Neutrinos in the Universe

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in memory of
György Marx
(1926 - 2002)

Eötvös Cosmo Course, Balatonfüred
27 June 2003

How far back can we look into the past?

→ Optical photons : Hubble deep field
$z \sim \mathcal{O}(1)$, $t \sim 1$ billion years

→ Microwave photons : Last scattering surface
$z \sim 1000$, $t \sim 300000$ years

→ Neutrinos : Decoupling
$z \sim 10^{12}$, $t \sim 1$ sec

... deepest probe of material universe

→ coincides with primordial nucleosynthesis era – boundary of standard cosmology

most abundant particles in the universe

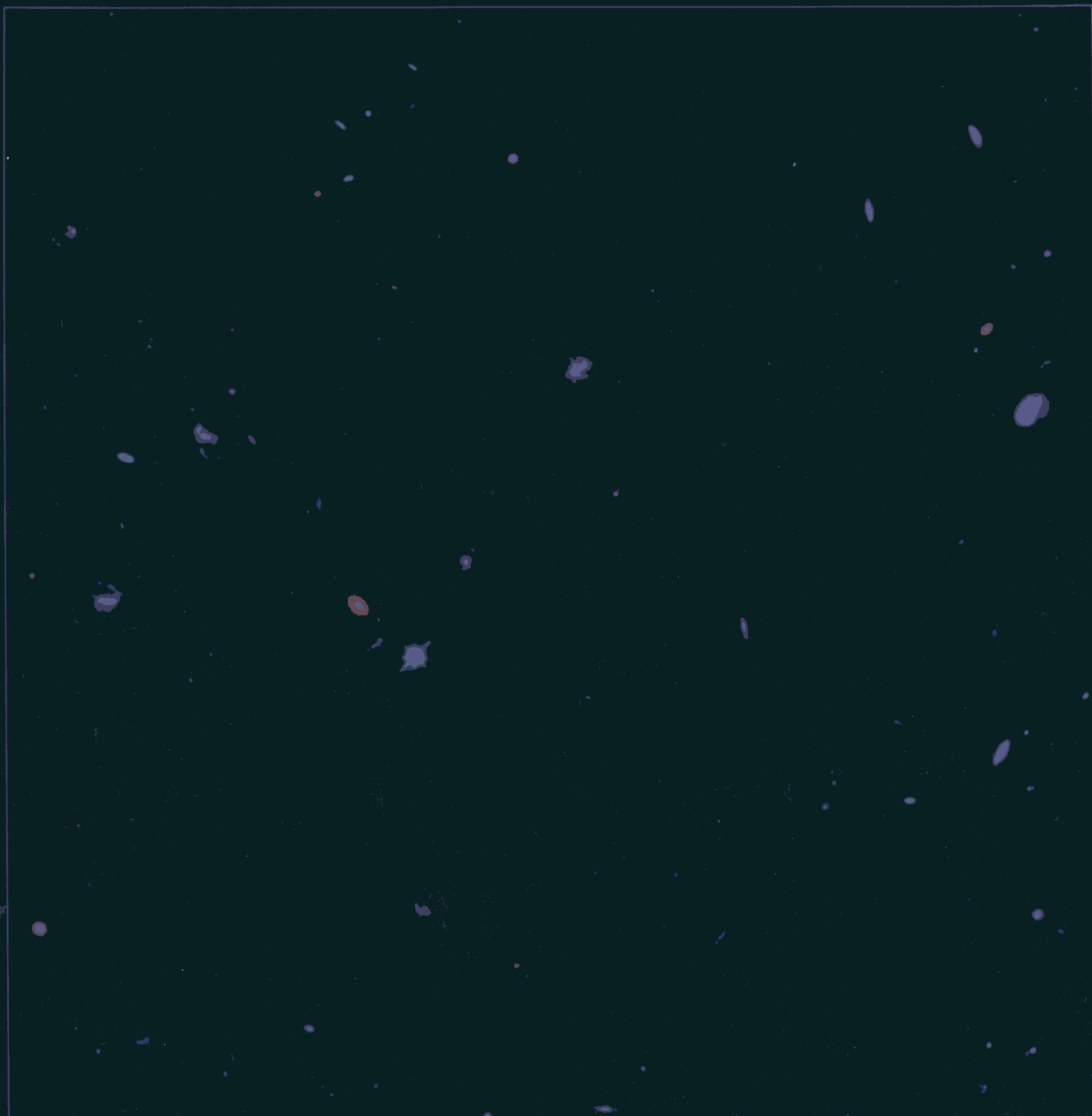
Can they constitute the dark matter in galaxies?
(Cowsik & McClleland 1972, Szalay & Marx 1972)

Pauli exclusion principle ⊕ Liouville's theorem require:

$$m_\nu > 120\ \text{eV} \left(\frac{\sigma}{100\ \text{km s}^{-1}}\right)^{-1/4} \left(\frac{r_{ce}}{\text{kpc}}\right)^{-1/2}$$

So neutrinos of mass $\mathcal{O}(\text{eV})$ will not cluster but can constitute 'dark energy'

→ probe through observations of large-scale structure



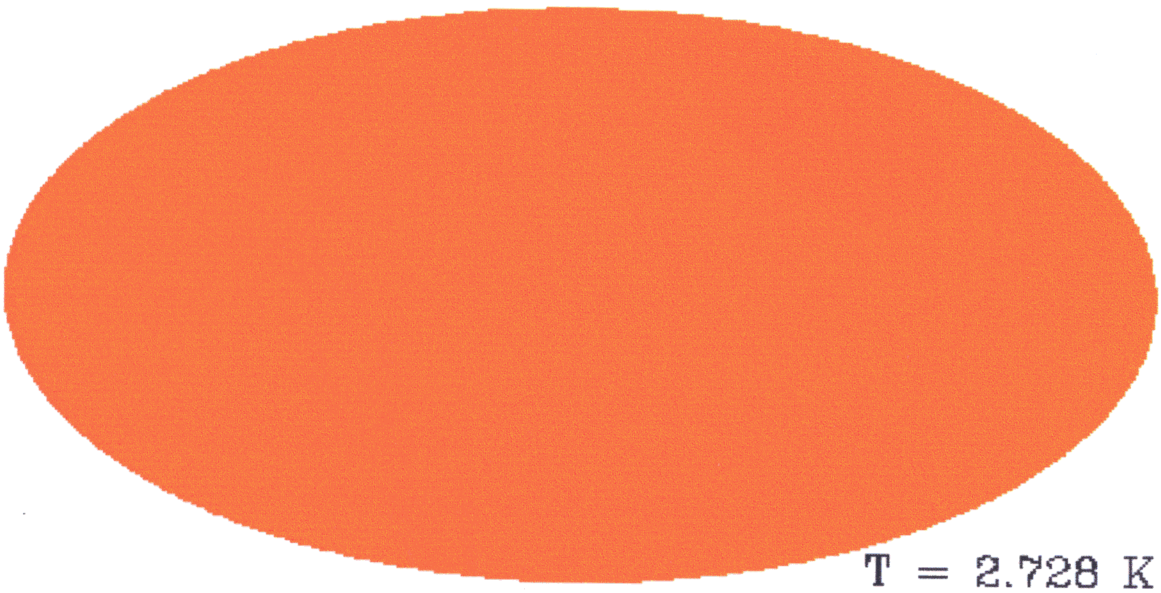
Hubble Deep Field

Hubble Space Telescope • WFPC2

PRC96-01a • ST ScI OPO • January 15, 1995 • R. Williams (ST ScI), NASA

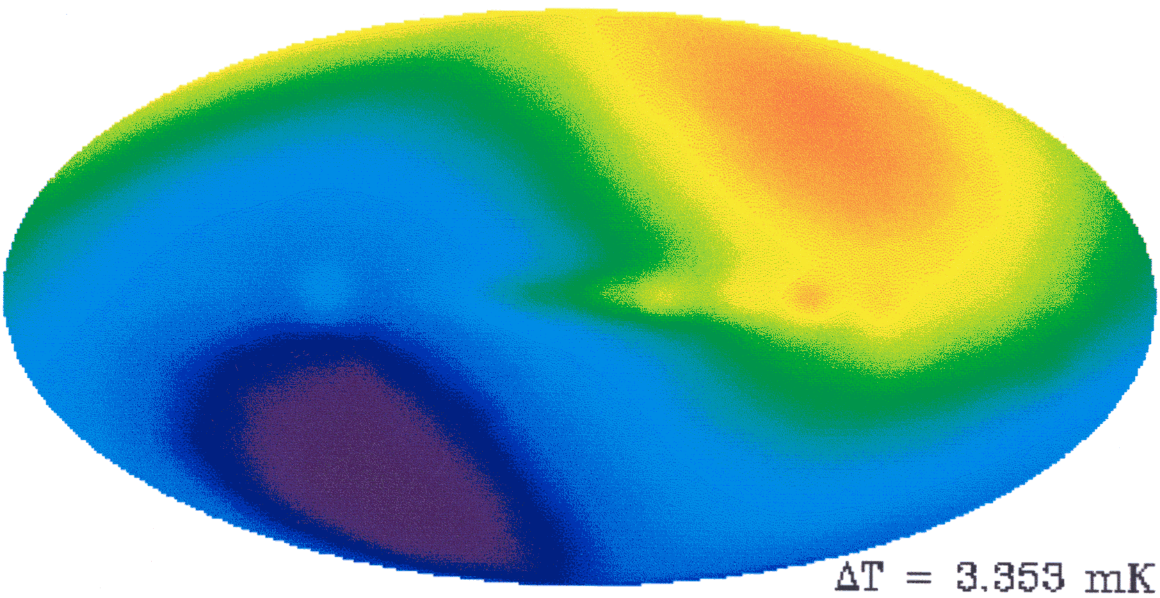
COBE DMR Microwave Sky at 53 GHz

monopole



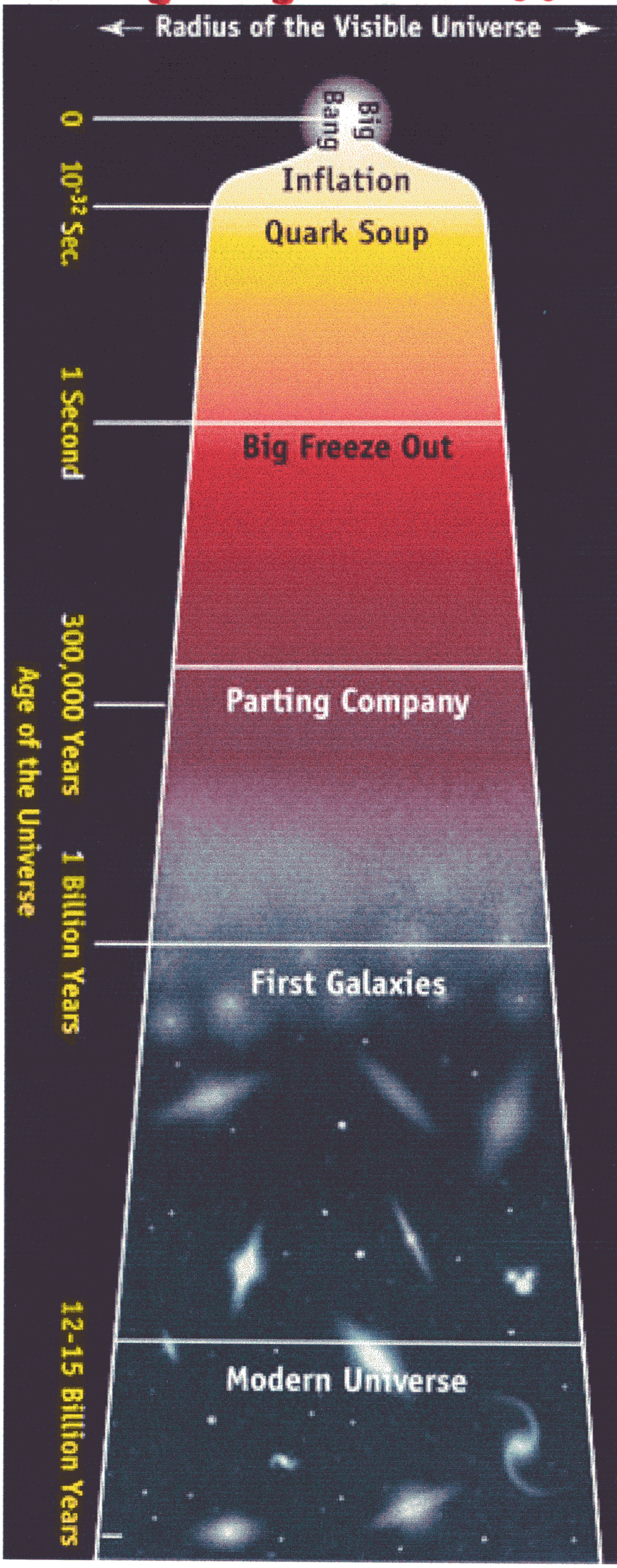
$T = 2.728$ K

dipole



$\Delta T = 3.353$ mK

The Big Bang Cosmology



initial singularity?
homogeneity/isotropy?
phase transitions?
baryogenesis?
dark matter?
dark energy?

nucleosynthesis of light elements

decoupling of microwave background

formation of structure

Neutrino decoupling occurs when the interaction rate $\Gamma \sim n \langle \sigma v \rangle$ (with $n \sim T^3$, $\langle \sigma v \rangle \sim G_F^2 T^2$) falls behind the Hubble expansion rate $H \sim \sqrt{G_N \rho}$ (with $\rho \sim T^4$)

at $T_{dec} \sim (G_N^{1/2} G_F^{-2})^{1/3} \sim \mathcal{O}(\text{MeV})$

(A precise calculation gives $T_{dec}(\nu_e) \simeq 3.1$ MeV, $T_{dec}(\nu_\mu, \nu_\tau) \simeq 2.1$ MeV)

→ at decoupling $n_\nu + n_{\bar\nu} = \frac{3}{4} n_\gamma$

→ after e^+e^- annihilation $= \frac{4}{11} \cdot \frac{3}{4} n_\gamma$

Thus today: $n_\nu + n_{\bar\nu} = \frac{3}{11} \frac{2\zeta(3)}{\pi^2} \left(\frac{T_0}{2.728\text{K}}\right)^3 \simeq 112 \text{ cm}^{-3}$

so if neutrinos have mass then $\Omega_\nu = \frac{m_\nu n_\nu}{\rho_{crit}} = \frac{\sum_i (m_{\nu_i}/\text{eV})}{93 h^2}$

i.e. $m_\nu \sim 0.07$ eV implies $\Omega_\nu \sim 0.002$

cf. $\Omega_{luminous} \sim 0.006$ } for $h = 0.65$

For massless ($m_\nu < T_0$) neutrinos, $f(p,T) = f^{eq} = \left[e^{E_\nu/T_\nu} + 1\right]^{-1}$

with present temperature $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \simeq 1.9$ K

More precisely, must solve $\left[\frac{\partial}{\partial t} - H p \frac{\partial}{\partial p}\right] f(p,T) = I^{\text{collisions}}$

... the energy dependence of the scattering # section implies the spectral distortion: $\frac{f - f^{eq}}{f^{eq}} \simeq 3 \times 10^{-4} \frac{E}{T} \left(\frac{11}{4}\frac{E}{T} - 3\right)$ for ν_e

