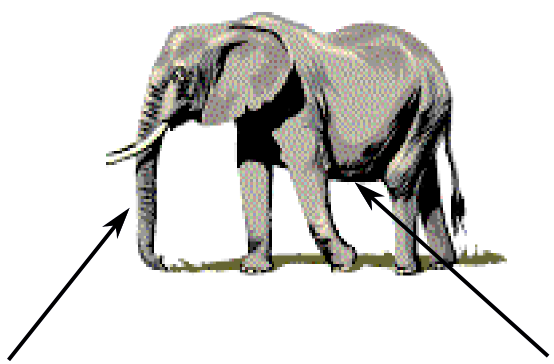
# Non-Perturbative String Physics

W. Lerche, CERN AcTr, 12/2002
Part 3

- We have seen that there are 5 superstring theories in D=10, leading to very many different D=4 compactifications

- But it turns out that thinking in terms of perturbation theory only, we are effectively blindfolded...



Approximate description in terms of cylinder geometry

Description in terms of cylinder geometry is not useful here

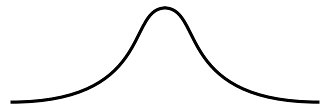
(And yes, I had the elephant before Brian Greene...)



# Non-perturbative Equivalences

- Map **solitonic** (non-perturbative, non-local extended) degrees of freedom to **elementary** (perturbative, local) ones, and vice versa

"local" • $\Phi(x)$ elementary field $\longleftrightarrow$ "non-local" collective excitation ("Soliton")



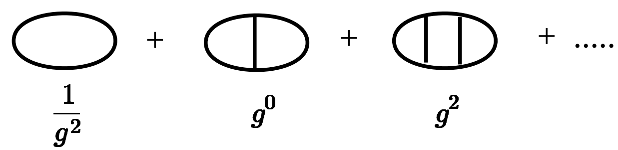
These are simply two ways to describe one and the same physical degree of freedom

Simplest example is 2d Ising model:

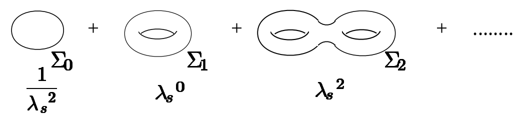
fermion $\Psi \longleftrightarrow :e^{i\Phi}:$ soliton

- Intrinsically a quantum phenomenon !

- Duality typically **maps weak** to **strong coupling**: $g \to \frac{1}{g}$




string theory:




Therefore it cannot be captured in perturbation theory !

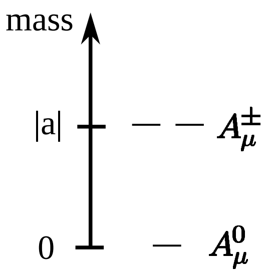
Usual QFT, Lagrange formalisms fail and must be **abandoned** ..

# Montonen-Olive Duality

- Cons. SU(2) gauge theory with Higgs field in the adjoint representation

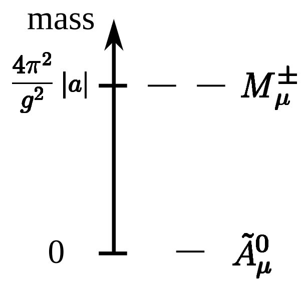
  For non-zero Higgs VEV: $a \equiv \langle \phi \rangle$ the symmetry is broken to U(1), and the charged gauge fields get mass due to the Higgs mechanism

Perturbative spectrum:




elementary gauge fields

Non-perturbative spectrum:




solitonic magnetic monopoles

- Duality transformation:

$$
\begin{aligned}
F_{\mu\nu} &\rightarrow \tilde{F}_{\mu\nu} \quad (\vec{E} \rightarrow \vec{B},\ \vec{B} \rightarrow -\vec{E}) \\
q &\rightarrow m \quad (\text{charges}) \\
a &\rightarrow a_D = \frac{4\pi}{g^2} a \quad (\text{Higgs VEVs}) \\
\frac{4\pi}{g^2} &\rightarrow \frac{g^2}{4\pi} \quad (\text{couplings})
\end{aligned}
$$

In the dual theory, the magnetic monopoles behave like the gauge bosons in the original theory, and are massive via the dual Higgs field

> Montonen-Olive:
> The theory might be **invariant** under this non-perturbative transformation !

