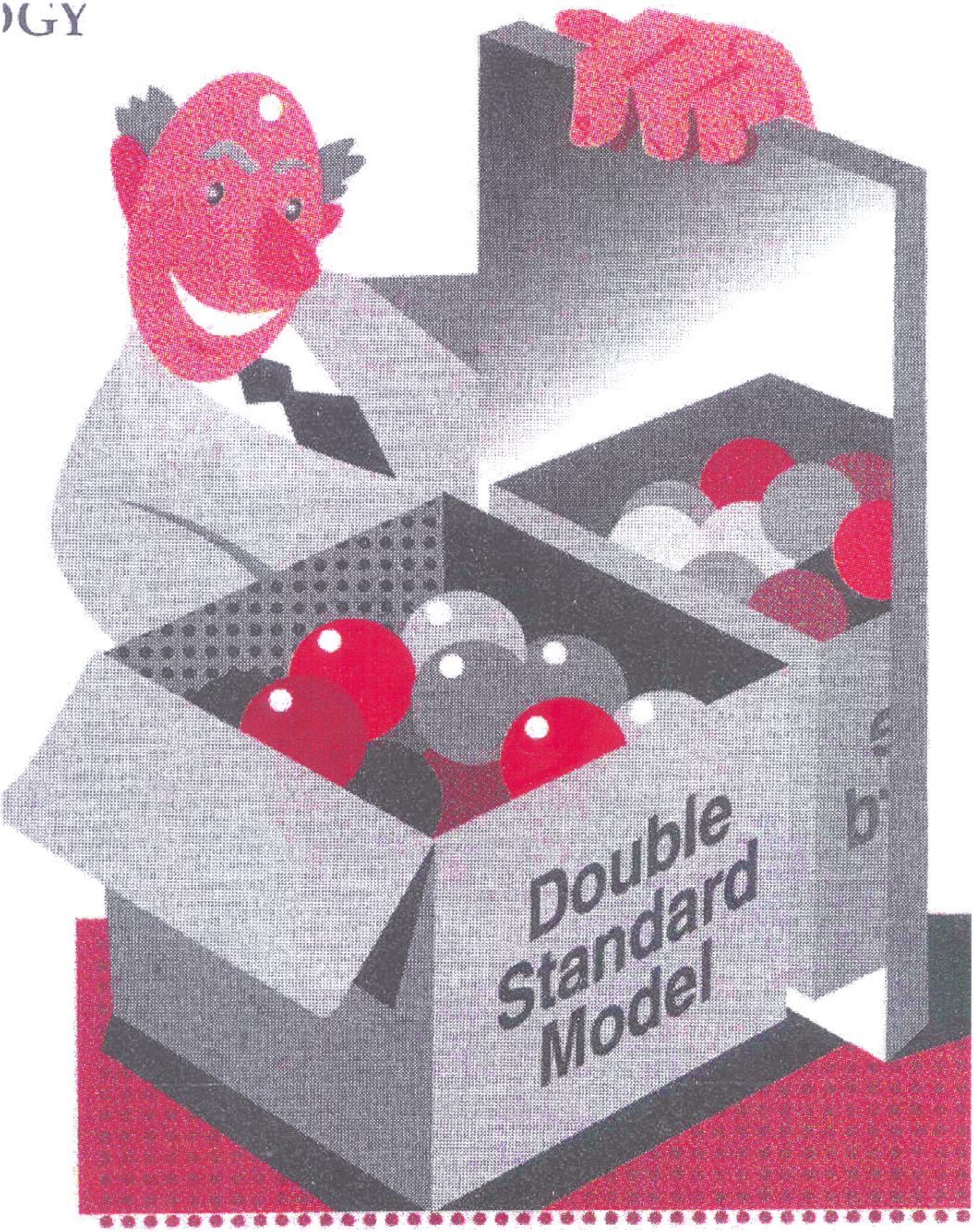
CHIRAL SF's

H HIGGS — FUND'L OF SU(2)

$\downarrow$

$\tilde{H}$ HIGGSINO — NEED 2 HIGGS DOUBLETS (ANOMALY, YUKAWA)

)GY



FROM THE ECONOMIST

- BUT: UNBROKEN SUSY

$$\left.\begin{array}{l} m_e = m_{\tilde{e}} \\ m_q = m_{\tilde{q}} \\ \vdots \end{array}\right\} \text{NOT OBSERVED}$$

- SUSY (IF EXISTS) MUST BE A BROKEN SYMMETRY

  HOW SHOULD IT BE BROKEN?

