RECENT PROGRESS IN QCD AT HIGH T AND μ

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- THE MANY PHASES OF QCD
- THE FALL AND RISE OF PERTURBATIVE METHODS
- EXCURSION TO FINITE μ_q

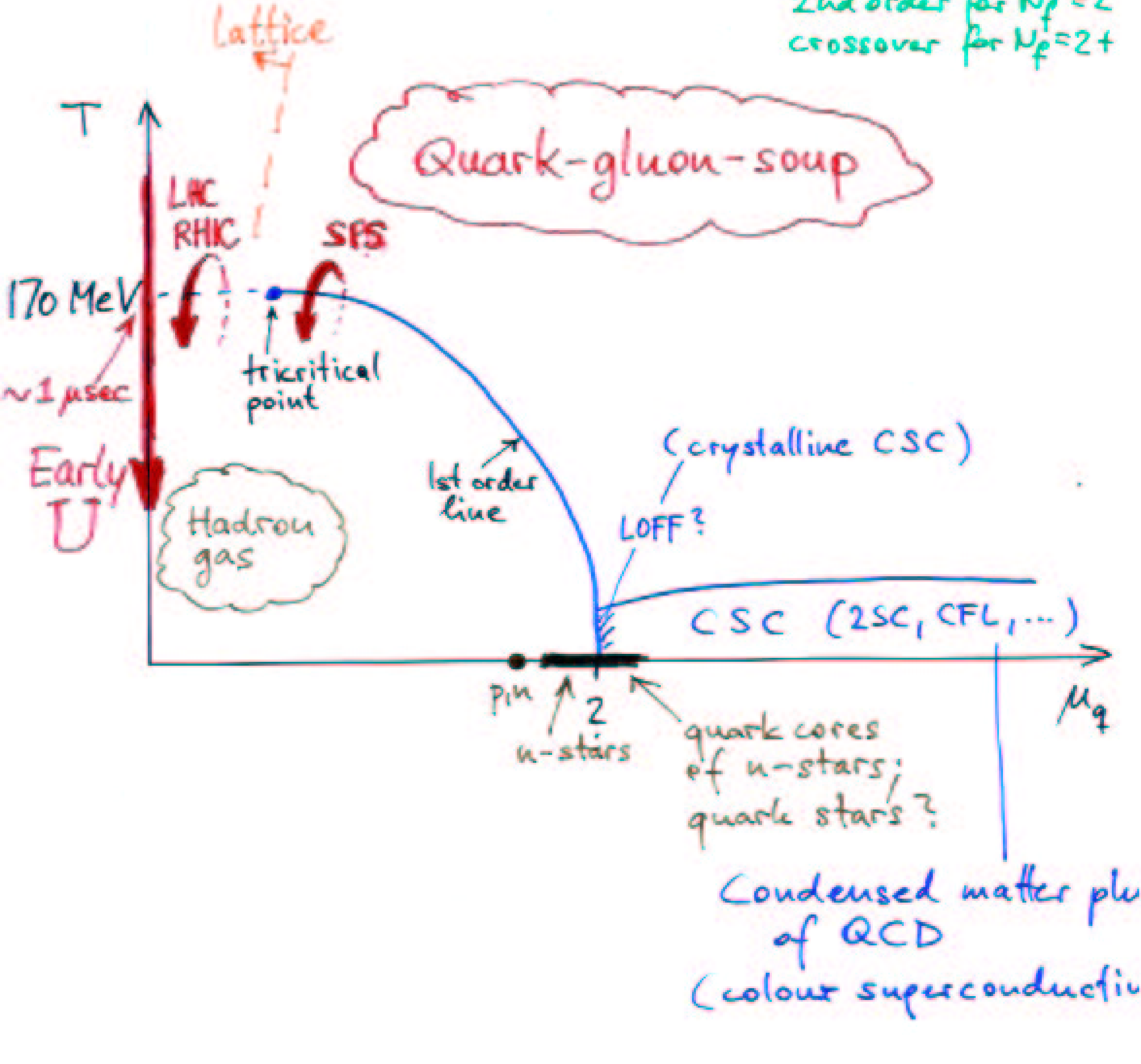
collab's: J.-P. BLAIZOT, E. IANCU — SACLAY

A. IPP, P. ROMATSCHKE — VIENNA

Phases of QCD

Early U @ $T \approx 170$ MeV, $\mu_q \approx 0$:
rapid crossover*) from quark-gluon-plasma
to hadron gas (π, p, n)

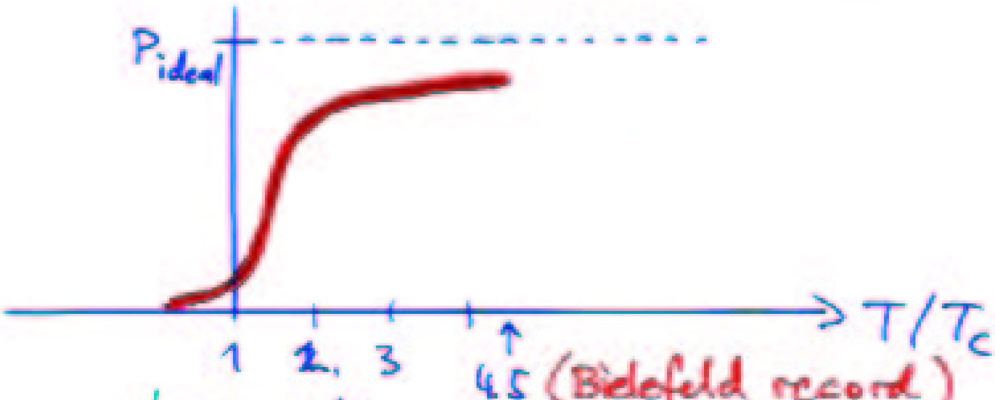
*) lattice:
1st order for $N_f = 0$
2nd order for $N_f = 2$
crossover for $N_f = 2+$



