Towards Multi-leg amplitudes at NLO

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- Formalism for Hexagon calculations
- Examples: $\phi\phi \to \phi\phi\phi\phi$, $\gamma\gamma \to \phi\phi\phi\phi$, $gg \to \gamma\gamma g$
- Automatic computation of general one-loop amplitudes:
  The GOLEM project
  [General One-Loop Evaluation of Matrix Elements]
- Summary/Outlook

Jet rates at the LHC

Number of jets:	3	4	5	6	7	8
σ/nb	91.4	6.54	0.46	0.032	0.002	0.0002

$p_T(jet) > 60$ GeV, $\theta_{ij} > 30^0$, $|\eta_j| < 3$ [Draggiotis,Kleiss,Papadopoulos, hep-ph/0202201]

Multi–particles/jet production plays a very important role !!!

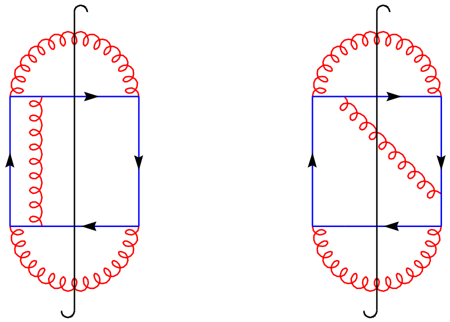
Problems with leading order predictions:

- Scale dependence: N–jet cross sections behave $\sim \alpha_s(\mu)^N$ $\Rightarrow$ To have predictions for jet rates NLO corrections have to be included

- Peripheral phase space regions: degenerate partonic configurations at LO are sensitive to extra parton emission $\Rightarrow$ estimates for backgrounds — especially after severe cuts — may be considerably underestimated

- Jet structure: the more partons are in the amplitudes the more precise the jet structure is described

QCD amplitudes with N partons at one–loop

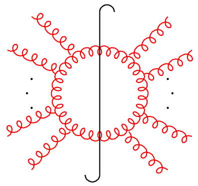


- N=4: $2 \to 2$ parton scattering [R.K. Ellis, J.C. Sexton 1986]
  ... massless scalar four point integrals ("boxes"), reduction of tensor integrals à la Passarino–Veltman, $1 \to 2$ phase space integrals

- N=5: $2 \to 3$ parton scattering [Bern,Dixon,Kosower 1993; Kunszt,Signer,Trócsányi 1994]
  ... massless five point scalar integrals ("pentagons"), reduction of tensor integrals in Feynman parameter space ...
  New methodes: "stringy methods", colour ordered sub–amplitudes, susy relations, helicity formalism at NLO ...

- N=6: $2 \to 4$ parton scattering [Bern,Dixon,Kosower,Dunbar,Chalmers,Morgan,... 1994]
  New approach: gluing of off–shell tree amplitudes. Non-ambiguous only for supersymmetric amplitudes! $\Rightarrow$ 6 gluon amplitudes are known for $\mathcal{N} = 4$ super Yang–Mills.

$$\mathcal{A} = \int d\mathcal{PS}$$

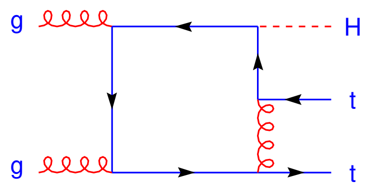


!!! The non–supersymmetric case remains to be solved !!!

Multi particle production in the electro-weak sector

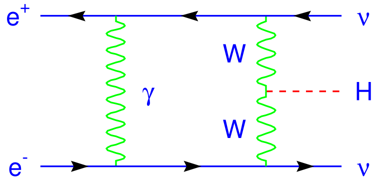
- The analytic approach becomes very complex for electro–weak processes if $N > 4$ ($\Rightarrow$ numeric approach !!!).
  Very important for TESLA!!!
- Amplitudes for $2 \to 3$ processes are very rare:

$PP \to t\bar{t}H$:



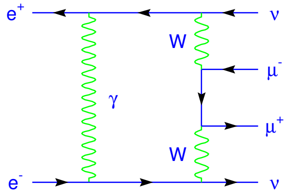
Beenaker, Dittmeier, Krämer, Plümper, Spira, Zerwas, hep–ph/0107081; S. Dawson, C. Jackson, L.H. Orr, L. Reina, D. Wackeroth, hep–ph/0305087.

$e^+e^- \to \nu\bar{\nu}H$:



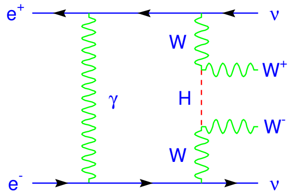
Tarasov, Jegerlehner, “FIRCLA” hep–ph/0212004; Bélanger, Boudjema, Fujimoto, Ishikawa, Kaneko, Kato, Shimizu, “GRACE” hep–ph/0211268; A. Denner, S. Dittmaier, M. Roth, M.M. Weber, hep–ph/0302198.