  EXPLICITELY? NO

  SPONTANEOUSLY? YES

- FROM SPONTANEOUS BREAKING CAN ONLY GET CERTAIN TYPE OF ~~SUSY~~ OPERATORS:

  "SOFT SUSY BREAKING"

- THE SOFT BREAKING TERMS:
  - SCALAR MASS $\varphi_i \varphi_i^*$
  - GAUGINO MASS $\lambda\lambda + h.c.$
  - REAL SCALAR MASS + CUBIC $\varphi^2 + h.c.$ $\varphi^3 + h.c.$

- "HOW COME WE HAVE SEEN HALF OF PARTICLES ONLY?"

EXACTLY THE YET UNSEEN SUPERPARTNERS CAN GET A MASS FROM ~~SUSY~~. NOT UNNATURAL, DICTATED BY STRUCTURE OF SUSY & ~~SUSY~~.

- USUAL APPROACH: DO NOT SPECIFY WHERE SUSY IS COMING FROM, JUST TREAT SOFT SUSY TERMS AS FREE PHENOMENOLOGICAL PARAMETERS.

THE Minimal Supersymmetric Standard Model

CHIRAL SF	SU(3)	SU(2)	U(1)	B	L
L_i	1	2	$-1/2$	0	1
$\bar{E}_i$	1	1	1	0	-1
Q_i	3	2	$1/6$	$1/3$	0
$\bar{U}_i$	$\bar{3}$	1	$-2/3$	$-1/3$	0
$\bar{D}_i$	$\bar{3}$	1	$1/3$	$-1/3$	0
H_1	1	2	$-1/2$	0	0
H_2	1	2	$1/2$	0	0

- SUPERPOTENTIAL OF THE MSSM:

$$W = \left(\lambda_u^{ij} Q^i H_2 \bar{U}^j + \lambda_d^{ij} Q^i H_1 \bar{D}^j + \lambda_e^{ij} L^i H_1 E^j + \mu H_1 H_2\right)$$

USUAL YUKAWA + HIGGS MASS

$$+ \left(\alpha_1^{ijk} Q^i L^j \bar{D}^k + \alpha_2^{ijk} L^i L^j E^k + \alpha_3^{i} L^i H_2 + \alpha_4^{ijk} \bar{D}^i \bar{D}^j \bar{U}^k\right)$$

VIOLATES B, L

VERY BAD TERMS, CAUSE PROTON DECAY, LEPTON # VIOLATION, ...

- SM : B, L CONSERVATION FOR FREE ALL RENORMALIZABLE OPERATORS AUTOMATICALLY PRESERVE B, L

- MSSM: NOT TRUE. NEED ADDITIONAL SYMMETRY TO FORBID THESE TERMS "R-PARITY"

- R-PARITY Z_2 GLOBAL SYMMETRY IMPOSED ON $\mathcal{L}$

(ORDINARY PARTICLE) $\to$ (ORD. PART)

(SUPERPARTNER) $\to$ $-$ (SUPERPARTNER)

- FORBIDS EXACTLY UNWANTED OPERATORS FROM SUPER POT.

R-PARITY ESSENTIAL PART OF MSSM

- CONSEQUENCE OF R-PARITY:

LIGHTEST SUPERPARTNER STABLE. COULD BE DARK MATTER! CAN BE PAIR PRODUCED IN EQUILIBRIUM, FREEZES OUT, CAN NOT DECAY ...