Lattice gauge theory calculations successfully describe phase transition / crossover to quark-gluon plasma

best: $N_f = 0$

$\rightarrow$ high-precision, continuum-extrapolated data on thermodynamical potentials ($P = -\mathcal{F}$)



only accessible by nonperturbative methods (lattice)

$\rightarrow$ accessible by perturbative QCD ?

running α_s:

renormalization scale, $\bar{\mu} \propto T$ to avoid large logarithms

$$\bar{\mu} \frac{d\alpha_s(\bar{\mu})}{d\bar{\mu}} = \beta(\alpha_s) < 0$$

$$\Downarrow$$

$$\alpha_s(\bar{\mu}) \propto \frac{1}{\ln \bar{\mu}/\Lambda_{QCD}}$$

$\Lambda_{QCD} \sim T_c$

$\underbrace{2\pi T}$ $\gtrsim$ GeV when $\bar{\mu} \gtrsim T_c$ $\Rightarrow \alpha_s \lesssim 0.1$

(lowest Matsubara frequency)

HALL OF FAME

Perturbative calculations of thermodynamic potential of QCD at high T/μ :

\# of gluons · \# of quark flavours

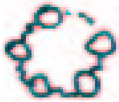
$$P = \frac{8\pi^2}{45}T^4\Big\{ 1 + \frac{21}{32}N_f$$

PLANCK 1900

$$- \frac{15}{4}\left(1+\frac{5}{12}N_f\right)\frac{\alpha_s}{\pi}$$

SHURYAK / CHIN 1978

$$+ 30\left(1+\frac{N_f}{6}\right)^{\frac{3}{2}}\left(\frac{\alpha_s}{\pi}\right)^{\frac{3}{2}}$$



KAPUSTA 1979

$$+ \frac{135}{2}\left(1+\frac{N_f}{6}\right)\ln\left[\frac{\alpha_s}{\pi}\left(1+\frac{N_f}{6}\right)\right]\cdot\left(\frac{\alpha_s}{\pi}\right)^2$$

TOIMELA 1983

$$+ \Big\{237.2 + 15.97\,N_f - 0.413\,N_f^2 - \frac{165}{8}\left(1+\frac{5}{12}N_f\right)\left(1-\frac{2}{33}N_f\right)\ln\frac{\bar\mu}{2\pi T}\Big\}\left(\frac{\alpha_s}{\pi}\right)^2$$

ARNOLD & ZHAI 1995

$$+ \left(1+\frac{N_f}{6}\right)^{\frac{1}{2}}\Big\{-799.2 - 21.96\,N_f - 1.93\,N_f^2 + \frac{495}{2}\left(1+\frac{N_f}{6}\right)\left(1-\frac{2}{33}N_f\right)\ln\frac{\bar\mu}{2\pi T}\Big\}\left(\frac{\alpha_s}{\pi}\right)^{\frac{5}{2}}$$

ZHAI & KASTENING 1995

BRAATEN & NIETO 1996

$$+ \Big\{1134.8 + 65.89\,N_f + 7.653\,N_f^2 - \frac{1485}{2}\left(1+\frac{N_f}{6}\right)\left(1-\frac{2}{33}N_f\right)\ln\frac{\bar\mu}{2\pi T}\Big\}\ln\frac{\pi}{\alpha_s}\left(\frac{\alpha_s}{\pi}\right)^3$$

KAJANTIE, LAINE, RUMMUKAINEN & SCHRÖDER 2002

$\mu_q \neq 0$: VUORINEN 2003

$$+ \Big\{ ?! + \boxed{?}\,N_f + \boxed{?}\,N_f^2 + C(\bar\mu)\,N_f^3\Big\}\left(\frac{\alpha_s}{\pi}\right)^3 + O\left(\frac{\alpha_s}{\pi}\right)^{7/2}\Big\}$$