$\Rightarrow \delta\rho_{\nu_e} / \rho_{\nu_e} \simeq 0.9\%$

{ Dolgov & Fukugita '92
Dodelson & Turner '92

... and ~twice as weak an effect for $\nu_{\mu,\tau}$

ORDER OF MAGNITUDE EFFECTS FOR ν EXPERIMENTS

EXPERIMENTS AT REACTORS

$\bar{\nu}_e e \to \bar{\nu}_e e$ $\qquad$ $\sigma \sim 10^{-44}\,\mathrm{cm}^2$

$F_{\bar{\nu}} \sim 10^{13}\,/\,\mathrm{cm}^2\,/\,\mathrm{sec}$

$$\begin{aligned} \text{RATE} &\sim \sigma \cdot F \cdot N \\ &\sim 10^{-44} \times 10^{13} \times N \simeq N \cdot 10^{-31} \\ N &\sim 10^{30} \quad \text{for 1 TON} \\ \text{RATE} &\sim 10^{-1}\,\mathrm{sec}^{-1} \implies 10^{6}\,/\,\text{year} \end{aligned}$$

RELIC NEUTRINOS

$\nu_e + e \to \nu_e + e$ $\qquad$ $\sigma \sim 10^{-62}\,\mathrm{cm}^2$

$E_e \simeq 10^{-4}\,\mathrm{eV}$ $\qquad$ $F_\nu \sim 10^{12}/\mathrm{cm}^2/\mathrm{sec}$

$$\text{RATE} \sim 10^{-62} \times 10^{12} \cdot N = 10^{-50} \cdot N$$

$$\text{For RATE} \sim 10^{3}\,/\,\text{year}\ N = 10^{44} \quad [\sim\sim 10^{14}\ \text{TONS}]$$

TWO WAYS TO IMPROVE $\sigma \to 10^{-44}\,\mathrm{cm}^2$ MAGNETIC MOMENT

OR

COHERENT EFFECT

$$R \sim \sigma \cdot F \cdot N^2 \simeq 10^{-50} N^2$$

(Smith 1983)

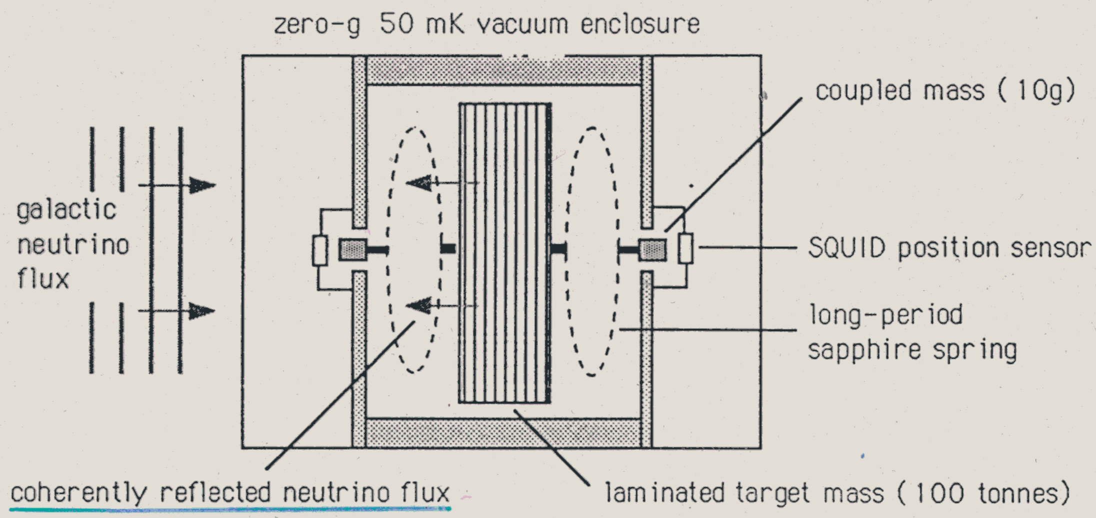


Fig 4.1 Hypothetical Galactic neutrino detector based on measurement of macroscopic forces from coherent reflection.

$$\text{acceleration} = 8\times10^{-24}\ \frac{\text{cm}}{\text{sec}^2}\left(\frac{A-Z}{A}\right)^2\left(\frac{v_{sun}}{10^{-3}c}\right)^2\left(\frac{n_\nu}{10^7\,\text{cm}^{-3}}\right)\left(\frac{\rho}{20\ \text{gm cm}^{-3}}\right)$$

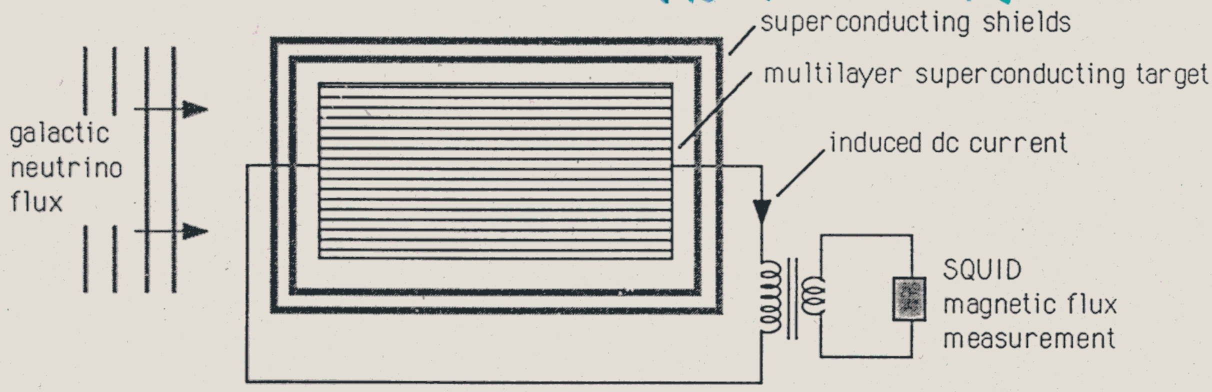


Fig 4.2 Hypothetical Galactic neutrino detector based on coherent momentum transfer to superconducting electrons.

(Smith & Lewin 1983)

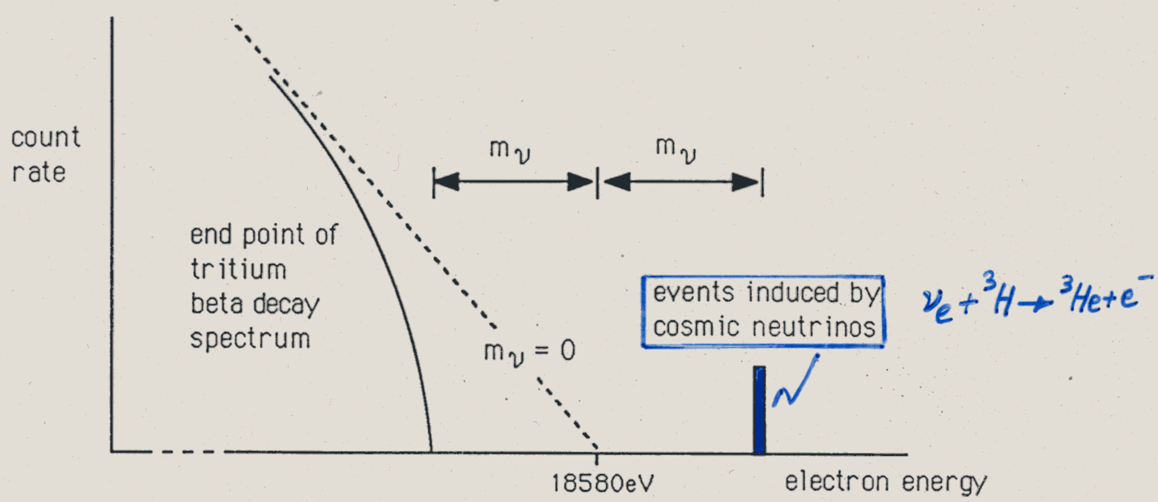


Fig 4.3 Possible detection principle for Galactic neutrinos based on induced beta decay in tritium (from [4.5]).

(Weinberg 1962)

Present status

Anomaly	Solar	Atmospheric
first hint	1968	1986
confirmed	2002	1998
evidence	9σ	17σ
for	$\nu_e \to \nu_{\mu,\tau}$	$\nu_\mu \to \nu_\tau$
seen by	Cl,2Ga,SK,SNO,KL	SK,Macro, K2K
disappearance	seen	seen
appearance	seen	partly seen
oscillations	not yet	partly seen
$\sin^2 2\theta$	0.86 ± 0.04	1.00 ± 0.04
Δm^2	$(7.1 \pm 0.6)10^{-5}\,\mathrm{eV}^2$	$(2.7 \pm 0.4)10^{-3}\,\mathrm{eV}^2$
sterile?	5σ disfavoured	7σ disfavoured

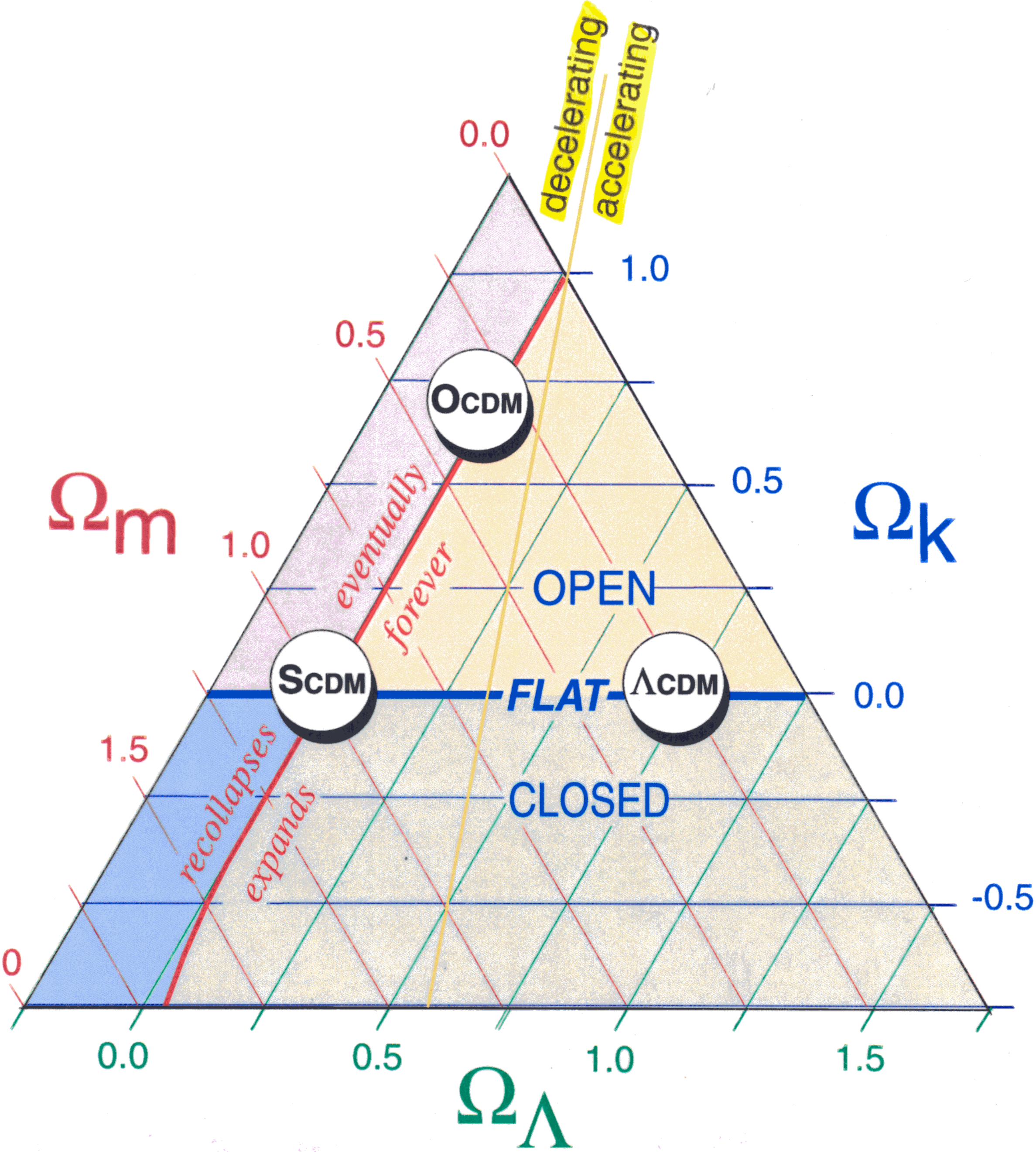
(extra unconfirmed hints from LSND, $0\nu2\beta$, NuTeV, GZK)

Strumia, CAPP 2003

The Cosmic Triangle

Sum rule: $\Omega_m + \Omega_K + \Omega_\Lambda = 1$

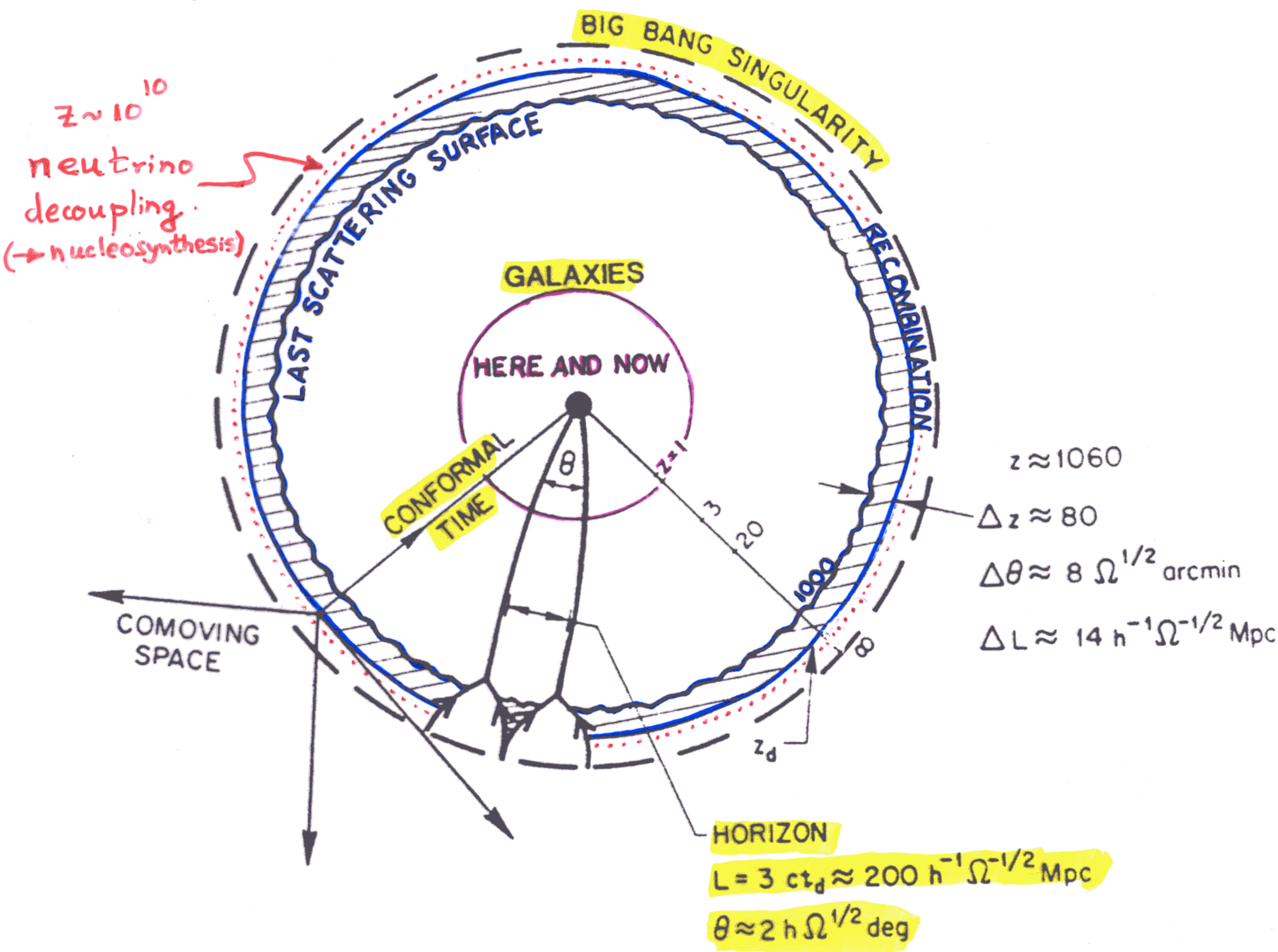
$\rho_m / \frac{3H_0^2}{8\pi G}$ $\quad -K/a_0^2 H_0^2$ $\quad \Lambda/3H_0^2$



