Gravitational radiation reaction in compact binary systems:

Quadrupole-monopol and magnetic dipole-magnetic dipole interaction

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Dynamics:

Lagrangian:

$$L = L_N + L_{QM} + L_{DD},$$

$$L_N = \frac{\mu v^2}{2} + \frac{Gm\mu}{r},$$

$$L_{QM} = \frac{G\mu m^3}{2r^5}\sum_{i=1}^{2} p_i\left[3\left(\hat{\mathbf{S}}_i \cdot \mathbf{r}\right)^2 - r^2\right],$$

$$L_{DD} = \frac{1}{r^3}[3(\mathbf{n}\cdot\mathbf{d}_1)(\mathbf{n}\cdot\mathbf{d}_2) - \mathbf{d}_1\cdot\mathbf{d}_2],$$

p_i **quadrupole scalar (axial symmetry)**

d_i **magnetic dipole vector**

Identities:

$$v^2 = \frac{2E_N}{\mu} + \frac{2Gm}{r},$$

$$\dot{r}^2 = \frac{2E_N}{\mu} + \frac{2Gm}{r} - \frac{L_N^2}{\mu^2 r^2}.$$

Solution of the radial equation

Defination of turning points:

$$\boxed{\begin{array}{ll} r_{min} = r(0)\,, & \dot{r}^2(0) = 0\,, \\ \hline r_{max} = r(\pi)\,, & \dot{r}^2(\pi) = 0\,. \end{array}}$$

Defination of χ :

$$\boxed{\frac{2}{r} = \left(\frac{1}{r_{min}} + \frac{1}{r_{max}}\right) + \left(\frac{1}{r_{min}} - \frac{1}{r_{max}}\right)\cos\chi\,,}$$

$$\boxed{r = \frac{\bar{L}^2}{\mu(Gm\mu + \bar{A}\cos\chi)} + \frac{G\mu^2 m^3}{4\bar{A}\bar{L}^2(Gm\mu + \bar{A}\cos\chi)^2}\sum_{i=1}^{2} p_i\Lambda^i + \frac{\mu d_1 d_2 \Lambda}{2\bar{A}\bar{L}^2(Gm\mu + \bar{A}\cos\chi)^2}\,,}$$

where:

$$\boxed{\begin{array}{l} \Lambda^i = \bar{A}[\bar{A}^2(\alpha_0^i + 4\beta_0^i) + (Gm\mu)^2(3\alpha_0^i + 10\beta_0^i)] + Gm\mu[\bar{A}^2(3\alpha_0^i + 11\beta_0^i) + (Gm\mu)^2(\alpha_0^i + 3\beta_0^i)]\cos\chi\,, \\ \hline \Lambda = \bar{A}\left[(3G^2m^2\mu^2 + \bar{A}^2)\mathrm{A}_0 + (G^2m^2\mu^2 + \bar{A}^2)\mathrm{B}_0\right] + Gm\mu\left[(G^2m^2\mu^2 + 3\bar{A}^2)\mathrm{A}_0 + 2\bar{A}^2\mathrm{B}_0\right]\cos\chi\,, \end{array}}$$

