Cosmological Perturbation Theory and Wave Mechanics

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Outline

- Introduction: Standard approach: Bolztmann/Euler/BBKGY eqs
- Wave mechanical approach
- Calculations in the correspondence limit
- Results
- Future work

Standard Approach

- Based on the Liouville (collisionless Boltzmann or Vlasov) eq.

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{p}{m} - \frac{\partial f}{\partial p} m \nabla V = 0$$

- f = # of particles in a phase space volume

$$f = f(x, p, t)$$

Euler's Equations

- Arise from taking the first two velocity moments of the Vlasov Equations
- Describes the evolution until shell crossing
- Non-linear approximations: Zeldovich, Frozen Flow, Linear Potential, Adhesion
- Alternatives: Langrangian description, BBKGY equations, N-body

BBKGY Equations

- Infinite hierarchy derived from the Vlasov eq. or the N-dim Liouville eq.
- The time derivative of the N-distribution depends on the N+1 distribution
- Difficult to deal with unless the hierarchy can be cut off at a reasonable level (e.q. plasma physics)

Schrodinger Eq.

- 1-particle in expanding background

$$i\hbar\left[\dot{\psi}+\frac{3}{2}H\psi\right]+\mathbf{H}\psi=0$$

- Self-consistent non-linear equation (similar to the Vlasov description)

$$\psi=\psi(\mathrm{x},\mathrm{t}),\quad |\psi|^2\propto\rho$$

Correspondence limit

$$i\hbar\left[\dot{\psi}+\frac{3}{2}H\psi\right]+\frac{\hbar^2}{2ma^2}\nabla^2\psi-mV\psi=0$$

$$\nabla^2 V=4\pi G\bar{\rho}a^2(\left|\psi\right|^2-1),\ (\Omega=1)$$

$$\Psi\propto\int e^{-(x-x')^2/2\eta^2-\frac{ip}{\hbar}(x'-x/2)}\psi(x,t),$$

$$f(x,p,t)\propto\left|\Psi\right|^2,\quad \frac{\hbar}{p}=\lambda<<\eta<<L$$

The Basic Equations

- Using
$$\psi = \psi_0 \left(\frac{a}{a_0} \right)^{-3/2} \exp\left(A(x,t) + iS(x,t)/\hbar \right)$$

$$\dot{A} = -\frac{1}{2ma^2}\left[\nabla^2 S + 2\nabla A \nabla S \right]$$

$$\dot{S} = \frac{\hbar^2}{2ma^2}\left[\nabla^2 A + (\nabla A)^\wedge 2 \right] - \frac{1}{2ma^2}(\nabla S)^\wedge 2 - mV$$

$$\nabla^2 V = 4\pi G \bar{\rho} a^2 \left(e^{2A} - 1 \right)$$

Fourier Transform

$$S_k \Rightarrow -\frac{S_k k^2}{2ma^2 H}, \quad \Psi = (A_{k,} S_k), \text{ and } z = \ln(a)$$

$$\dot{\Psi}_A = \Omega_{AB} \Psi_B + f_A(\Psi)$$

$$\Omega = \begin{Bmatrix} 0 & 1 \\ 3/2 & -1/2 \end{Bmatrix}$$

$$f = \begin{Bmatrix} 2(kA_k)*(k/k^2 S_k) \\ k^2(k/k^2 S_k)*(k/k^2 S_k) + 3\sum_{N\geq 2} \frac{2^{N-2}}{N!}(A_k)_*^N \end{Bmatrix}$$

Linear Propagator

$$\sigma(s)^{-1} = Es - \Omega = \begin{pmatrix} s & -1 \\ -3/2 & s+1/2 \end{pmatrix}$$

$$g_{AB} = \int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i} \sigma(s) e^{sz} =$$

$$e^{\lambda_1 z} \frac{1}{2(\lambda_1 - \lambda_2)} \begin{pmatrix} 2\lambda_1 - 1 & 2 \\ 3 & 2\lambda_1 \end{pmatrix} + \left(\lambda_1 \Leftrightarrow \lambda_2\right)$$

$$\lambda_{1,2} = (1, -3/2)$$

Growing mode

$$\Psi = \sum \Psi^{(N)} e^{n\lambda_1 z}, \text{ where } \Psi^{(1)} = A^i_k (1,1)$$

