Log Likelihood

$$L = -\frac{1}{2} x^T C^{-1} x - \frac{1}{2} \ln |C|$$

- Estimate the log likelihood in the KL basis, by rotating into the diagonal eigensystem, and rescaling with the square root of the eigenvalues
- Then C=1 at the fiducial basis
- We recompute C around this point – always close to a unit matrix
- Fisher matrix also simple

Quadratic Estimator

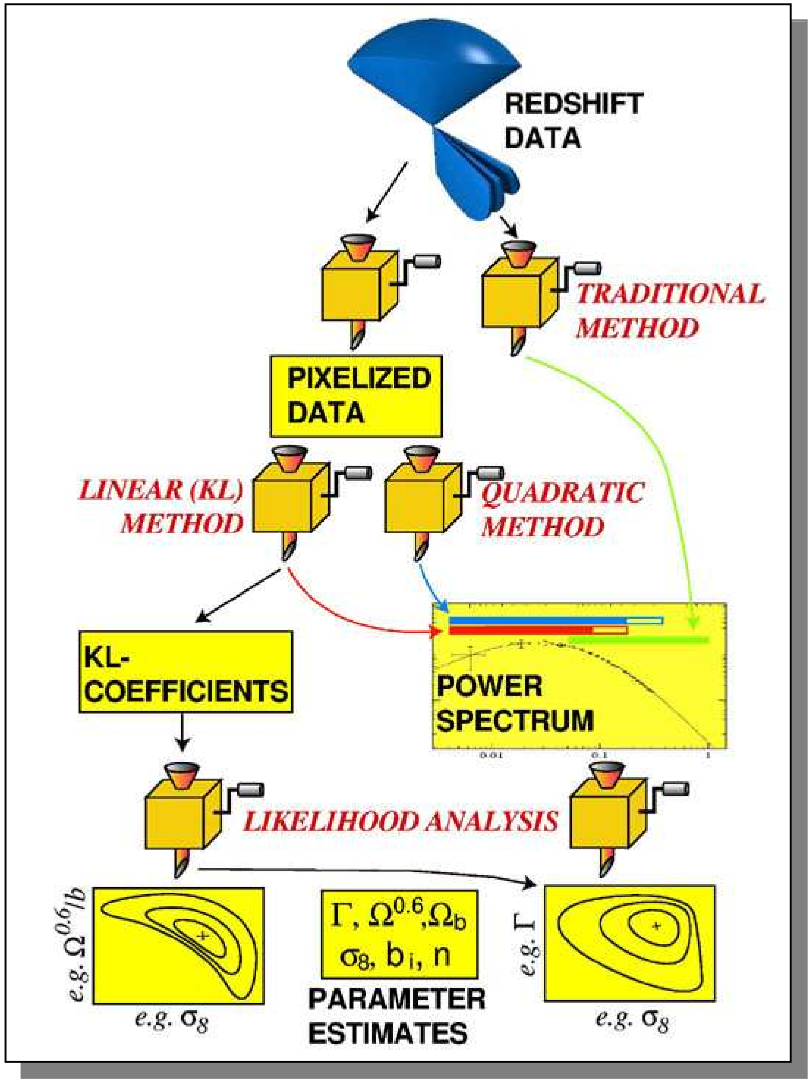
- One can compute the correlation matrix of

$$C(k,k') = \left\langle \hat{P}(k)\hat{P}(k') \right\rangle$$

- P is averaged over shells, using the rotational invariance
- Used widely for CMB, using the degeneracy of a_{lm}'s
- Computationally simpler
- But: includes 4th order contributions – more affected by nonlinearities
- Parameter estimation is performed using $x_i = \hat{P}(k_i)$

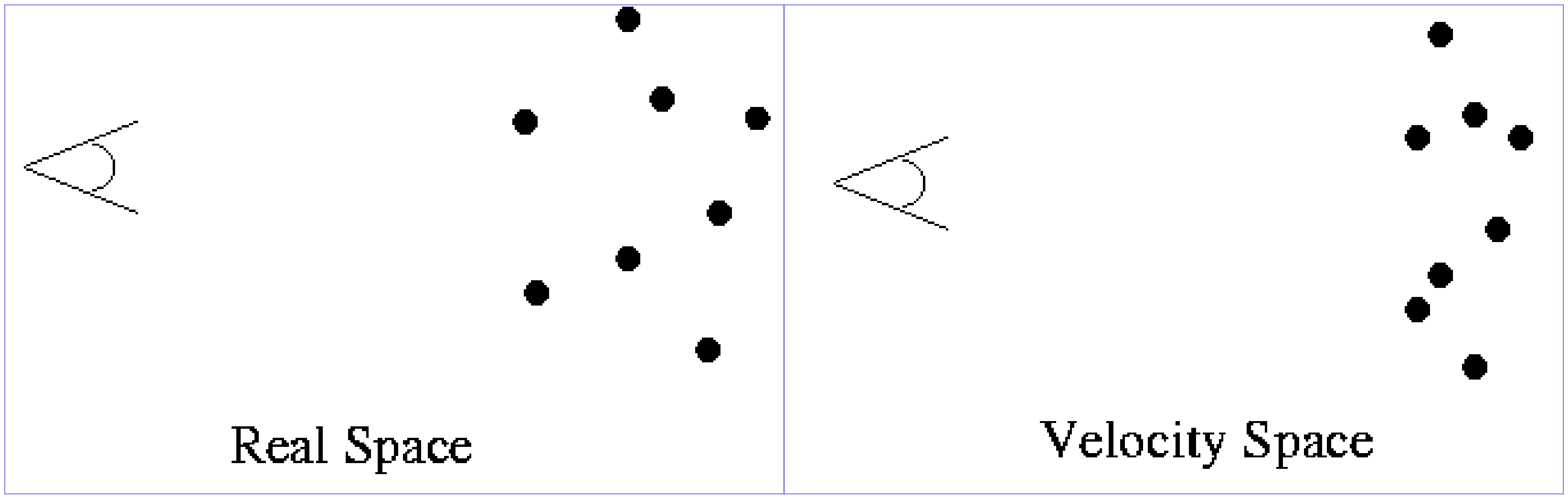
$$L = -\frac{1}{2}x^T C^{-1} x - \frac{1}{2}\ln|C|$$

Parameter Estimation



Distance from Redshift

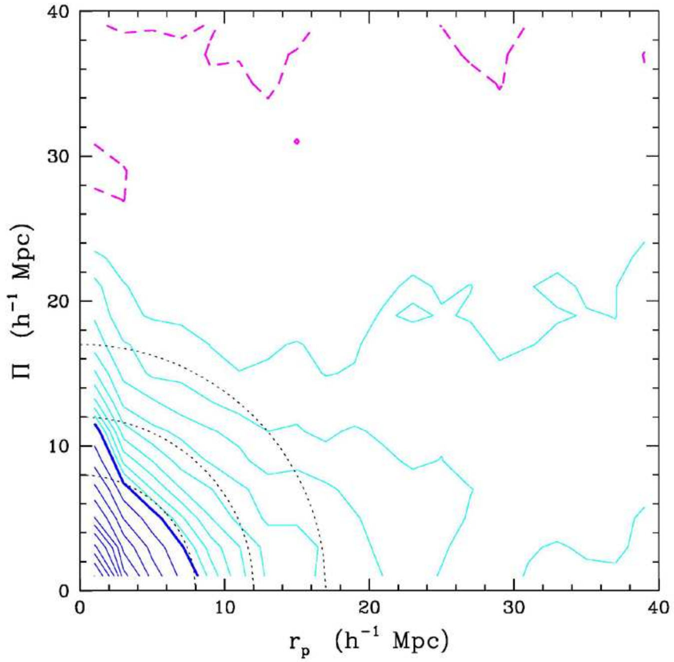
- Redshift measured from Doppler shift
- Gives distance to zeroth order
- But, galaxies are not at rest in the comoving frame:
  - *Distortions along the radial directions*
  - *Originally homogeneous isotropic random field, now anisotropic!*



Redshift Space Distortions

Three different distortions

- Linear infall (large scales)
  - *Flattening of the redshift space correlations*
  - *L=2 and L=4 terms due to infall (Kaiser 86)*
- Thermal motion (small scales)
  - *'Fingers of God'*
  - *Cuspy exponential* $P(v_{12}) \propto e^{-|v_{12}|/\sigma}$
- Nonlinear infall (intermediate scales)
  - *Caustics (Regos and Geller)*



Power Spectrum

- Linear infall is coming through the infall induced mock clustering
- Velocities are tied to the density via

$$\beta = \left(\frac{\dot{D}}{D}\right) / \left(\frac{\dot{a}}{a}\right) = \frac{\Omega^{0.6}}{b}$$