A priori, unlikely to be true due to quantum corrections.....



# Supersymmetry and BPS-States

- Supersymmetry in itself may not be not fundamentally important, but it allows us do to non-trivial exact computations, by virtue of its **non-renormalization properties** that **protect** many quantities from perturbative corrections.

- In particular, quantities related to "BPS"-states:

$$Q_\alpha |\text{BPS}\rangle = 0$$

From the algebra of supersymmetry charges

$$\{Q_\alpha, Q_\beta\} = \gamma^{\mu}_{\alpha\beta} P_\mu + \delta_{\alpha\beta} Z$$

("central charge" Z can be eg. U(1) charge)

follows for such BPS-states that their mass is **exactly** given by their charge:

$$m^2 = |Z|^2$$

- Idea: Find that in semi-classical approximation some state is BPS - this implies it has less degress of freedom than a generic state ("short SUSY multiplet")

  But under smooth perturbative and non-perturbative corrections, the number of degrees of freedom cannot jump

  ➡ The state is BPS also in the full quantum theory, and in particular its mass is **exactly** known !

> The BPS property is the quintessential basis of our modern non-perturbative techniques.

## S-Duality in N=4 SUSY Gauge Theory

- N=4 SUSY: no quantum corrections to gauge coupling, and BPS masses:

  Montonen-Olive duality is indeed exact !

- But there is more structure: include the theta-angle to define a complexified gauge coupling,

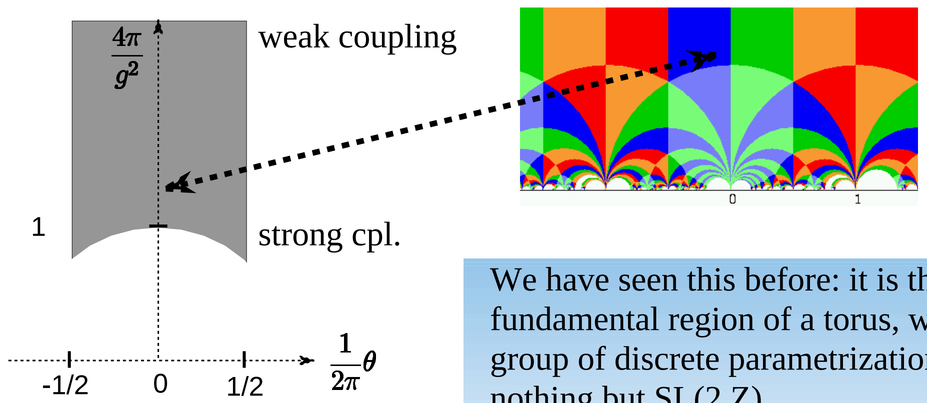
$$\tau \equiv \frac{1}{2\pi}\theta + 2\pi i \frac{1}{g^2}$$

This combines the MO-duality and then theta-shift symmetry:

$$\tau \longrightarrow -\frac{1}{\tau}, \qquad \tau \longrightarrow \tau + 1$$

These transformations generate the non-abelian, discrete "**S-duality**" group, SL(2,Z) !

- This non-perturbative symmetry group implies a non-trivial phase structure, governed by the fundamental domain:




We have seen this before: it is the fundamental region of a torus, whose group of discrete parametrizations is nothing but SL(2,Z) .....

What's the significance of this fact ???



## Duality in N=2 SUSY Gauge Theories

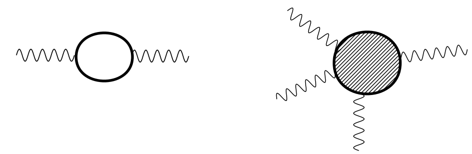
- In N=2 SYM theory, the monople masses do get renormalized, however both the gauge fields (elementary) and the magnetic monopoles (solitonic, non-local) are still BPS.

