String Theories in Lower Dimensions

W. Lerche, CERN AcTr, 12/2002
Part 2

Recall:
Perturbative constructions (based on 2d conformal field theory on Riemann surfaces) , subject to certain consistency requirements, lead to

- 5 supersymmetric consistent string theories in D=10:

Combination	Name	Gauge group
$S \otimes \bar{S}^{\dagger}$	Type IIA	$U(1)$
$S \otimes \bar{S}$	Type IIB	–
$S \otimes \bar{B}$	Heterotic	$E_8 \times E_8$
$S \otimes \bar{B}'$	Heterotic′	$SO(32)$
$(S \otimes \bar{S})/Z_2$	Type I (open)	$SO(32)$

- Spectra are highly restricted by anomaly cancellations (guaranteed by modular invariance)

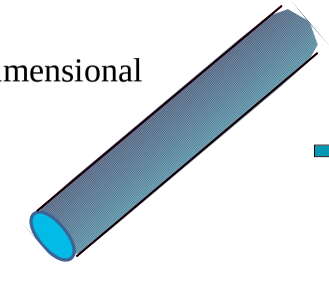
- They have very different perturbative spectra in 10d; Naively, all reason to believe that they are different theories... !

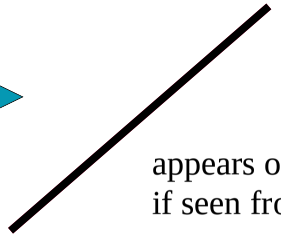
- But we don't live in D=10 but in D=4.....

"Compactification" of Dimensions

Rolling up:

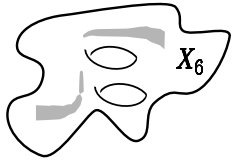
three-dimensional object





appears one-dimensional, if seen from a distance !

compact six-dimensional manifold



appears zero-dimensional, if seen from a distance !

- assume space-time has form:

$$\mathbb{R}^{10} \longrightarrow \mathbb{R}^4 \otimes X_6$$

where X_6 is some 6-dim manifold of very small size

10-dim fields

$$A_M \longrightarrow \{A_\mu, \phi_i\}$$
$$g_{MN} \longrightarrow \{g_{\mu\nu}, A_{\mu i}, \phi_{ij}\}$$

4-dim fields

At large enough distances, or low energies, the extra dimensions are hidden and the theory effectively looks four-dimensional !

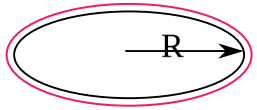
Toy Model: Compactification on Circle S^1

Hamiltonian:

$$H = \frac{m^2}{R^2} + n^2 R^2 - 1 \,, \qquad m, n \in \mathbf{Z}$$

discrete momentum states
(like in particle QFT,
"Kaluza-Klein"
spherical harmonics)

winding states
(specifically stringy)



become light as $R \to \infty$

become light as $R \to 0$

Important: theory is invariant under exchange

$$R \leftrightarrow \frac{1}{R} \,, \qquad m \leftrightarrow n$$

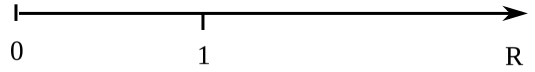
"T-Duality"

Large-Small Radius "T"-Duality

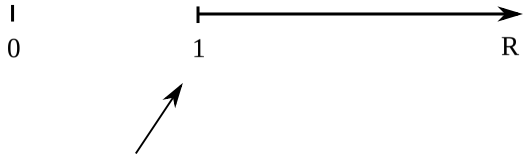
No **physical distinction** between large or small compactification radius !

Compare parameter ("**moduli**") spaces of inequivalent vacua:

- Particle QFT on circle with radius R:



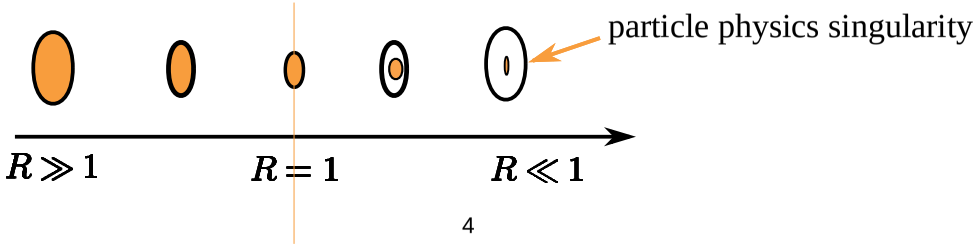
- String theory on circle with radius R:



effective minimal length scale

String theory defines a novel kind of geometry: **"Stringy Geometry"**

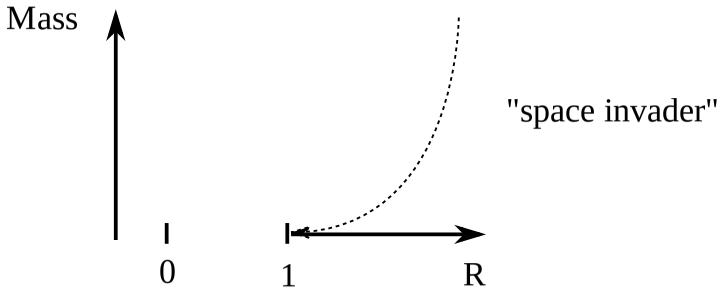
(here: geometrically distinct circles
become identified in string theory)



Extra Gauge Symmetry

- Typically, interesting phenomena arise at special (boundary) points of parameter space:

Consider the mass of momentum-winding states with $(m, n) = (1, \pm 1)$



extra massless states at self-dual radius R=1

.... SU(2)xSU(2) gauge fields

changing the radius away from R=1, gives these gauge fields a non-zero mass

→ Stringy Higgs effect

- General principles:

compactification induces additional states and geometrical parameters

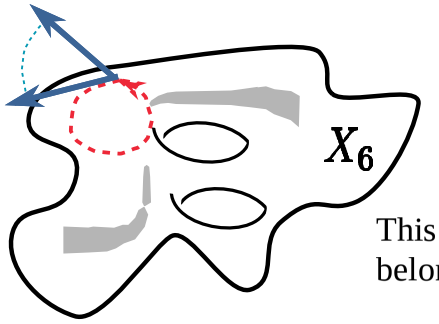
these correspond to undetermined field theory VEV's $R \sim \langle \Phi \rangle$

fixed points of duality transformations: extra gauge symmetries

Holonomy and Supersymmetry

N=1 Supersymmetry is phenomenologically desirable, and technically required for having a tractable theory with a stable ground state.

Consider looping a tangent vector on the 6-dimensional compactification manifold:



This generically induces a rotation which belongs to the "**holonomy**" group SO(6)

Condition for supersymmetry:
Existence of a covariantly constant spinor

A priori, spinors on some X6 transform as the 4-dimensional spin representation of SO(6).
Assume a complex "Kahler" manifold with holonomy group $\mathcal{H}(X_6) \simeq SU(3)$ and thus $\mathbf{4} \rightarrow \mathbf{3} \oplus \mathbf{1}$

The singlet component is supposedly covariantly constant and represents the unbroken supercharge:

$$\nabla \Psi = 0 \longrightarrow \gamma^k R_{ik} \Psi = 0$$

Represents definition of a "**Calabi-Yau**" manifold:

"complex Kahler manifold with vanishing first Chern class"

$$c_1 = R_{ik} = 0$$

Calabi-Yau Compactifications

To ensure supersymmetry in D dimensions, X must be a multi-torus, or a **Calabi-Yau**-manifold with holonomy

$$\mathcal{H} \subset SU(5-D/2)$$

Possibilities for having N supersymmetries in various dimensions:

D	X_{10-D}	$\mathcal{H}$	Type II string on X_{10-D}	heterotic string on X_{10-D}
8	T_2	1	$N=2$	$N=1$
6	T_4	1	$N=4$	$N=2$
	$K3$	$SU(2)$	$N=2$	$N=1$
4	T_6	1	$N=8$	$N=4$
	$K3 \times T_2$	$SU(2)$	$N=4$	$N=2$
	Calabi-Yau	$SU(3)$	$N=2$	$N=1$

Phenomenologically most interesting (can have chiral fermions)

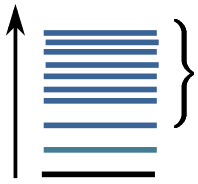
Unfortunately, plenty of possibilities....

Supersymmetry Breaking

in perturbative heterotic string compactifications

- As a dogma, one likes approximate supersymmetry, spontaneously broken only at the TeV scale in order to protect the weak scale from renormalization
- Generic string prediction:
  Modular invariance implies that SUSY breaking scale is scale of compact dimensions !

Mass



KK momentum excitations with large multiplicities..