- Using the continuity equation we get

$$P^{(s)}(k) = P(k)(1+\beta\mu^2)^2$$

- Expanded: we get $P_2(\mu)$ and $P_4(\mu)$ terms
- Fourier transforming:

$$\xi(r,\mu) = \sum_{L=0,2,4} a_L \xi_L(r)$$

$$\xi_L(r) = \frac{1}{\sqrt{2\pi^2}} \int_0^\infty dk\, k^2 j_L(kr) P(k)$$

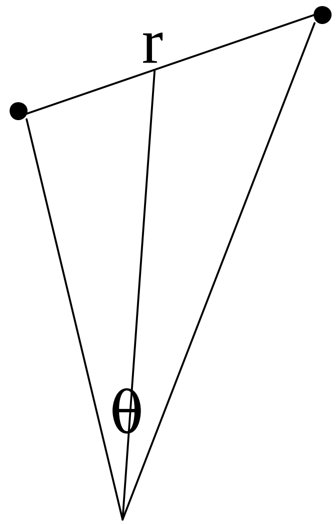
Angular Correlations

- Limber's equation

$$w(\theta) = \int dr_1 r_1^2 dr_2 r_2^2 \frac{\phi(r_1)}{F(r_1)} \frac{\phi(r_2)}{F(r_2)} \xi(r_{12})$$

$$s = \frac{r_1 + r_2}{2}, \quad p = r_1 - r_2$$

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$



$$r^2 = s^2\theta^2 + p^2 = s^2\left(\theta^2 + \frac{p^2}{s^2}\right) = s^2\left(\theta^2 + y^2\right)$$

$$w(\theta) = r_0^\gamma \int ds\, s^{5-\gamma} \left[\frac{\phi(s)}{F(s)}\right]^2 \int dy (\theta^2 + y^2)^{-\gamma/2}$$

$$w(\theta) = r_0^\gamma \theta^{1-\gamma} \int ds\, s^{5-\gamma} \left[\frac{\phi(s)}{F(s)}\right]^2 \int dt (1 + t^2)^{-\gamma/2} = r_0^\gamma \theta^{1-\gamma} H_\gamma A_w$$

Applications

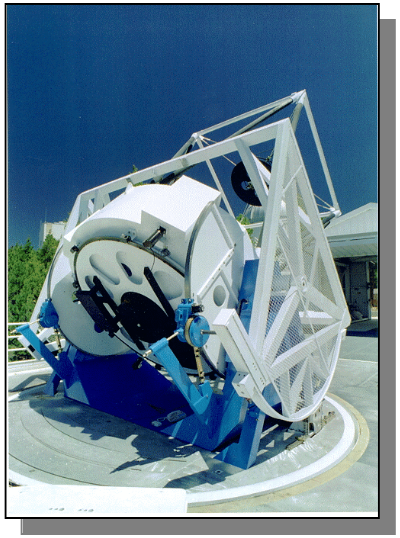
- Angular clustering on small scales
- Large scale clustering in redshift space

The Sloan Digital Sky Survey

Special 2.5m telescope, at Apache Point, NM
- *3 degree field of view*
- *Zero distortion focal plane*

Two surveys in one
- *Photometric survey in 5 bands*
  - *detecting 300 million galaxies*
- *Spectroscopic redshift survey*
  - *measuring 1 million distances*

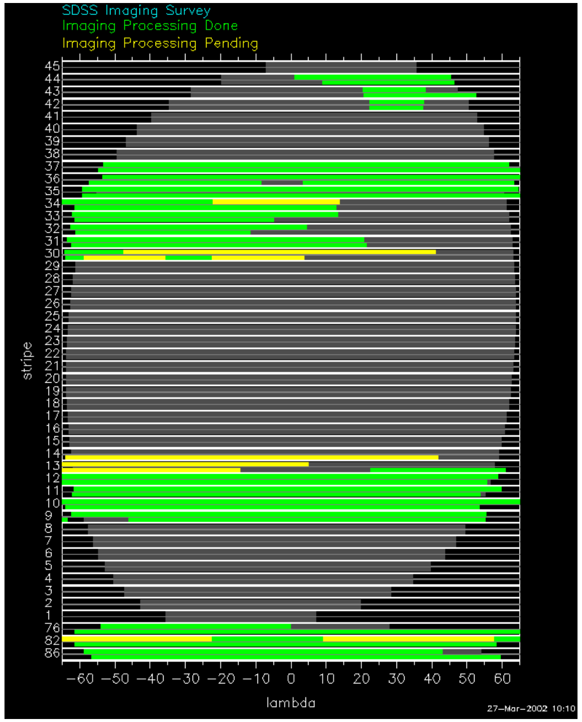
Automated data reduction
- *Over 120 man-years of development*
- *(Fermilab + collaboration scientists)*



Very high data volume
- *Expect over 40 TB of raw data*
- *About 2 TB processed catalogs*
- *Data made available to the public*

Current Status of SDSS

- As of this moment:
  - *About 4500 unique square degrees covered*
  - *500,000 spectra taken (Gal+QSO+Stars)*
- Data Release 1 (Spring 2003)
  - *About 2200 square degrees*
  - *About 200,000+ unique spectra*
- Current LSS Analyses
  - *2000-2500 square degrees of photometry*
  - *140,000 redshifts*



w(θ) with Photo-z

T. Budavari, A. Connolly, I. Csabai, I. Szapudi, A. Szalay,
S. Dodelson, J. Frieman, R. Scranton, D. Johnston
and the SDSS Collaboration

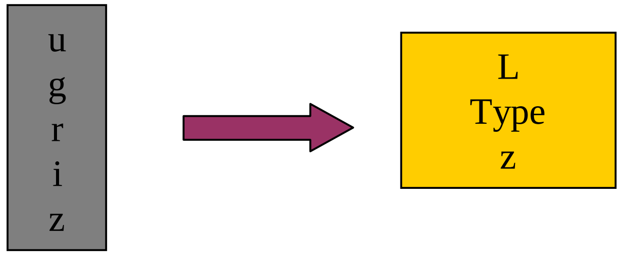
- Sample selection based on rest-frame quantities
- Strictly volume limited samples
- Largest angular correlation study to date
- Very clear detection of
  - *Luminosity dependence*
  - *Color dependence*
- Results consistent with 3D clustering

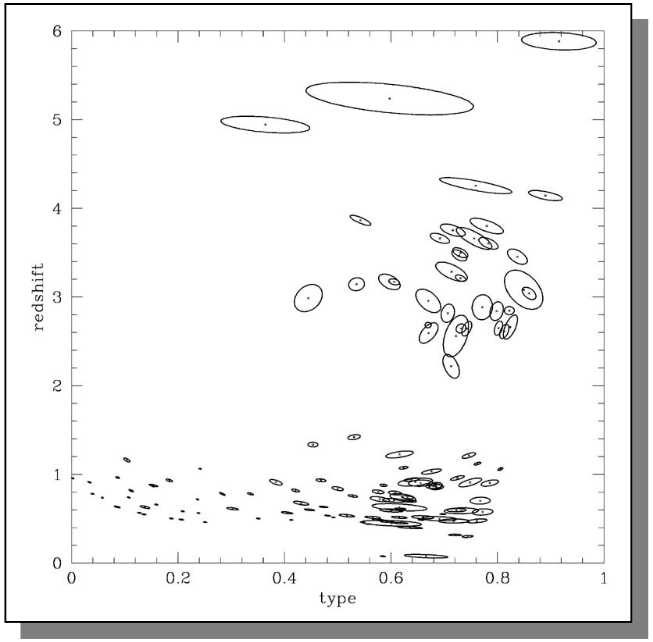
Photometric Redshifts

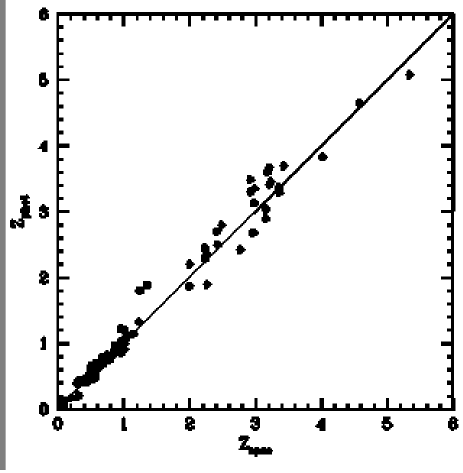
- Physical inversion of photometric measurements!

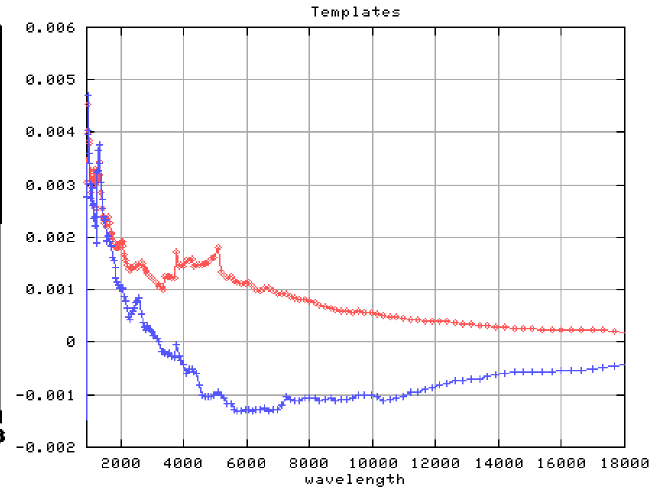
  Adaptive template method (Csabai etal 2001, Budavari etal 2001, Csabai etal 2002)