MAJOR PROPERTIES OF MSSM

- **LIGHT HIGGS** REASON:

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$\rightarrow m_H \sim \sqrt{\lambda}\,\mu$, μ FIXED BY M_Z.

IN SUSY THEORY λ CAN ONLY COME FROM D-TERMS $\Rightarrow \lambda \sim g$

TREE LEVEL $m_H \lesssim M_Z$ (WOULD BE BAD)

CORRECTIONS FROM TOP-STOP LOOPS $m_H \lesssim 135\text{-}140$ GeV

- **RADIATIVE ELECTROWEAK SYMMETRY BREAKING**

NEED TO SIMPLIFY MANY NEW PARAMETERS FOR SOFT BREAKING

<u>ASSUME</u> : AT HIGH SCALE ($\sim M_{GUT}, M_{Pl} \sim 10^{16}\text{-}10^{19}$ GeV) UNIVERSAL SOFT BREAKING TERMS

AT HIGH SCALE

$$-\mathcal{L}_{SOFT}\Big|_{M_P} = m_0^2 \sum_{i=Q_i,\dots} |\phi_i|^2$$

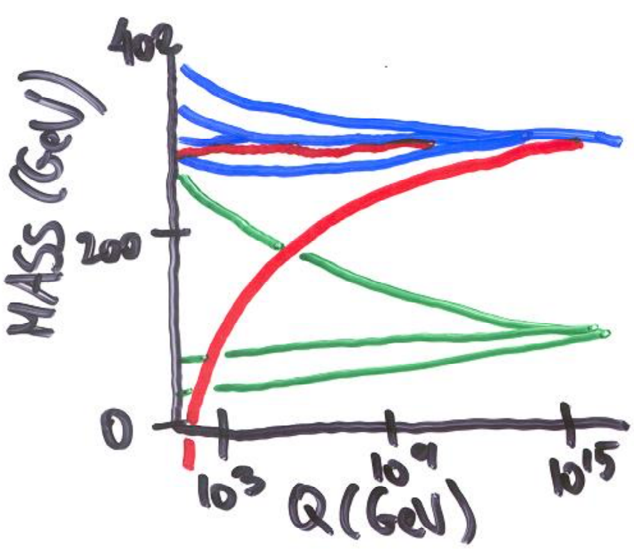
$$+ \Big[M_{1/2} \sum_{i=1,2,3} \lambda_i \lambda_i - B_\mu H_1 H_2$$

$$+ A_0 \Big(\sum_{ij} \lambda_u^{ij} Q^i H_2 \bar{u}^j + \sum_{ij} \lambda_d^{ij} Q^i H_1 \bar{D}^j$$

$$+ \sum_{ij} \lambda_e^{ij} L^i H_1 \bar{E}^j \Big) + h.c. \Big]$$

RUN IT DOWN TO LOW SCALE

MOST IMPORTANT EFFECT FROM TOP YUKAWA COUPLING ($\lambda_t \sim 1$)



ONE HIGGS WILL GET NEGATIVE $(mass)^2$ BY RUNNING

RADIATIVE EWSB!

UNIFICATION OF GAUGE COUPLINGS

- LIKE ALL PARAMETERS IN QFT'S, COUPLINGS ENERGY DEPENDENT
- IF WE ASSUME THEY UNIFY, CAN PREDICT ONE OF 3 COUPLINGS

$$\alpha_s^{SM} \sim 0.07 \qquad \text{BAD}$$

$$\alpha_s^{MSSM} \sim 0.12 \qquad \text{GOOD}$$

- HINTS AT SUSY GUT!