- Effective gauge coupling gets renormalized, and dependent on the Higgs field:

$$\tau(\phi) \equiv \frac{1}{2\pi}\theta(\phi) + 2\pi i \frac{1}{g^2(\phi)}$$

One knows beforehand the general form of the quantum corrections:

$$\tau(\phi) = \underbrace{\frac{2\pi i}{g_0{}^2}}_{\text{bare coupling}} + \underbrace{\frac{i}{\pi}\log\left[\frac{\phi}{\Lambda^2}\right]}_{\text{one-loop}} - \underbrace{\frac{i}{\pi}\sum_{\ell=1} c_\ell \left(\frac{\Lambda}{\phi}\right)}_{\text{instanton correct}}$$

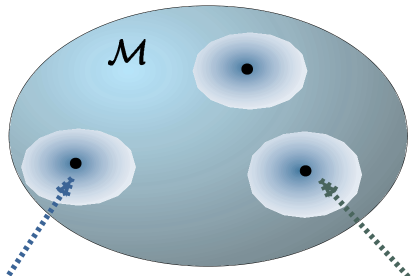


- How to determine the instanton coefficients $c_l$ ?

**Seiberg-Witten 1994**: found a surprising solution, starting from the topology of the parameter space !

# Quantum Moduli Space of N=2 Gauge Theory

- Moduli (parameter) space $\mathcal{M}$: VEV of complex Higgs field $\phi$ (or dual $\phi_D$ )




$\langle \phi \rangle \to \infty$ , $\tau_{\rm eff}(\phi) \to 0$

$\langle \phi_D \rangle \to 0$ , $\tau^D_{\rm eff}(\phi_D) \equiv \frac{-1}{\tau_{\rm eff}(\phi)} \to 0$

gauge fields weakly coupled;
monopoles strongly coupled

$$\tau_{\rm eff}(\phi) = \frac{1}{\pi}\log\left[\frac{\phi^2}{\Lambda^2}\right] - \frac{1}{\pi}\sum_{\ell=1}^{\infty} c_\ell \left(\frac{\Lambda}{\phi}\right)^{4\ell}$$

1-loop ... instanton corr

SU(2) gauge theory with instanton corrections

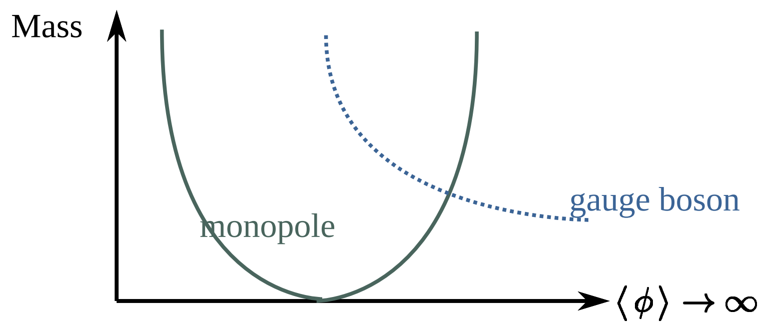
gauge fields strongly coupled;
massless monopoles weakly coupled,
effectively look like electrons;

$$\tau^D_{\rm eff}(\phi) = \frac{-1}{2\pi}\log\left[\frac{\phi_D^2}{\Lambda^2}\right] - \frac{1}{\pi}\sum_{\ell=1}^{\infty} c^D_\ell \left(\frac{\phi_D}{\Lambda}\right)^{2\ell}$$

1-loop ... non-pert corr

U(1) gauge theory with extra electrons

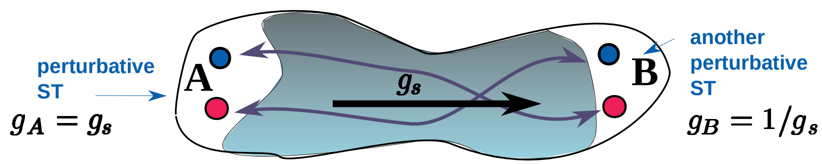
Resummation of non-perturbative corrections



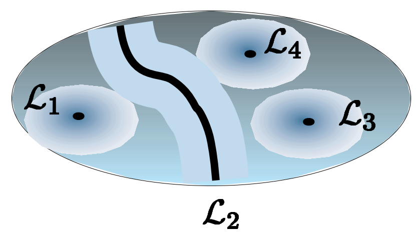



# A general lesson we can abstract from this:

- In general, there is no global description that would be valid throughout the whole moduli space; **no particular lagrangian is more fundamental** than the other ones.