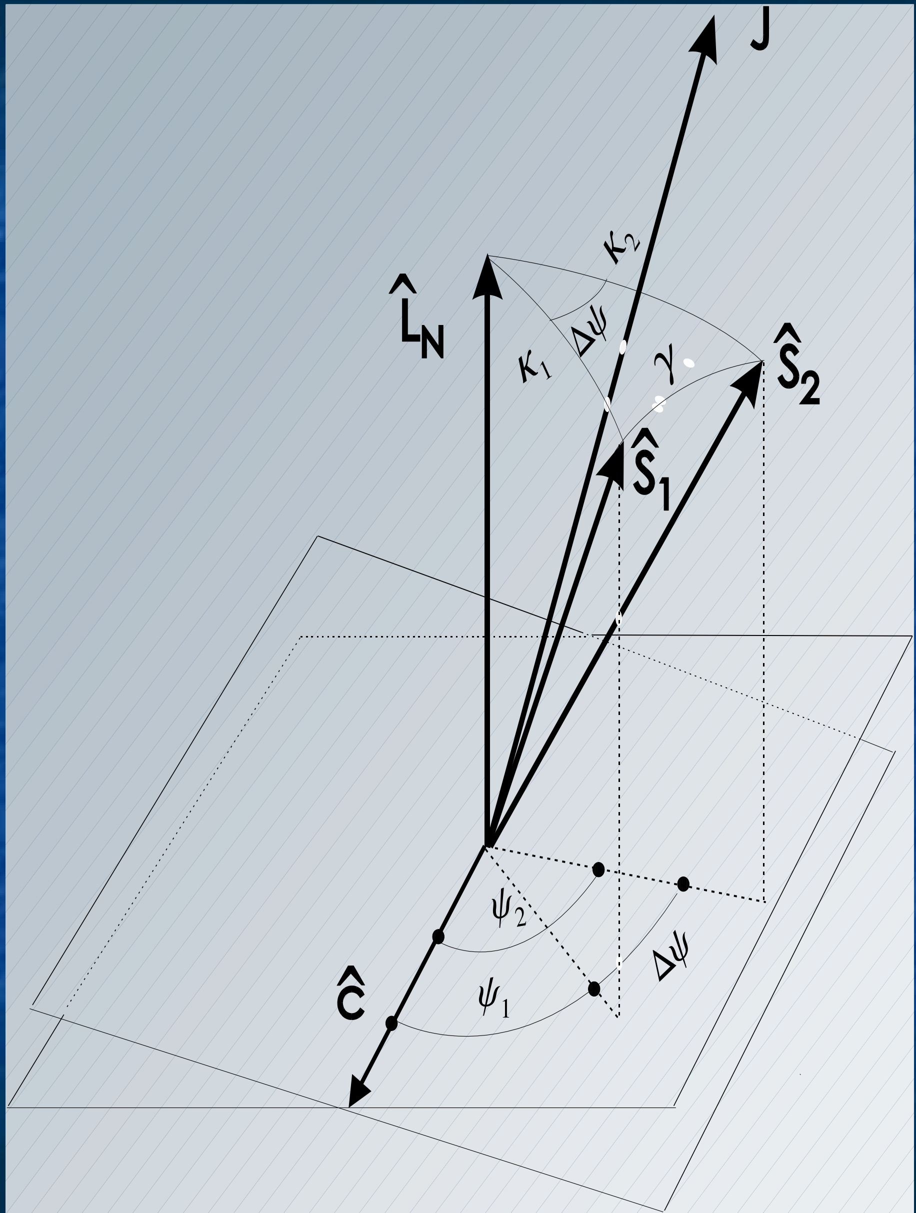
Defination of ξ :