SUSY breaking scale = gravitino mass ~ 1 TeV

10d parameters: string length $1/\ell_s = m_s$

string coupling $\lambda_s = e^{<\Phi>}$

4d parameters: Planck scale $m_{\rm planck} \cong 10^{19}{\rm GeV}$

gauge coupling $1/g^2 \cong 1/25$

Compare: $m_s = g\, m_{\rm planck}$

$\lambda_s = g \dfrac{\sqrt{{\rm Vol}(X_6)}}{\ell_s^{\,3}}$ string coupling grows for large X_6 !

If we have 1/TeV-sized compact dimensions, heterotic strings must be strongly coupled !

Perturbative description breaks down... need to use non-perturbative dualities....

Doing away with Supersymmetry ?

- Supersymmetry is **not** an intrinsic prediction of string theory !

  It has been invented to remedy renormalization properties of particle QFT, ensure the cosmological constant to vanish, etc.

  However, string theory is more clever than QFT, and naive particle physics intuition can be very misleading.....

- Consider eg vanishing of 1-loop vacuum energy (cosmolog. const)

  - Particle theory:

$$\mathcal{A}_{\text{part}} = \int_t \text{Tr}[e^{-tH}] = \int \sum \equiv - \equiv \stackrel{!}{=} 0$$

bos fer

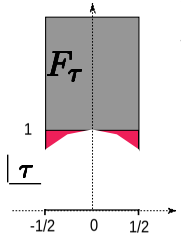
(level-by-level cancellation)

  - String theory:

$$\mathcal{A}_{\text{string}} = \int_{F_\tau} \text{Tr}[e^{-tH} e^{2\pi i s R}] = \mathcal{A}_{\text{part}} + \text{stringy stuff} \stackrel{!}{=} 0$$

May vanish only after modular integration, without integrand being zero

Has no analog in particle theory



Amplitudes can vanish even without supersymmetry !

Topology and Zero-Modes

- Since the excitation spectrum is typically 10^19GeV, we are mainly interested in the **massless zero-modes**.

  ...these probe the global, topological properties of X_6

- Expand 10-dim field on $\mathbb{R}^4 \otimes X_6$:

$$\Phi = \sum_i \phi_i^{(4)} \omega_i^{(6)}$$

Laplace operator:

$$\Delta^{(10)} = \Delta^{(4)} + \Delta^{(6)}$$

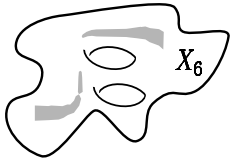
(mass term in 4d wave equation)

The 4-dim fields $\phi^{(i)}$ are massless if $\omega^{(i)}$ are **harmonic differential** forms on X_6 :

$$d\omega_i^{(6)} = d^*\omega_i^{(6)} = 0$$

- Such forms play a crucial role in algebraic geometry, and indeed reflect the topology ("cohomology") of the compactification space...

Their numbers are (roughly!) given by the numbers of higher-dimensional "holes" within X_6



More precisely, the spectrum is given by the topological "**Hodge numbers**" associated with every Calabi-Yau X_6:

$$h^{pq} = \dim H_{\bar\partial}^{p,q}(X_6, \mathbf{C})$$

of which only h^{11} and h^{21} are independent

- For the heterotic string compactified on some Calabi-Yau manifoldX6, we typically get the an effective N=1 supergravity theory plus various extra gauge and matter ("chiral") super-fields:

$$\Phi_i \equiv (\phi_i, \psi_i)$$

- graviton and gravitino, $g_{\mu\nu}, \Psi_{\mu\alpha}$
  dilaton-axion superfield S,

- gauge bosons and gauginos corresponding to gauge group $E_6 \times E_8$,

- h^{21} matter superfields in the $\underline{27}$ of E_6,

  h^{11} matter superfields in the $\underline{27}^*$ of E_6

  h^{21} matter superfields: complex structure (shape) moduli,

  h^{11} matter superfields: Kahler (size) moduli,

  H^1(End T) matter superfields: gauge singlets,

- Net # of left- minus right-handed families = 1/2 "Euler number" :

$$\frac{1}{2}\left|h^{11}(X_6) - h^{21}(X_6)\right| \equiv \frac{1}{2}|\chi(X_6)|$$

This gives a natural **repetitive structure** of "particle generations" !

Generic Properties of D=4 Compactifications

On typical Calabi-Yau manifolds, heterotic strings provide thus effective particle field theories in D=4 with:

- Gravity
- Gauge symmetries
- Chiral fermions
- Repetitive generation structure
- Higgs mechanism
- (Supersymmetry)

... the **generic** features of what we do see in nature, all coupled together in a truly consistent manner !