- Lagrangian description makes sense only in "**local coordinate patches**" covering the parameter space $\mathcal{M}$ :




- These describe **different local approximations** of the same theory **in terms of different weakly coupled physical degrees of freedom** (eg, electrons or monopoles)

- The perturbative physics (local QFT) looks completely different in the various local patches (eg, different gauge groups)

➡ Concept of "fundamental degrees of freedom" is questionable, at least ambiguous...

# Solitons in String Theory

- What are the **analogs** of **magnetic monopoles** in string theory ?
  Depending on the string model, there are various
  p-dimensional solitonic "**p-branes**"
  (p=0: particle, p=1: string, p=2: membrane,.....),
  which are not visible in perturbation theory.

- Recall gauge theory EM duality in D=4:
  **1-form gauge field:** $A_\mu \equiv A^{(1)}$

  **gauge boson** $A^{(1)} \rightarrow F^{(2)} \overset{4}{\longleftrightarrow} \tilde{F}^{(2)} \rightarrow \tilde{A}^{(1)}$ **magn monopole**

- Generalized EM duality in D dimensions:

$$F^{(p)} \overset{D}{\longleftrightarrow} \tilde{F}^{(D-p)}$$

➡ Check what generalized gauge fields there are (D=10 Type II strings):

**2-form:** $B^{(2)}$ **RR tensor fields:** $A^{(p+1)}$ $\begin{cases} p = 2k & \text{IIA} \\ p = 2k+1 & \text{IIB} \end{cases}$

**1. fund. string**
**string** $B^{(2)} \rightarrow F^{(3)} \overset{10}{\longleftrightarrow} \tilde{F}^{(7)} \rightarrow \tilde{A}^{(6)}$ **"NS" 5-brane**

**2. RR tensors**
**"D" p-brane** $A^{(p+1)} \rightarrow F^{(p+2)} \overset{10}{\longleftrightarrow} \tilde{F}^{(8-p)} \rightarrow \tilde{A}^{(7-p)}$ **"D" (6-p)-brane**

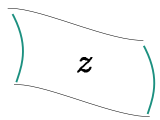
- Typically, some of those branes are BPS and we may hope to be able to do exact computations with them !



# D-Branes as Dirichlet Boundary Conditions

- It was shown that sources for the RR tensor gauge fields are provided simply by Dirichlet boundary conditions for open strings

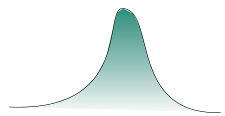
$X(z)$

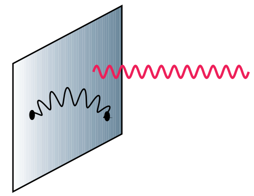



$\partial_\perp X(z) = 0$ $\delta p = 0$ **Neumann**

$\partial_{||} X(z) = 0$ $\delta X = 0$ **Dirichlet**

D-branes can thus simply be described as regions on which open strings can end.
As such, they provide a perturbative description in terms of conformal field theory.





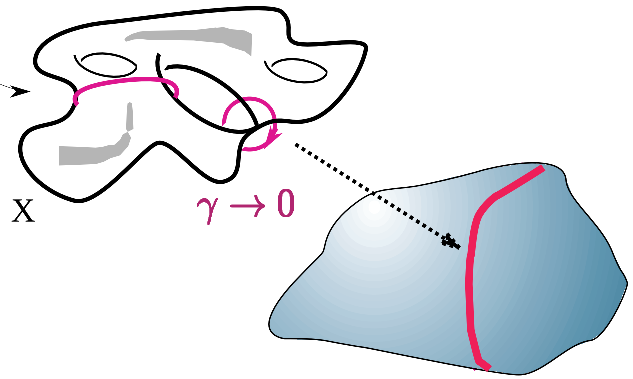
| Closed string (Type II) p-brane soliton | ⟷ | Open string (Type I) elementary D-p-brane |
|---|---|---|
| non-perturbative | | perturbative |

- D-branes are thus string analogs of the elementary electrons into which the magnetic monopoles transform under S-duality.