$$\boxed{2r = (r_{max} + r_{min}) - (r_{max} - r_{min})\cos\xi\,,}$$

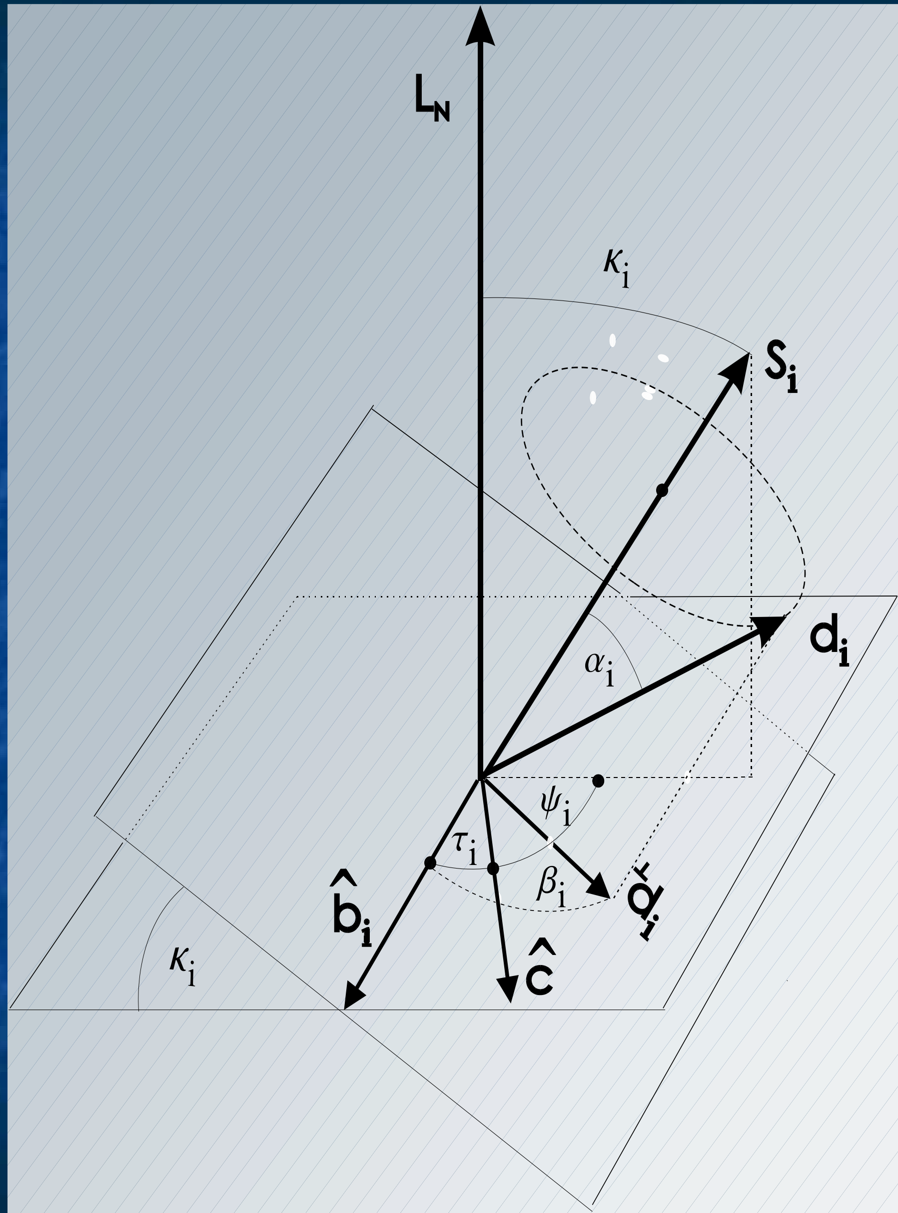
$$\boxed{r = \frac{Gm\mu - \bar{A}\cos\xi}{-2E} + \frac{G\mu^2 m^3}{4\bar{A}\bar{L}^2}\sum_{i=1}^{2} p_i\Xi^i + \frac{\mu d_1 d_2}{2\bar{A}\bar{L}^2}\left\langle \bar{A}(\mathrm{A}_0 + \mathrm{B}_0) + Gm\mu\mathrm{A}_0\cos\xi \right\rangle\,,}$$

where:

$$\boxed{\Xi^i = \bar{A}(\alpha_0^i + 4\beta_0^i) + Gm\mu(\alpha_0^i + 3\beta_0^i)\cos\xi\,.}$$



J
$\hat{L}_N$
κ_2
κ_1
$\Delta\psi$
γ
$\hat{S}_2$
$\hat{S}_1$
ψ_2
ψ_1
$\Delta\psi$
$\hat{c}$