- Covariance of parameters

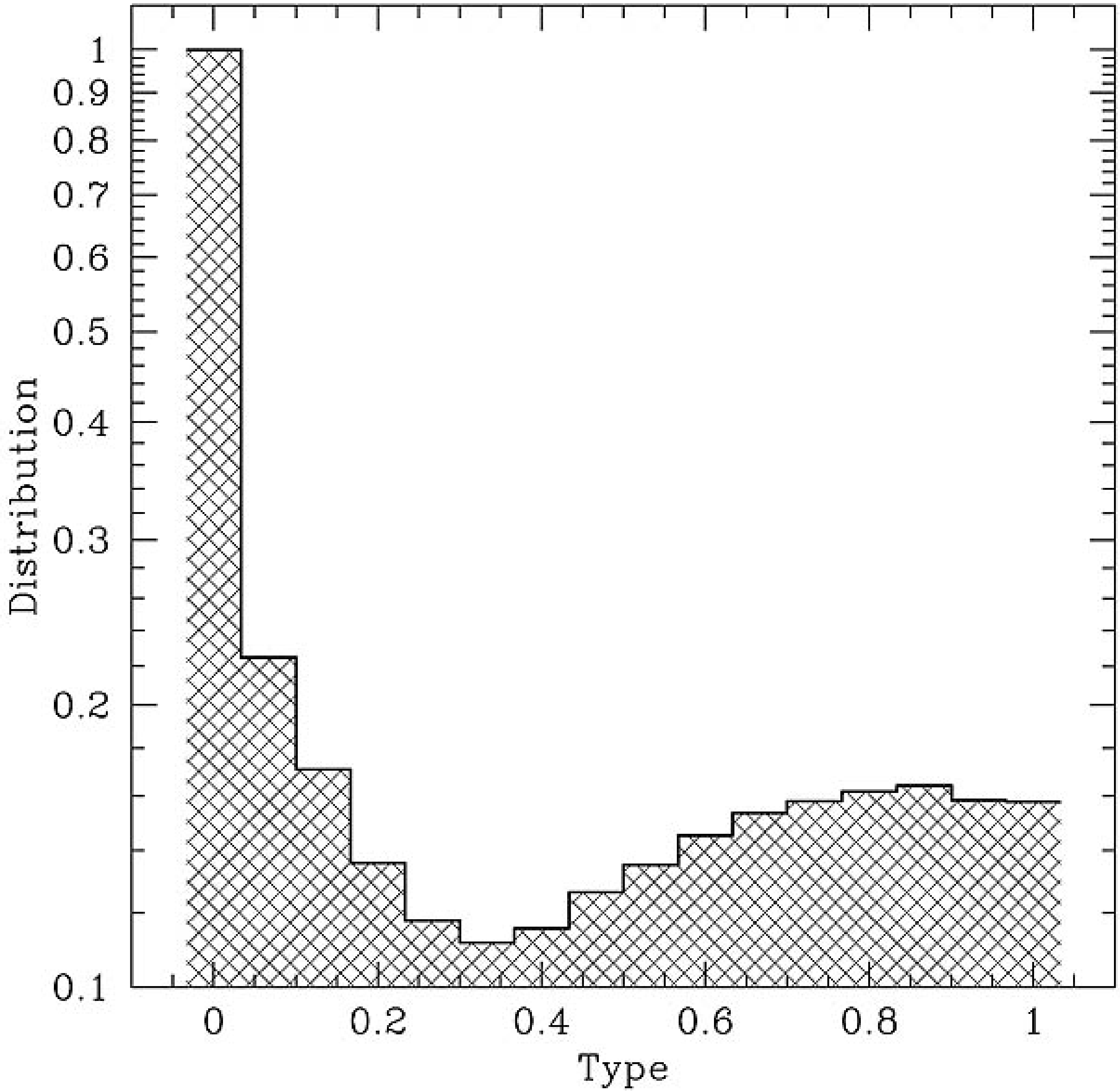




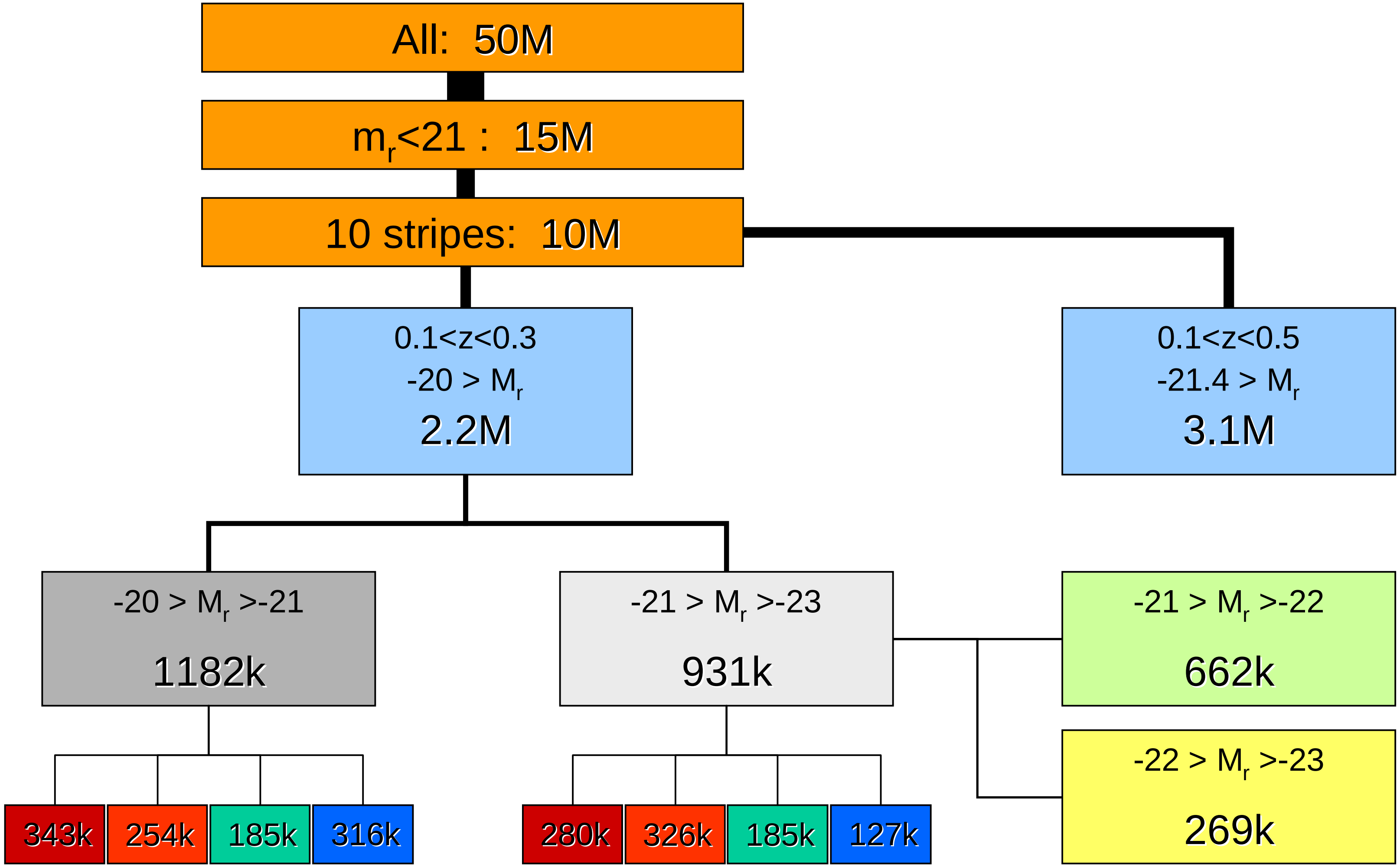




Distribution of SED Type

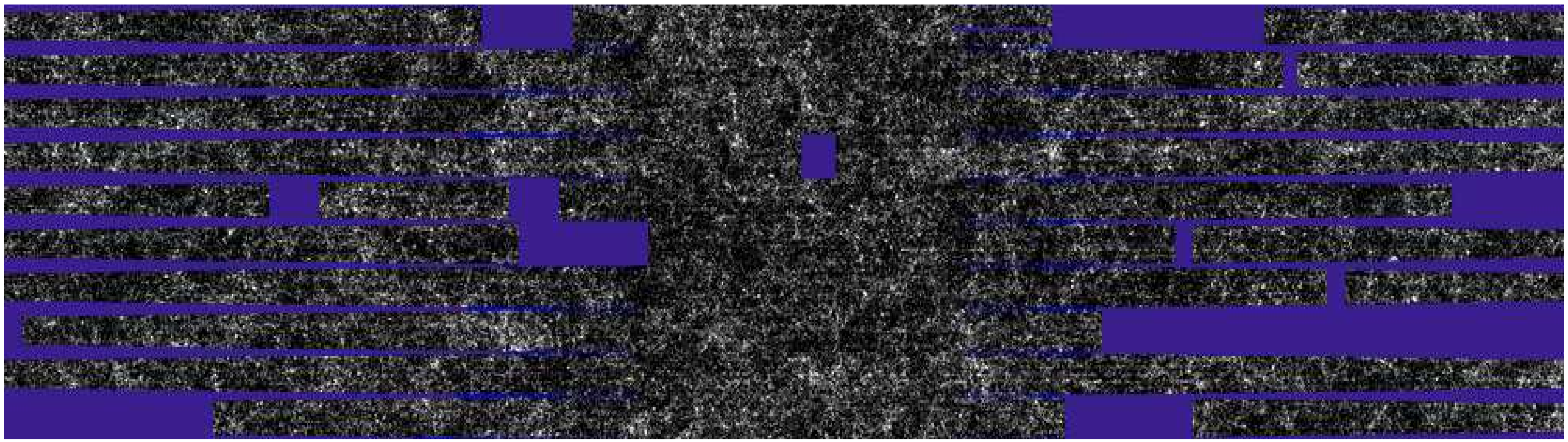


The Sample


All: 50M
m_r<21 : 15M
10 stripes: 10M
0.1<z<0.3
-20 > M_r
2.2M
0.1<z<0.5
-21.4 > M_r
3.1M
-20 > M_r >-21
1182k
-21 > M_r >-23
931k
-21 > M_r >-22
662k
-22 > M_r >-23
269k
343k
254k
185k
316k
280k
326k
185k
127k