Bahcall et al.
(astro-ph/9906463)

The standard cosmological model

... maximally symmetric (simply connected) space-time containing 'ideal fluids' (dust, radiation, ...)



conformal time : $d\tau \equiv \dfrac{dt}{a(t)}$, $1+z \equiv \dfrac{\lambda_0}{\lambda_{em}} = \dfrac{a(t_0)}{a(t_{em})}$

FRW metric : $ds^2 = -dt^2 + a^2(t)\left[\dfrac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$

Einstein equations : $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$\Rightarrow \quad H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G\rho_m}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} = H_0^2\left[\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda\right]$$

Since $\Delta m^2_\odot \simeq 7 \times 10^{-5}\,\mathrm{eV}^2$ and $\Delta m^2_{atm} \simeq 3 \times 10^{-3}\,\mathrm{eV}^2$

... can assume all 3 masses to be degenerate

for $\sum m_\nu \gtrsim 0.4\,\mathrm{eV}$

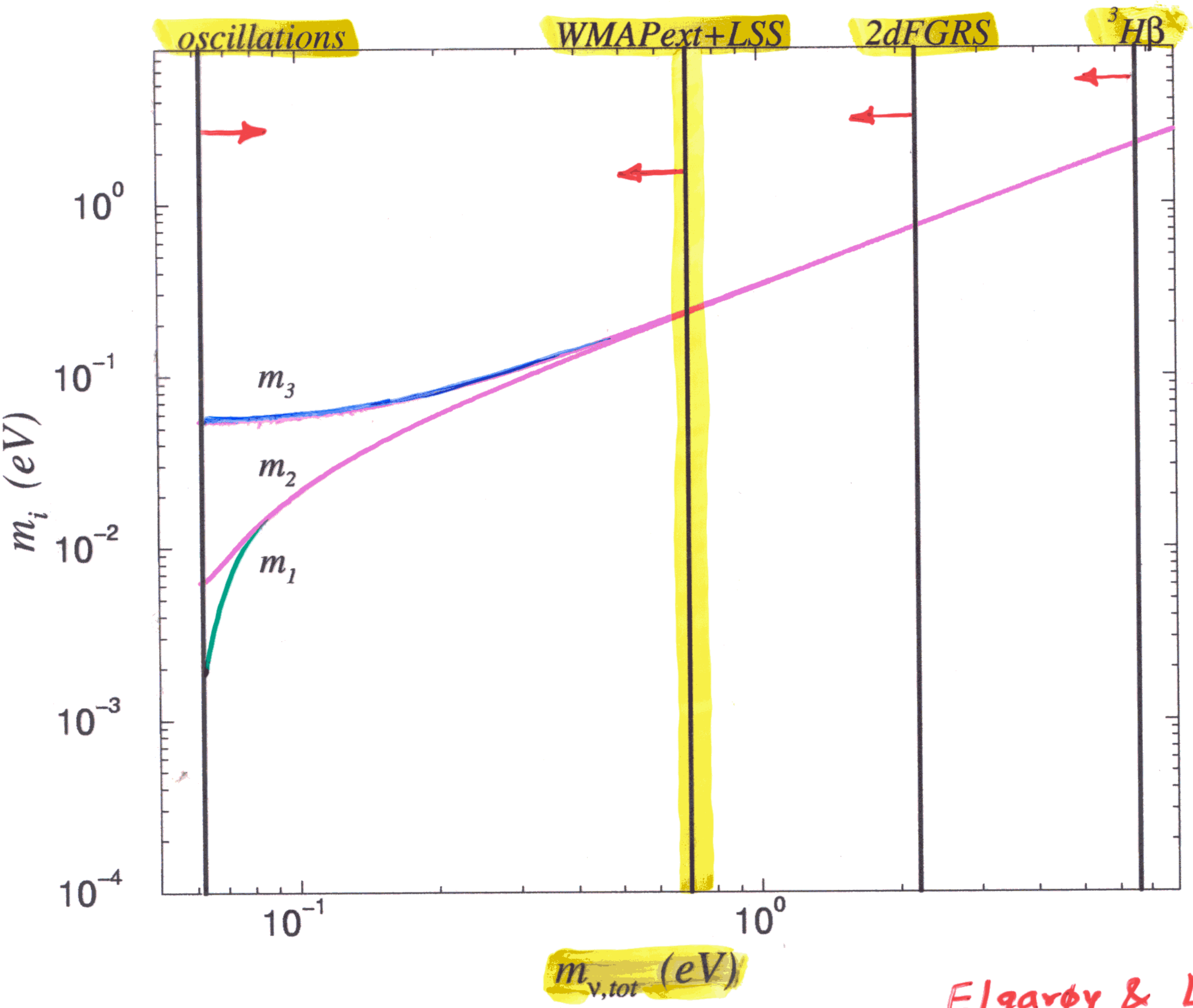
Laboratory bound from ^3H β-decay : $\sum m_\nu < 6.6\,\mathrm{eV}$ @ 95% c.l. (Troisk + Mainz)

cf.

Cosmological bound from observations of large-scale structure : $\sum m_\nu < 2.2\,\mathrm{eV}$ (2dFGRS)

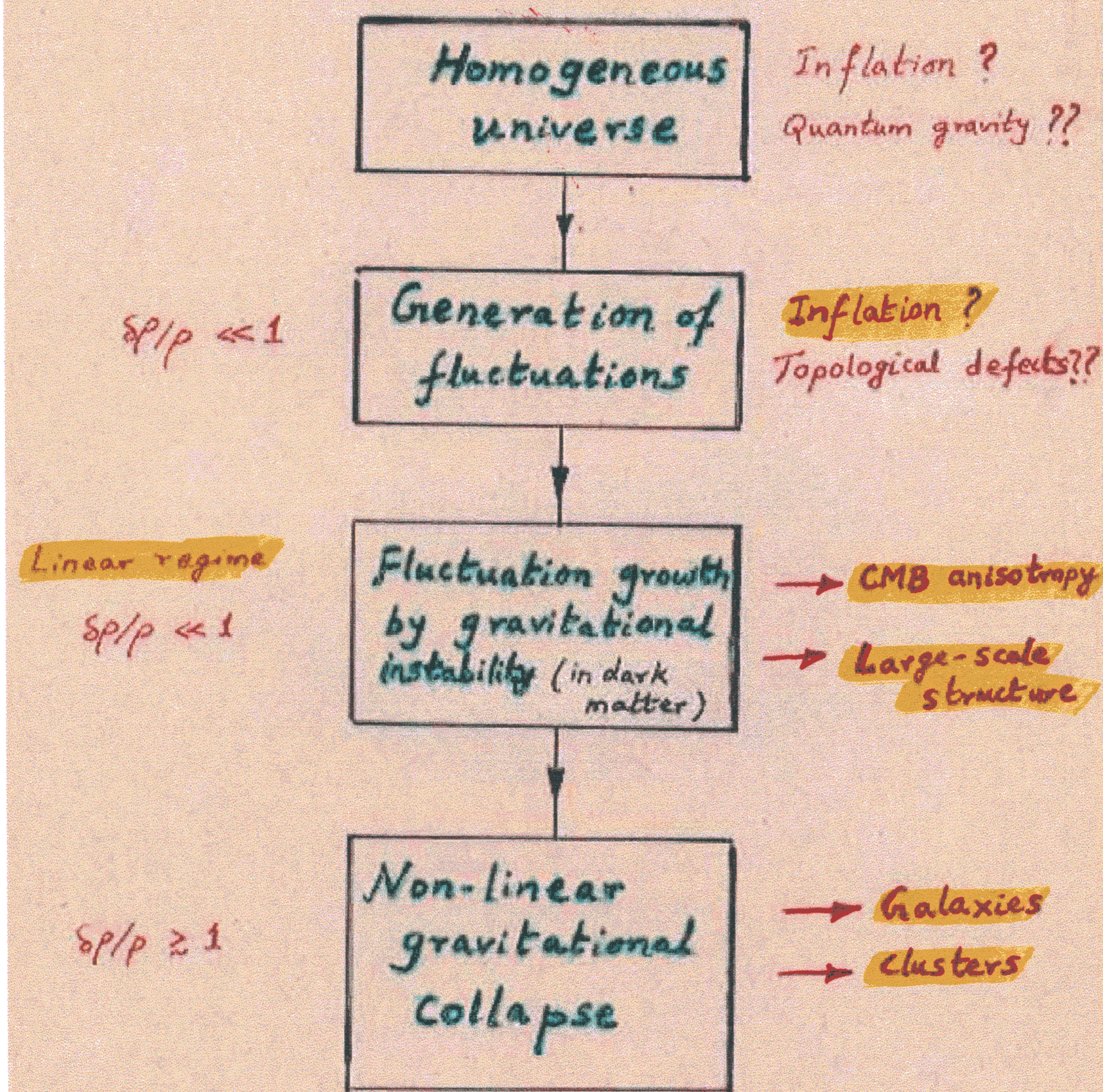
⋮

$< 0.71\,\mathrm{eV}$ ("WMAP")

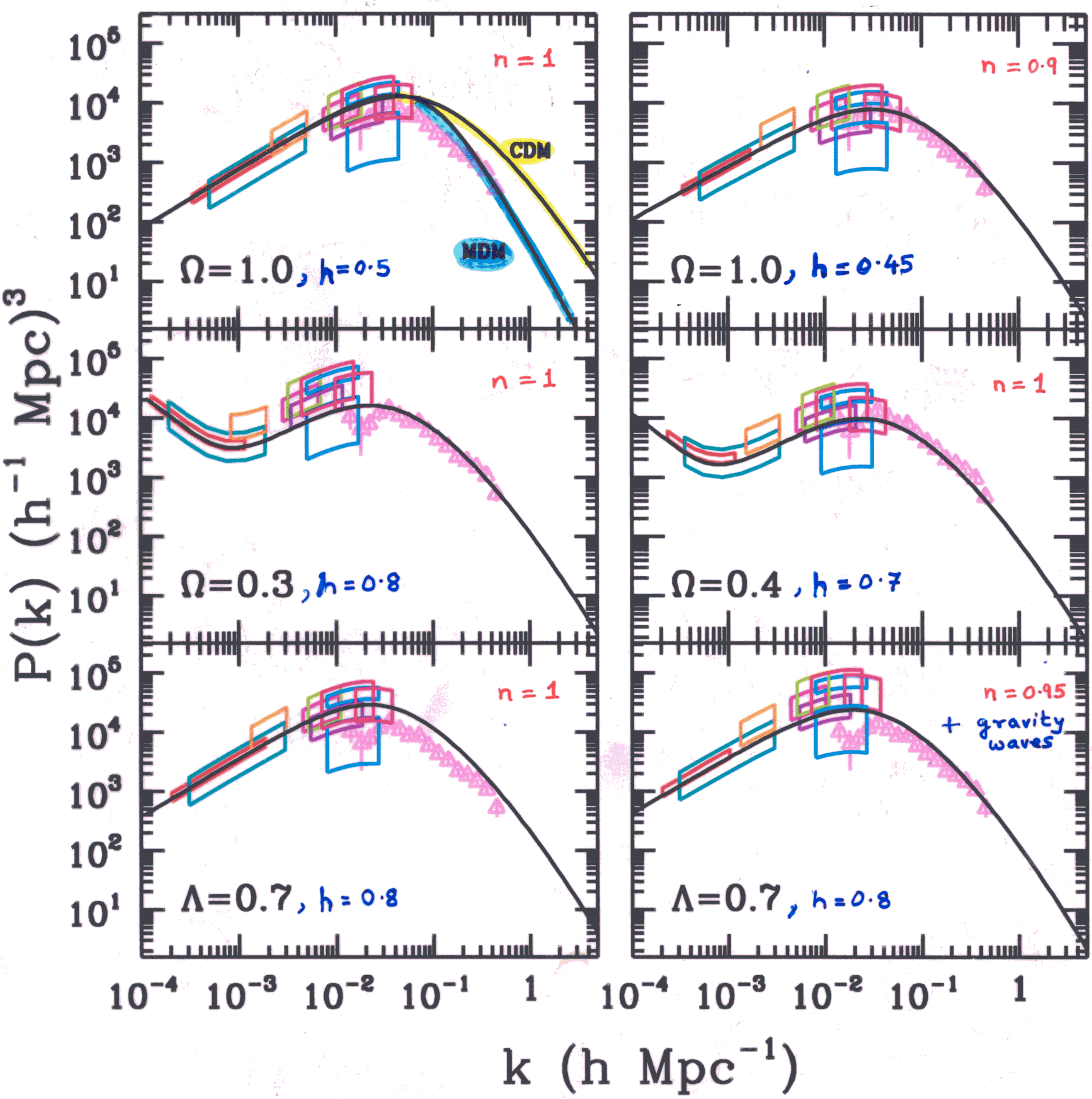


Elgarøy & Lahav (astro-ph/0303089)

Formation of Structure in the Universe

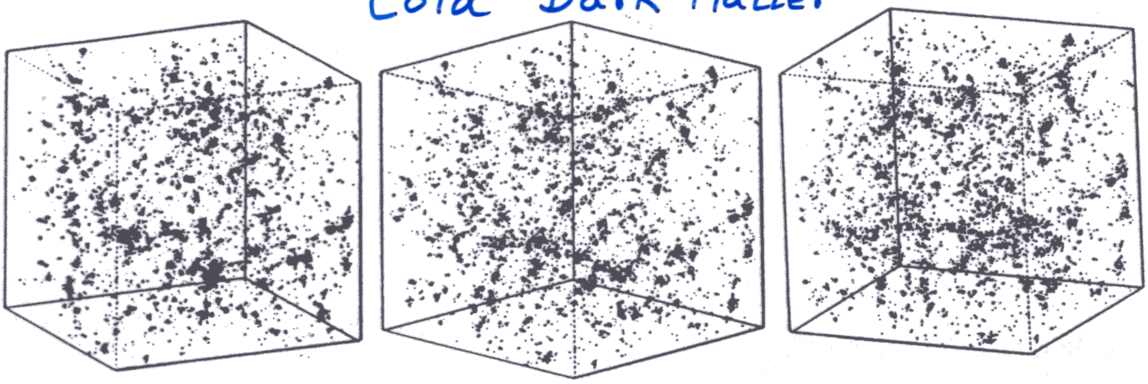


The matter power spectrum for modified CDM models

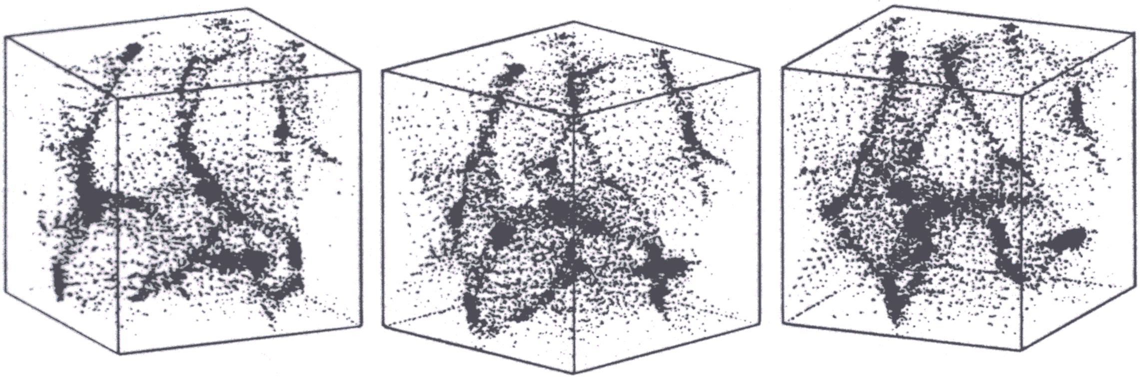


Scott, Silk, White
(astro-ph/9505015)