$e^+e^- \to f\bar{f}f'\bar{f}'$:



A. Vicini, . . . in progress (?)

$e^+e^- \to f\bar{f}VV$:



???

!!! paradigm process !!!

Backgrounds for Higgs boson search

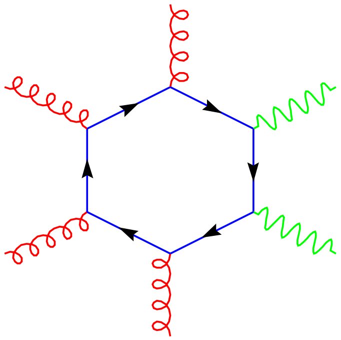
To make a reliable estimate of backgrounds for the relevant signal signatures

– $PP \to H$ by Gluon Fusion (NNLO), $H \to \gamma\gamma$, $H \to WW^{(*)}$

– $PP \to Hj$ (NLO $m_{Top} \to \infty$)

– $PP \to Hjj$ by Weak Boson Fusion (NLO), $H \to \tau^+\tau^-$, $H \to WW^{(*)}$, $H \to \gamma\gamma$

the following NLO computations are needed:



– $PP \to \gamma\gamma + 0, 1, 2\,\text{jets}$

– $PP \to WW^* + 0, 1, 2\,\text{jets}$

– $PP \to t\bar{t} + 0, 1, 2\,\text{jets}$

– $PP \to \tau^+\tau^- + 0, 1, 2\,\text{jets}$

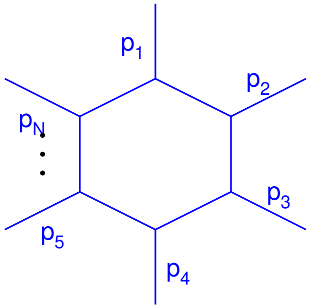
Computation needs a GOLEM: General One-Loop Evaluator of Matrix Elements

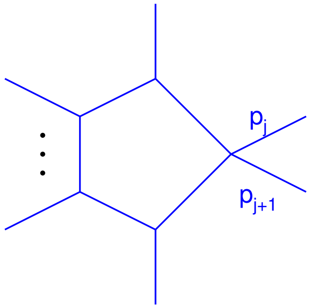
...combination of EW and QCD methods
...isolation of IR poles (→subtractions)
...(maybe) development of numeric routines for finite reminders
...optimal amplitude representations (→numerically stable)

Reduction of scalar integrals

[T.B., J.P. Guillet, G. Heinrich, Nucl. Phys. B572 (2000)]

$$
\begin{aligned}
I_N^n &= (-1)^N \Gamma(N - \frac{n}{2}) \int_0^1 d^N z \frac{\delta(1 - \sum_{j=1}^N z_j)}{\left(\frac{1}{2}\sum_{i,j=1}^N S_{ij}\, x_i\, x_j\right)^{N-\frac{n}{2}}} \\
S_{ij} &= G_{ij} - G_{ii}/2 - G_{jj}/2 + m_i^2 + m_j^2 \\
G_{ij} &= 2\, r_i \cdot r_j \qquad , \quad r_j = p_1 + \ldots + p_j \qquad , \quad n = 4 - 2\,\epsilon
\end{aligned}
$$

$I_N^n =$ 

$= \sum_{j=1}^N B_j$ 

$$
+ \begin{cases} -(1+2\epsilon)\frac{\det(G)}{\det(S)}\, I_N^{n+2} & , N = 4 \\ \mathcal{O}(\epsilon) & , N = 5 \\ 0 & , N \geq 6 \end{cases}
$$

$$
\sum_{j=1}^N S_{ij} B_j = -1 \quad \Leftrightarrow \quad B_i = -\sum_{j=1}^N S_{ij}^{-1}
$$

Any N point integral can be represented by n–dimensional triangle functions and (n+2) dimensional box functions. The latter are infrared finite.

Five and six point functions

- Hexagons/pentagons are expressed by the functions I_3^n and I_4^{n+2}
- The isolation of infrared singularities is straight forward!!!

$$\begin{aligned}
I_5^n = \; & (B_1B_{12}+B_2B_{21})I^n_{3,12}+(B_1B_{13}+B_3B_{31})I^n_{3,13}+B_1(B_{12}+B_{13}+B_{14}+B_{15})I^{n+2}_{4,1}\\
&+(B_2B_{23}+B_3B_{32})I^n_{3,23}+(B_2B_{24}+B_4B_{42})I^n_{3,24}+B_2(B_{21}+B_{23}+B_{24}+B_{25})I^{n+2}_{4,2}\\
&+(B_3B_{34}+B_4B_{43})I^n_{3,34}+(B_3B_{35}+B_5B_{53})I^n_{3,35}+B_3(B_{31}+B_{32}+B_{34}+B_{35})I^{n+2}_{4,3}\\
&+(B_4B_{45}+B_5B_{54})I^n_{3,45}+(B_4B_{41}+B_1B_{14})I^n_{3,14}+B_4(B_{41}+B_{42}+B_{43}+B_{45})I^{n+2}_{4,4}\\
&+(B_5B_{51}+B_1B_{15})I^n_{3,15}+(B_5B_{52}+B_2B_{25})I^n_{3,25}+B_5(B_{51}+B_{52}+B_{53}+B_{54})I^{n+2}_{4,5}
\end{aligned}$$