$$SU(5) \supset SU(3) \times SU(2) \times U(1)$$

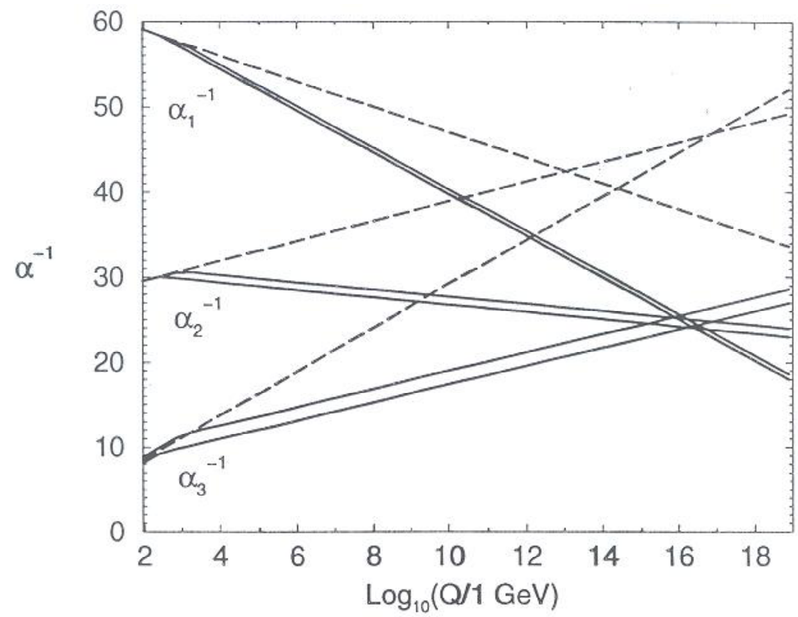
BUT $\left[\begin{array}{c|c} 3 & X,Y \\ \hline X,Y & 2 \end{array}\right]$

X,Y MEDIATE PROTON DECAY.

SUSY: HIGHER UNIF. SCALE $M_X \sim 2 \cdot 10^{16}$ GeV

OK.

UNIFICATION OF GAUGE COUPLINGS IN THE MSSM VS. THE SM



FROM S.MARTIN: A SUSY PRIMER

Figure 13: RG evolution of the inverse gauge couplings $\alpha_a^{-1}(Q)$ in the Standard Model (dashed lines) and the MSSM (solid lines). In the MSSM case, $\alpha_3(m_Z)$ is varied between 0.113 and 0.123, and the sparticle mass thresholds between 250 GeV and 1 TeV. Two-loop effects are included.

UNIFICATION IN MSSM

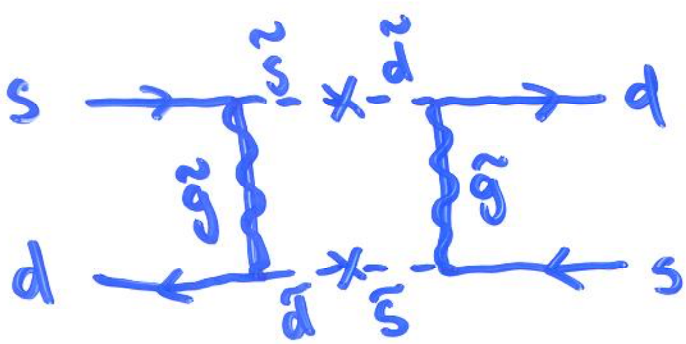
- MUCH BETTER
- AT HIGHER SCALE (GOOD FOR PROTON DECAY)

SUSY FLAVOR PROBLEM

- IN SM ABSENCE OF FCNC's WAS AUTOMATIC

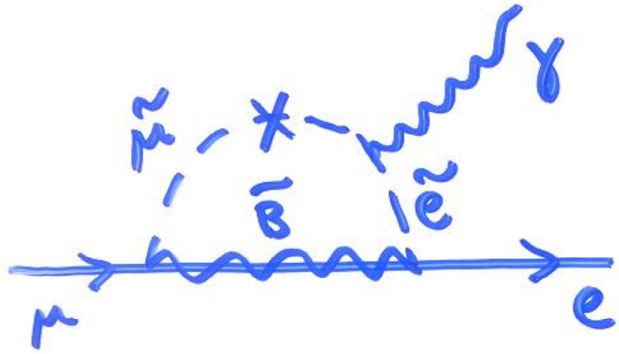
- IN MSSM NOT SO.