## Wrapped Branes: Non-Perturbative Extra States

- String compactification: **proliferation of physical degrees of freedom**, obtained from wrapping strings (p=1), membranes (p=2), general p-branes around non-contractible p-cycles of X

At a given singularity in the parameter space $\mathcal{M}(X)$, a compactification manifold X becomes singular in that some p-dimensional "**vanishing cycle** $\gamma$ " **shrinks to zero size**:




- This typically implies a BPS p-brane to become massless, when wrapped around $\gamma$:

$$m^2_{\mathrm{p-brane}} = |\int_\gamma \Omega(X)|^2 \longrightarrow 0 \quad \text{if} \quad \gamma \longrightarrow 0$$

In an appropriate situation, the remnant of this in D=4 space-time is simply an "extra" massless particle of some kind, e.g., a Seiberg-Witten monopole or dual quark, a gauge field, quark, Higgs field....

The existence of such extra solitonic states, not seen in naive perturbative string theory, is the basis for non-trivial equivalences of string theories !



## "Stringy Geometry"

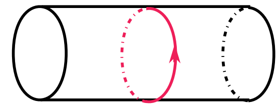
.... transcends ordinary geometry

String theory A compactified on $X_A$ can be dual, ie, quantum equivalent, to string theory B compactified on $X_B$, where the manifolds $X_A$ and $X_B$ are completely different !

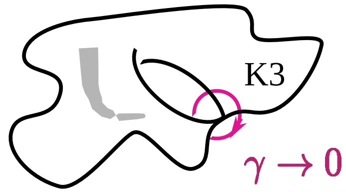
- Example in D=6: Heterotic($T_4$) = Type IIA(K3) = Type I (K3)

One and the same massless SU(2) gauge boson has the following representations in the different theories:

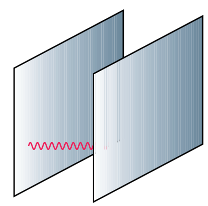
- In the heterotic string model, as a fundamental heterotic string wrapped around a cylinder of radius R=1 (perturbative):



- In the Type IIA string theory, as a 2-brane wrapped around a collapsing 2-cycle (non-perturbative):



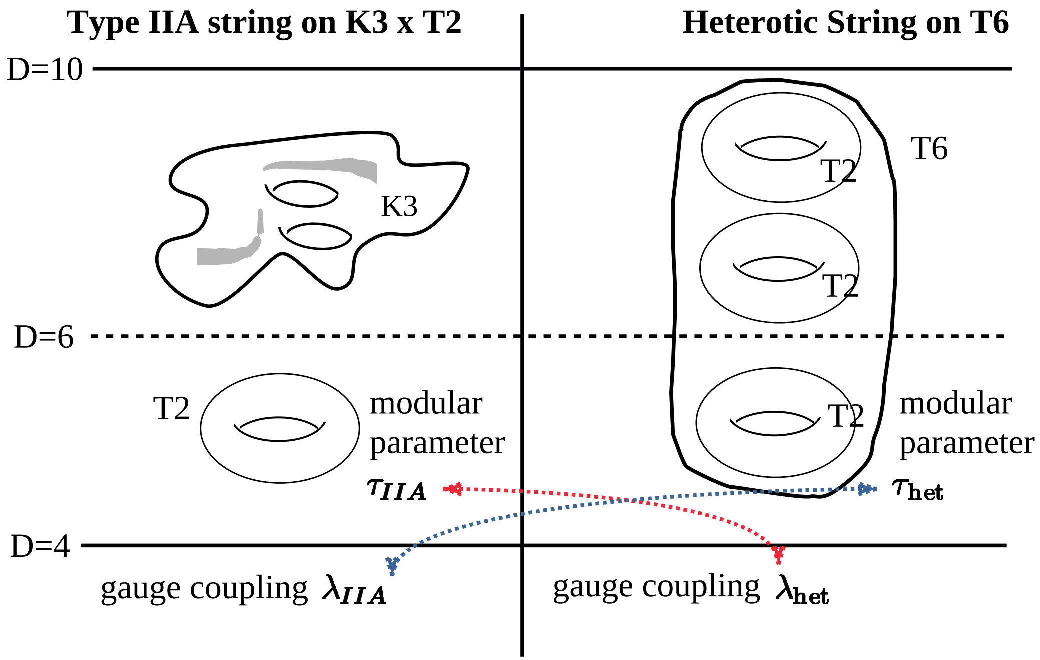

- In the Type I string model, as a fundamental open string stretched between D-branes, in the limit of coinciding D-branes (perturbative):