$$\begin{aligned}
I_6^n = \;&\\
&\{[B_1(B_{12}B_{123}+B_{13}B_{132})+B_2(B_{21}B_{123}+B_{23}B_{231})+B_3(B_{31}B_{132}+B_{32}B_{231})]\,I^n_{3,123}+5\text{ c.p.}\}\\
&+\{[B_1(B_{12}B_{124}+B_{14}B_{142})+B_2(B_{21}B_{214}+B_{24}B_{241})+B_4(B_{41}B_{412}+B_{42}B_{421})]\,I^n_{3,124}+5\text{ c.p.}\}\\
&+\{[B_1(B_{13}B_{134}+B_{14}B_{143})+B_3(B_{31}B_{314}+B_{34}B_{341})+B_4(B_{41}B_{413}+B_{43}B_{431})]\,I^n_{3,134}+5\text{ c.p.}\}\\
&+\{[B_1(B_{13}B_{135}+B_{15}B_{153})+B_3(B_{31}B_{315}+B_{35}B_{351})+B_5(B_{51}B_{513}+B_{53}B_{531})]\,I^n_{3,135}+1\text{ c.p.}\}\\
&+\{(B_1B_{12}+B_2B_{21})(B_{123}+B_{124}+B_{125}+B_{126})I^{n+2}_{4,12}+5\text{ c.p.}\}\\
&+\{(B_1B_{13}+B_3B_{31})(B_{132}+B_{134}+B_{135}+B_{136})I^{n+2}_{4,13}+5\text{ c.p.}\}\\
&+\{(B_1B_{14}+B_4B_{41})(B_{142}+B_{143}+B_{145}+B_{146})I^{n+2}_{4,14}+2\text{ c.p.}\}
\end{aligned}$$

Subtraction formalism for tensor integrals

For numeric approach sufficient to isolate IR part from tensor integrals:

$$I_N^{\mu_1,\dots\mu_R} \sim \sum \tau_{l_1,\dots l_R}^{\mu_1,\dots\mu_R} \; I_N^n(l_1,\dots,l_R)$$

$$I_N^n(l_1,\dots,l_R) \sim I_N^{n+2}(l_1,\dots,l_{R-1}) \oplus I_{N-1}^n(l_1,\dots,l_{R-1})$$

- Integrales $\sim I_N^{n+2}$ are IR finite $\Rightarrow$ numerical evaluation!
- Iteration for integrals $\sim I_{N-1}^n$ till analytical evaluation possible.

From the numerical point of view:

$$I_N^{n+2}(l_1,\dots,l_R) \sim I_N^{n+2}$$

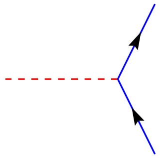
Numerical methods for scalar graphs already developed!

$\rightarrow$ see Nikolas Kauer's talk.

Six scalar scattering in the Yukawa model

[T.B., J.P. Guillet, G. Heinrich, C. Schubert, Nucl. Phys. B615 (2001)]

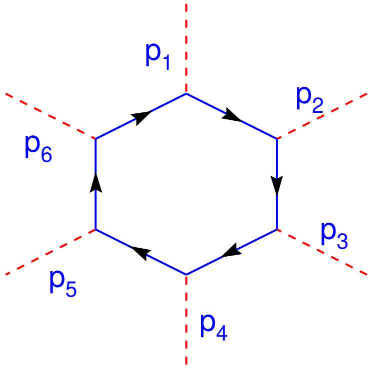
$$\mathcal{L}_{\text{Yukawa}} = \bar{\psi}\,[\,i\,\partial\!\!\!/ - \lambda\,\phi\,]\,\psi + \mathcal{L}_\phi \quad , \quad \mathcal{L}_{\phi\psi\bar{\psi}} = i\lambda$$

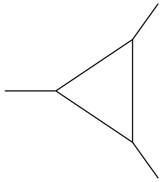
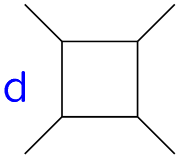


$$\mathcal{M}(\phi\phi \to \phi\phi\phi\phi) = -\frac{\lambda^6}{(4\pi)^2\, 6} \sum_{\pi \in \mathcal{S}_6} G(\pi_1, \ldots, \pi_6)$$

$$G(1,2,3,4,5,6) = \int \frac{d^n k}{i\pi^{n/2}} \frac{tr(\,q\!\!\!/_1 \ldots \,q\!\!\!/_6)}{q_1^2 \ldots q_6^2} =$$

$$q_j = k - p_1 - \ldots - p_j$$



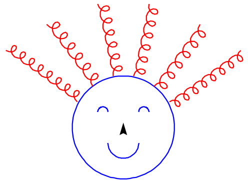
- Tensor structure can be simplified
- each diagram is IR,UV finite $\Rightarrow$ expressible by  and  $\Big|_{n+2}$