This is the main achievement of string theory

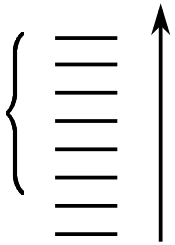
But there are also quite a few unpleasant features and unsolved problems, eg:

- Vacuum state indeterminacy
- plenty of scalar fields, massless "dilaton" field
- How to get rid off supersymmetry
- Why is the cosmological constant (almost) zero

Predictivity

- Are there more than those generic predictions of string theory ?

In principle: infinitely many predictions !
(spectrum very tightly constrained by consistency)



In practice: almost no predictions in zero mode sector

Properties of massless sector

⟷

Properties of compactification space

= **choice of vacuum state** $R \sim \langle \Phi \rangle$

.. not much determined by 10D string theories !

Analogous to spontaneously chosen direction of magnetization in a ferro-magnet, which is also not determined by fundamental principles....

The specific properties of the standard model may not have any particular reason at all ...
they simply might be "frozen historical accidents"

The Vacuum Degeneracy Problem

The lack of predictivity at low energies is the most serious challenge for the credibility of string theory !

- There are plenty of Calabi-Yau spaces, like 10^4, and it is not clear why any one should be singled out. Nor why D=4 would be preferred at all ....

- On top of that there are 5 theories in D=10, all of which give four-dimensional theories upon appropriate compactification.

If one is the fundamental theory,
what is the meaning of the others ?

- Each Calabi-Yau space leads in general to a different spectrum in D=4, and, even worse, has a huge **parameter space** by itself.

We don't have good answers for the vacuum degeneracy problem, but have made exciting progress in understanding the second question.... see lecture 3.

Geometrization of Coupling Constants

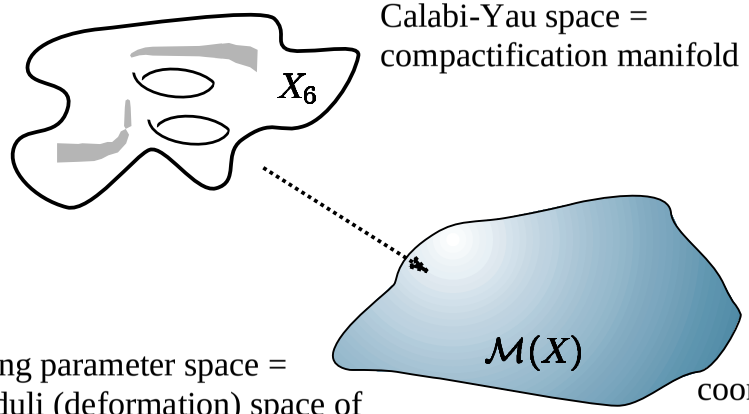
- In D=10, there is essentially just one free parameter, namely the string coupling; it corresponds to the VEV of the dilaton field:

$$\lambda_s = e^{<\Phi>}$$

- Compactification to lower dimensions makes theories much **more complex** than in D=10 !

(Recall compactification on circle: radius gives masses of gauge bosons)

The geometrical parameters ("moduli") that govern the **shape** of X_6 become free physical vacuum VEV's (like in couplings $\langle\Phi_i\rangle\psi\psi$), which are not determined by the 10d theory.



Calabi-Yau space = compactification manifold

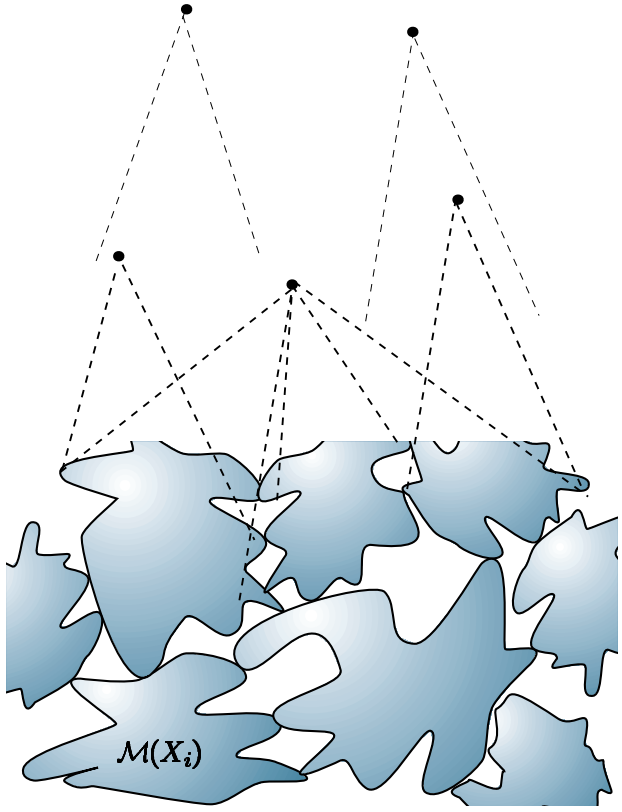
String parameter space = moduli (deformation) space of compactification manifold

coordinates = moduli fields $\langle\Phi_i\rangle$

Typically, $\dim\mathcal{M}(X_6) \cong 100$

- Almost every coupling of the effective 4 dimensional theory has a geometric interpretation rooted in the properties of X_6