Cold Dark Matter



Hot Dark Matter



Computer simulations of structure formation in the cold dark-matter (top) and hot dark-matter (bottom) scenarios (assuming random overdensities act as the seeds). Galaxies form first and cluster later in cold dark-matter models; with hot dark matter, by contrast, clustering occurs first at large scales, followed later by fragmentation and galaxy formation.

$$\frac{\delta\rho(\tilde{x},t)}{\rho} = \frac{1}{(2\pi)^3}\int d^3k\;\delta_{\tilde{k}}(t)\,e^{-i\tilde{k}\cdot\tilde{x}} \quad ; \quad \langle \delta_{\tilde{l}}\,\delta_{\tilde{m}} \rangle = \langle |\delta_k|^2 \rangle (2\pi)^3\,\delta^{(3)}(\tilde{l}-\tilde{m})$$

Plane-wave expansion

$$\langle |\delta_k|^2 \rangle \equiv P(k) = Ak^n$$

$n=1 \Rightarrow$ Scale-invariant

Harrison-Zeldovich Spectrum

$\rightarrow$ "expected" from inflation

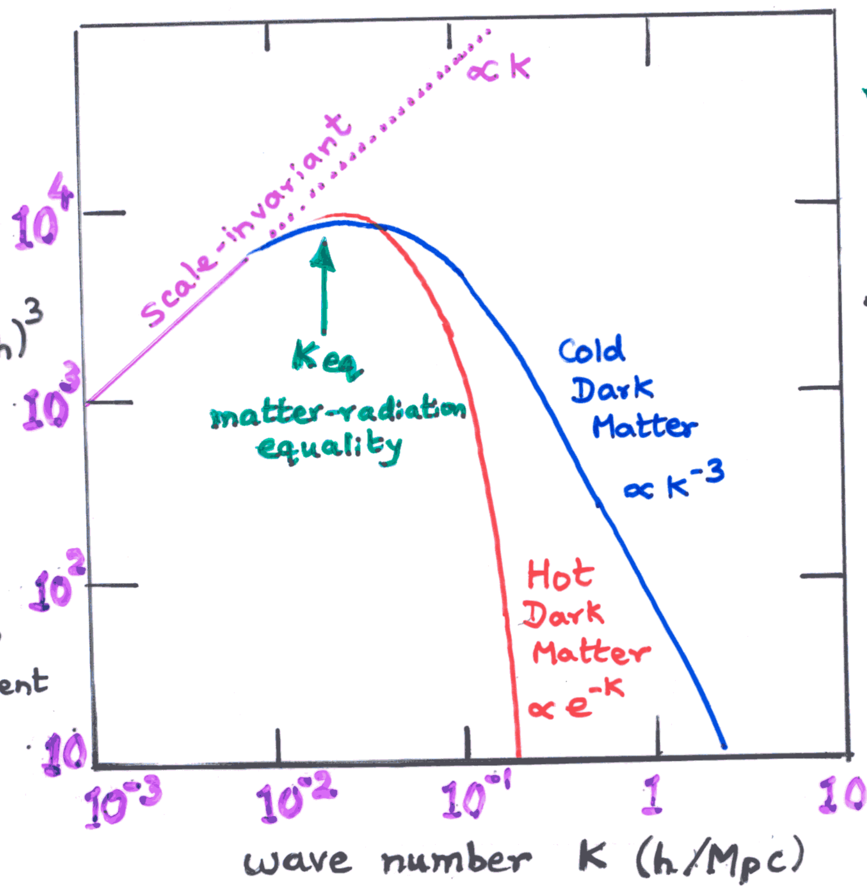
Power

$P(k)\ (\mathrm{Mpc}/h)^3$

$= Ak^n \otimes T^2(k)$

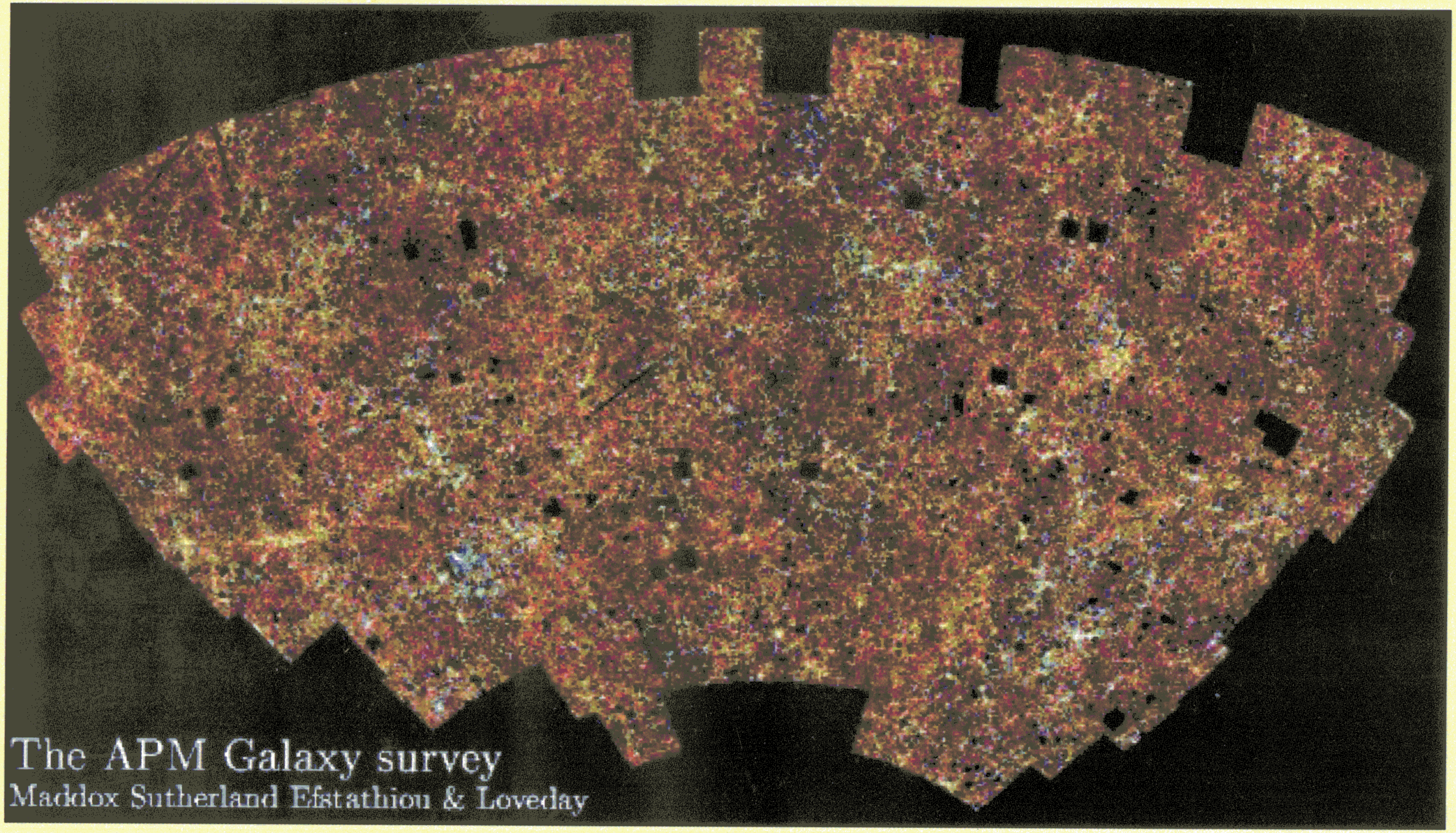
"transfer fn."

...dependent on (dark) matter content

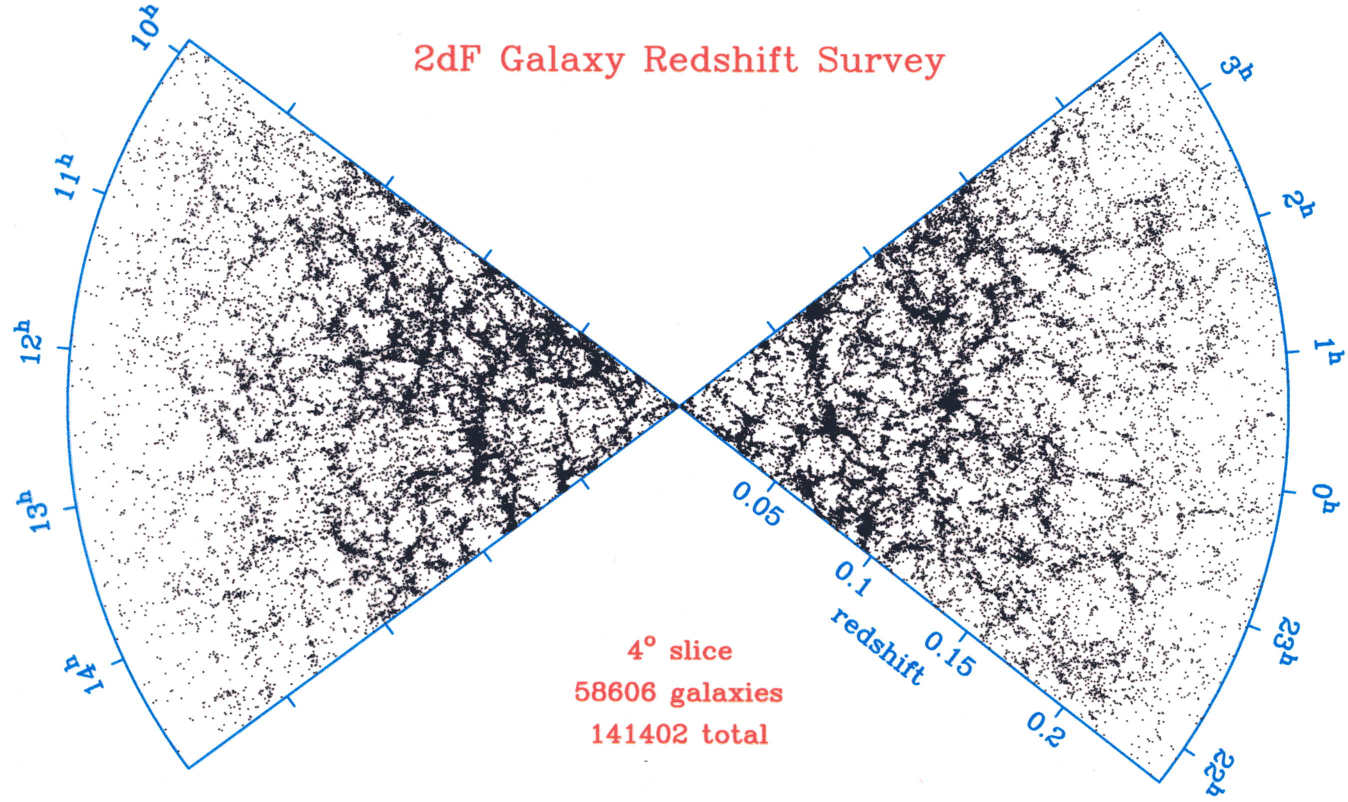


2dFGRS input catalogue

- Galaxies: $b_J \leq 19.45$ from revised APM



- Total area on sky ~ 2000 deg^2
- 250,000 galaxies in total, 93% sampling rate


2dF Galaxy Redshift Survey
10h
11h
12h
13h
14h
3h
2h
1h
0h
23h
22h
0.05
0.1
0.15
0.2
redshift
4° slice
58606 galaxies
141402 total

Bound on neutrino mass from 2dF galaxy survey

$$f_\nu \equiv \frac{\Omega_\nu}{\Omega_m} < 0.16 \quad @ 95\% \text{ c.l.}$$

$$\Rightarrow \sum m_\nu < 2.2 \text{ eV} \qquad (\text{for } \Omega_m h^2 = 0.15)$$

... given the 'priors': $h = 0.7 \pm 0.07$; $\Omega_b h^2 = 0.02 \pm 0.002$

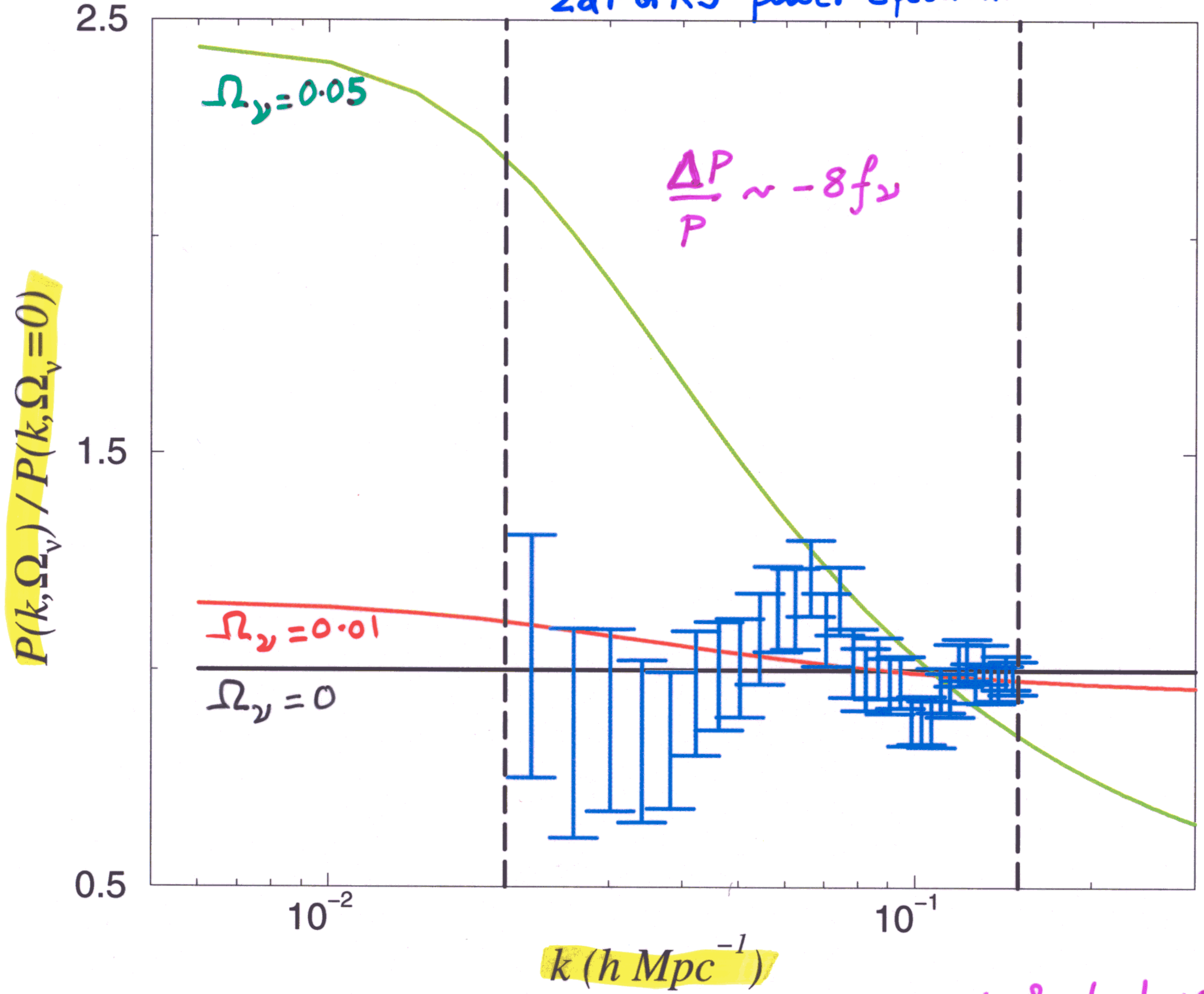
$0.1 < \Omega_m < 0.5$, $n = 1.0 \pm 0.1$