Result for $\mathcal{M}(\phi\phi \to \phi\phi\phi\phi)$:

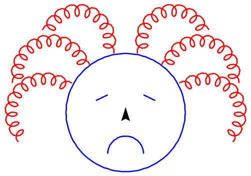
$$\mathcal{M}(\phi\phi \to \phi\phi\phi\phi) = -\frac{\lambda^6}{(4\pi)^2} \sum_{\pi \in \mathcal{S}_6} \mathcal{A}(p_{\pi_1}, p_{\pi_2}, p_{\pi_3}, p_{\pi_4}, p_{\pi_5}, p_{\pi_6})$$

$$\begin{aligned}
\mathcal{A}(p_1,p_2,p_3,p_4,p_5,p_6) = {} & \frac{2}{3} I_3^n(p_{12}, p_{34}, p_{56}) \\
& + \left\{ \frac{B_1 \, [\mathrm{tr}(6123) - 2 s_{61}(s_{123} - s_{12})]}{s_{123}s_{345} - s_{12}s_{45}} + \frac{B_2 \, [\mathrm{tr}(1234) - 2 s_{34}(s_{123} - s_{23})]}{s_{234}s_{123} - s_{23}s_{56}} \right\} \\
& \quad \times \log\left(\frac{s_{12}}{s_{123}}\right) \log\left(\frac{s_{23}}{s_{123}}\right) \\
& - \left\{ B_1 - \frac{B_2 \, [\mathrm{tr}(1234) - 2 s_{34}(s_{123} - s_{23})]}{2[s_{234}s_{123} - s_{23}s_{56}]} - \frac{B_6 \, [\mathrm{tr}(5612) - 2 s_{56}(s_{345} - s_{61})]}{2[s_{345}s_{234} - s_{61}s_{34}]} \right\} \\
& \quad \times \left[\log\left(\frac{s_{12}}{s_{234}}\right) \log\left(\frac{s_{56}}{s_{234}}\right) + \log\left(\frac{s_{34}}{s_{234}}\right) \log\left(\frac{s_{12}}{s_{56}}\right) \right]
\end{aligned}$$

First non–trivial non–supersymmetric hexagon amplitude ...



... probably the most simple one



Automated computation of multi-parton processes at NLO

- $2 \to 2$ easy, $2 \to 3$ hard, $2 \to 4$ examples unknown
- Standard Packages: FF, FeynArts, FeynCalc, FormCalc, Loop-Tools,...,

$2 \to N$ at NLO needs:

- Real $2 \to N+1$ corrections: $d\sigma^{R}_{NLO}[2 \to N+1]$
- Virtual $2 \to N$ corrections: $d\sigma^{V}_{NLO}[2 \to N]$
- explicit cancellations of IR divergencies

  [for an alternative see: D. Soper]

> Question: Is it feasible to construct an automatic program which produces finite, numerically stable NLO amplitudes for up to $2 \to 4$ partonic processes in terms of FORTRAN, C,... functions which can be used to build NLO Monte Carlo programs?

Our aim is the construction of a ...

G eneral

O ne

L oop

E valuator for

M atrix Elements

... for QCD $\oplus$ EW processes !

Tree $2 \to 5$ matrix elements for SM processes...

- Helicity amplitudes sorted in irreducible colour structures!

...do not seem to be a problem...

- Several packages exist:
  MadGraph, GRACE, CompHep, Omega, Amegic++, Alpgen,...

...up to the usual trouble with resonances of unstable particles.

One-loop $2 \to N$, $N \leq 4$ matrix elements for SM processes

- Sort amplitudes by helicity, colour and analyticity properties!
- Generate graphs for a given amplitude (e.g. FeynArts)
- Make tensor decomposition of scattering amplitude
  $\Rightarrow$ Finite number of types of amplitudes ($6V$,$4Vf\bar{f}$,$2V2Sf\bar{f}$,...)
- Project onto tensor coefficients
  $\Rightarrow$ Lorentz indices saturated,
  tensor reduction of 5, 6 point functions not problematic ($k \cdot p_j$ reducible)
  $\Rightarrow$ no high powers of Gram determinants, good for numerical stability
- Tensor reduction of 1- to 4-point functions: use form factors in $n-$dimensions (!)
  (or use numeric approach as outlined above!)
- Sort into basis set of two, three, four point functions
  $\Rightarrow$ Basis set for each amplitude can be determined automatically
  $\Rightarrow$ IR, UV divergent part can be isolated explicitly!
- Simplify coefficients analytically using algebraic programs