L_N
κ_i
S_i
d_i
α_i
ψ_i
τ_i
β_i
$d_i^{\perp}$
$\hat{b}_i$
$\hat{c}$
κ_i

Averaged radiative losses

$$\left\langle \frac{dE}{dt} \right\rangle = \left\langle \frac{dE}{dt} \right\rangle_N + \left\langle \frac{dE}{dt} \right\rangle_{QM} + \left\langle \frac{dE}{dt} \right\rangle_{DD},$$

$$\left\langle \frac{dE}{dt} \right\rangle_N = -\frac{G^2 m(-2E\mu)^{3/2}}{15c^5\bar{L}^7}(148E^2\bar{L}^4 + 732G^2m^2\mu^3E\bar{L}^2 + 425G^4m^4\mu^6),$$

$$\left\langle \frac{dE}{dt} \right\rangle_{QM} = \frac{G^2\mu m^3(-2E\mu)^{3/2}}{30c^5\bar{L}^{11}}\sum_{i=1}^{2} p_i[C_1 \sin^2\kappa_i \cos 2\delta_1 + C_2(2 - 3\sin^2\kappa_i)],$$

$$\left\langle \frac{dE}{dt} \right\rangle_{DD} = \frac{G^2 d_1 d_2(-2E\mu)^{3/2}}{15c^5\bar{L}^{11}}(C_1 \mathrm{B}_0 + C_2 \mathrm{A}_0),$$

C_k coefficients:

$$C_1 = -\mu\bar{A}^2(948E^2\bar{L}^4 + 8936G^2E\bar{L}^2m^2\mu^3 + 8335G^4m^4\mu^6),$$

$$C_2 = 708E^3\bar{L}^6 + 10020G^2E^2\bar{L}^4m^2\mu^3 + 18865G^4E\bar{L}^2m^4\mu^6 + 8316G^6m^6\mu^9.$$

$$\left\langle \frac{dL}{dt} \right\rangle = \left\langle \frac{dL}{dt} \right\rangle_N + \left\langle \frac{dL}{dt} \right\rangle_{QM} + \left\langle \frac{dL}{dt} \right\rangle_{DD},$$

$$\left\langle \frac{dL}{dt} \right\rangle_N = -\frac{4G^2 m(-2\mu E)^{3/2}}{5c^5\bar{L}^4}(14E\bar{L}^2 + 15G^2m^2\mu^3),$$

$$\left\langle \frac{dL}{dt} \right\rangle_{QM} = \frac{G^2 m^3\mu(-2\mu E)^{3/2}}{10c^5\bar{L}^8}\sum_{i=1}^{2} p_i[D_1 \sin^2\kappa_i \cos 2\delta_i + D_2(2 - 3\sin^2\kappa_i)],$$

$$\left\langle \frac{dL}{dt} \right\rangle_{DD} = \frac{G^2 d_1 d_2(-2E\mu)^{3/2}}{5c^5\bar{L}^8}[D_1 \mathrm{B}_0 + D_2 \mathrm{A}_0],$$

D_k coefficients:

$$D_1 = -6\mu\bar{A}^2(31E\bar{L}^2 + 90G^2m^2\mu^3),$$

$$D_2 = 252E^2\bar{L}^4 + 1200G^2E\bar{L}^2m^2\mu^3 + 805G^4m^4\mu^6.$$

$$\left\langle \frac{d\gamma}{dt} \right\rangle = 0 \quad ,$$

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$$\left\langle \frac{d\kappa_i}{dt} \right\rangle = \left\langle \frac{d\kappa_i}{dt} \right\rangle_{QM} + \left\langle \frac{d\kappa_i}{dt} \right\rangle_{DD} \quad ,$$

$$\left\langle \frac{d\kappa_i}{dt} \right\rangle_{QM} = \frac{3G^2 m^3 \mu (-2\mu E)^{3/2}}{10 c^5 \bar{L}^9} \sum_{j=1}^{2} p_j \sin 2\kappa_j [V_1 \cos(\delta_i + \delta_j) + V_2 \cos(\delta_i - \delta_j)] \, ,$$