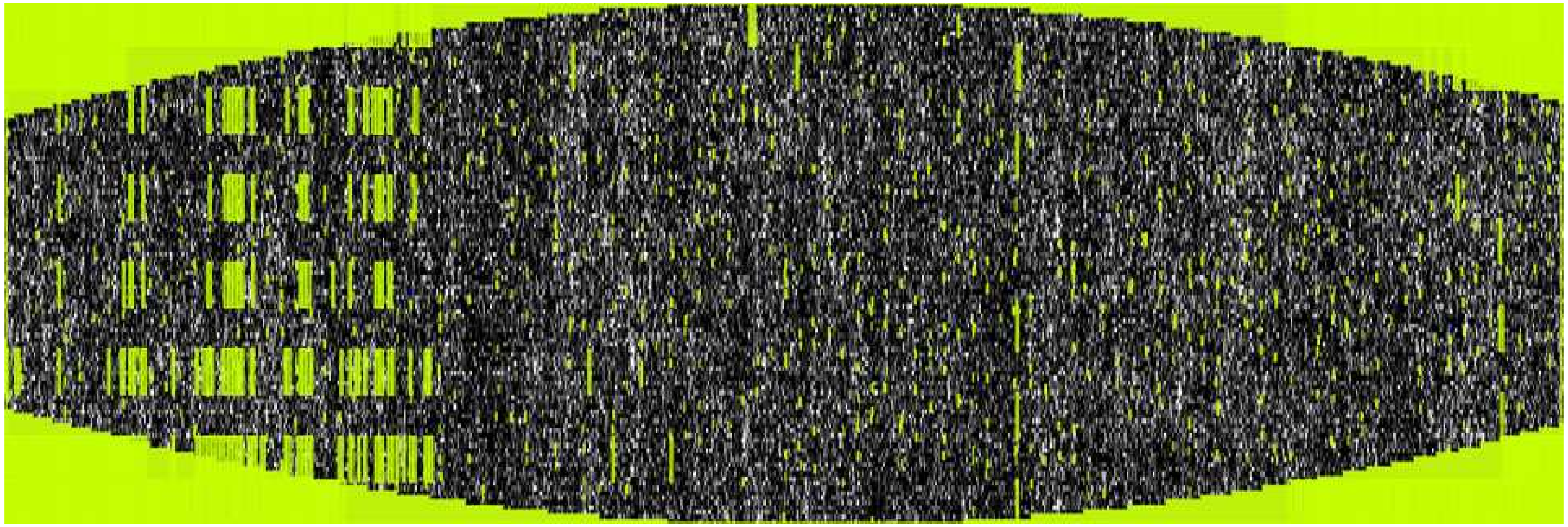
The Stripes

- 10 stripes over the SDSS area, covering about 2800 square degrees
- About 20% lost due to bad seeing
- Masks: seeing, bright stars



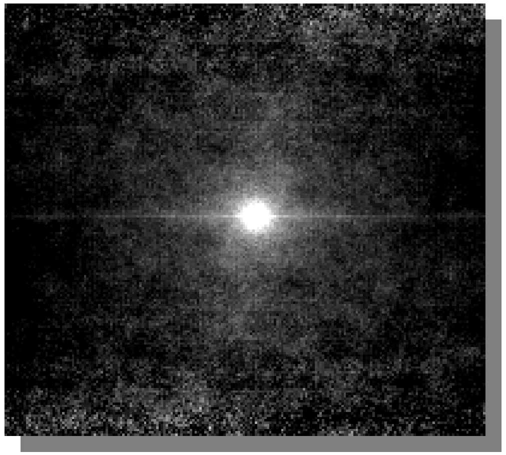
The Masks

- Stripe 11 + masks
- Masks are derived from the database
  - *bad seeing, bright stars, satellites, etc*



The Analysis

- eSpICE : I.Szapudi, S.Colombi and S.Prunet
- Integrated with the database by T. Budavari
- Extremely fast processing:
  - *1 stripe with about 1 million galaxies is processed in 3 mins*
  - *Usual figure was 10 min for 10,000 galaxies => 70 days*
- Each stripe processed separately for each cut
- 2D angular correlation function computed
- w(θ): average with rejection of pixels along the scan
  - *Correlations due to flat field vector*
  - *Unavoidable for drift scan*



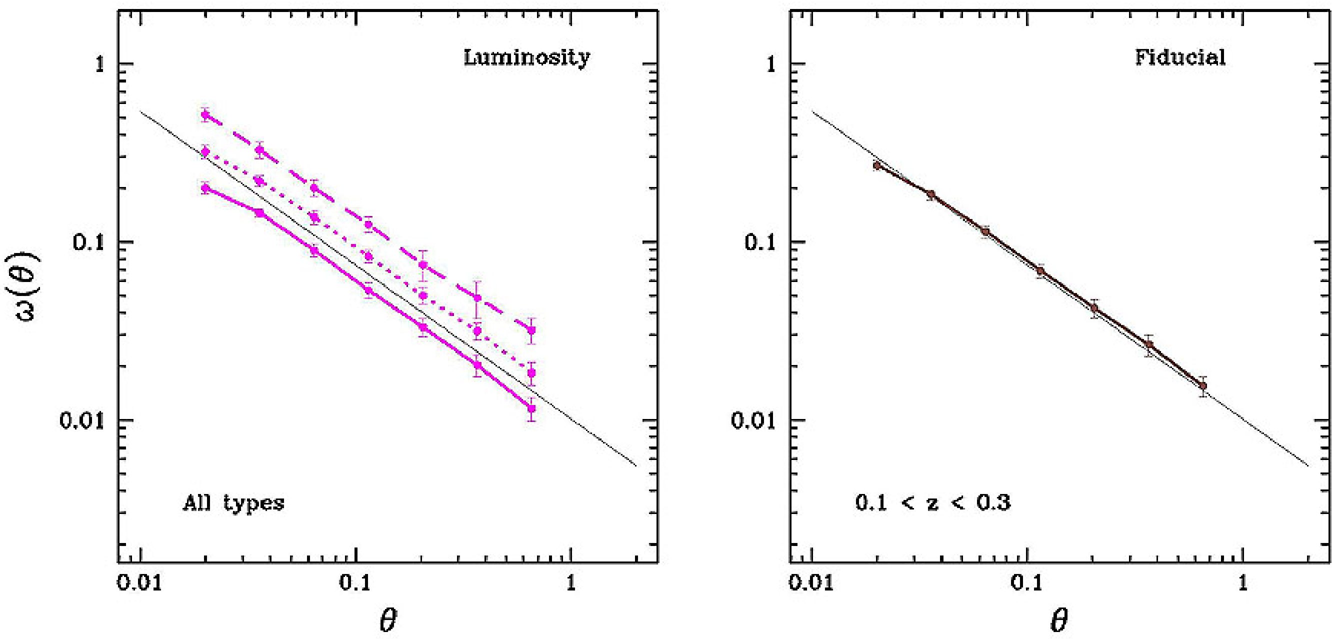
Angular Correlations I.

- Luminosity dependence: 3 cuts

  -20> M > -21

  -21> M > -22

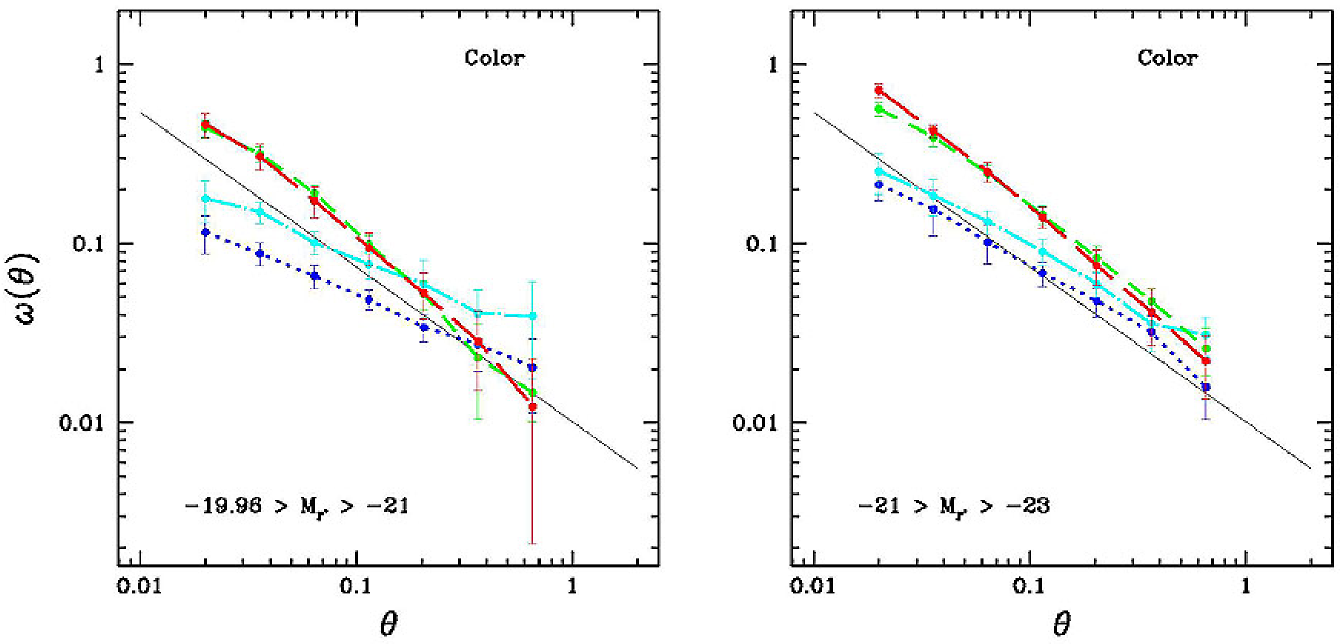
  -22> M > -23



Angular Correlations II.