→ **These different "mathematical" geometries represent the same physical model !** (here, the SU(2) Higgs model)

## Geometrization of non-perturbative Dualities

- Consider duality between compactifications with N=4 SUSY in D=4:




- The string duality maps geometrical compactification moduli into gauge coupling constants, and vice versa:

$$\tau_{IIA} = \lambda_{het}$$
$$\lambda_{IIA} = \tau_{het}$$

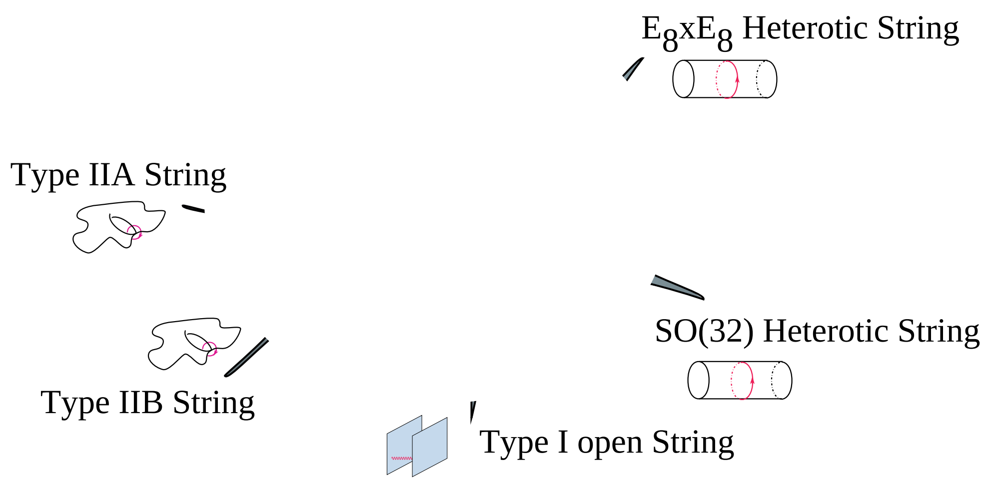
- In this way, the non-perturbative S-duality symmetry SL(2,Z) of the type II string is mapped to a perturbative T-duality of the heterotic string (and v.v), explaining the appearence of the non-perturbative modular geometry!

Recall that the coupling constant $\lambda$ governs the perturbation expansion of quantum corrections - this means the duality maps between classical and quantum objects....
**No unique distinction as to what classical and what quantum effects are !**



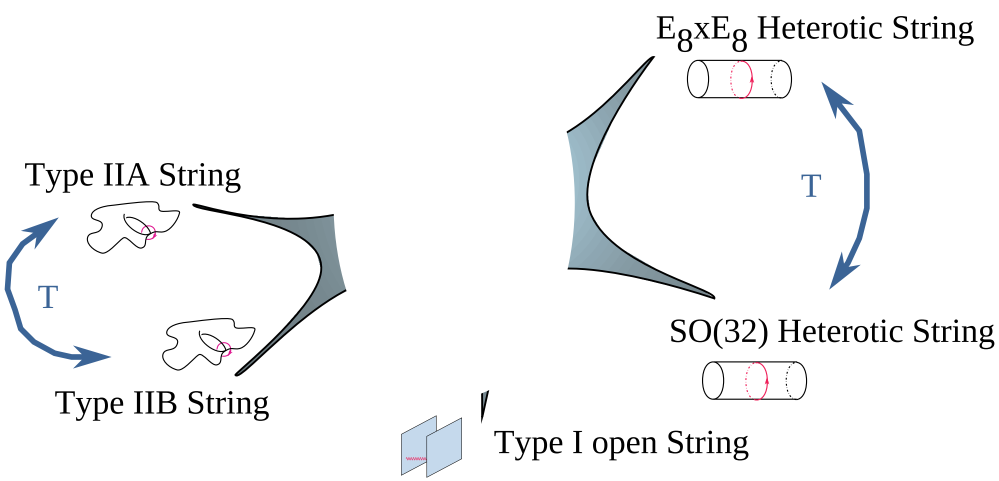
## 10-D String Theories Revisited

Recall we had five string theories in D=10 -
how are they interrelated in view of the dualities ?



