SMALL SCALE STRUCTURE OF CDM

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MOTIVATION

- Mass of the 1st gravitationally bound CDM system:

$M_{mc} \sim 10^{-12} M_\odot$ for Axion minicluster

(Kolb and Tkachev 1996, ApJ 460, L25-L28)

$M_{min} \sim 10^{-06} M_\odot$ for Bino clouds

(Hofmann et al. 2001, PRD 64, 083507;
Berezinsky et al. 2003, astro-ph/0301551)

- Resolution of N-body simulations

$M_{res} \sim 10^{-05} M_{tot}$

substructures is growing with resolution

(Moore et al. 1998, ApJ 499, L5-L8)

- SSS formation is sensitiv on CDM particle candidates!

- Consistent initial conditions for N-body simulations?

⇒ understand structure formation on smallest scales

- Observations: Direct and indirect CDM searches, e.g. γ-ray flux from CDM annihilations

$$\phi_\gamma = \text{diffuse flux} + \underbrace{\text{line contribution}}_{\propto \left\langle \left(\frac{\delta\rho}{\rho}\right)^2 \right\rangle_{\substack{\text{CDM} \\ \text{halo}}}}$$

(Bergström et al. 2001, PRL 87, 251301; Ullio et al. 2002, PRD 66, 123502)

- knowledge of local CDM overdensities

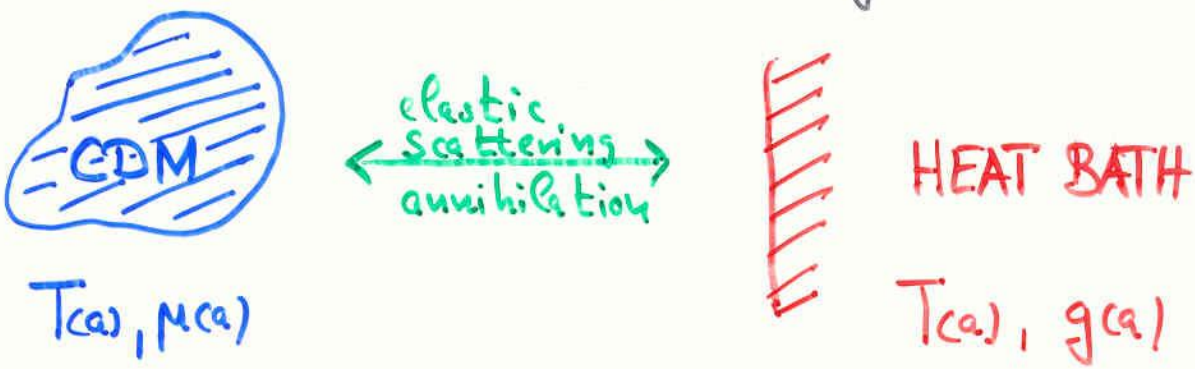
⊃ knowledge of CDM structures on smallest scales is mandatory for designing and interpreting Experiments

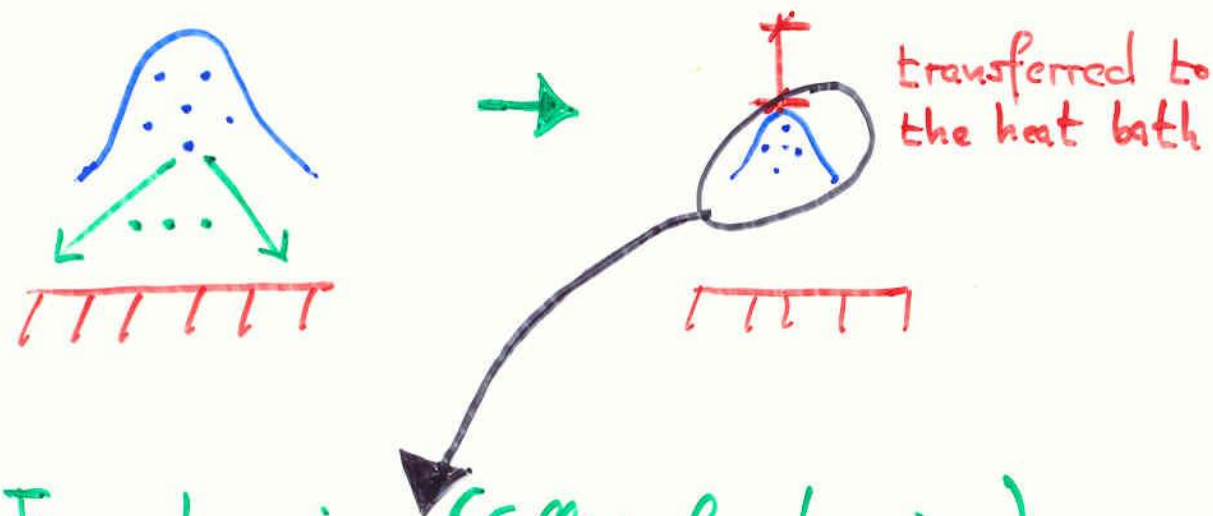
EXISTENCE OF SSS

CDM structure formation is affected by

- Collisional damping

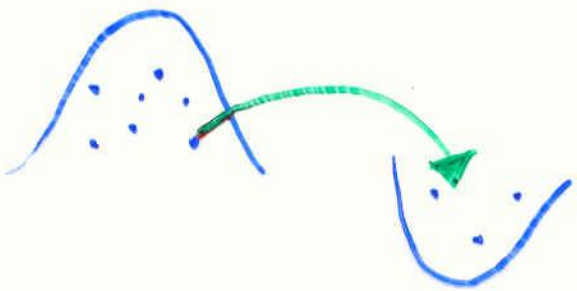
(Silk 1968, ApJ 151, 459; Weinberg 1971, ApJ 168, 175)