$0.040\ldots$ @ $\bar\mu = \pi T$ — A. IPP & AR... 2003

 + ∞'ly many more

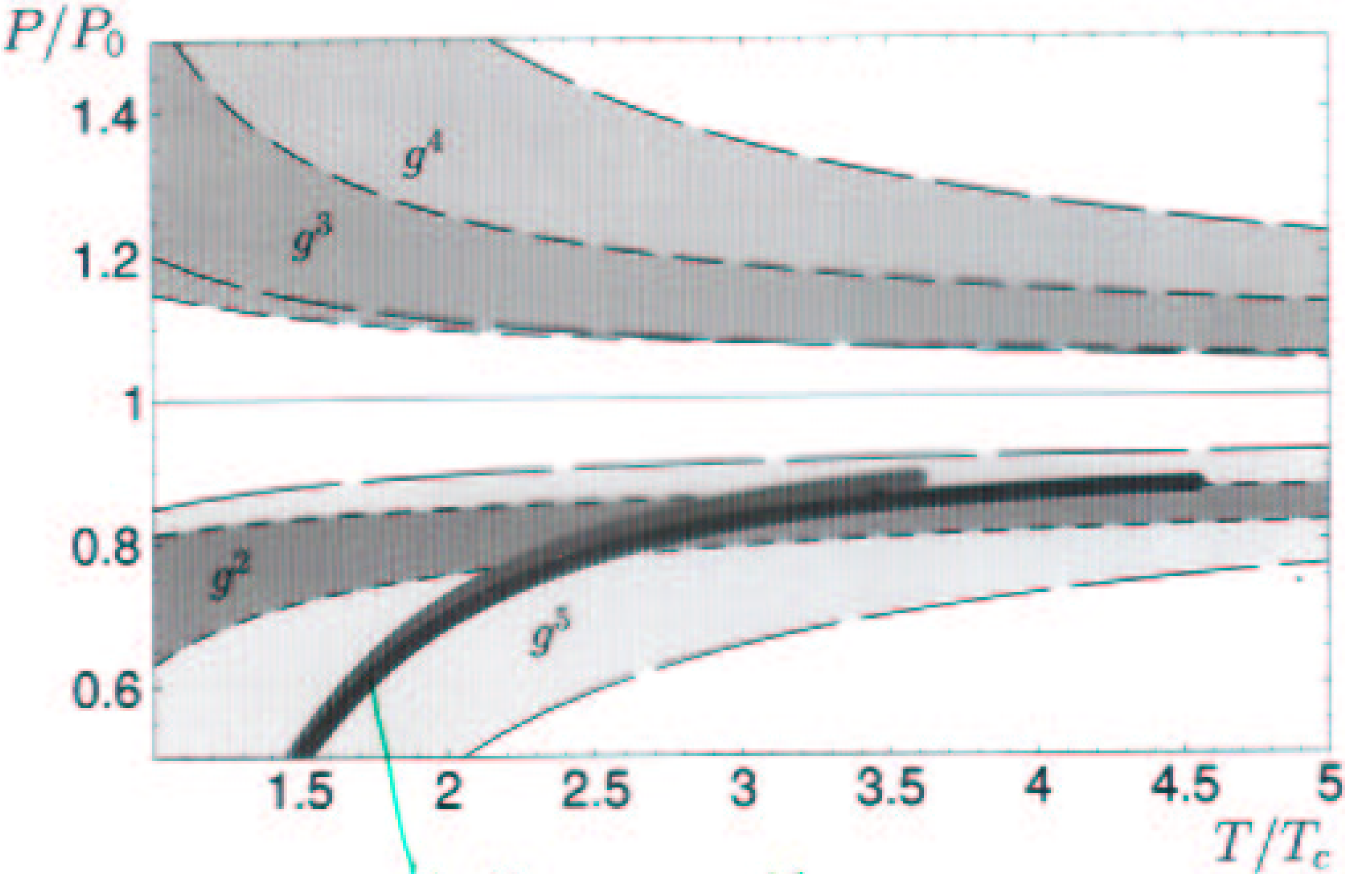
completely non-perturbative (LINDE 1980, GROSS, PISARSKI & YAFFE 1980)

ALAS!

Only order-g^2 result reasonable

Large $\bar{\mu}$ ($= \pi T \dots 4\pi T$) dependence
at order g^3 and beyond,
which even increases with loop order!

Need $T > 10^5 T_c$ in order that result stops oscillating around P_0, but then $P - P_0 \ll P_0$



Lattice results
(Boyd et al., 1996; Okamoto et al., 1999)

$\longrightarrow$ Problem for any perturbative treatment at all T of interest

BUT Lattice only good for static quantities @

"Numerological" attempts to improve convergence:

Padé approximants, Borel transforms (Hatsuda, Kastening; Parwani; ...)

More promising:

SCREENED PERTURBATION THEORY

→ optimized/variational perturbation theory with screening mass as variational parameter ← PMS

(Karsch, Patkós, Petrecky '97; Chiku, Hatsuda '98; Andersen, Braaten, Strickland '01) ← WORKS GREAT

e.g. scalar φ^4:

$$\mathcal{L} = \underbrace{\mathcal{L}_0 - \tfrac{1}{2}m^2\varphi^2}_{\mathcal{L}_0'} + \underbrace{\mathcal{L}_{int} + \tfrac{1}{2}m^2\varphi^2}_{\mathcal{L}_{int}'}$$

analogously for QCD: $-\frac{1}{2}m^2\varphi^2 \longrightarrow \mathcal{L}_{\underline{HTL}} \propto m_D^2 \to m^2_{variational}$

"HTLpt"

1-loop: ABS, PRL83(1999), ...

