HTL quasiparticle models of deconfined QCD at finite chemical potential

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Contents

Introduction

- Assuming an SU(N) plasma of gluons and N_f light quarks in thermodynamic equilibrium can be described as a weakly interacting gas of massive quasiparticles with residual interaction B, the pressure of the system is given by

$$P(T,\mu) = \sum_{i=g,q} p_i(T,\mu_i,m_i^2) - B(m_g,m_q) \qquad (1)$$

- Using the stationarity of the thermodynamic potential under variation of the self-energies and Maxwell's relations, $\mathrm{d}/\mathrm{d}\mu - \mathrm{d}\mathcal{N}/\mathrm{d}T = 0$, one obtains a partial differential equation for G^2,

$$a_T \frac{\partial G^2}{\partial T} + a_\mu \frac{\partial G^2}{\partial \mu} = b \qquad (2)$$

- Given a valid boundary condition (e.g. $G^2(T,0)$), a solution for $G^2(T,\mu)$ is found by solving the above flow equation by the method of characteristics. Once G^2 is thus known in the T,μ plane, the pressure is fixed completely up to a constant B_0

- In practice, as boundary condition for G^2 we use the *ansatz*

$$G^2(T,0) = \frac{24\pi^2}{(11N - 2N_f)\ln \frac{T+T_s}{T_c}\lambda} \qquad (3)$$

and fit T_s, λ and B_0 from lattice data

HTL quasiparticle model

- The HTL model ansatz for the QP pressure for gluons (p_g) and quarks p_q reads

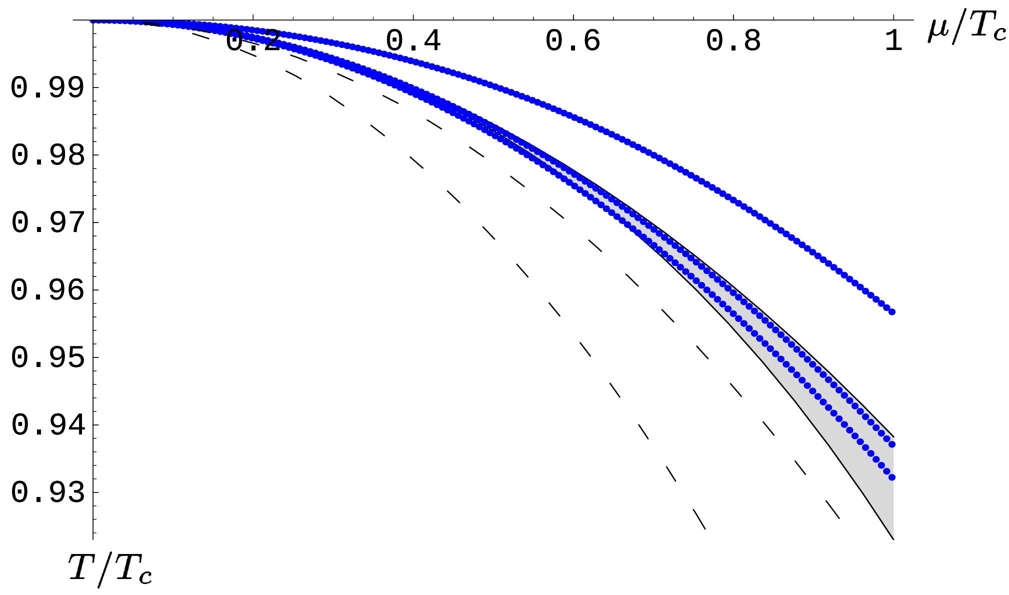
$$
\begin{aligned}
p_g &= -d_g \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} n(\omega) \Big[2\mathrm{Im}\ln\left(-\omega^2 + \mathrm{k}^2 + \hat{\Pi}_\mathrm{T}\right) \\
&\quad -2\mathrm{Im}\hat{\Pi}_\mathrm{T}\mathrm{Re}\hat{\mathrm{D}}_\mathrm{T} + \mathrm{Im}\ln\left(\mathrm{k}^2 + \hat{\Pi}_\mathrm{L}\right) + \mathrm{Im}\hat{\Pi}_\mathrm{L}\mathrm{Re}\hat{\mathrm{D}}_\mathrm{L} \Big] \\
p_q &= -d_q \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} \tilde{f}(\omega) \Big[\mathrm{Im}\ln\left(\mathrm{k} - \omega + \hat{\Sigma}_+\right) \\
&\quad -\mathrm{Im}\hat{\Sigma}_+\mathrm{Re}\hat{\Delta}_+ + \mathrm{Im}\ln\left(\mathrm{k} + \omega + \hat{\Sigma}_-\right) + \mathrm{Im}\hat{\Sigma}_-\mathrm{Re}\hat{\Delta}_- \Big],
\end{aligned}
$$