- Color Dependence

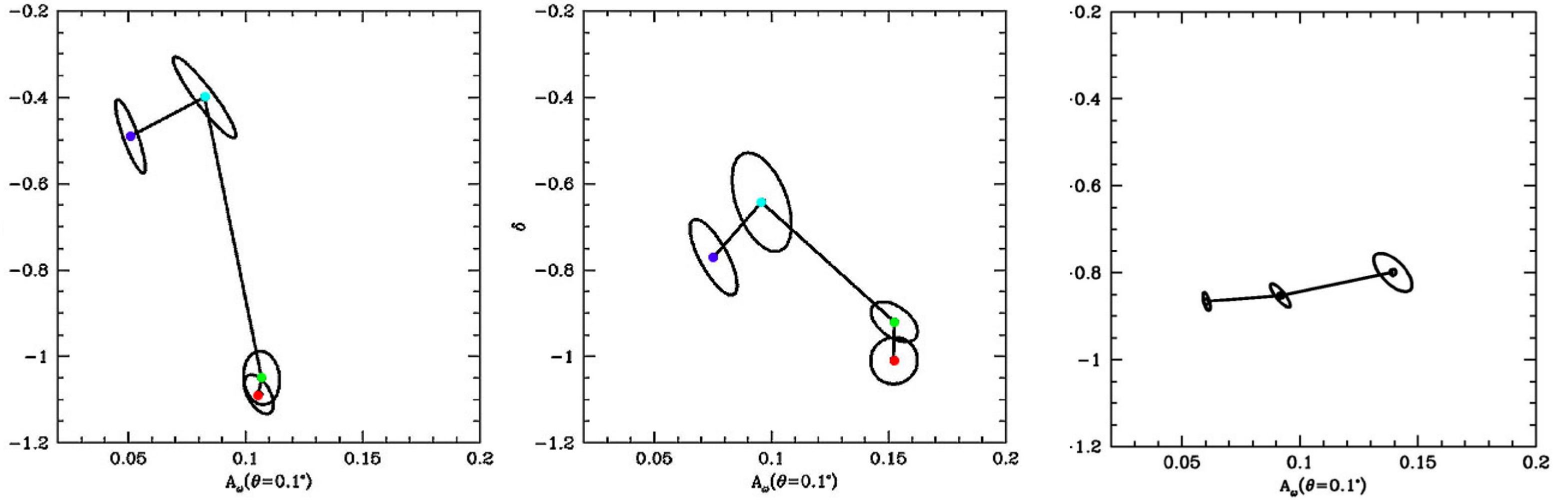
4 bins by rest-frame SED type



Power-law Fits

- Fitting

$$w(\theta) = A_\omega (\theta / 0.1^o)^\delta$$



Bimodal w(θ)

- No change in slope with L cuts
- Bimodal behavior with color cuts
- Can be explained, if galaxy distribution is bimodal (early vs late)
  - *Correlation functions different*
  - *Bright end (-20>) luminosity functions similar*
  - *Also seen in spectro sample (Glazebrook and Baldry)*
- In this case L cuts do not change the mix
  - *Correlations similar*
  - *Prediction: change in slope around -18*
- Color cuts would change mix
  - *Changing slope*

Redshift distribution

- The distribution of the true redshift *(z)*, given the photoz *(s)*
- Bayes' theorem

$$P(z|s) = \frac{P(s|z)P(z)}{P(s)}$$

- Given a selection window W(s)

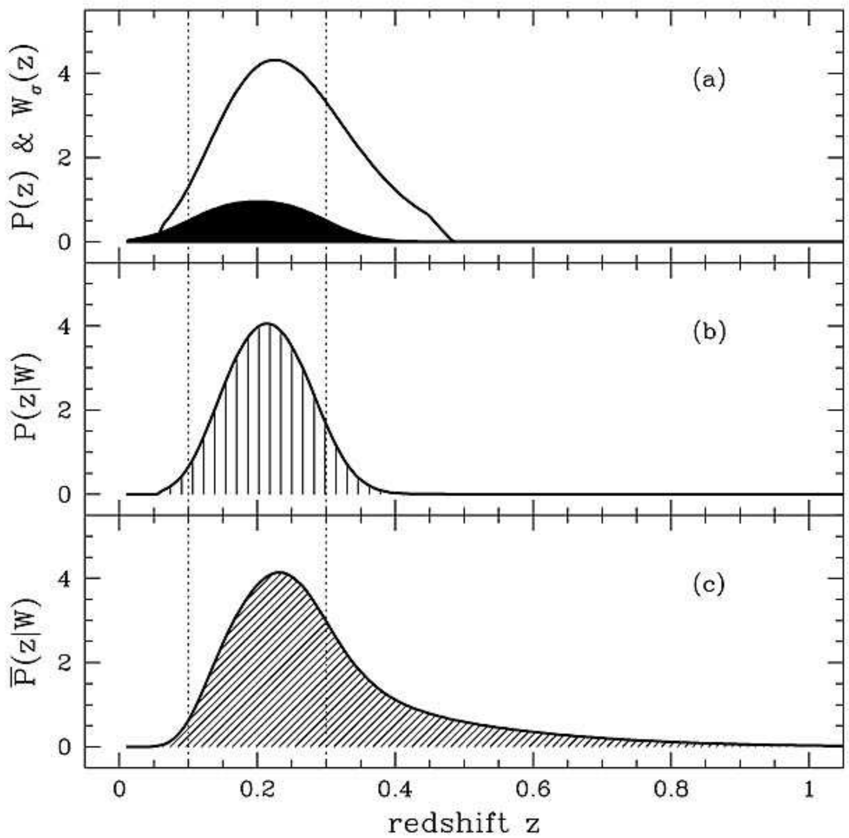
$$P_w(z) = \int ds P(s) W(s) \frac{P(s|z)P(z)}{P(s)}$$