Explicit cancellations of IR divergences

- Use Dipole formalism for massive partons

[S. Catani, S. Dittmaier, M.H. Seymour, Z. Trocsányi, hep–ph/0201036]

$$\begin{aligned}\sigma_{NLO} &= \sigma_{NLO}[2 \to N+1] + \sigma_{NLO}[2 \to N] \\ &= \int\limits_{N+1} \left[(\mathrm{d}\sigma_{\mathrm{R}})_{\epsilon=0} - \left(\sum_{\mathrm{Dipoles}} \mathrm{d}\sigma_{\mathrm{B}} \otimes \mathrm{dV}_{\mathrm{Dipole}} \right)_{\epsilon=0} \right] \\ &\quad + \int\limits_{N} \left[\mathrm{d}\sigma_{\mathrm{V}} + \mathrm{d}\sigma_{\mathrm{B}} \otimes \mathbf{I}(\epsilon) \right]_{\epsilon=0}\end{aligned}$$

- Analytical work done!

Example: $\Gamma(\gamma\gamma \to \phi\phi\phi\phi)$ in the gauged Yukawa Model

Tensor decomposition of amplitude:

$$\Gamma^{\lambda_1\lambda_2}(\gamma\gamma \to \phi\phi\phi\phi) = \epsilon_1^{\mu,\lambda_1}\epsilon_2^{\nu,\lambda_2}\mathcal{M}_{\mu\nu} \quad , \quad \mathcal{M}_{\mu\nu} = A\, g_{\mu\nu} + \sum_{l,j=1}^{5} B_{lj}\, r_l^\mu r_j^\nu$$

Projection on tensor coefficients:

$$\mathcal{P}^{\mu\nu} = \frac{1}{n-5}[g^{\mu\nu} - 2\sum_{l,j=1}^{5} H_{lj} r_l^\mu r_j^\nu] \quad , \quad \mathcal{R}_l^\mu = 2\sum_{j=1}^{5} H_{lj}\, r_j^\mu \,, \qquad H = G^{-1}\,,\ G_{ij} = 2 r_i \cdot r_j$$

$\Rightarrow$

$$\tilde{A}(\mathcal{M}) = \mathcal{P}^{\mu\nu}\mathcal{M}_{\mu\nu} = A \quad , \quad \tilde{B}_{ij}(\mathcal{M}) = \left[\mathcal{R}_i^\mu \mathcal{R}_j^\nu - 2H_{ij}\mathcal{P}^{\mu\nu}\right]\mathcal{M}_{\mu\nu} = B_{ij}$$

Transversality: $\epsilon_1 \cdot p_1 = \epsilon_2 \cdot p_2 = 0$

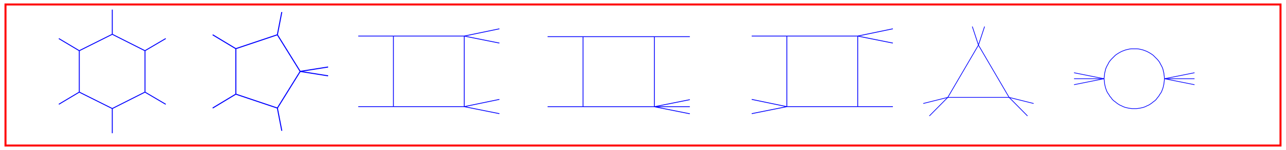
Gauge invariance: solve Ward identities $\mathcal{M}^{p_1\epsilon_2} = \mathcal{M}^{\epsilon_1 p_2} = 0$
or work in helicity basis $\epsilon_1 \cdot p_1 = \epsilon_1 \cdot p_2 = \epsilon_2 \cdot p_1 = \epsilon_2 \cdot p_2 = 0$

$\Rightarrow$ Only 9 out of 26 tensor coefficients needed!

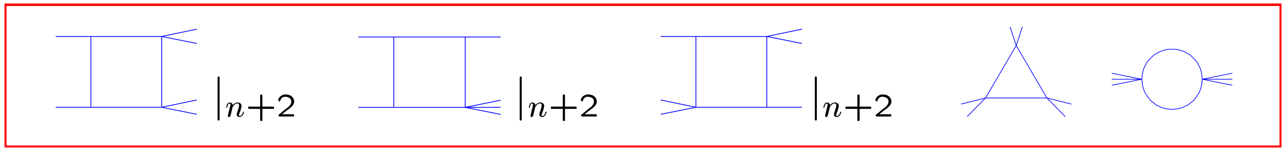
$\Gamma(\gamma\gamma \to \phi\phi\phi\phi)$ in the gauged Yukawa Model

A, B_{ij} are linear combinations of contracted Tensor integrals or equivalently Feynman parameter integrals with non trivial numerators.