# T-Duality and Mirror Symmetry

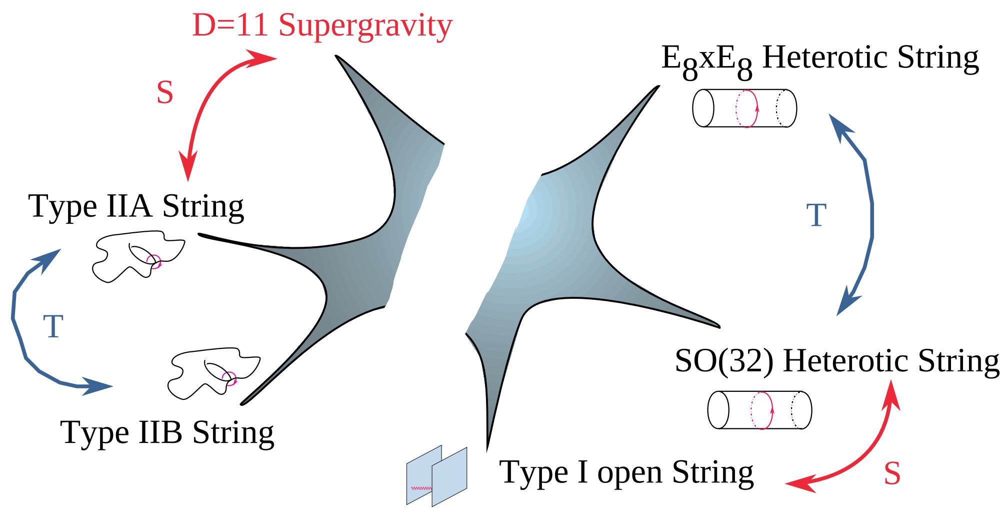
Staying completely within perturbative CFT, we know how to continuously deform some theories in the following way:






# Adding S-Duality

Staying completely within perturbative CFT, we know how to continuously deform some theories in the following way:



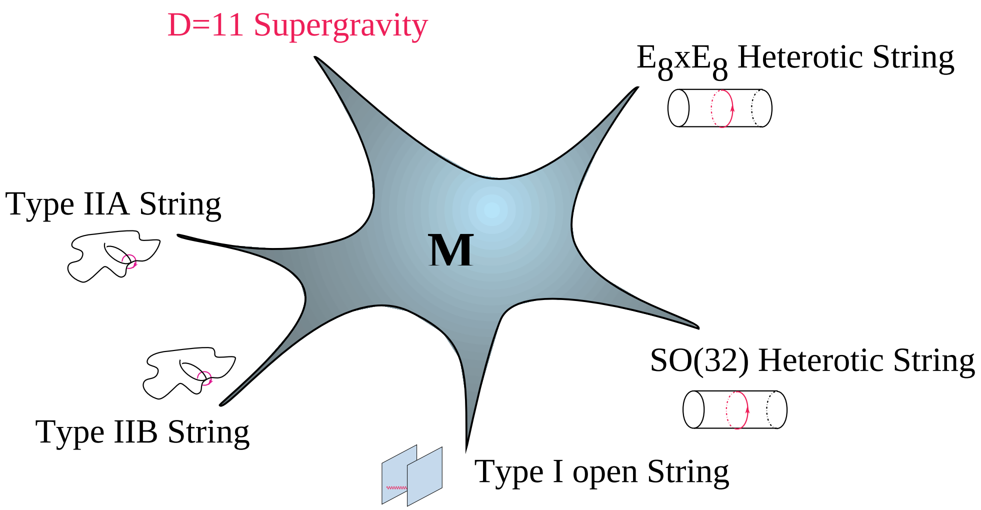

Performing strong coupling limits induces new non-perturbative relationsships !

**Surprise**: taking the strong coupling limit in the Type IIA string, non-perturbative states ("D0-branes") generate an **11th dimension** !

D=11 supergravity in not related to a string theory, rather is related to **supersymmetric membranes**.....

# The Grand Picture

Adding certain brane backgrounds finally links all theories together:




All five string theories in D=10 appear as particular perturbative approximations of **one theory** !

- Just like in N=2 SYM theory, there are various parametrizations, each of which prefers certain physical excitations being as "fundamental" and weakly coupled.