- A convolution with the selection window

$$P_w(z) = P(z) \int ds W(s) P(s|z)$$

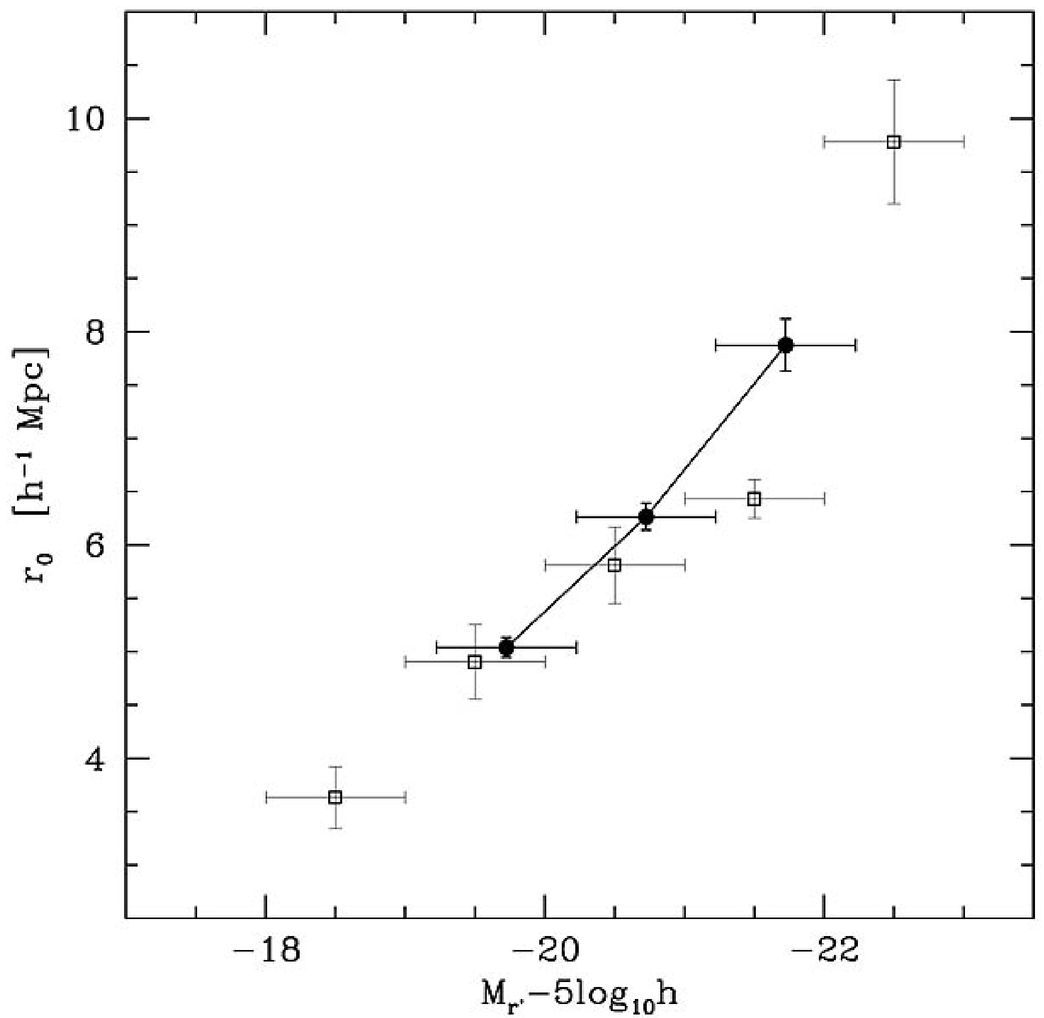
Detailed modeling

- Errors depend on S/N
- Final dn/dz summed over bins of m_r



Inversion to r_0

From (dn/dz) + Limber's equation => r_0

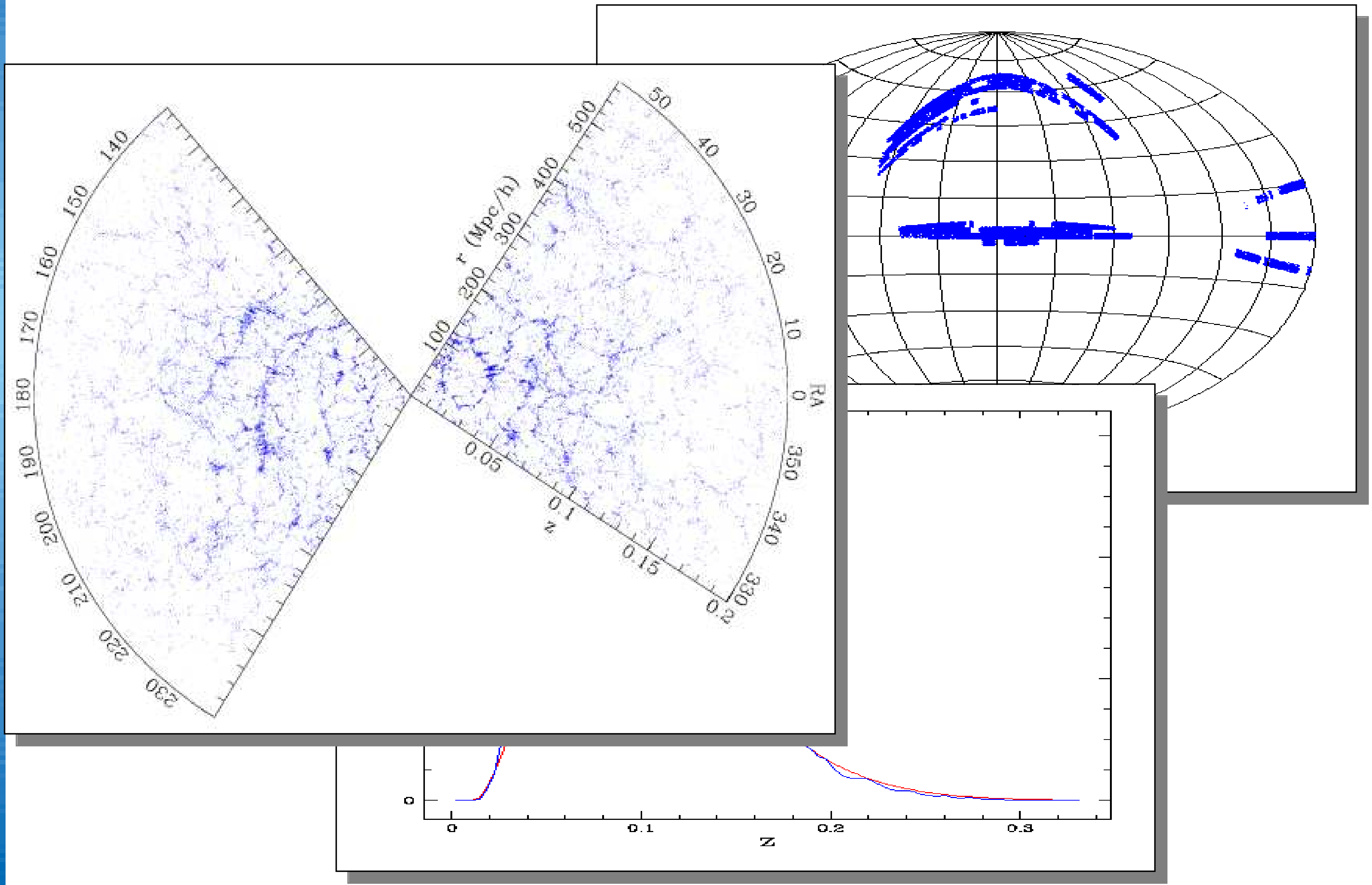


Redshift-Space KL

Adrian Pope, Takahiko Matsubara, Alex Szalay,
Michael Blanton, Daniel Eisenstein, Bhuvnesh Jain
and the SDSS Collaboration

- Michael Blanton’s LSS sample 9s13:
  - *SDSS main galaxy sample*
  - *-23 < M_r < -18.5, m_r < 17.5*
  - *120k galaxy redshifts, 2k degrees^2*
- Three “slice-like” regions:
  - *North Equatorial*
  - *South Equatorial*
  - *North High Latitude*

The Data


r (Mpc/h)
100
200
300
400
500
z
0.05
0.1
0.15
0.2
RA
0
10
20
30
40
50
350
340
330
140
150
160
170
180
190
200
210
220
230
0
0.1
0.2
0.3
Z

Pixelization

- Originally: 3 regions
  - *North equator: 5174 cells, 1100 modes*
  - *North off equator: 3755 cells, 750 modes*
  - *South: 3563 cells, 1300 modes*
  - *Likelihoods calculated separately, then combined*
- Most recently: 15K cells, 3500 modes
- Efficiency
  - *sphere radius = 6 Mpc/h*
  - *150 Mpc/h < d < 485 Mpc/h (80%): 95k*
  - *Removing fragmented patches: 70k*
  - *Keep only cells with filling factor >74%: 50k*

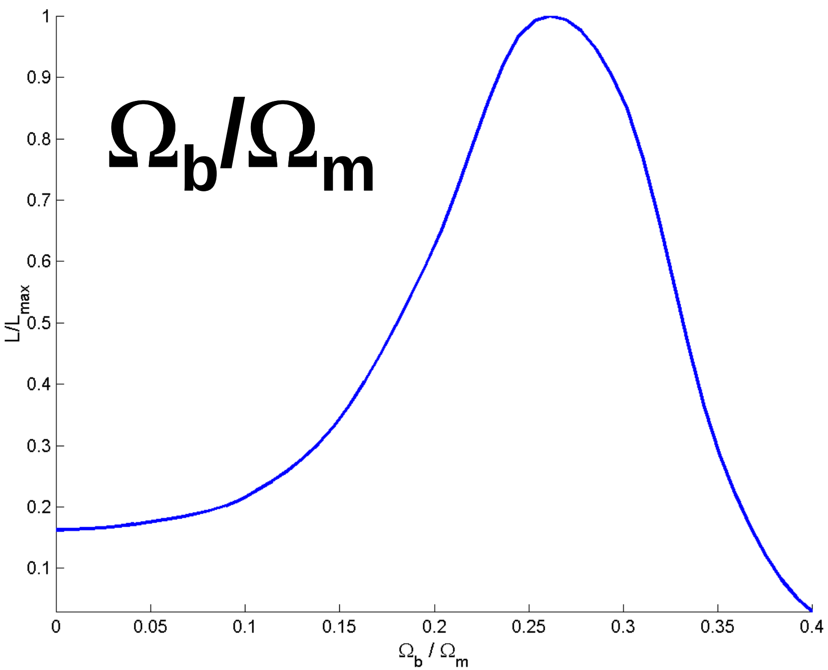
Redshift Space Distortions

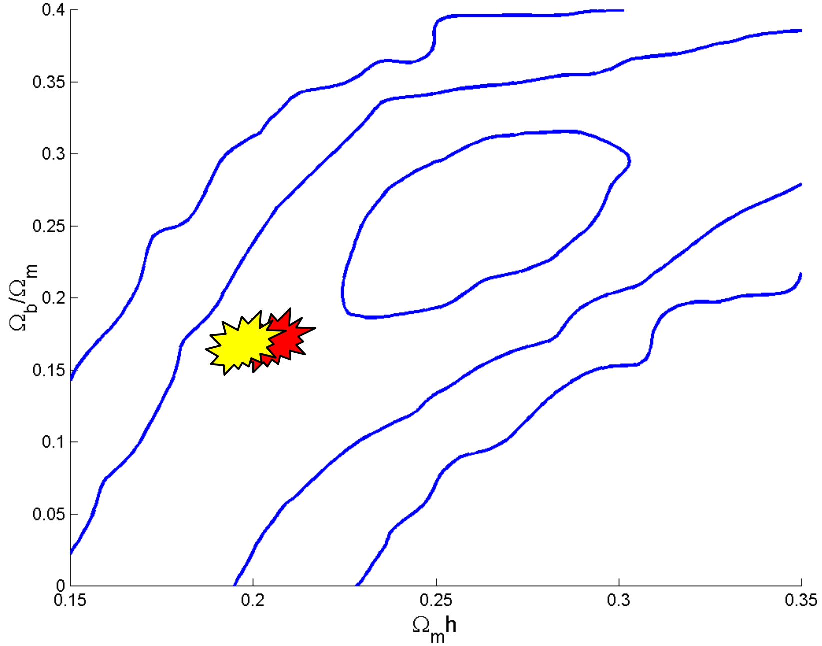
- Expand correlation function

$$\xi^{(s)}(r_1, r_2) = \sum_L C_{nL} \xi_L^{(n)}(r)$$

$$\xi_L^{(n)}(r) = \frac{1}{\sqrt{2\pi^2}} \int_0^\infty dk k^{2-n} j_L(kr) P(k)$$