$$\Psi^{(N)} = \int d^3k_1 \ldots d^3k_N \delta(k = k_1 + \ldots + k_N) \begin{pmatrix} F^N \\ G^N \end{pmatrix} A^i_{k_1} \ldots A^i_{k_N}$$

$$NF^N = G^N + 2\sum_S \alpha F^S G^{N-S}$$

$$\left(N + 1/2\right) G^N = \sum_S \beta G^S G^{N-S} + \sum_{M, s_{1,\ldots,} s_M} \frac{2^{M-2}}{M!} \delta(N = \sum s_i) F^{s_1} \ldots F^{s_M}$$

$$\alpha = \frac{q_1 q_2}{q_2^2}, \quad \beta = q^2 \frac{q_1 q_2}{q_1^2 q_2^2}, \quad F^1 = G^1 = 1$$

Tree-level PT

$$F^{2}(k_1,k_2)=\frac{3}{7}+\frac{10}{7}\alpha(k_1,k_2)+\frac{2}{7}\beta(k_1,k_2)$$

$$\delta = e^{2A}-1 \Rightarrow \delta^{(1)}=2A^{(1)}, \delta^{(2)}=2(A^{(2)}+A^{(1)2})\ldots$$

$$<\delta^{3}>=3<\delta^{(2)}\delta^{(1)2}>=\frac{34}{7}<\delta^{(1)2}>^{2}$$

- All 3rd order results agree with EPT

Cumulants-Angular Averaging

$$\nu_N = N! < F^N >, \mu_N = N! < G^N >,$$

$$< \alpha >= 0, < \beta >= 0$$

$$N \nu_N = \mu_N$$

$$\nu_N = \frac{2}{(2N+1)N-3} \sum_{s=1}^{N-1} \frac{2}{3} \binom{N}{s} s(N-s) \nu_s \nu_{N-s} +$$

$$3N! \sum_{M \geq 2} \frac{2^{M-2}}{M!} \sum_{s_1, \ldots, s_M} \delta(N = \sum s_i) \prod_j \frac{\nu_j}{s_j!}$$

Cumulants in log-space

$$\nu_2 = \frac{26}{21}, \quad \nu_3 = \frac{568}{189}, \quad \nu_4 = \frac{473744}{43659}$$

$$S_N^A = \langle A^N \rangle / \langle A^2 \rangle^{N-1}$$

$$S_3^A = 3\nu_2 = \frac{26}{7}$$

$$S_4^A = 4\nu_3 + 12\nu_2^2 = \frac{40240}{1323}, \quad S_5^A = \frac{119609680}{305613}$$

Tree-level projection to δ-space

- Fry & Gaztanaga (1993)

$$\delta = e^{2A} - 1 = \sum b_k \frac{A^k}{k!},$$

$$\text{with } b = 2, c_k = b_k / b = 2^{N-1}$$

$$S_3 = b^{-1}(S_3^A + 3c_2) = \frac{34}{7}$$

$$S_4 = b^{-2}(S_4^A + 12c_2 S_3^A + 4c_3 + 12c_2^2) = \frac{60712}{1323}$$

$$S_5 = \frac{200575880}{305613}$$

Higher order projection

- Variance

$$\sigma^2 = 4 < A^2 > + 4 < A^2 >^2 (2 S_3^A + 6) + \ldots$$

from which

$$\sigma^2 = \sigma_L^2 + \frac{S_3^A}{2} \sigma_L^4 + \ldots$$

$$S_3^A / 2 = 13/7 = 1.857$$

EPT loop corrections: 1.82

Effects of smoothing

$$\text{Using } S_3^A \cong 26/7 - 2(n+3)$$

$$\sigma^2 = \sigma_L^2 + (-1.14 - n)\sigma_L^4 + \ldots$$

n	Tree-SPT	1-Loop PT
-3	1.86	1.82
-2	0.86	0.88

Summary

- Introduced SPT
- The log of the field naturally arises
- Agreement with EPT at tree level
- WKB limit equations at tree level give 98% of 1-loop corrections of EPT
- PT kernels and cumulants were given through a recursion which will be the basis of all future calculations

Future work

- Quantum corrections/shell crossing
- Smoothing/higher order incomplete loop corrections
- General cosmological background
- Data/ simulation analysis in log-space
- Non-perturbative techniques
- Applications: N-point correlations/N-1 spectra, PDF, genus, velocities, lensing, etc.