ARBITRARY SOFT BREAKING MASSES CAUSE LARGE FCNC's



LARGE IF OFF-DIAGONAL SOFT MASSES

$m_0^2 \propto \mathbb{1}$ IS GOOD



AGAIN LARGE FOR OFF-DIAGONAL MASSES

NEED UNIFICATION OF SOFT PARAMETERS AS WELL

EXPERIMENTAL SIGNATURE

- R-PARITY : LIGHTEST SUPERPARTNER (LSP)

STABLE WILL ESCAPE DETECTION

LOOK LIKE MISSING ENERGY

- USUALLY LSP = "NEUTRALINO"

$$\left(i\tilde{B} \quad i\tilde{W}_3 \quad \tilde{h}_1^0 \quad \tilde{h}_2^0 \right) \begin{bmatrix} -M_1 & 0 & \frac{g' v_1}{\sqrt{2}} & -\frac{g' v_2}{\sqrt{2}} \\ 0 & -M_2 & -g_2 \frac{v_1}{\sqrt{2}} & \frac{g v_2}{\sqrt{2}} \\ \frac{g' v_1}{\sqrt{2}} & -\frac{g v_1}{\sqrt{2}} & 0 & \mu \\ -\frac{g' v_2}{\sqrt{2}} & \frac{g_2 v_2}{\sqrt{2}} & \mu & 0 \end{bmatrix} \begin{bmatrix} i\tilde{B} \\ i\tilde{W}_3 \\ \tilde{h}_1^0 \\ \tilde{h}_2^0 \end{bmatrix}$$

neutral gauginos

neutral higgsinos

SOFT BREAKING MASSES

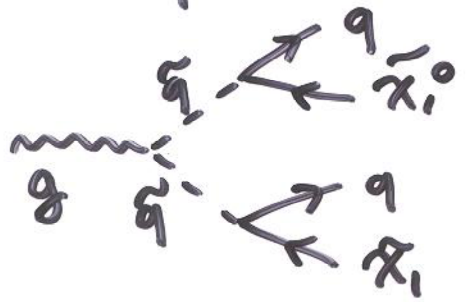
FROM INTERACTION BETWEEN HIGGS, HIGGSINO, GAUGINO

FROM μ-TERM

- FOR MOST OF PARAMETER SPACE
LIGHTEST NEUTRALINO = LSP

IF NLSP SQUARK

$$\tilde{q} \rightarrow q\,\tilde{\chi}_1^0$$



2 jets +
+ missing energy

IF NLSP GLUINO

$$\tilde{g} \rightarrow q\bar{q}\,\tilde{\chi}_1^0$$

2 jets +
missing energy

⋮

$m_{\tilde{g},\tilde{q}} \gtrsim 200\,\mathrm{GeV} - 300\,\mathrm{GeV}$ $\quad$ $m_{\tilde{e},\tilde{\mu},\tilde{\tau}} \gtrsim 80-90\,\mathrm{GeV}$

$m_{\chi_1^\pm} \gtrsim 100\,\mathrm{GeV}$ $\quad$ $m_{\tilde{t}} \gtrsim 90\,\mathrm{GeV}$

$m_{\chi_1^0} \gtrsim 20-50\,\mathrm{GeV}$

THE LSP AS DARK MATTER

- ASSUME STABLE PARTICLE χ (FOR US NEUTRALINO, BUT ARGUMENT GENERAL)

- WHEN $T > m_\chi$

$$l\bar{l} \rightarrow \chi\bar{\chi} \quad \text{and} \quad \chi\bar{\chi} \rightarrow l\bar{l}$$

KEEP χ IN THERMAL EQUILIBRIUM.