→ without any priors, the bound is relaxed to

$$f_\nu < 0.24$$

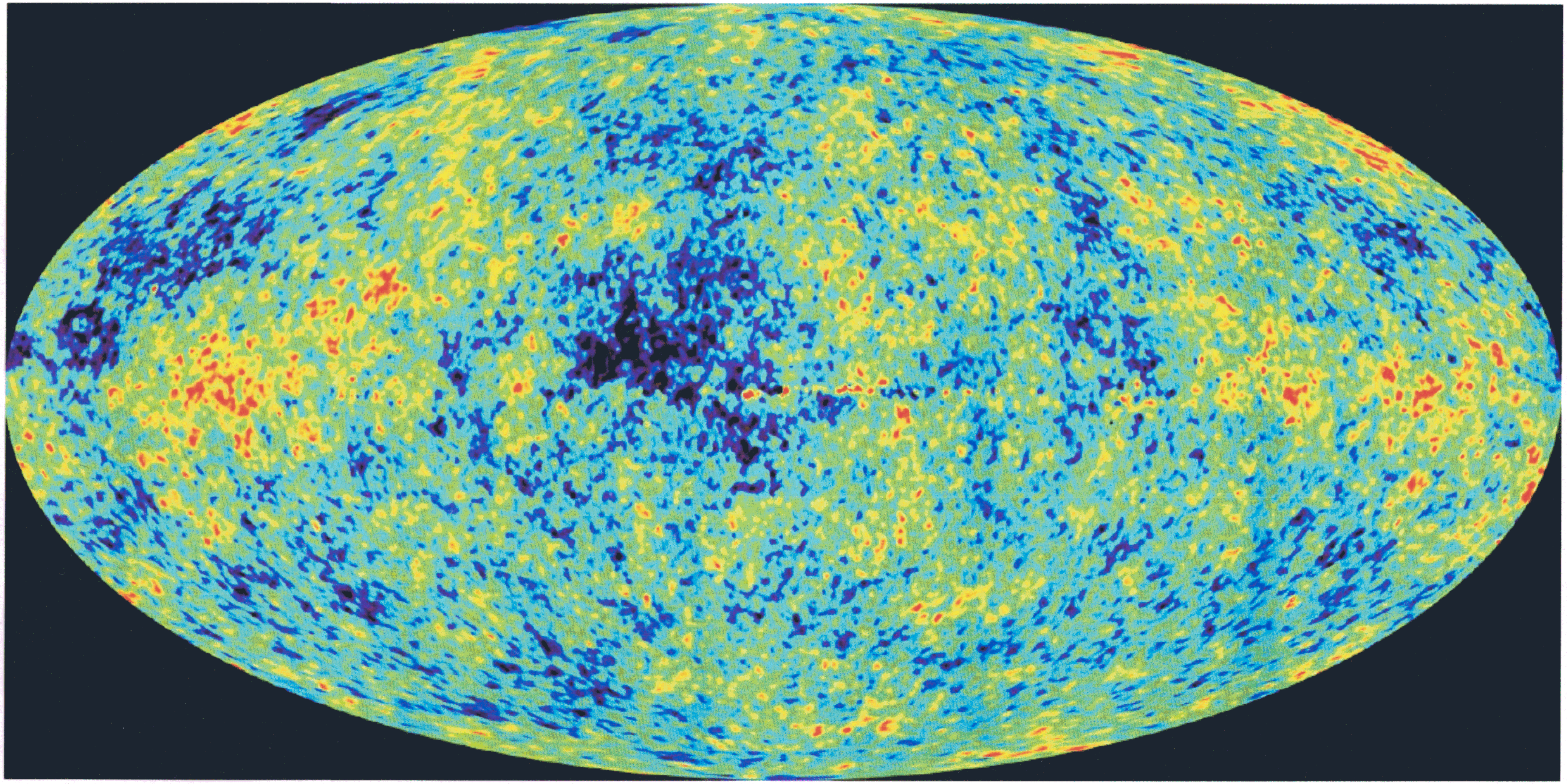
i.e. neutrinos can make up upto a quarter of dark matter → possibly important effect on dynamics

2dFGRS power spectrum



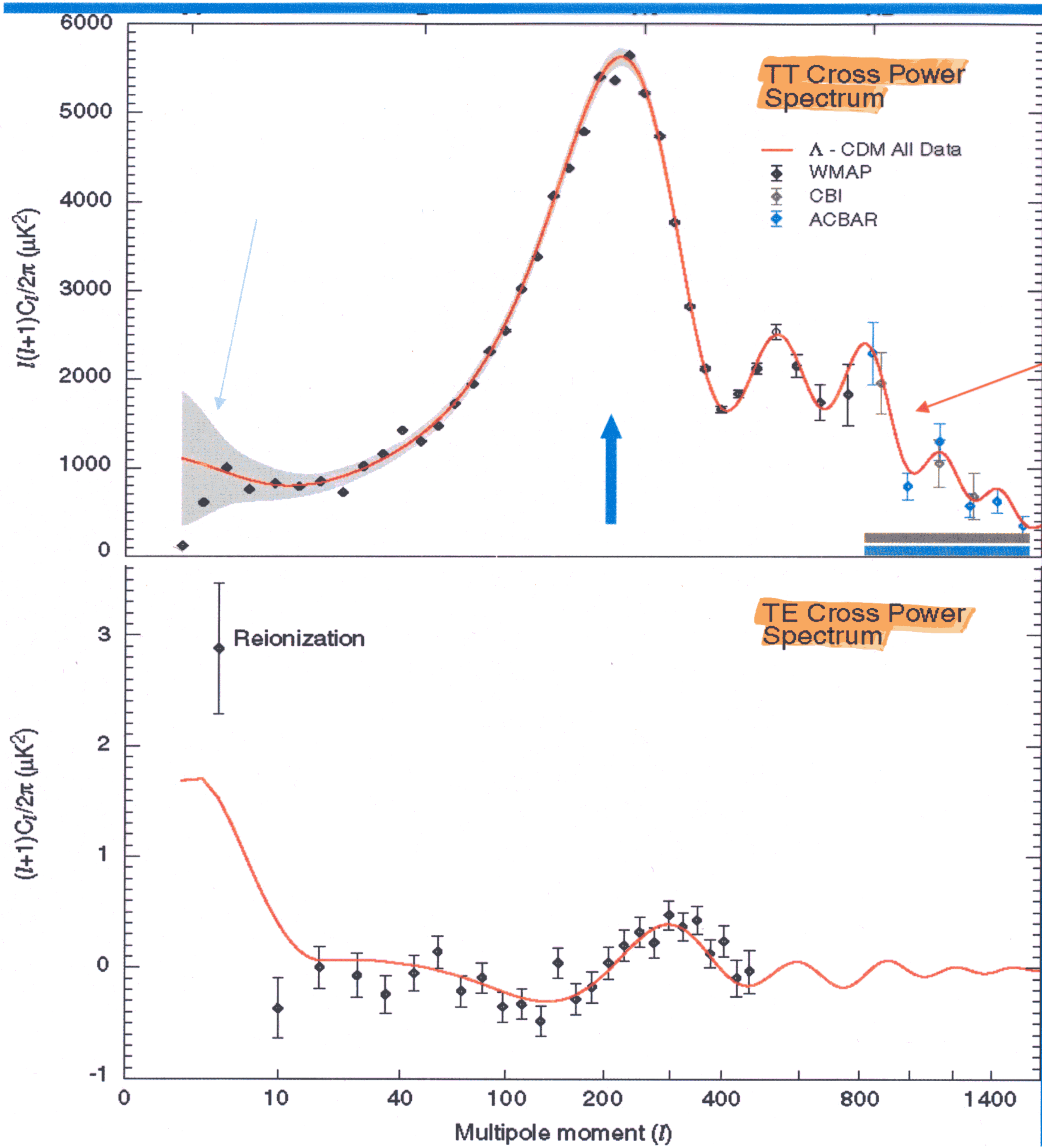
Elgarøy & Lahav (astro-ph/0303089)

Wilkinson Microwave Anisotropy Probe, 1^{st} yr data release



Bennett et al
(astro-ph/0302208)

Spergel, CAPP 2003



Temperature

85% of sky

Best fit model

$n=0.99$

$\sigma_8 = 0.9$

$\Omega_b h^2 = 0.024$

$\Omega_x h^2 = 0.126$

$H_0 = 72$

$\tau = 0.17$

Temperature-polarization

Spergel, CAPP 2003

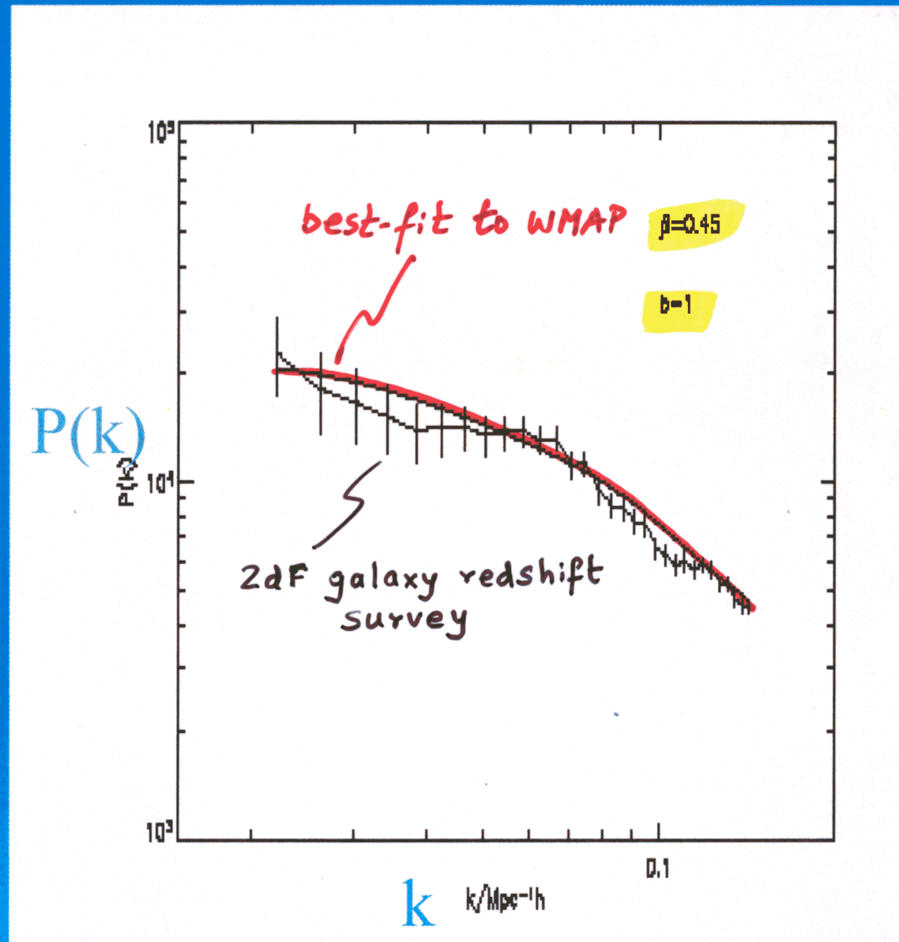
CMB + External Data

- Supernova: $D_A(z)$
- Large Scale Structure
  - Shape of transfer function sensitive to $\Omega_m h$ and $\Omega_b h$
  - Three point function $\rightarrow$ bias $\rightarrow$ σ_8
  - Clustering & Velocity Field $\rightarrow$ $\sigma_8 \Omega^{0.6}$
- Lyman α forest
  - Sensitive to n, $\Omega_m h$ and $\Omega_b h$

Spergel, CAPP 2003

Consistent Cosmological Model

- Consistent with BBN estimate of baryon density
- HST measurements of expansion rate
- Stellar evolution estimates of stellar ages
- Estimates of density fluctuations
  - Gravitational lensing
  - Clusters
  - Large scale structure
  - Lyman α forest