- Free streaming (Collisionless damping)

(Bond and Szalay 1983, ApJ 274, 443)



OCCURENCE OF SSS

Structure formation in CDM begins at

$$T_{kd} \approx \left[\frac{10^2}{M_{Pl}} M_{\tilde{\chi}}^{\alpha} \left(M_{\tilde{F}}^2 - M_{\tilde{\chi}}^2 \right)^2 \right]^{\frac{1}{3+\alpha}}$$

$\alpha = 0$: $\tau_{relax} \equiv \tau_{coll}$ ↯

$\alpha = 1$: $\tau_{relax} \equiv N_{relax}\, \tau_{coll}$ ✓

$$\langle t \rangle = -\frac{1}{\sigma_{el}} \int_{-(2E_f)^2}^{0} dt\, t\, \frac{d\sigma_{el}}{dt} = 2E_f^2 \rightsquigarrow N_{relax} \approx \frac{M_{\tilde{\chi}}}{T} \gg 1$$

For the MSSM

$$T_{kd} \in [10, 100]\ \text{MeV} \;\hat{=}\; t_{kd} \approx 1\ \text{ms}$$

For $T \lesssim T_{kd}$ non equilibrium processes defuel SSS:

$$T_{COM} = \underbrace{\rho_{COM}\, u \otimes u - P_{COM}\, h}_{\text{equilibrium}} + \tau^{(d)}$$

$$\tau^{(d)} = \zeta \left(\rightarrow \boxed{\leftrightarrow} \right) + \eta \left(\rightarrow \boxed{\uparrow} \right) + \chi \left(\rightarrow \boxed{\uparrow} \right)$$

bulk viscosity — shear viscosity — heat conduction

COLLISIONAL DAMPING

Fluctuations in CDM

$$\delta\{n, T, \varrho, P, u\}_{CDM} \sim e^{\pm i\omega t}\; e^{-i k\cdot x}$$

"in medium"

$$\mathrm{Im}\,\omega(k, 3, \eta) \neq 0 \qquad \text{damping}$$

The damping during $[0, t_{kd}]$ is given by

$$\delta(k) = \delta_p(k)\, \exp\left(-\int_0^{t_{kd}} dt\; \mathrm{Im}\,\omega(k, 3, \eta)\right)$$

$$\boxed{\delta(M(k)) = \delta_p(M(k))\, \exp\left[-\left(M_d / M(k)\right)^{2/3}\right]}$$

Characteristic damping mass

$$\boxed{M_d = 2.6 \cdot 10^{-8}\, \frac{GeV}{\sqrt{M_{\tilde{x}} T_{kd}}}\, \omega_{\tilde{x}}\, M_\odot}$$

COLLISIONLESS DAMPING

Characteristic length scale for free streaming

$$\ell_{fs} = a \int_{\eta_{kd}}^{\eta} d\eta' \, V(\eta')$$

$$\approx \ln\left[\frac{a_{kd}}{a} \frac{a/a_{eq} + 2\left(1 - \sqrt{1 + a/a_{eq}}\right)}{a_{kd}/a_{eq} + 2\left(1 - \sqrt{1 + a_{kd}/a_{eq}}\right)} \right]$$

Damping of CDM fluctuations

$$\delta(M) = \delta_p(M) \exp\left[-(M_d/M)^{2/3} \right] \times$$

$$\times \left[1 - \frac{2}{3} (M_{fs}/M)^{4/3} \right] \exp\left[-(M_{fs}/M)^{4/3} \right]$$

with the free streaming mass scale

$$M_{fs} = \frac{4\pi}{3} \left(\frac{T_{kd}}{\frac{1}{2} M_{\tilde{\chi}} V_{kd}^2} \right)^{3/2} \rho_{\tilde{\chi}} \, \ell_{fs}^{\,3}$$

RESULTS

- The CDM power spectrum has a small scale cut-off: $\delta(M_{fs}) = 0$

  ⇓

  1st purely gravitationally bound $\tilde{\chi}$- clouds have to have masses

  $$M_{min} > M_{fs} \approx 10^{-6} M_\odot > M_d \approx 10^{-3} M_\odot$$

  (Hofmann et al. 2001, PRD64, 083507; Berezinsky et al. 2003, astro-ph/0301551)

- Small Scale Structure formation is sensitive to different CDM candidates

- The CDM power spectrum has a maximum: $\frac{d}{dM} \delta(M_{max}) = 0$

  ⇓

  1st structures entering the nonlinear regime. These are the most promising fluctuations for direct and indirect CDM searches.