2-loop: ABS+Petitgirard, hep-ph/0205085

↳ variational ✓ $\frac{\partial F}{\partial m^2} = 0$ w/ solutions, $\neq m_{pe}^2, m_D^2$

in contrast to old/standard HTL pert. th.:

- resummation of HTL's not only at soft momenta $\sim gT$, but throughout

– HTL's no longer L.O. for $\omega, k \sim T$ in general

→ additional UV subtractions and additional RS dependencies at any finite order

↑ suppressed by powers of g^2 only @ ≥ 3 loop order recap:

in contrast to LO thermal mass of φ^4-scalar, in gauge th.:

HTL quasiparticles have

momentum-dependent masses given by

poles of gluon/quark (photon/electron) propagators:

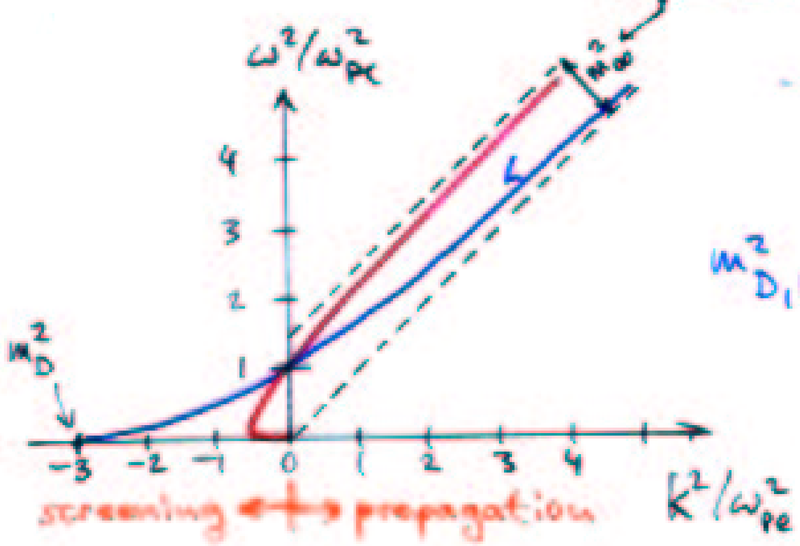
e.g. gauge bosons:

Landau damping cut

2 branches:

$$(L) \quad \omega^2 - k^2 = m^2_{D,LO}\left(1 - \frac{\omega}{2k}\log\frac{\omega+k}{\omega-k}\right) \equiv \hat{\Pi}_L$$

$$(T) \quad \omega^2 - k^2 = \hat{\Pi}_T = \underbrace{\frac{1}{2} m^2_{D,LO}}_{m^2_\infty} + \frac{\omega^2-k^2}{2k^2}\hat{\Pi}_L$$

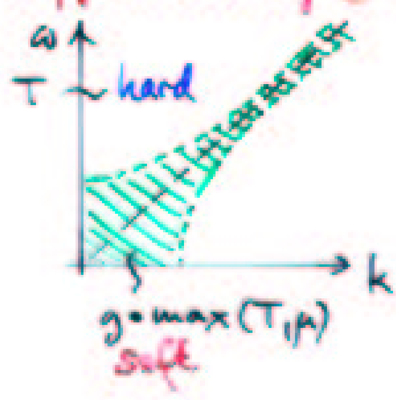


$$\omega^2_{pe,LO} = \frac{1}{3} m^2_{D,LO}$$

$$m^2_{D,LO} = \left(N + \frac{N_f}{2}\right)\frac{g^2T^2}{3} + \sum_f \frac{g^2\mu_f^2}{2\pi^2}$$

NLO corrections in HTL pert. th.

applicability (Gelis



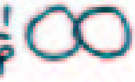
$\delta\omega^2_{pe}/\omega^2_{pe} \approx -0.18\sqrt{N}\, g$ (Schulz '93)

$\delta m^2_D / m^2_D = +(\sqrt{3N}/2\pi)\, g \log\frac{c}{g}$ (AR '93)

$\delta m^2_\infty / m^2_\infty < 0$, momentum-dependent!

Alternative HTL resummation (BLAIZOT, IANCU, AR '99)
based on 2PI formalism (Φ-derivable approx.):
(Luttinger + Ward, Baym 1962)

$$\mathcal{F}[D] = \tfrac{1}{2}\,\mathrm{Tr}\,\ln D^{-1} - \tfrac{1}{2}\,\mathrm{Tr}\,\Pi D + \underbrace{\Phi[D]}$$

$-\frac{1}{12}$  $-\frac{1}{8}$  $-\frac{1}{48}$  ...

(2PI)

$$\frac{\delta \mathcal{F}[D]}{\delta D} = 0 \;\rightarrow\; \tfrac{1}{2}\Pi[D] = \frac{\delta \Phi}{\delta D}$$

2-loop truncated Φ, self-consistently resummed

$\rightarrow$ simple entropy expression
can be used for HTL resummation
without having to correct for
overcounting through thermal counterterms

$$\frac{S^{2\text{-loop}}}{V} = -\int\frac{d^4k}{(2\pi)^4}\frac{\partial n}{\partial T}\,\mathrm{Im}\log D^{-1} + \int\frac{d^4k}{(2\pi)^4}\frac{\partial n}{\partial T}\,\mathrm{Im}\,\Pi\cdot\mathrm{Re}\,D$$

* describes quark-gluon plasma in terms
of weakly interacting quasi-particles

LO: HTL quasiparticles

NLO: corrections to HTL dispersion relations
calculable through standard HTL p.th.