2dfGRS

Bound on neutrino masses from

WMAP (+ACBAR + CBI) + 2dFGRS

with the 'priors': $h = 0.72 \pm 0.05$, $n = 0.99 \pm 0.04$

$\Omega_m h^2 = 0.14 \pm 0.02$

$\sigma_8 \Omega_m^{0.6} = 0.44 \pm 0.10$

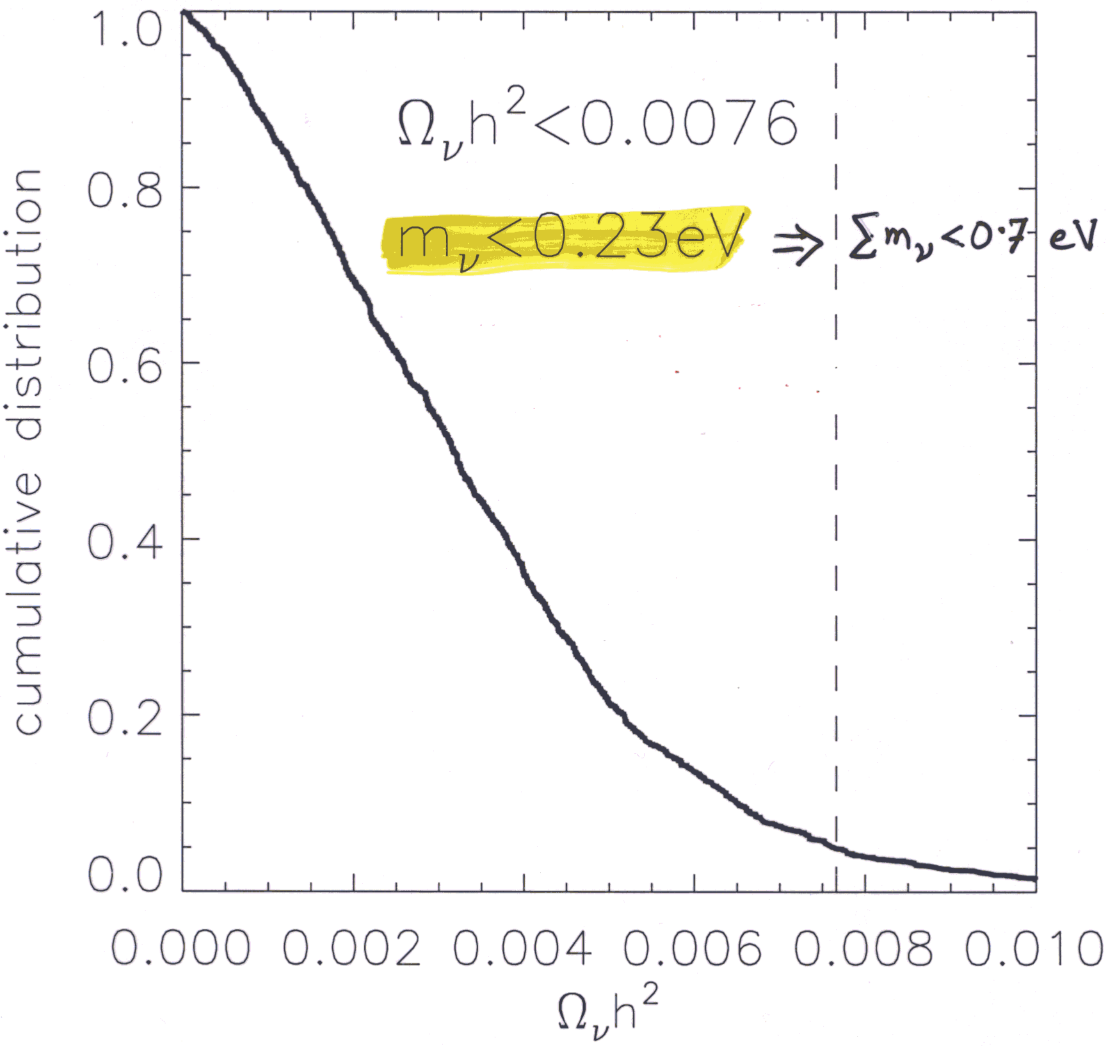
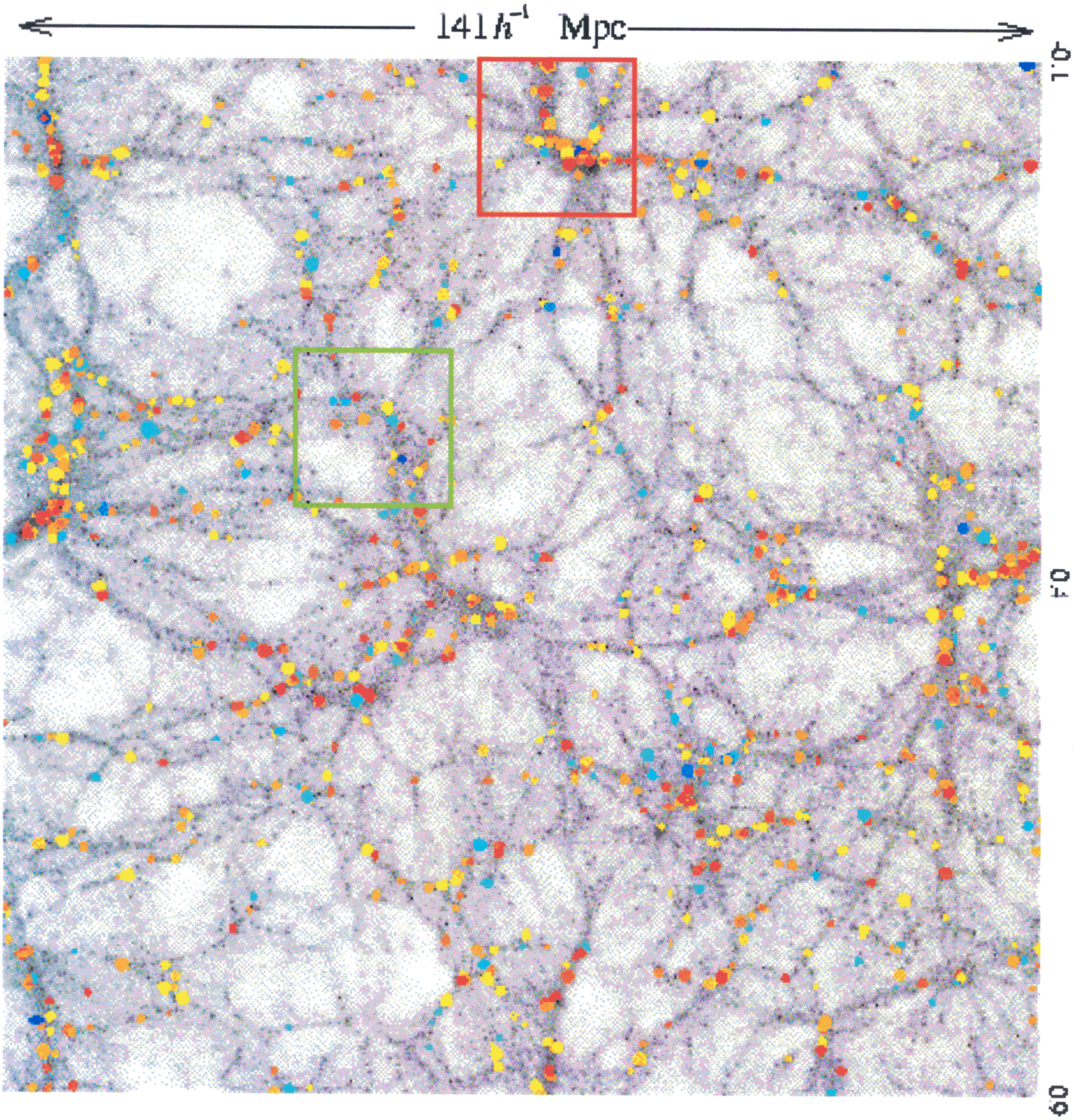


Fig. 14.— This figure shows the marginalized cumulative probability of $\Omega_\nu h^2$ based on a fit to the WMAPext+ 2dFGRS data sets. (assuming no bias between galaxies and dark matter distribution) $b = 1.04 \pm 0.11$

Spergel et al (astro-ph/0302209)

Do galaxies trace the dark matter?



VIRGO Collaboration
AP^3M simulation

Galaxy Clustering varies with Galaxy Type

How are each of them related to the underlying Mass distribution?

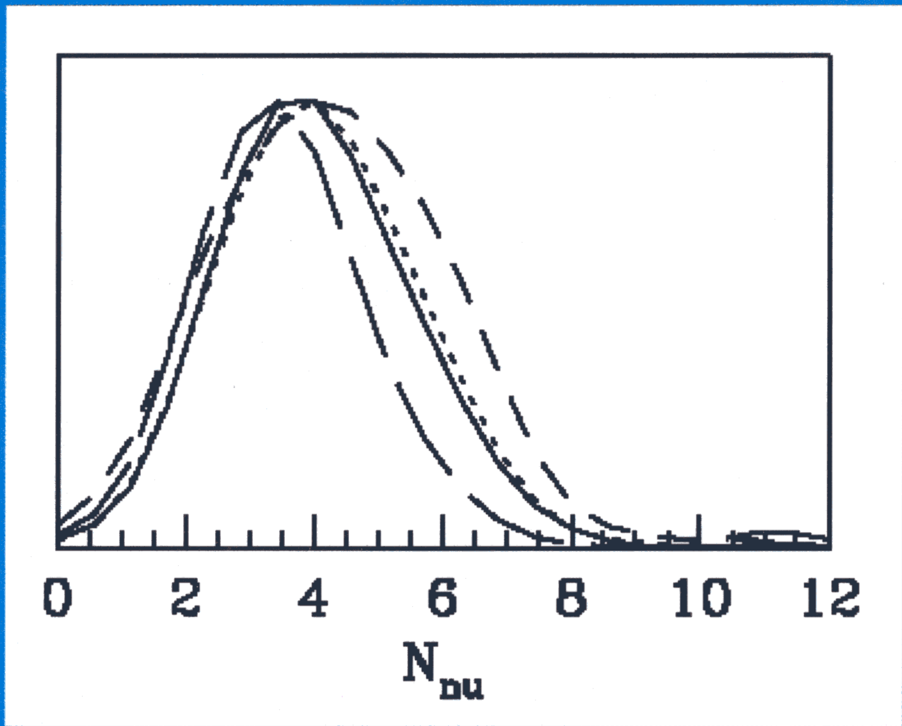
Bias depends upon Galaxy Color & Luminosity

Caveat for inference of Cosmological Parameters from LSS

Fréeman, CAPP 2003

Spergel, CAPP 2003

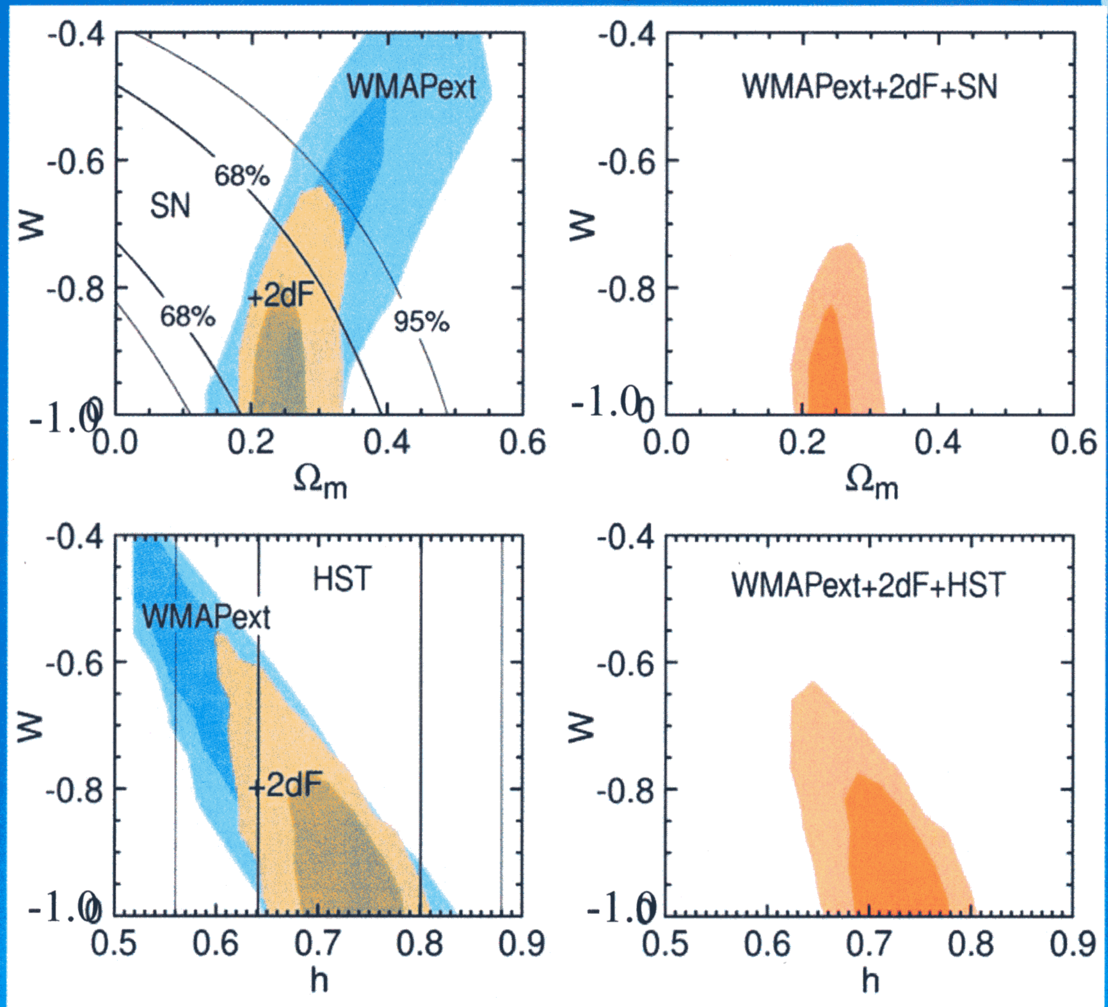
Constraining Composition of Universe



- Varying energy density in relativistic species alters evolution of CMB fluctuations: $1.6 < N\nu < 8$ (Hannested 2003; Pierpaoli 2003)
- Growth of structure constrains neutrino mass $m\nu < 0.23$ eV
- Dark matter must be non-baryonic
- Evidence for Dark Energy independent of Supernovae
- Rules out "standard CDM"

Spergel, CAPP 2003

Beyond the Standard Model: Dark Energy



CMB data consistent with other data sets if w is near -1

(dark energy is a cosmological constant)

"What I say three times is true"

$\Omega_\Lambda \sim \Omega_m \sim \mathcal{O}(1) \Rightarrow$ energy density $\sim (10^{-3}\,\mathrm{eV})^4 \sim 10^{-120} M_P^4$

- if $\Omega_\Lambda = 0$ ... then must understand why different contributions to Λ cancel so accurately
- if $\Omega_\Lambda \simeq 10^{-120} M_P^4$ ... then must also understand why $\Omega_\Lambda \sim \Omega_m$ today

... models of 'quintessence' (evolving scalar field) which track the energy density of matter, address the second problem, not the first

- Vacuum energy is real (Casimir effect)

$\oplus$

- Vacuum energy gravitates (otherwise construct perpetual motion machine!)

$\rightarrow$ no solution to problem in field theory

Recent suggestions:

- Possible UV $\leftrightarrow$ IR connection for FT in curved space-time
  'holographic principle'?
- 'self-tuning' of cosmological constant $\rightarrow 0$ in "brane-world" constructions
- GR cannot be quantised (Hilbert space of finite dimension) unless embedded in a more complete theory

⋮

may be possible to understand why $\Lambda = 0$

... harder to understand $\Omega_\Lambda \sim \Omega_m$ today

Situation so bad that 'anthropic' arguments have begun to be invoked!

Fitting cosmological models to data

Do we know how many parameters we need?

cf.

Standard $SU(3)_c \times SU(2)_L \times U(1)_Y$ Model

(effective field theory valid upto $E < \Lambda$)

super-renormalisable

$\phi^2 \Lambda^2$, Λ^4

Solve by (softly) broken supersymmetry

(another $\mathcal{O}(100)$ parameters)

renormalisable

(19 parameters)

non-renormalisable

neutrino mass
proton decay
FCNC
⋮

huge 'cosmological constant' when coupled to gravity

... no solution known!

(how many parameters will it have?)

Moral: The "simplest" cosmological models may not be adequate to describe the real universe

Studies of structure formation usually assume a Harrison-Zeldovich spectrum for the primordial density perturbation : $P(k) \propto k^n$, $n=1$

... but inflationary models generically predict (logarithmic) departures from scale-invariance

$$\delta_H^2(k) \propto \frac{P(k)}{k} \propto \left.\frac{V(\phi)^3}{V'^2}\right|_{k=H}$$

$$\Rightarrow n(k) = 1 + 2\frac{V''}{V} - 3\left(\frac{V'}{V}\right)^2$$

→ Since $V(\phi)$ steepens towards the end of inflation there will be a scale-dependent spectral 'tilt'

$$\delta_H^2 \propto \left[51 + \ln\left(\frac{k^{-1}}{3000h^{-1}\,\mathrm{Mpc}}\right)\right]^\alpha$$

e.g. $\alpha = 4$ for $V \propto \phi^3 \Rightarrow n \simeq 0{\cdot}9$

Adams, Ross & Sarkar (hep-ph/9608336)

... however the spectrum can be very close to scale-invariant for an exponential potential ('power-law' inflation) or in 'hybrid' inflation (where the dynamics of a second field ends inflation)

But in multi-field models, can even generate features in the spectrum — 'bumps', 'steps' ...