After tensor reduction (only needed for up to 4-point functions) each graph is written as linear combination of N-point scalar integrals with $N \le 6$



Apply scalar reduction formulas



Each graph IR finite $\Rightarrow$ 3-point functions with massless legs $\sim 1/\epsilon, 1/\epsilon^2$ cancel!
Each graph UV finite $\Rightarrow$ $\sim 1/\epsilon$ poles of 2-point functions cancel!

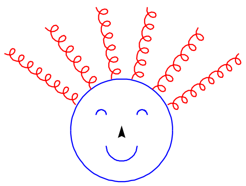
All tensor coefficients are expressible in this function basis $\to$ decomposition, combine to form helicity amplitudes (cancellations at each step)

$$\boxed{\begin{aligned} \Gamma^{++} &= A\,\epsilon_1^+ \cdot \epsilon_2^+ + \sum B_{lj}\, r_l \cdot \epsilon_1^+\, r_j \cdot \epsilon_2^+ \\ \Gamma^{+-} &= A\,\epsilon_1^+ \cdot \epsilon_2^- + \sum B_{lj}\, r_l \cdot \epsilon_1^+\, r_j \cdot \epsilon_2^- \end{aligned}}$$

Result for $\mathcal{M}(\gamma\gamma \to \phi\phi\phi\phi)$:

$$\mathcal{M}^{++}(\gamma\gamma \to \phi\phi\phi\phi) = -\frac{\lambda^4 g^2}{(4\pi)^2}\frac{\varepsilon_1^+ \cdot \varepsilon_2^+}{s_{12}} \sum_{\pi \in S_4} \{C(1,\pi_3,2,\pi_4,\pi_5,\pi_6)$$
$$\times[F_1(s_{\pi_4\pi_5}, s_{\pi_5\pi_6}, s_{12\pi_3}) + F_1(s_{1\pi_3}, s_{2\pi_3}, s_{12\pi_3}) - 2\,F_1(s_{1\pi_3}, s_{3\pi_4}, s_{1\pi_3\pi_4})$$
$$+2\,F_2(s_{12\pi_3}, s_{2\pi_4\pi_5}, s_{1\pi_3}, s_{\pi_4\pi_5}) - F_2(s_{1\pi_3\pi_4}, s_{2\pi_4\pi_5}, s_{1\pi_3}, s_{2\pi_5}) - F_2(s_{1\pi_3\pi_4}, s_{2\pi_3\pi_4}, s_{\pi_3\pi_4}, s_{\pi_5\pi_6})]$$
$$+2\,F_2(s_{1\pi_3\pi_4}, s_{2\pi_3\pi_4}, s_{\pi_3\pi_4}, s_{\pi_5\pi_6}) + (1 \leftrightarrow 2)\}$$

$$\begin{aligned}
C(1,2,3,4,5,6) &= 2 + \frac{\mathrm{tr}^+(3456)B_5 + \mathrm{tr}^+(1234)B_1}{s_{34}} + \frac{\mathrm{tr}^+(6123)B_2 + \mathrm{tr}^+(4561)B_4}{s_{16}} \\
F_1(s_1,s_2,s_3) &= -\log\left(\frac{s_1}{s_3}\right)\log\left(\frac{s_2}{s_3}\right) \\
F_2(s_1,s_2,s_3,s_4) &= -\frac{1}{2}\log^2\left(\frac{s_3}{s_4}\right) - \mathrm{Li}_2\left(1 - \frac{s_1 s_2}{s_3 s_4}\right) \\
\mathrm{tr}^+(jl\ldots) &= \mathrm{tr}((1+\gamma_5)\,\not{p}_j \not{p}_l \ldots)/2
\end{aligned}$$



Non–linear constraintes between Mandelstam variables are represented <u>linearly</u> through reduction coefficients B_j !!!

$\mathcal{M}^{+-}$ no short representation found ...

The $\gamma\gamma \to ggg$ amplitude

- Relevant for $\gamma\gamma +$ jet background for Higgs+jet production
- Amplitude implicitly known from $gg \to ggg$
- no direct computation

Independent helicity structures: $\Gamma^{+++++}, \Gamma^{++++-}, \Gamma^{+++--}, \Gamma^{+-+++}, \Gamma^{+-++-}, \Gamma^{--+++}$

Exercise: calculate helicity amplitudes in a modular way

- Box, pentagon topologies
- Only one colour structure: $\sim f^{abc}$
- Sort by analytical structure: $1, I_2^n, I_{3,1\text{mass}}^n, I_{3,2\text{mass}}^n, I_{4,1\text{mass}}^{n+2}$