- $c_{nL} = \Sigma_k f_k(\text{geometry}) \beta^k$
  - *$\beta = \Omega^{0.6}/b$ redshift distortion*
  - *b is the bias*
- Closed form for complicated anisotropy
  => computationally fast

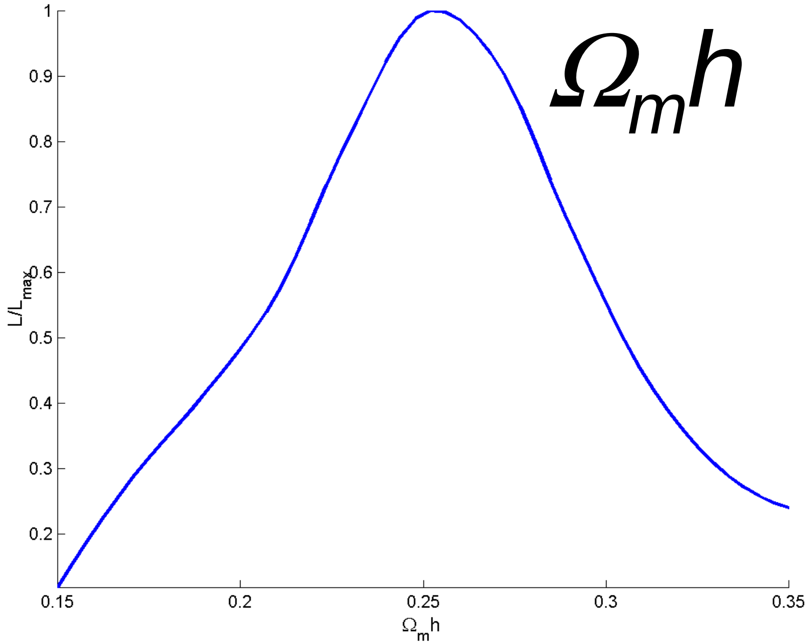


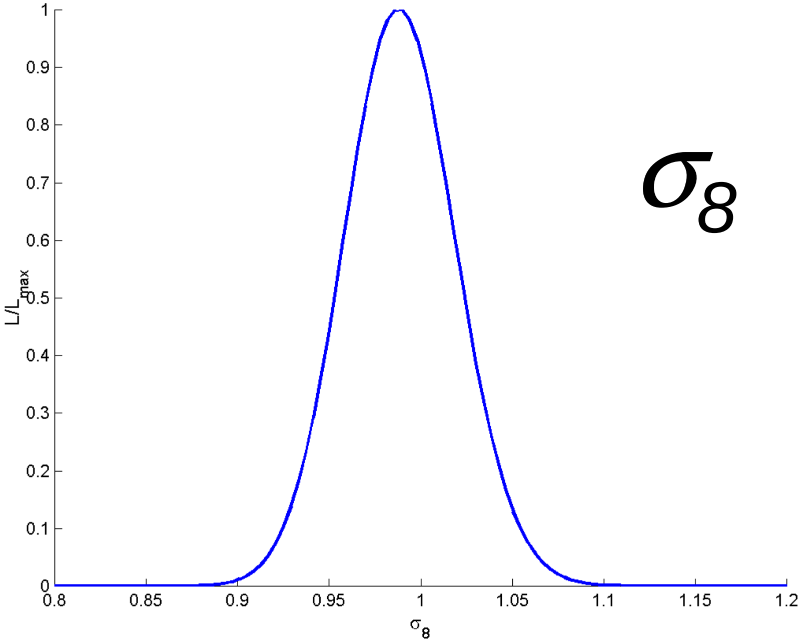


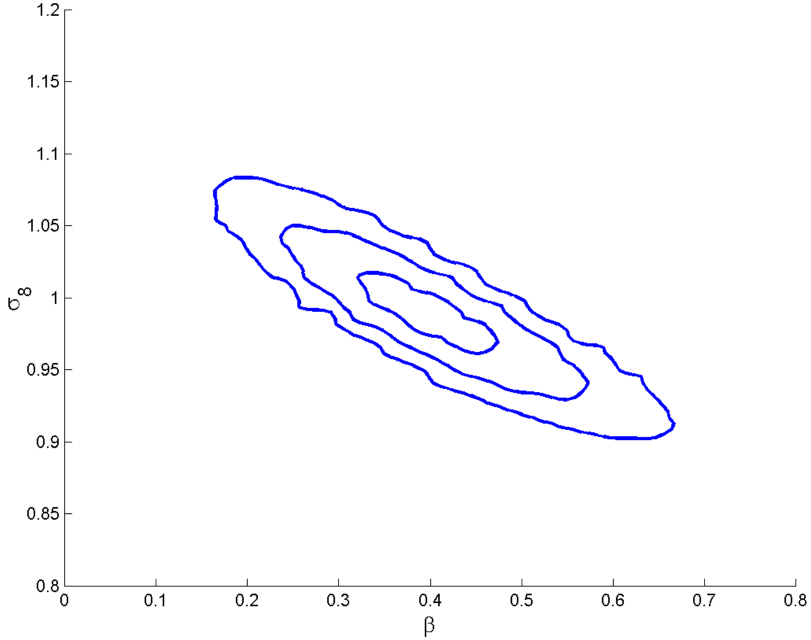
Shape

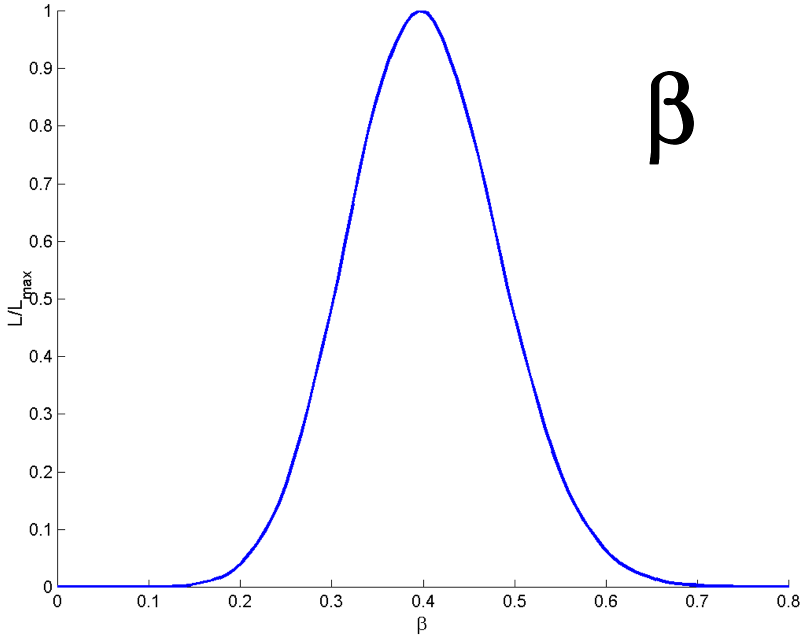
$\Omega_m h = 0.25 \pm 0.04$

$f_b \quad = 0.26 \pm 0.06$









Both depend on *b*

$\beta = 0.40 \pm 0.08$

$\sigma_8 = 0.98 \pm 0.03$

Parameter Estimates

- Values and STATISTICAL errors:

$\Omega h = 0.25 \pm 0.05$

$\Omega_b/\Omega_m = 0.26 \pm 0.06$

$\beta = 0.40 \pm 0.05$

$\sigma_8 = 0.98 \pm 0.03$

- 1σ error bars overlap with 2dF

$\Omega h = 0.20 \pm 0.03$

$\Omega_b/\Omega_m = 0.15 \pm 0.07$

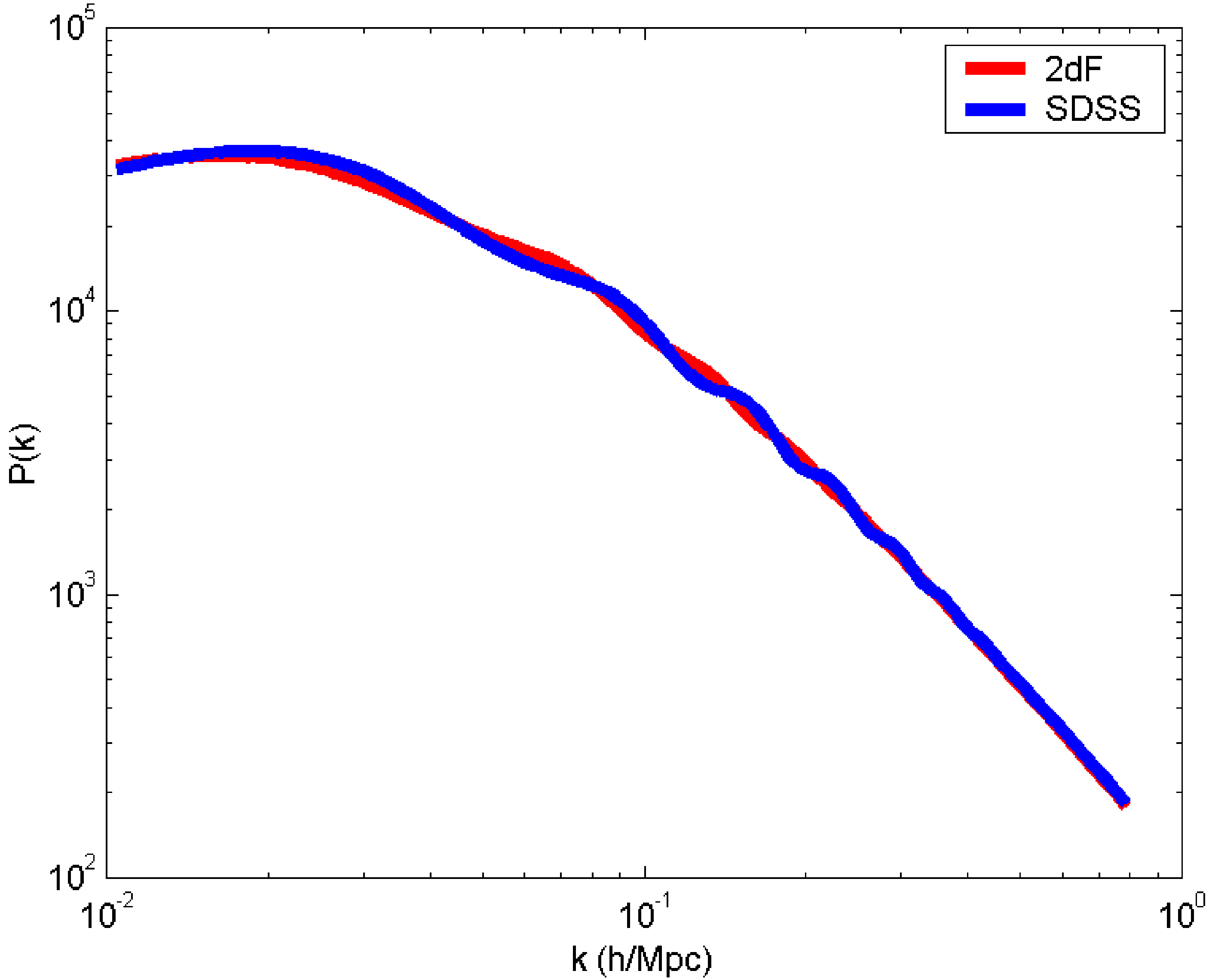
With h=0.71
$\Omega_m = 0.35$
$b = 1.33$
$\sigma_{8m} = 0.73$

Degeneracy:
$\Omega h = 0.19$
$\Omega_b/\Omega_m = 0.17$
also within 1σ

With h=0.7
$\Omega_m = 0.27$
$b = 1.13$
$\sigma_{8m} = 0.86$

WMAP
$\sigma_{8m} = 0.84$

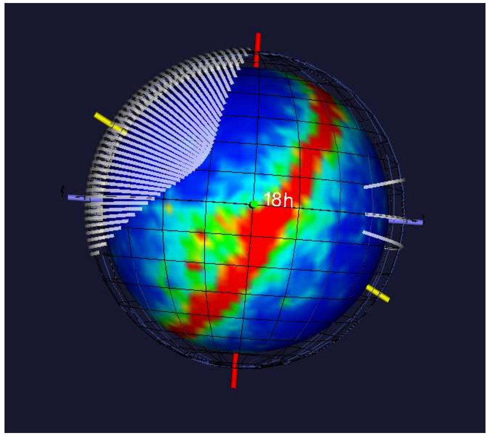
Shape of P(k)



Technical Challenges

- Large linear algebra systems
  - *KL basis: eigensystem of 15k x 15k matrix*
  - *Likelihood: inversions of 5k x 5k matrix*
- Hardware / Software
  - *64 bit Intel Itanium processors (4)*
  - *28 GB main memory*
  - *Intel accelerated, multi-threaded LAPACK*
- Optimizations
  - *Integrals: lookup tables, symmetries, 1D numerical*
  - *Minimization techniques for likelihoods*

Systematic Errors



- Main uncertainty:
  - *Effects of zero points, flat field vectors result in large scale, correlated patterns*
- Two tasks:
  - *Estimate how large is the effect*
  - *De-sensitize statistics*
- Monte-Carlo simulations:
  - *100 million random points, assigned to stripes, runs, camcols, fields, x,y positions and redshifts => database*
  - *Build MC error matrix due to zeropoint errors*
- Include error matrix in the KL basis
  - *Some modes sensitive to zero points (# of free pmts)*
  - *Eliminate those modes from the analysis => projection Statistics insensitive to zero points afterwards*

SDSS LRG Sample

- Three redshift samples in SDSS
  - *Main Galaxies*
    - 900K galaxies, high sampling density, but not very deep
  - *Luminous Red Galaxies*
    - 100K galaxies, color and flux selected
    - $m_r < 19.5$, $0.15 < z < 0.45$, close to volume-limited
  - *Quasars*
    - 20K QSOs, cover huge volume, but too sparsely sampled
- LRGs on a “sweet spot” for cosmological parameters:
  - *Better than main galaxies or QSOs for most parameters*
  - *Lower sampling rate than main galaxies, but much more volume (>2 Gpc^3)*
  - *Good balance of volume and sampling*

LRG Correlation Matrix

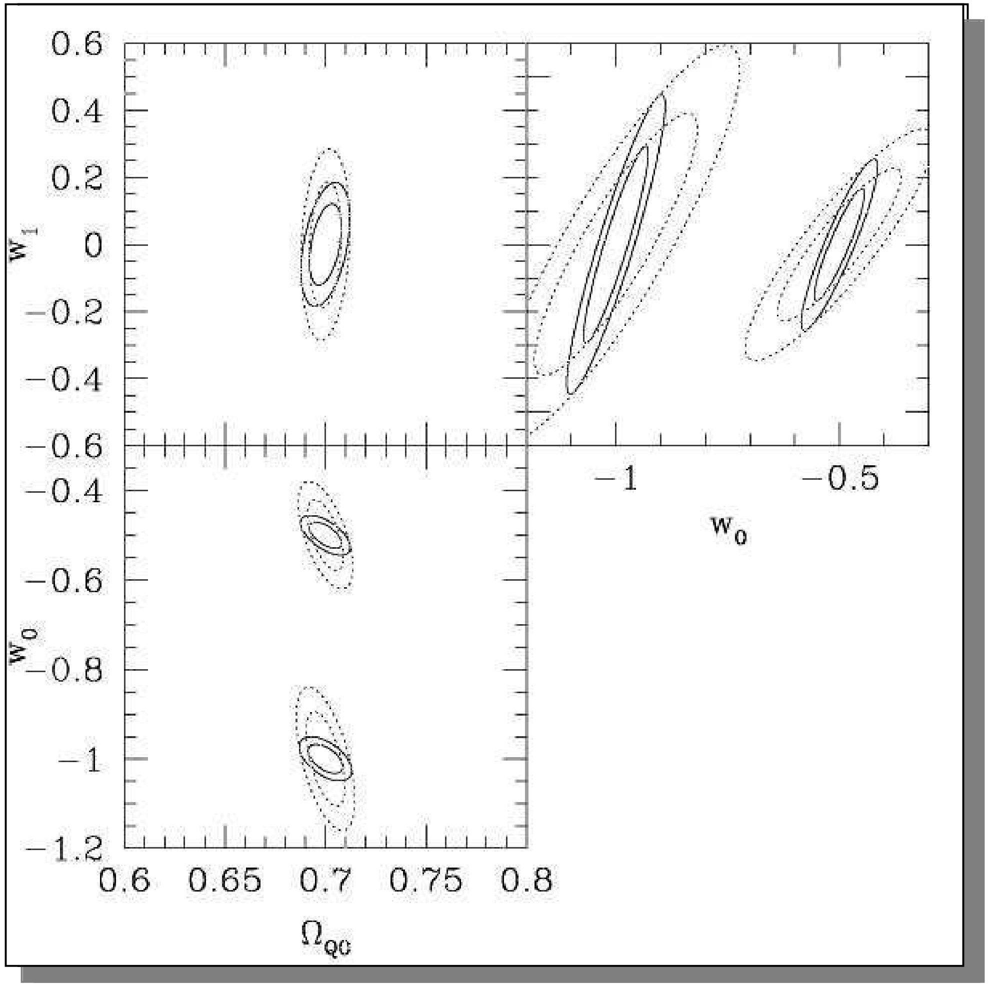
- Curvature cannot be neglected
  - *Distorted due to the angular-diameter distance relation (Alcock-Paczynski) including a volume change*
  - *We can still use a spherical cell, but need a weighting*
  - *All reduced to series expansions and lookup tables*
  - *Can fit for Ω_Λ or w!*
  - *Full SDSS => good constraints*
- β and σ_8 no longer a constant

  $\beta = \beta(z) = \Omega(z)^{0.6} / b(z)$

  - *Must fit with parameterized bias model,*
    cannot factor correlation matrix same way (non-linear)

Fisher Matrix Estimators

- SDSS LRG sample
- Can measure Ω_Λ to ± 0.05
- Equation of state: $w = w_0 + z\, w_1$



Matsubara & Szalay (2002)

Summary

- Large samples, selected on rest-frame criteria
- Excellent agreement between redshift surveys and photo-z samples
- Global shape of power spectrum understood
- Good agreement with CMB estimations
- Challenges:
  - *Baryon bumps, cosmological constant, equation of state*
  - *Possible by redshift surveys alone!*
  - *Even better by combining analyses!*
- We are finally tying together CMB and low-z