Adams, Ross, Sarkar (hep-ph/9704286)

An alternative to the ΛCDM model

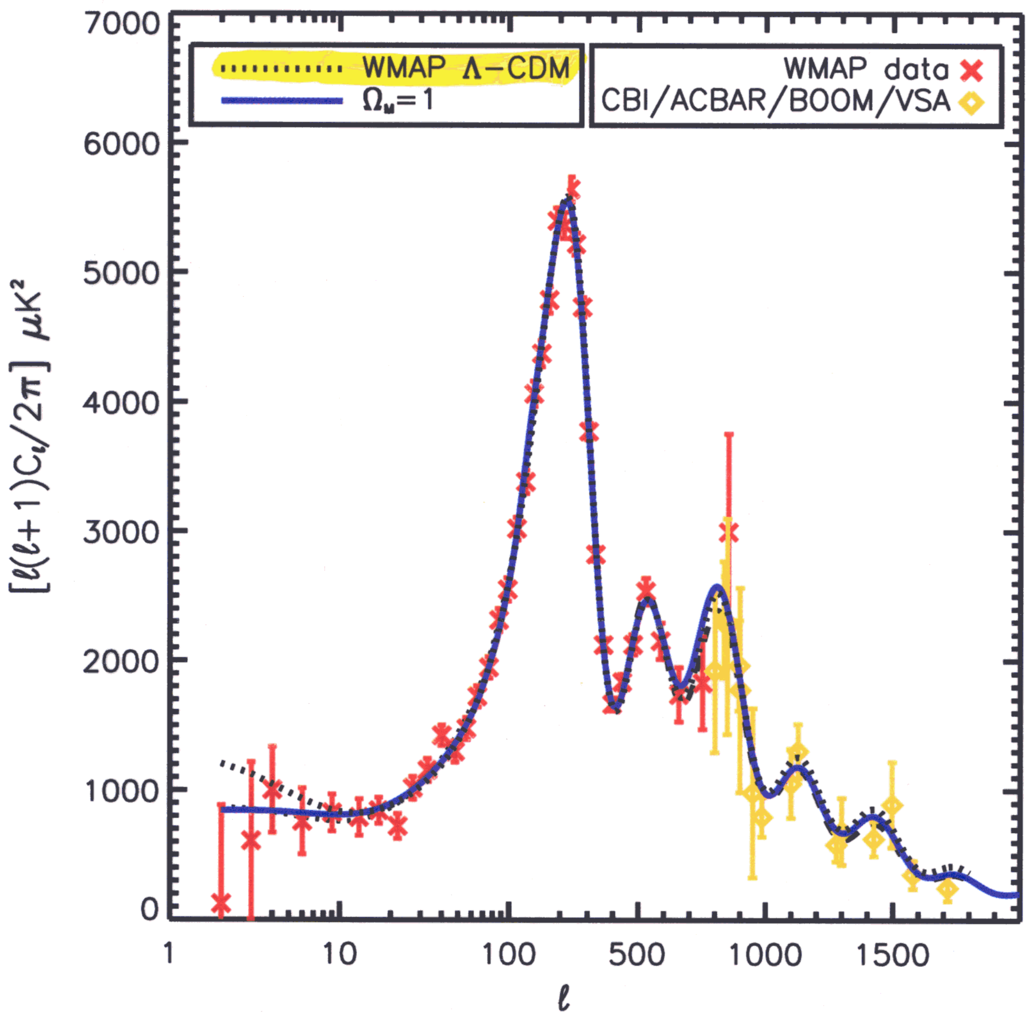
WMAP 'concordance' model: $\Omega_\Lambda = 0.73$, $\Omega_m = 0.27$, $h = 0.72$, $n = 0.99$

Our E-deS model: $\Omega_\Lambda = 0$, $\Omega_m = 1$, $h = 0.46$

$n = 1.02$, for $k < k_1 = 0.0096\ \mathrm{Mpc}^{-1}$

$= 0.81$, for $k > k_1$

... fits even better !

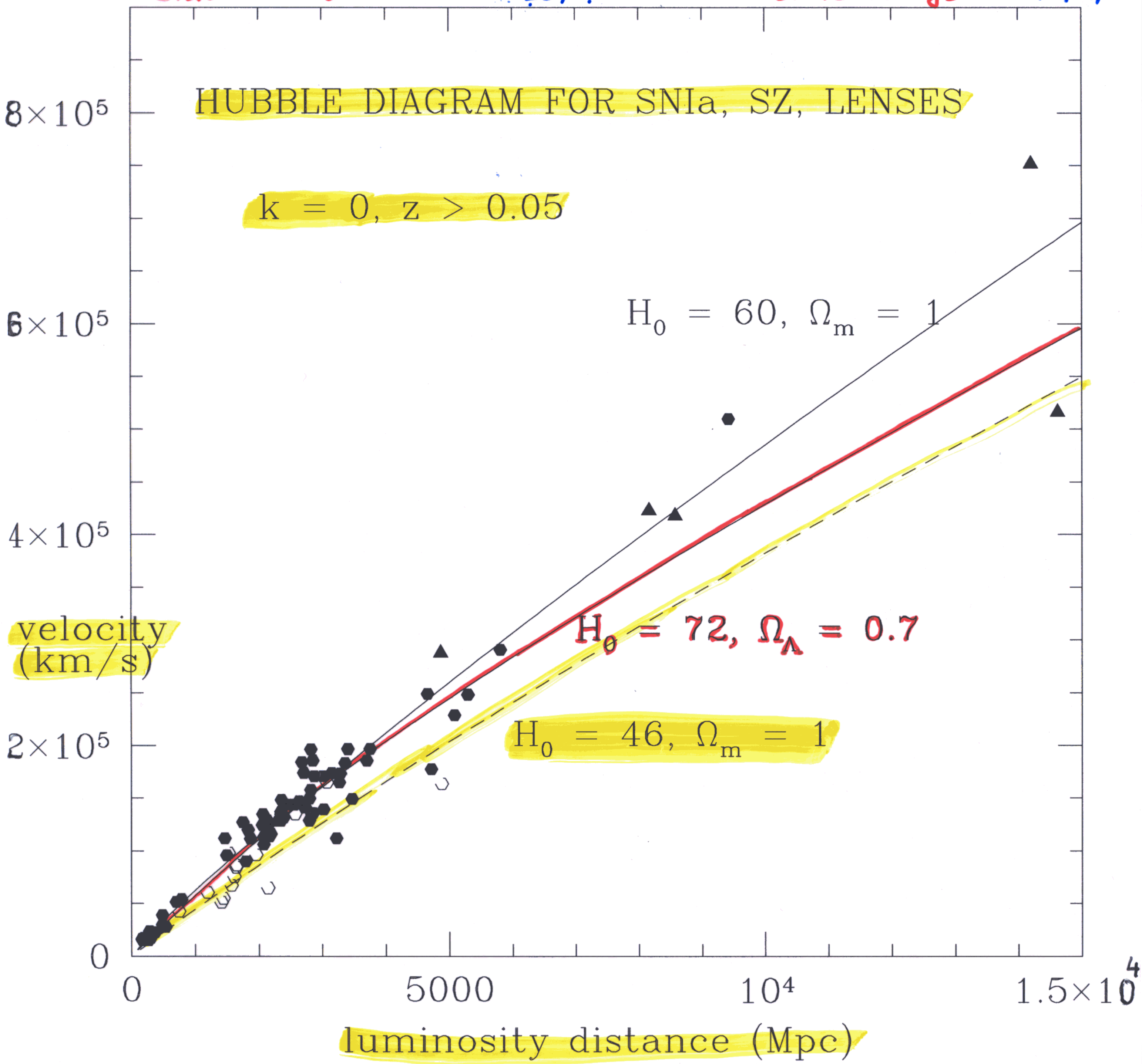


Blanchard, Douspis, Rowan-Robinson, S.S.
(astro-ph/0304237)

$H_0 = 46$ km/s/Mpc is inconsistent with the Hubble Key Project value (72 ± 8 km/s/Mpc)

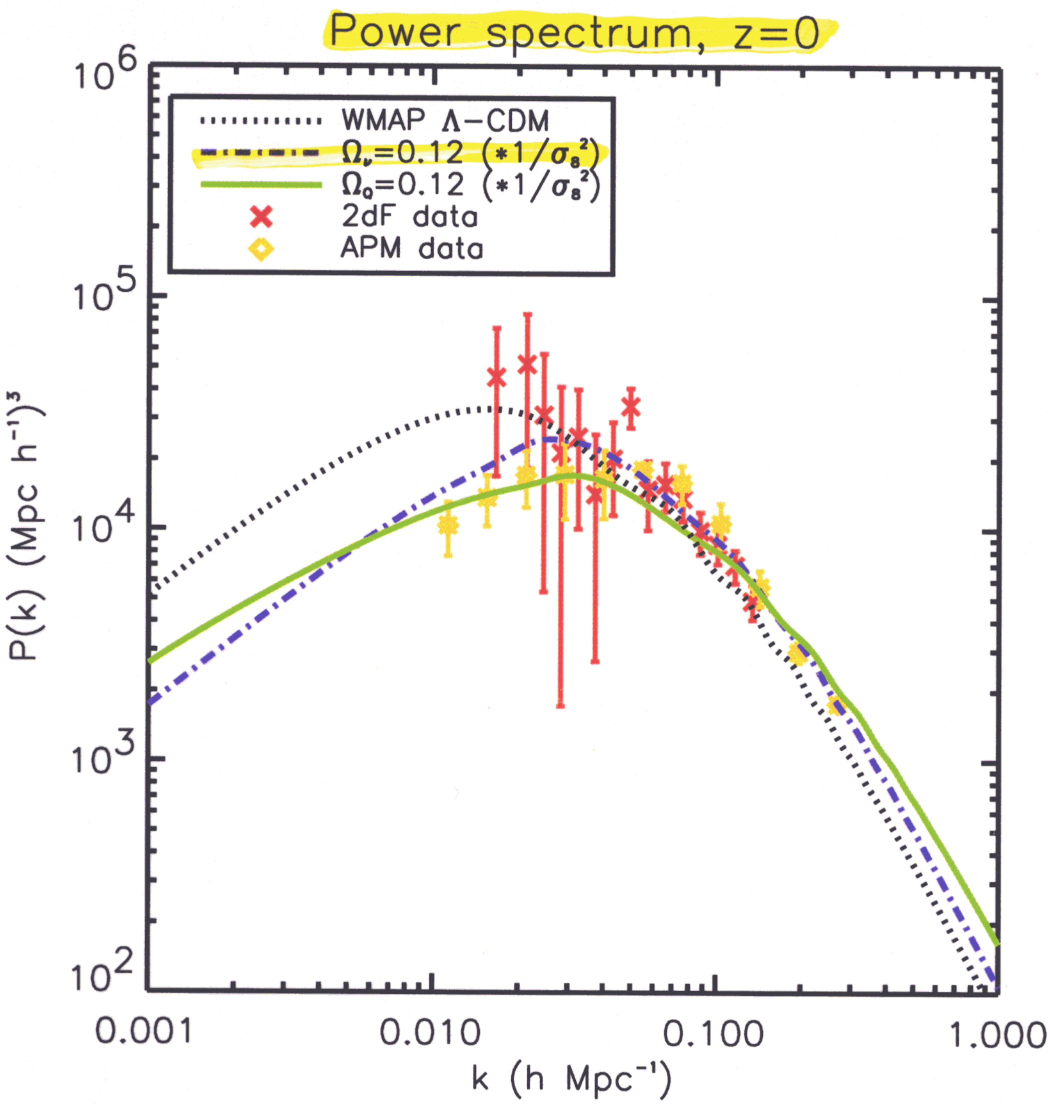
... but not with direct (and deeper) methods:

Sunyaev-Zeldovich cluster distances (54 ± 4 km/s/Mpc) -20%?, gravitational lens time delays ($48 \pm 3 \pm ?$ km/s/Mpc)



→ need further work on the distance scale (e.g. metallicity effects on Cepheid calibration ...)

Blanchard et al. (astro-ph/0304237)



→ on smaller scales, clustering of matter would be excessive ... unless damped by e.g. a hot (neutrino) dark matter component

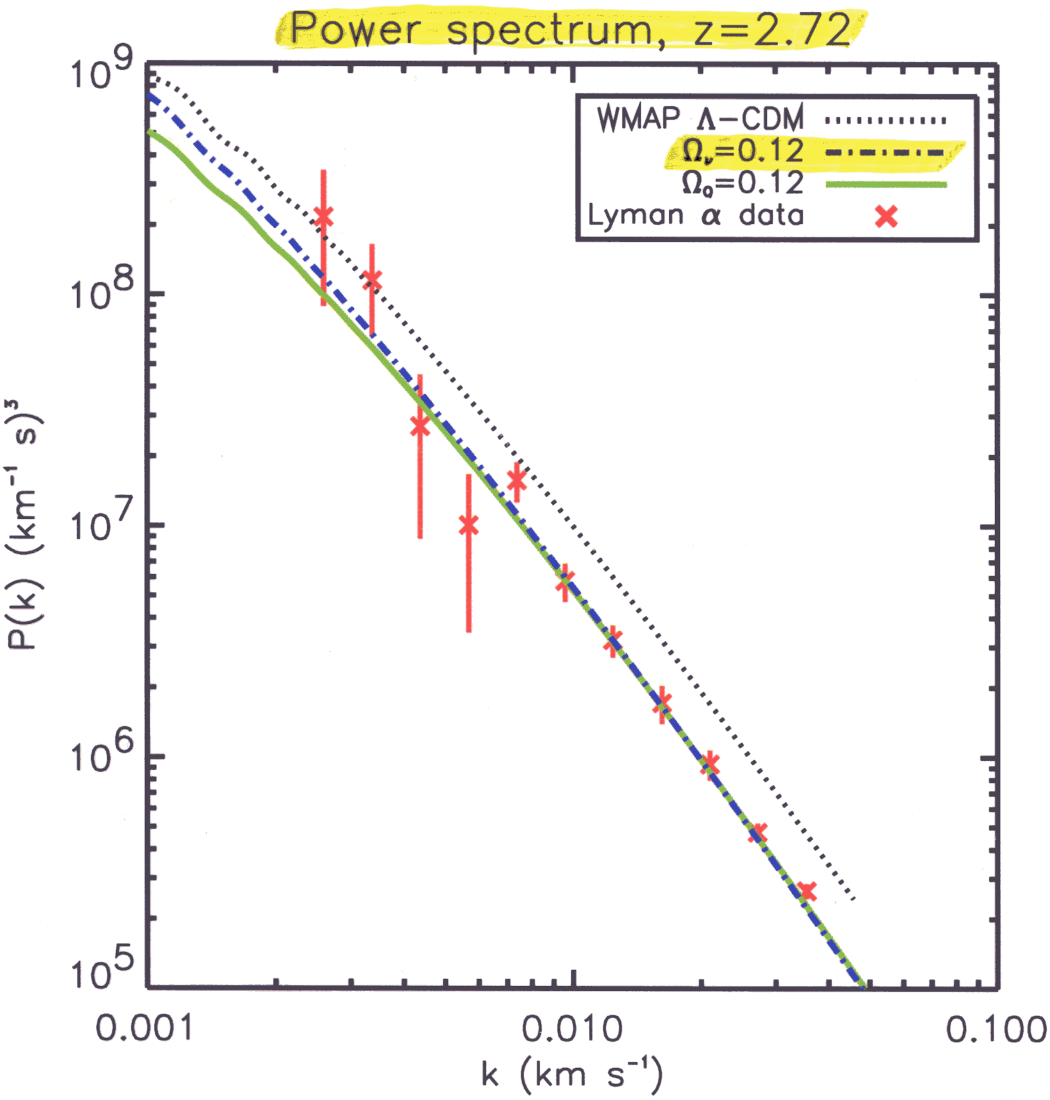
Obtain good fit to large-scale structure data with 3 quasi-degenerate neutrinos of mass ~0.8 eV

$\Rightarrow \Omega_\nu = 0.12$ (NB: Well above WMAP 'bound'!)

and $\Omega_B h^2 = 0.021$ (in agreement with BBN value)

$\Rightarrow$ baryon fraction in clusters of ~15% (acceptable?)

and $\sigma_8 = 0.64$ (consistent with weak lensing determination)



... with a bias factor $b \simeq 1/\sigma_8$, can also fit power spectrum of Lyman-alpha forest (if amplitude is reduced by $\sim 1\sigma$ calibration uncertainty)
$\Rightarrow 20\%$.

→ in these fits, the optical depth to last scattering is $\tau \simeq 0.1$ ... easier to accomodate with our understanding of star formation in CDM cosmogony ...