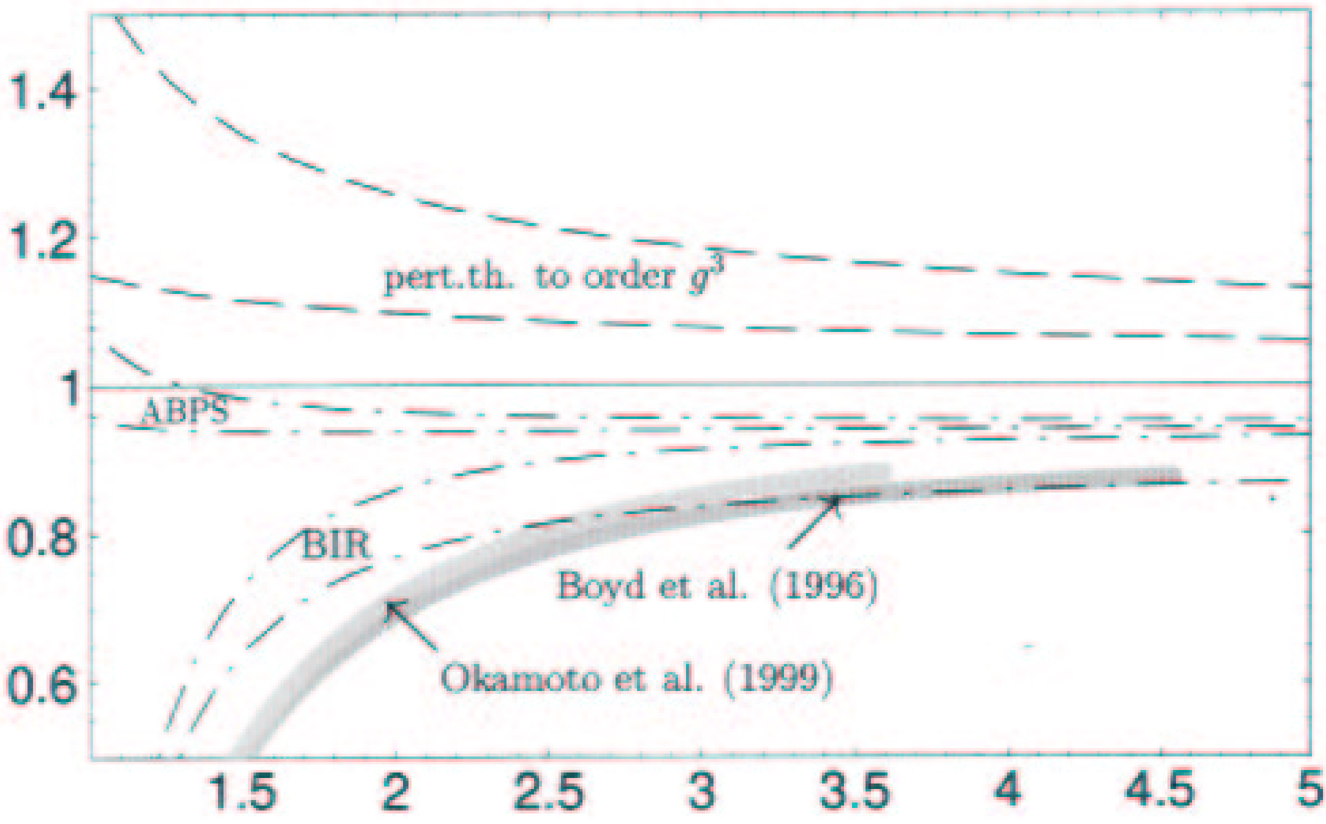
(improvement over older "simple" quasiparticle
models using constant masses given by m_∞)