The amplitude $\Gamma^{+++++}[\gamma\gamma ggg \to 0]$

With the reference momenta p_5, p_1, p_2, p_3, p_4 products of $\epsilon_j \cdot p_k$ can be treated uniformly for all graphs $2\text{tr}^-(p_j, \ldots) = \text{tr}(p_j) - \text{tr}(\gamma_5, p_j, \ldots)$:

$$\prod_{j=1}^{5} \epsilon_j \cdot q_j = \frac{\text{tr}^-(p_1, q_1, \ldots, p_5, q_5)}{4\sqrt{2} < 51 >< 12 >< 23 >< 34 >< 45 >} \tag{1}$$

$$\Gamma^{+++++} = \frac{16}{\sqrt{2} < 51 >< 12 >< 23 >< 34 >< 45 >} \text{tr}^+(1, 2, 3, 5) \frac{s_{12}}{s_{35}}$$

$F_j^{\mu\nu} = p_j^{\mu} \epsilon_j^{\nu} - p_j^{\nu} \epsilon_j^{\mu}$

$$\boxed{\Gamma^{+++++} = 32 f^{abc} \frac{\text{tr}(F_1^+ F_2^+) \text{tr}(F_3^+ F_4^+ F_5^+)}{s_{34} s_{45} s_{35}}}$$

The amplitude $\Gamma^{+++++-}[\gamma\gamma \to ggg]$

With the reference momenta p_3, p_4, p_1, p_1, p_1 and has:

$$\prod_{j=1}^{5} \epsilon_j \cdot q_j = \frac{\text{tr}^-(245 q_5 1 q_3 3 q_1 1 q_4 4 q_2)}{4\sqrt{2} < 41 >< 32 >< 13 >< 14 > [51] < 24 > [45]} \tag{2}$$

$$\Gamma^{++++-} = f^{abc} \frac{A + 4\,\epsilon(p_1, p_2, p_3, p_4)\, B}{4\sqrt{2} < 41 >< 32 >< 13 >< 14 > [51] < 24 > [45]}$$

$F_j^{\mu\nu} = p_j^\mu \epsilon_j^\nu - p_j^\nu \epsilon_j^\mu$

$$\boxed{\Gamma^{++++-} = f^{abc} \frac{\text{tr}(F_1^+ F_3^+)\text{tr}(F_2^+ F_4^+)[C_A\, p_1 \cdot F_5^- \cdot p_4 + C_B\, p_1 \cdot F_5^- \cdot p_3]}{2 s_{13}^2 s_{24}^2 s_{14} s_{45} s_{15}}}$$

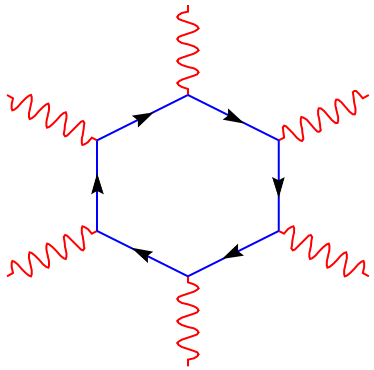
$\rightarrow$ Ward identities manifestly fullfilled

$\rightarrow$ Bose symmetry $1 \leftrightarrow 2$, $3 \leftrightarrow 4$ strong check

$$
\begin{aligned}
C_A \quad = \quad & +(-64\, s_{35}\, s_{14}\, (s_{14}\, s_{35} - s_{35}^2 + s_{12}\, s_{35} - s_{35}\, s_{34} + s_{14}\, s_{34})/s_{23}) \\
& +(64\, (s_{14} + s_{12})\, (s_{12} - s_{34})\, (-s_{12} + s_{23} + s_{34} - s_{14})\, (s_{23}\, s_{12} - s_{14}\, s_{34})/s_{12}/s_{35}) \\
& +(64\, s_{14}\, s_{35}\, (s_{12} - s_{35})\, (-s_{14} + s_{23} + s_{35})\, (s_{23} - s_{14} + s_{35} - s_{12})/s_{12}/s_{34}) \\
& +(-64\, s_{14}\, (-6\, s_{35}\, s_{14}\, s_{34} - 3\, s_{34}\, s_{14}\, s_{23} + s_{34}\, s_{23}^2 + 4\, s_{35}\, s_{34}\, s_{23} \\
& -4\, s_{34}^2\, s_{14} + 3\, s_{35}^3 + 4\, s_{35}^2\, s_{34} + 3\, s_{35}\, s_{34}^2 + s_{34}^3 - 5\, s_{14}\, s_{35}^2 \\
& +4\, s_{23}\, s_{35}^2 + 2\, s_{34}^2\, s_{23} - 3\, s_{35}\, s_{14}\, s_{23} + s_{23}^2\, s_{35} + 2\, s_{35}\, s_{14}^2 + 2\, s_{34}\, s_{14}^2)/s_{12}) \\
& +(320\, s_{35}\, s_{14}\, s_{34} + 64\, s_{34}\, s_{23}^2 - 576\, s_{35}\, s_{14}^2 + 256\, s_{35}\, s_{34}\, s_{23} - 64\, s_{12}^3 \\
& -192\, s_{12}\, s_{34}^2 - 128\, s_{23}^2\, s_{12} + 192\, s_{35}\, s_{34}^2 + 64\, s_{34}\, s_{14}\, s_{23} - 192\, s_{12}\, s_{35}^2 \\
& +192\, s_{35}^2\, s_{34} + 192\, s_{34}^2\, s_{14} + 256\, s_{35}\, s_{14}\, s_{23} - 320\, s_{23}\, s_{35}\, s_{12} \\
& +64\, s_{23}^2\, s_{35} + 128\, s_{23}\, s_{35}^2 + 64\, s_{35}^3 - 64\, s_{23}\, s_{14}^2 + 192\, s_{12}^2\, s_{35} \\
& +192\, s_{12}\, s_{14}\, s_{23} - 384\, s_{12}\, s_{23}\, s_{34} - 64\, s_{12}\, s_{14}\, s_{34} - 128\, s_{12}\, s_{14}\, s_{35} \\
& -384\, s_{34}\, s_{14}^2 + 384\, s_{14}\, s_{35}^2 - 384\, s_{12}\, s_{35}\, s_{34} + 256\, s_{23}\, s_{12}^2 + 192\, s_{12}^2\, s_{34} \\
& +128\, s_{14}^3 + 128\, s_{34}^2\, s_{23} + 192\, s_{14}^2\, s_{12} + 64\, s_{34}^3)
\end{aligned}
$$