Blanchard et al
(astro-ph/0304237)

Primordial Nucleosynthesis

- Weak interactions ($ne^+ \leftrightarrow p\bar{\nu}_e$, $pe^- \leftrightarrow n\nu_e$, $n \leftrightarrow pe^-\bar{\nu}_e$) keep neutrons and protons in equilibrium in the early universe, until the reaction rate ($\Gamma \sim T^5/\tau_n$) becomes smaller than the expansion rate ($H \sim g_*^{1/2}\, T^2/M_P$), at a temperature, $T_f \simeq \left(\frac{g_*^{1/2}\tau_n}{M_P}\right)^{1/3} \sim 1$ MeV, and the neutron-to-proton ratio 'freezes-out' with the value,

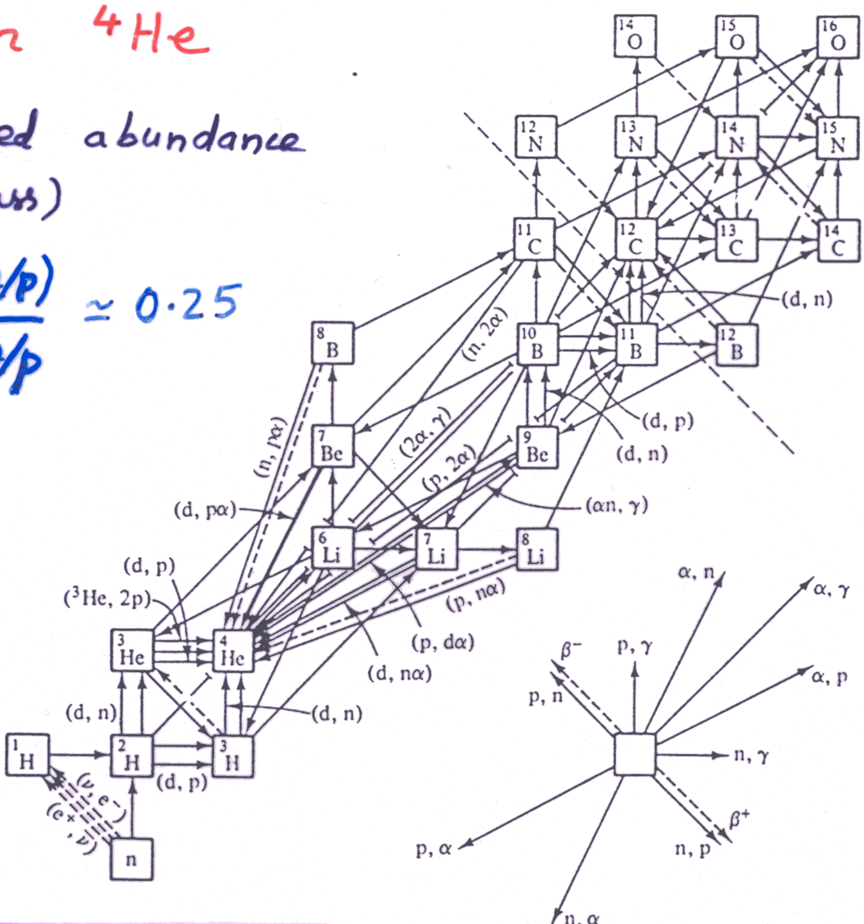
$$\left(\frac{n}{p}\right)_{T_f} = e^{-(m_n - m_p)/T_f} \simeq \frac{1}{6} \qquad \left(\text{reduced to } \tfrac{1}{7} \text{ by subsequent } \beta\text{-decay}\right)$$

[Alpher, Follin, Herman '53, ... Bernstein, Brown, Feinberg '88]

- Nuclear reactions begin when the universe cools to $T \sim 0.1$ MeV ... almost all neutrons get bound in ^{4}He

... predicted abundance (by mass)

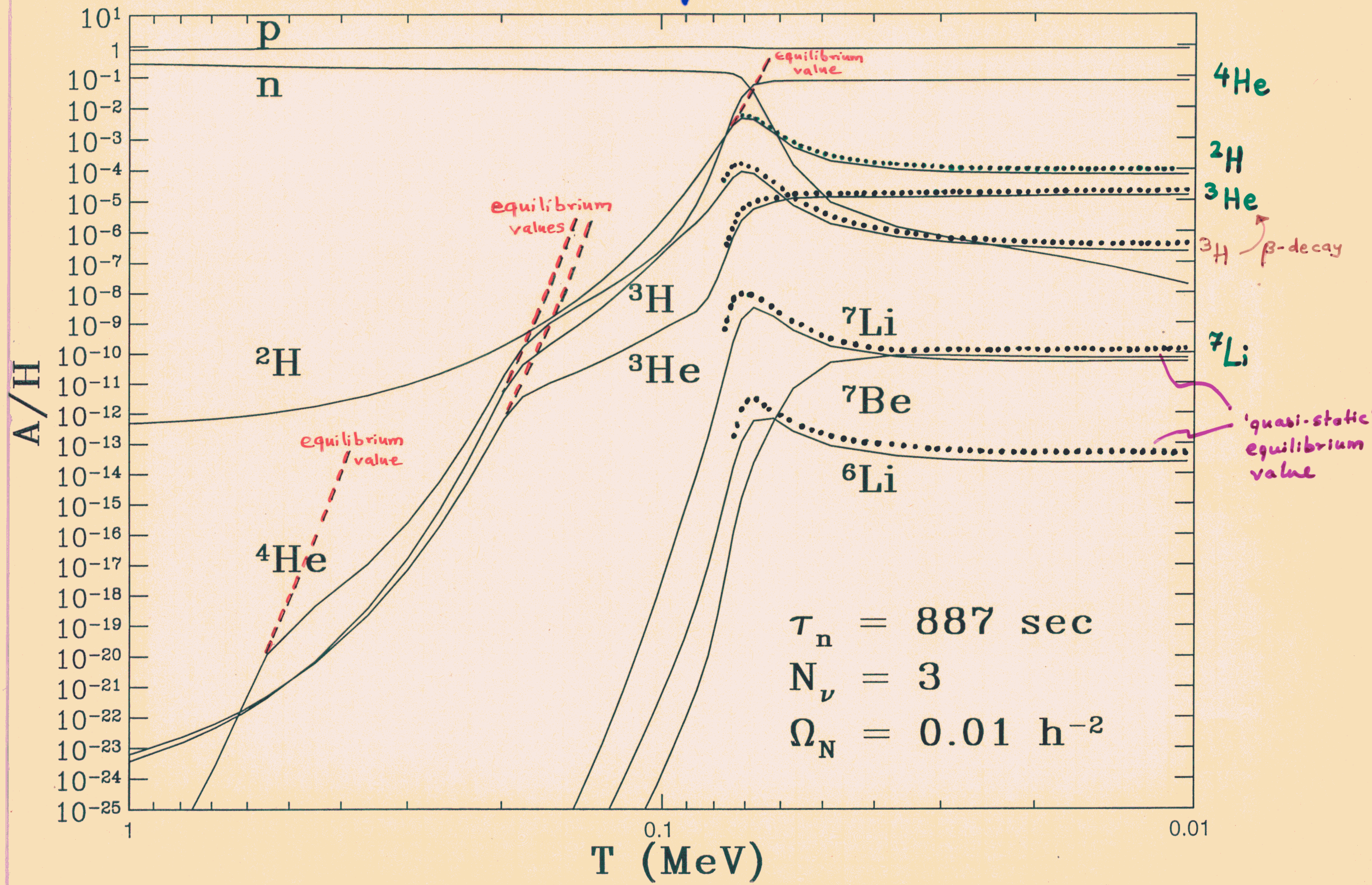
$$Y(^4\text{He}) = \frac{2(n/p)}{1 + n/p} \simeq 0.25$$



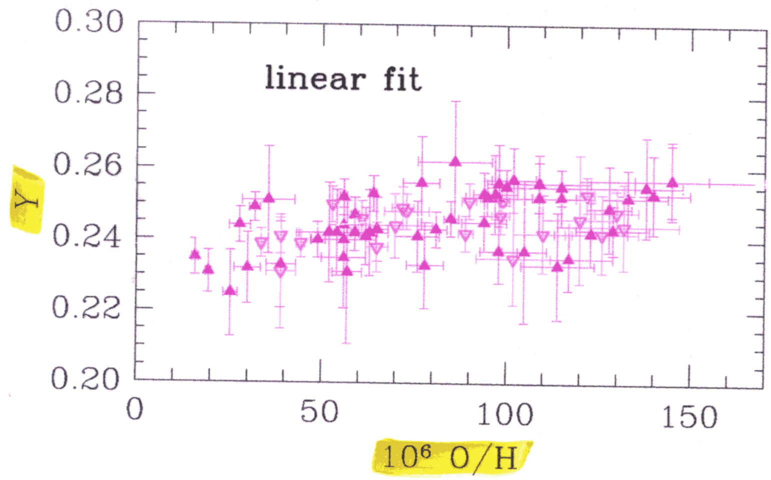
[Wagoner, Fowler, Hoyle '67 ... Esmailzadeh, Starkman, Dimopoulos '91]

- 'left-over' abundances of D, ^{3}He & ^{7}Li determined by rate of nuclear reactions ($\propto n_N^2$) ... depend sensitively on $\eta \equiv n_N/n_\gamma$

The first three minutes

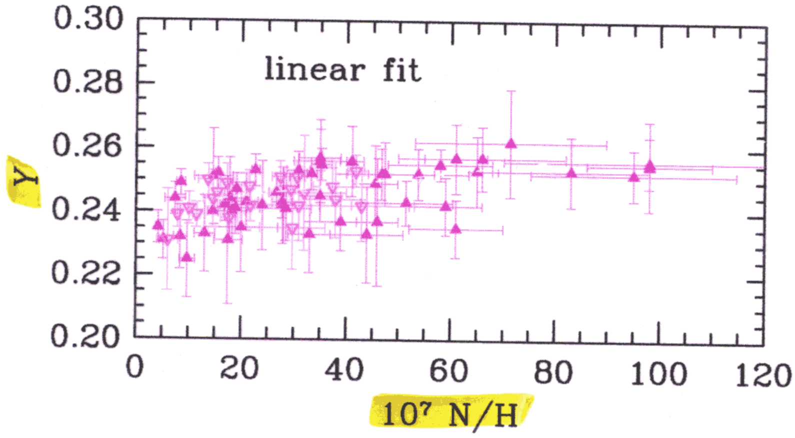


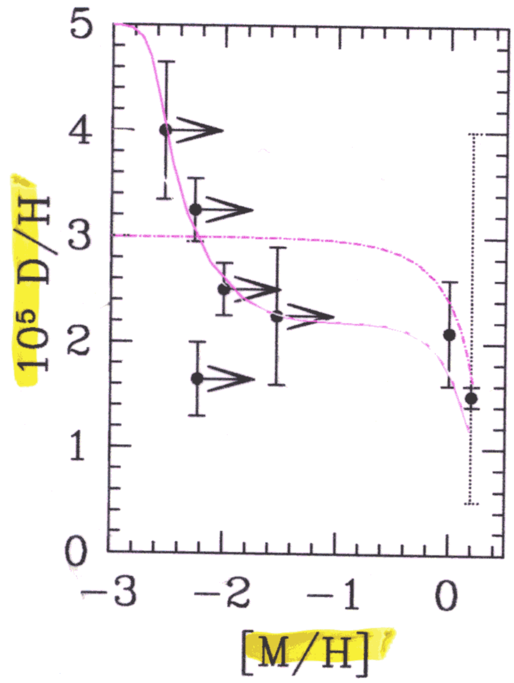
Inferred primordial abundances (PDG 2002)



$Y_p(^4He)$

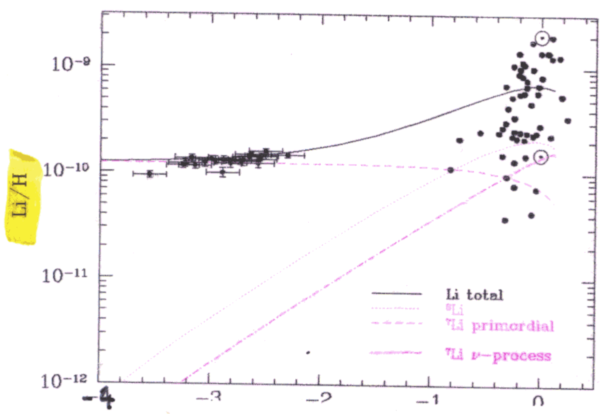
$= 0{\cdot}238 \pm 0{\cdot}002 \pm 0{\cdot}005$





$1{\cdot}3\times10^{-5} < D/H|_p < 9{\cdot}7\times10^{-5}$

$[\text{cf. } D/H = (3{\cdot}0 \pm 0{\cdot}4)\times10^{-5}]$



$^7Li/H|_p$

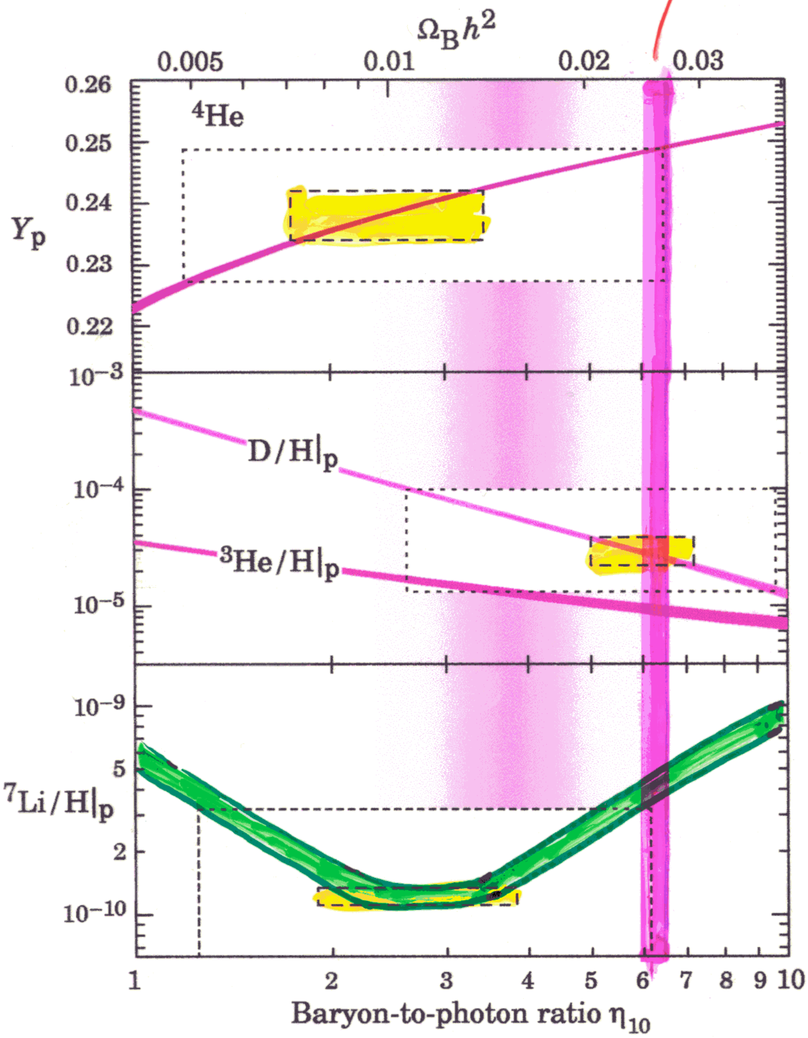
$= (1{\cdot}23 \pm 0{\cdot}06^{+0{\cdot}68}_{-0{\cdot}32}{}^{+0{\cdot}56}\,)\times10^{-10}$

light element abundances (allowing for systematic errors) imply : $2.6 \times 10^{-10} < n_B/n_\gamma < 6.2 \times 10^{-10}$

... marginally consistent with CMB determination (assuming scale-free spectrum of primordial fluctuations)

$\Rightarrow$ $2 \lesssim N_\nu \lesssim 4$ (conservative limits)

WMAP determination



Fields & Sarkar
(Review of Particle Properties
Phys. Rev D66 (2002) 010001)

Conclusions

- Direct detection of relic neutrinos
  ... remains an outstanding experimental challenge

- The existence of new neutrino types is severely constrained by primordial nucleosynthesis
  ... the observed light element abundances however have not fully stabilized

- Forthcoming precision measurements of galaxy clustering and CMB anisotropy have sensitivity to both new neutrino types and small neutrino masses
  ... however 'astrophysical' uncertainties remain

⇒ Astronomical observations may clarify many of the questions raised by the recent experimental evidence for neutrino oscillations

but "caveat emptor"!