- NUMBER DENSITY IN EQUILIBRIUM

# of internal DOF

$$n_\chi^{eq} = \frac{g}{(2\pi)^3} \int f(p)\, d^3p$$

FERM-DIRAC or BOSE-EINST. DISTRIBUTION

- FOR $T \gg m_\chi$

$$n_\chi^{eq} \sim T^3$$ (everything equal, # of χ ~ # of photons)

- FOR $T \ll m_\chi$

$$n_\chi^{eq} \simeq g \left(\frac{m_\chi T}{2\pi}\right)^{3/2} e^{-\frac{m_\chi}{T}}$$

EXPONENTIALLY SUPPRESSED.

IF COOLING HAPPENS WHILE MAINTAINING EQUILIBRIUM $\to$ BASICALLY NO REMAINING DENSITY OF χ's.

- BUT THERMAL EQUILIBRIUM <u>NOT</u> MAINTAINED

WHEN $T \lesssim m_\chi$ JUST BELOW

n_χ DROPS EXPONENTIALLY, AND ANNIHILATION RATE

$$\Gamma = \langle \sigma v \rangle n_\chi < H$$

- NO MORE ANNIHILATION, RELIC ABUND. REMAINS

- FREEZE-OUT OF PARTICLES GOVERNED BY BOLTZMANN EQUATION

$$\frac{dn_\chi}{dt} \underbrace{+3Hn_\chi}_{\text{Hubble expansion}} = -\langle\sigma v\rangle\Big[\underbrace{n_\chi^2}_{\text{depletion by annihilation}} - \underbrace{(n_\chi^{eq})^2}_{\text{creation of } \chi\text{'s}}\Big]$$

IF $\langle\sigma v\rangle = 0$ $\quad n_\chi \propto \frac{1}{a^3}$ AS IT SHOULD, ADDITIONAL TERMS DESCRIBE CREATION & ANNIHILATION OF χ.

A NAIVE ESTIMATE FOR THE RELIC DENSITY

- $H(T) = 1.66\, g_*^{1/2} \dfrac{T^2}{M_{Pl}}$

- $\Gamma(T_f) = H(T_f) \longrightarrow T_f \simeq \dfrac{m_\chi}{20}$

  TYPICAL NUMBER FOR WEAK-SCALE CROSS SECTIONS

- AFTER FREEZE-OUT COMOVING DENSITY ~ CONSTANT, AND SO IS COMOVING ENTROPY.

  $\dfrac{n_\chi}{s} = \text{const}$, $s \sim 0.4\, g_* T^3$

- $$\left(\frac{n_\chi}{s}\right)_0 = \left(\frac{n_\chi}{s}\right)_f = \frac{\dfrac{H(T_f)}{\langle\sigma v\rangle}}{s(T_f)} =$$

$$= \frac{10^{-8}}{\left(\dfrac{m_\chi}{\text{GeV}}\right) \dfrac{\langle\sigma v\rangle}{10^{-27}\,\frac{\text{cm}^3}{\text{s}}}}$$

- ENERGY DENSITY COMPARED TO CRITICAL DENS.:

$$\Omega_\chi h^2 = \frac{m_\chi \cdot n_\chi}{\rho_c} \simeq \frac{3 \cdot 10^{-27} \frac{cm^3}{s}}{\langle \sigma v \rangle}$$

- NOTE: INCREASING $\langle \sigma v \rangle$ ANNIHILATION RATE YIELDS IN SMALLER RELIC DENSITY SINCE FREEZE-OUT HAPPENS LATER

- WHY IS SUSY GOOD?

- INTERACTION STRENGTH ~ WEAK INT'S
- MASS SCALA ~ $\mathcal{O}$(100's GeV)

$$\langle \sigma v \rangle \sim \frac{\alpha^2}{(100\,GeV)^2} \sim 10^{-25} \frac{cm^3}{s}$$

RESULTS OF THE NUMERICAL SOLUTION FOR BOLTZMANN'S EQ.