  where $d_g = 2(N^2-1)$, $d_q = 2NN_f$ and $\hat{D}_{T,L}$, $\hat{\Delta}_\pm$ are the HTL propagators with $\hat{\Pi}_{T,L}$ and $\hat{\Sigma}_\pm$ the corresponding self-energies

- NLO extension to the HTL model includes the averaged $O(g^3)$ corrections to the self-energies in the hard momentum regime ($k \gg T$). Advantage: Full inclusion of the plasmon effect. Drawback: extra parameter $c_\Lambda \sim 1$

Results: small chemical potential

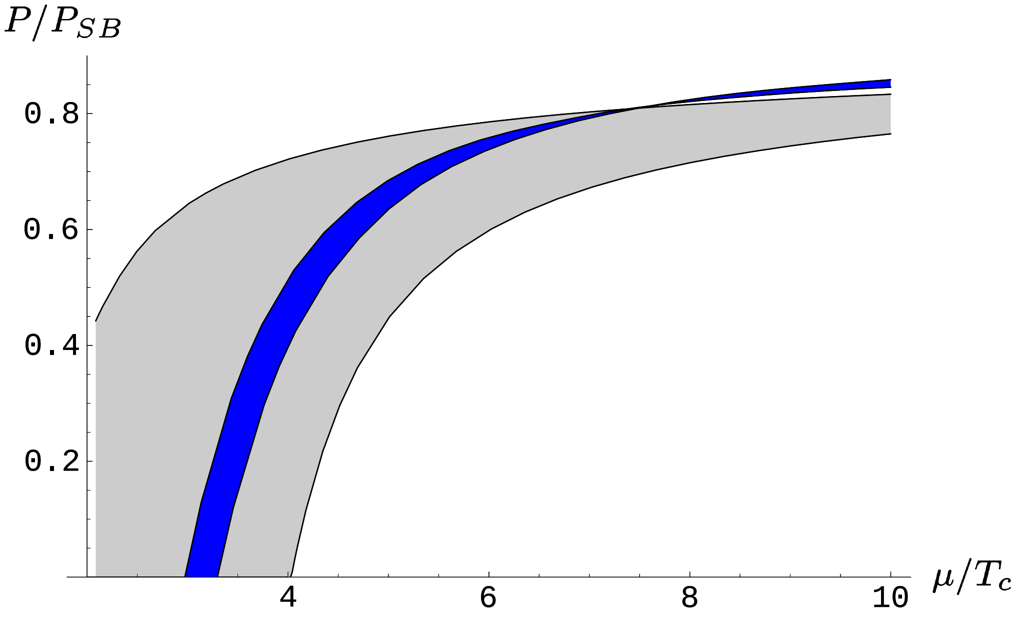
Using the approach outlined above one obtains the equation of state of deconfined QCD at arbitrary chemical potential. For small μ, these can be compared to recent lattice data, e.g. the lines of constant pressure,
$P(T,\mu) = P(T_c(\mu=0),0)$:



Shown are the results for our model for $N_f = 2$ (blue dotted lines corresponding to $c_\Lambda = 4, 1, 0.25$), lattice data for $N_f = 2+1$ [6] (light gray band) and $N_f = 2$ [5] (long dashed-lines).

Results: large chemical potential

Extending the lines of constant pressure from our models to very small temperatures we obtain a crude estimate of the phase transition line in this region of phase space.



Shown are the perturbative pressure (gray band) and our model for c_Λ from 1/4 to 4 (blue band).

By additionally calculating the number density we obtain an equation of state for cold deconfined matter which can be used to determine the mass-radius relations of non-rotating quark-stars.

Conclusions

- Using quasiparticle models and thermodynamic consistency, lattice data for the equation of state of deconfined QCD can be mapped to finite chemical potential
- The results are in agreement with recent lattice calculations which were done at small chemical potential
- One obtains an equation of state for deconfined QCD matter at arbitrary chemical potential (notably for cold dense)
- To eliminate c_Λ one would need to calculate the full momentum-dependent self-energies at $O(g^3)$, which is work in progress

References

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