N=2 SUSY String Compactifications in D=4



Five D=10 Theories

Compactification

D=4 string vacua

Each of the ~ 10^8 blobs corresponds to a continuous, ~100-parameter family of string theories in D=4

Consistency is restrictive primarily only for the high-dimensional string theories !

SUSY Effective Actions in d=4

- A computation of the general full string effective action is not feasible!

  However, in SUSY theories we can go pretty far:
  they are partially characterized by **holomorphic functions $f(\phi)$**
  of the massless moduli (scalar) fields.

  These are protected by non-renormalization theorems, and
  largely determined given by topological properties of X_6.

- Examples for such holomorphic functions:

$$\begin{aligned} N=4: &\quad \text{gauge coupling } \tau(\phi),\ \text{higher derivative terms} \\ N=2: &\quad \text{gauge coupling } \tau(\phi)=\partial_\phi{}^2 F(\phi),\ \text{(Prepotential)}\ \ldots \\ N=1: &\quad \text{Superpotential } W(\phi),\ \text{gauge coupling } \tau(\phi),\ \ldots \end{aligned}$$

- E.g., superpotential for the fields ϕ_a in the 27 of E_6

$$W(\phi) = \phi_a\phi_b\phi_c \int_{X_6} \omega_a^{1,1}\wedge\omega_b^{1,1}\wedge\omega_c^{1,1} + \text{corr.}$$

intersection #'s

non-perturbative instantion corrections - how to compute ?

World-Sheet Instanton Corrections

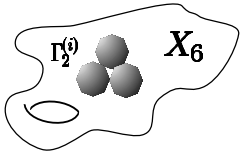
- Despite non-perturbative corrections, certain (holomorphic) quantities like superpotentials can often be computed exactly

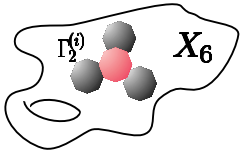
- eg gauge couplings in N=2 SUSY (type II strings on CY) depend on moduli fields t:

$$\tau_{\text{eff}}(t) \equiv \frac{1}{2\pi}\theta_{\text{eff}}(t) + 2\pi i \frac{1}{g_{\text{eff}}{}^2(t)} = \tau_0 + \sum_{\ell=1}^{\infty} c_\ell \log[1-e^{2\pi i \ell t}]$$

bare coupling = topological intersection number

Instanton corrections: string world-sheets wrapping around 2-cycles $\Gamma_2^{(i)}$





Nonperturbative in 2d, but tree-level in 4d

- How to determine the unknown coefficients c_ℓ ?

  Mathematically, this corresponds to summing up all maps from the string world-sheet into the Calabi-Yau:

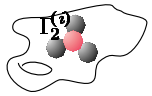
$$S^2 \longrightarrow X_6 \qquad e^{-S_{\text{inst}}} = e^{2\pi i t}$$

  ..which is an extremely hard problem in algebraic geometry!

Mirror Symmetry

- Is a first example of a very non-trivial duality, in fact it is a generalization of (perturbative) T-duality

type IIA string on CY ⟷ type IIB string on CY'



(CY' = a quite different "mirror manifold")

complicated world-sheet instantion corrections; 2-cycles Γ_2

classical tree-level intersection geometry of 3-cycles Γ_3

difficult

easy

boils down to computing certain integrals:

$$\tau_{\text{eff}} = \sum_{\ell=1}^{\infty} c_\ell \log[1 - e^{2\pi i \ell t}]$$

$$\tau_{\text{eff}}(t) = \partial_t \int_{\Gamma_3} \Omega^{(\text{CY}')}(t)$$

which determines all c_ℓ .

..and the mathematicians are delighted too ;-)

By mapping a **complicated** problem to a **simple** one, via duality, one can obtain highly non-trivial, exact results !

.. and as we will see, similar methods work at the non-perturbative level as well !

("**S-Duality**")