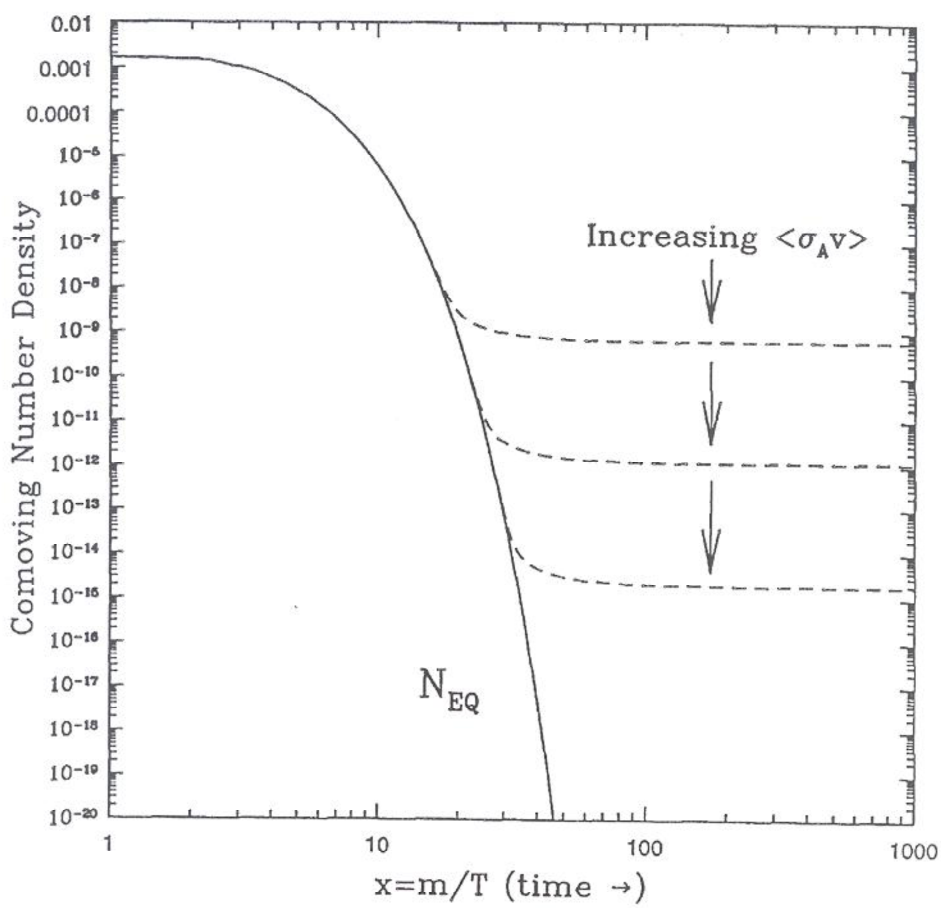


Fig. 4. Comoving number density of a WIMP in the early Universe. The dashed curves are the actual abundance, and the solid curve is the equilibrium abundance. From [31].

FROM KOLB & TURNER

- COULD GIVE $\Omega_X \sim \mathcal{O}(1)$ AS WOULD BE NEEDED FOR DARK MATTER

- GENERICALLY SUCH PARTICLES CALLED **W**EAKLY **I**NTERACTING **M**ASSIVE **P**ARTICLES

- NEED TO DO A DETAILED CALCULATION AS FUNCTION OF m_0, $M_{1/2}$, A, $\tan\beta$ TO FIND REGION OF MSSM THAT HAS RIGHT AMOUNT OF CDM.

  A MINI-INDUSTRY
  (ELLIS, OLIVE et al. ...)

SUMMARY OF LECTURE 2.

- SUPERSYMMETRY:
  - + ELEGANT SOLUTION TO HIERARCHY PROBLEM
  - + BIGGEST POSSIBLE SPACE-TIME SYMMETRY

- THE MSSM
  - + LIGHT HIGGS
  - + UNIFICATION OF COUPLINGS
  - + DARK MATTER
  - − SUSY FLAVOR PROBLEM
  - − PROTON STABILITY SOMEWHAT AD HOC