2-loop HTL-resummed results

BIR: Blaizot, Iancu + A.R. 1999

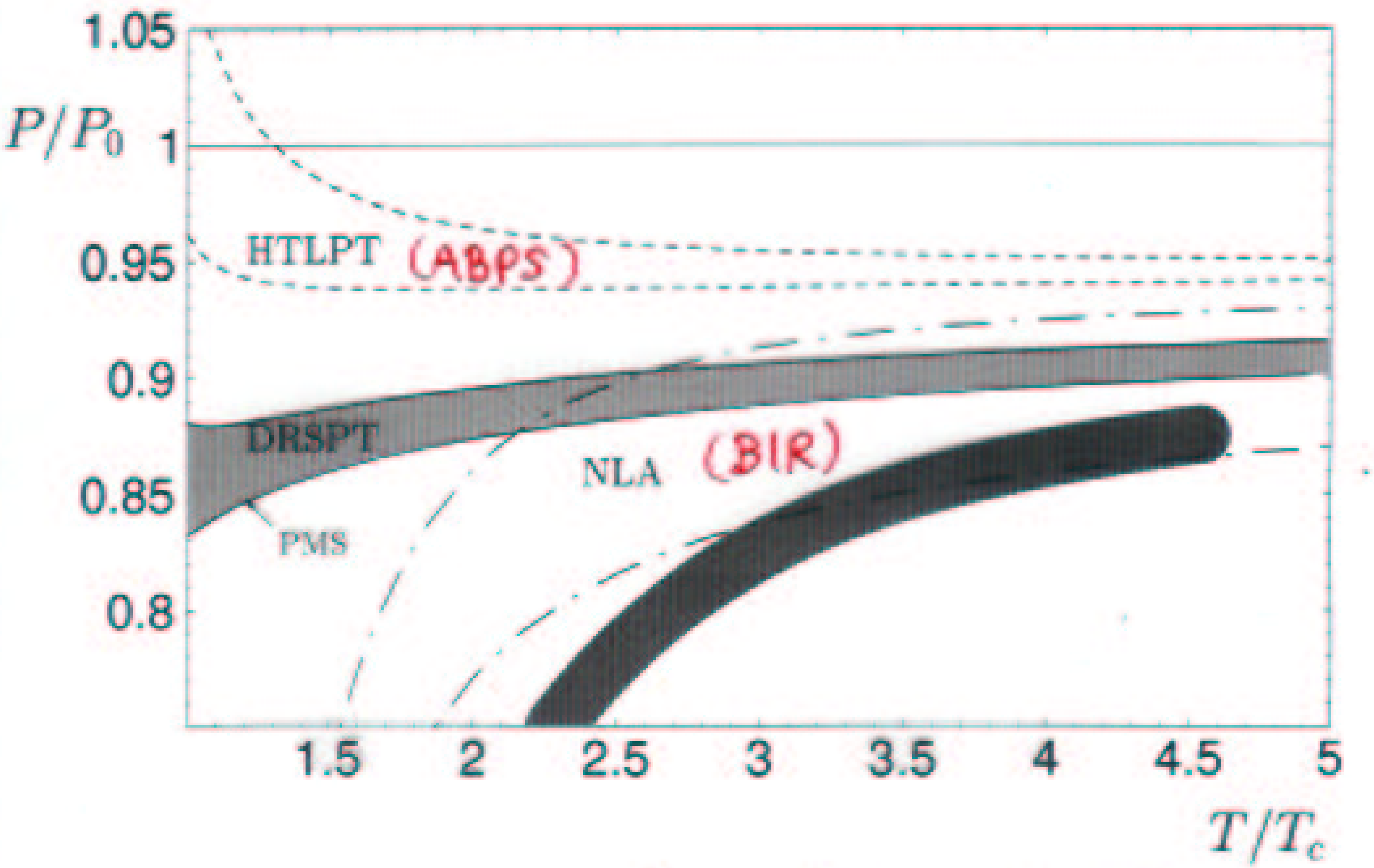
ABPS: Andersen, Braaten, Petitgirard + Strickland 2002

(1999: ABS 1-loop)



Discrepancy between HTLPT of ABPS and Φ-derivable approach indeed comes from kinematic regime where HTLPT has to correct for overcounting of hard contributions by thermal counterterms:

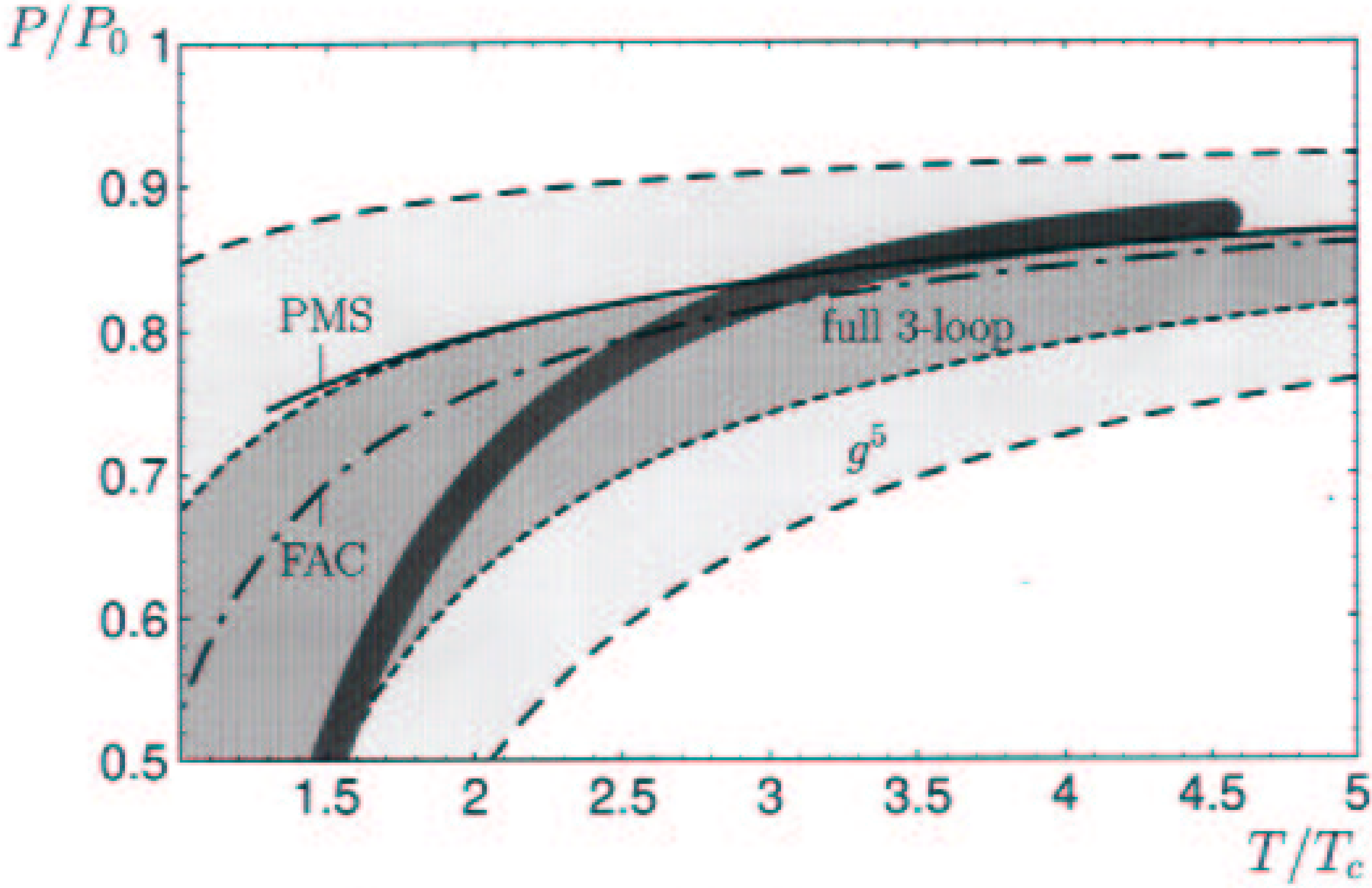
DRSPT = HTLPT in dimensional reduction (i.e. leaving hard sector untouched)