$$
\begin{aligned}
C_B \quad = \quad & +(-64\, s_{14}^2\, (s_{14}\, s_{35} - s_{35}^2 + s_{12}\, s_{35} - s_{35}\, s_{34} + s_{14}\, s_{34})/s_{23}) \\
& +(-64\, s_{14}\, (s_{34}^2 + s_{35}\, s_{34} + s_{35}^2)\, (s_{34} - s_{14} + s_{35} + s_{23})\, (s_{35}\, s_{34} \\
& -s_{14}\, s_{34} + s_{35}\, s_{23} - s_{14}\, s_{35} + s_{35}^2)/s_{12}/s_{35}/s_{34}) \\
& +(-64\, s_{14}\, (-2\, s_{34}^3 + s_{23}^2\, s_{12} + 4\, s_{34}^2\, s_{14} + 2\, s_{12}^2\, s_{14} - 5\, s_{12}\, s_{14}\, s_{34} \\
& -s_{12}\, s_{14}\, s_{23} + 4\, s_{12}\, s_{23}\, s_{34} - s_{23}\, s_{12}^2 - 3\, s_{34}^2\, s_{23} - s_{34}\, s_{23}^2 \\
& +s_{14}^2\, s_{12} + 2\, s_{34}\, s_{14}\, s_{23} - 3\, s_{12}^2\, s_{34} + s_{12}^3 - 2\, s_{34}\, s_{14}^2 + 4\, s_{12}\, s_{34}^2)/s_{35}) \\
& +(-64\, s_{14}\, (-s_{14} + s_{23} + s_{35})\, (s_{23}\, s_{12} - 2\, s_{35}\, s_{23} - s_{12}\, s_{14} + 3\, s_{12}\, s_{35} - 3\, s_{35}^2 + 2\, s_{14}\, s_{35} - s_{12}^2)/s_{34}) \\
& +(64\, s_{14}\, (-10\, s_{14}\, s_{35} + 2\, s_{23}^2 + 9\, s_{35}\, s_{23} - 9\, s_{12}\, s_{34} + 10\, s_{35}\, s_{34} \\
& -3\, s_{14}\, s_{23} + 8\, s_{35}^2 - 9\, s_{14}\, s_{34} - 6\, s_{23}\, s_{12} + 8\, s_{12}\, s_{14} \\
& +2\, s_{14}^2 - 8\, s_{12}\, s_{35} + 4\, s_{12}^2 + 8\, s_{34}\, s_{23} + 7\, s_{34}^2))
\end{aligned}
$$

Under construction: $\Gamma(\gamma\gamma \to \gamma\gamma\gamma\gamma)$:



- automatic FORM code generation
- modular programming for general 6–point kinematics
- n-dimensional reduction successful

Procedures are starting point of GOLEM

Summary:

- Multi–particle/jet production very important if $\sqrt{s} \sim$ TeV
  - $\Rightarrow$ new methods to calculate such processes at NLO
  - $\Rightarrow$ Encouraging results for the Yukawa model
  - $\Rightarrow$ Objectif: $PP \to 4$ jets at NLO
  - $\Rightarrow$ Numerical methods for the electro–weak sector at NLO
- Higgs searches need background calculations for $2 \to 3, 4$ processes
  - $\Rightarrow$ industrial production of NLO calculation needed
  - $\Rightarrow$ GOLEM project

Only a profound knowledge of the Standard Model will allow us to understand new and unexpected phenomena !!!