$$\left\langle \frac{d\kappa_i}{dt} \right\rangle_{DD} = \frac{3G d_1 d_2 (-2E\mu)^{3/2}}{5 c^5 \bar{L}^9} \sum_{j=1}^{2} \mu_{3-j|z} \{ V_1[\sigma_j \cos(\delta_i + \delta_j) - \rho_j \sin(\delta_i + \delta_j)] + V_2[\sigma_j \cos(\delta_i - \delta_j) + \rho_j \sin(\delta_i - \delta_j)] \} \, ,$$

V_{1-2} coefficients:

$$V_1 = 40E^2 \bar{L}^4 + 90 G^2 m^2 \mu^3 E \bar{L}^2 + 35 G^4 m^4 \mu^6 \quad ,$$

$$V_2 = 48E^2 \bar{L}^4 + 140 G^2 m^2 \mu^3 E \bar{L}^2 + 70 G^4 m^4 \mu^6 \, .$$

$$\left\langle \frac{d \cos\beta_i}{dt} \right\rangle_{DD} = \frac{G d_1 d_2 (-2E\mu)^{3/2}}{10 c^5 \bar{L}^9} \{ (6\rho_i Z_1 - 2\cos\beta_i D_1) B_0 + (\rho_i Z_2 - 2\cos\beta_i D_2) A_0 - \frac{6}{\sin\kappa_i} \sum_{j=1}^{2} \zeta_{3-j} \times$$

$$\{ V_1 [(\rho_i \sigma_j \cos\kappa_i - \sigma_j \cos\kappa_i \cos\beta_i + \rho_j \nu_i) \cos(\delta_i + \delta_j) + (-\rho_i \rho_j \cos\kappa_i + \rho_j \cos\kappa_i \cos\beta_i + \sigma_j \nu_i) \sin(\delta_i + \delta_j)]$$

$$+ V_2 [(\rho_i \sigma_j \cos\kappa_i - \sigma_j \cos\kappa_i \cos\beta_i - \rho_j \nu_i) \cos(\delta_i - \delta_j) + (\rho_i \rho_j \cos\kappa_i - \rho_j \cos\kappa_i \cos\beta_i + \sigma_j \nu_i) \sin(\delta_i - \delta_j)] \} \} \, ,$$

where:

$$Z_1 = 96E^2 \bar{L}^4 + 314 E G^2 \bar{L}^2 m^2 \mu^3 + 133 G^4 m^4 \mu^6 \, ,$$

$$Z_2 = 396E^2 \bar{L}^4 + 1380 E G^2 \bar{L}^2 m^2 \mu^3 + 735 G^4 m^4 \mu^6 \, .$$

Algebraic relations:

$$S_1 \sin\kappa_1 \cos\psi_1 + S_2 \sin\kappa_2 \cos\psi_2 = 0 \, ,$$

$$\cos\gamma = \cos\kappa_1 \cos\kappa_2 + \cos(\psi_2 - \psi_1) \sin\kappa_1 \sin\kappa_2 \, ,$$

$$\cos\lambda = (\rho_2 \sigma_1 - \rho_1 \sigma_2) \sin(\delta_1 - \delta_2) + (\rho_1 \rho_2 + \sigma_1 \sigma_2) \cos(\delta_1 - \delta_2) + \zeta_1 \zeta_2 \, .$$

REMARK:

***Circular orbit* limit**

of eccentric ENERGY LOSS agrees with earlier results:

(QM) Poisson, Phys Rev D **57**, 5287 (1998)

(Md - Md) Ioka & Taniguchi, Astrophys. J. **537**, 327 (2000)

FUTURE PROSPECTS:

- 2.5PN order 1th correction to SO
- Extension to angular motion (non-conservation of **L** requires a generalized Damour-Deruelle formalism)
- Orbital and signal frequency evolution for eccentric orbits