(Blaizot, Iancu, AR, 2003 PRD in press)

Large scale dependence of conventional pert. result is due to truncations of soft ($\sim gT$) contributions:

Keeping effective-field-theory parameters of dimensionally reduced theory untruncated reduces scale dependence



PMS: because of then nonlinear $\bar{\mu}$-dependence can fix it by $\frac{\partial P}{\partial \bar{\mu}} \stackrel{!}{=} 0$

Progress at finite μ_q:

"Sign problem" of lattice gauge theories
can be overcome/circumvented
by new clever techniques:

reweighting — FODOR + KATZ 2001
imaginary μ_q
and analytic continuation — FORCRAND + PHILIPSEN 2002

Taylor expansions — ALLTON et al. 2002

but always $\mu_q \lesssim T$

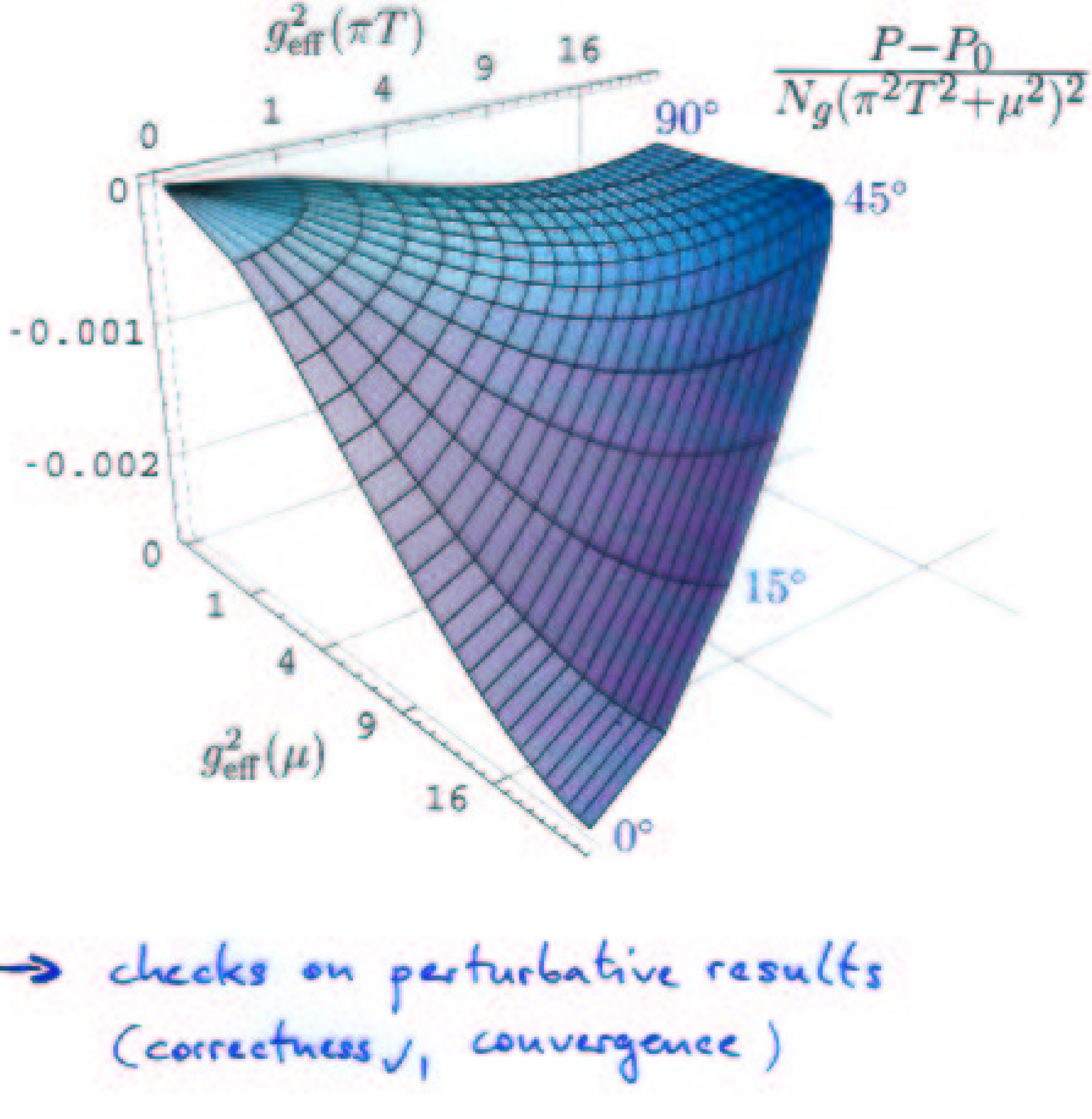
analytical calculations/approaches:

— dimensional reduction
at $\mu \lesssim T$: VUORINEN 2003

— HTL-based quasiparticle models
ROMATSCHKE + A.R. 2003
↳ next talk

A. IPP + A.R. 2003 (JHEP 06, 032):

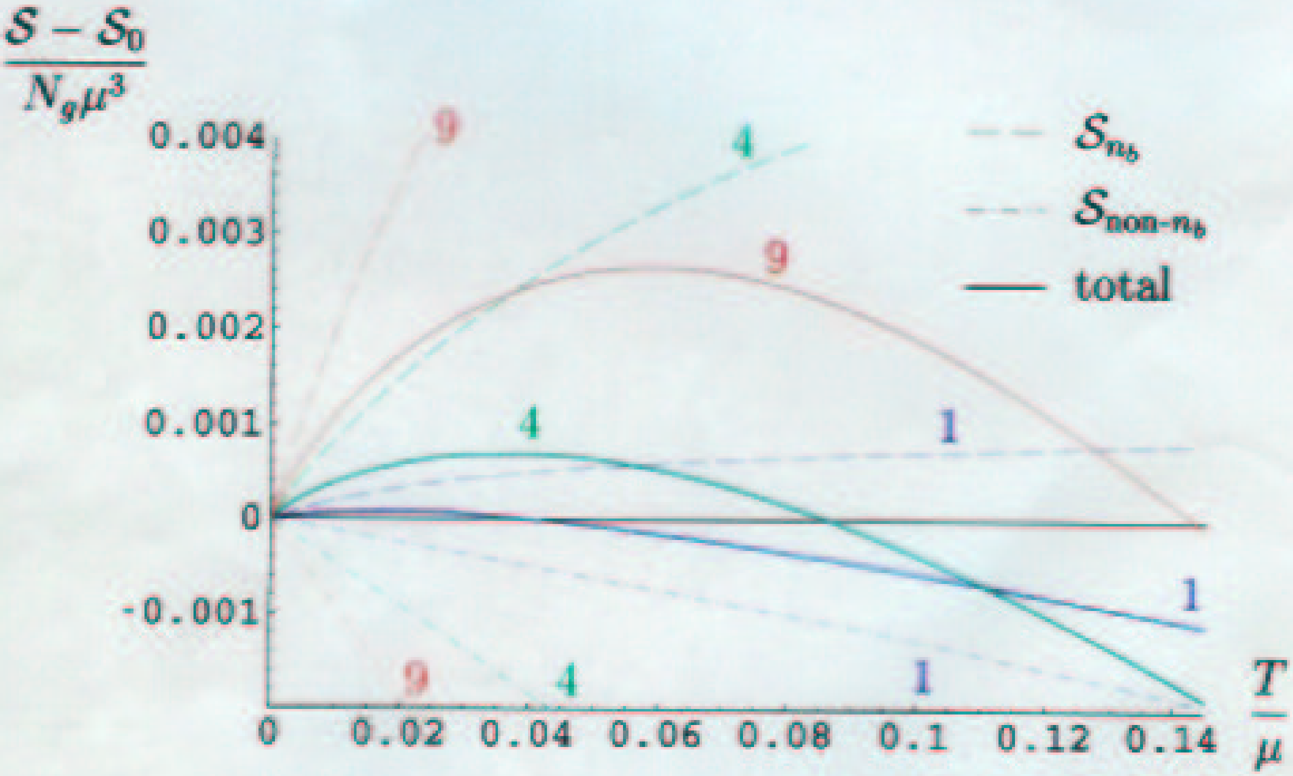
Exactly solvable large-N_f limit of QCD (G. Moore, 2002) can be extended to finite μ_q:



→ checks on perturbative results (correctness ✓, convergence)

→ qualitative changes at large μ_q/T

Non-Fermi-liquid behaviour
at $T/\mu \lesssim \alpha_s$
(S and c_v contain $T \ln T$ terms)



potentially relevant for
specific heat of n-stars
with normal quark-matter component
(Boyanovsky + de Vega (2001) —
conflict with Holstein, Norton + Pincus 1973)