**Dualities take us beyond string theory ! ... M-Theory ?**



# "M-Theory"

- Defined to be the theory that, upon compactification on a circle, gives Type IIA string theory:
  - for large R, strongly coupled Type IIA string
  - for R~0, weakly coupled Type IIA string

  Fundamental degrees of freedom: "**D-particles**" (Type IIA solitons)

- Dynamics described by large-N limit of SUSY quantum mechanics:

$$\mathcal{H} \; = \; R \,\mathrm{Tr}\left\{ (\partial X^i)^2 - [X^i, X^j]^2 + \Theta[\gamma X, \Theta] \right\}$$

= 10-D U(N) Yang-Mills theory reduced to 0+1 dimensions

  - X = NxN matrices: reflect non-commuting short-distance structure of space-time
  - Non-local; space-time is approximate, derived concept = moduli space of the QM model
  - Infinite momentum frame: not manifestly Lorentz covariant
  - Large-N Limit: gives D=11 supergravity, graviton scattering

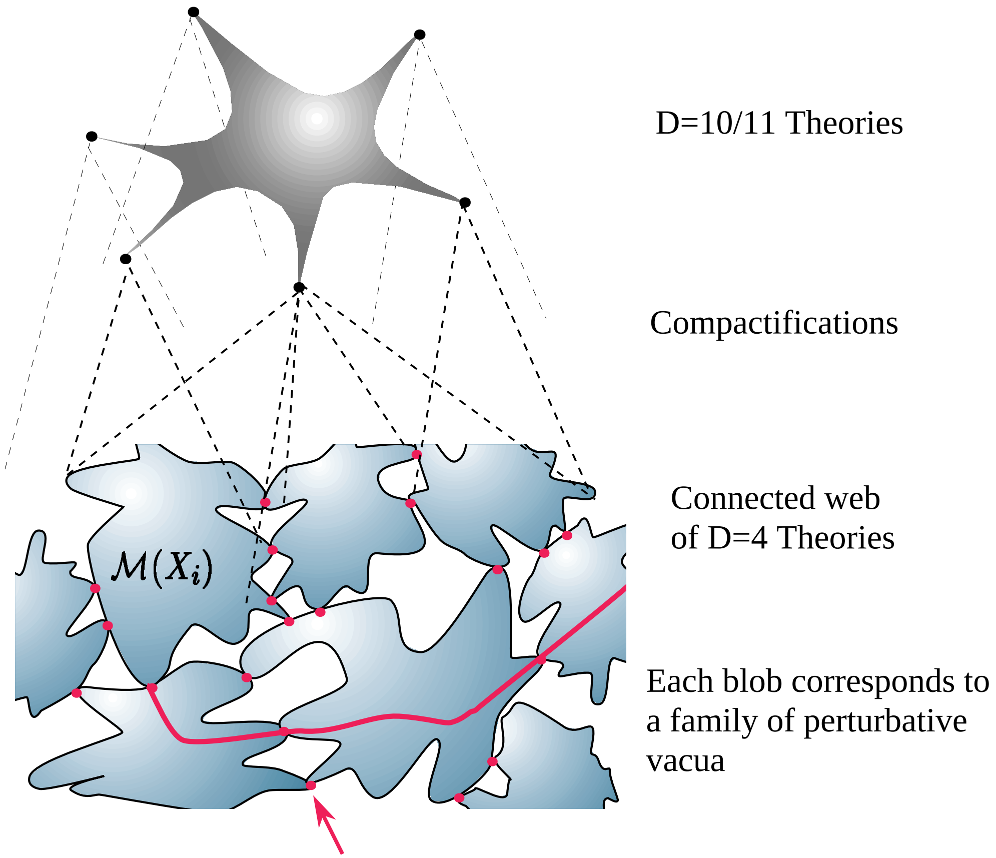
- Compactifications (eg on tori) reproduce known facts about the five D=10 string theories and their compactifications... highly non-trivial !

  Seem to provide non-perturbative formulation of type IIA and other string models.

- Incompletely understood, involves new concepts beyond quantum field theory and General Relativity .....

## The Grand Picture II: N=2 SUSY Strings in D=4

All vacua are **connected** by non-perturbative transitions, and so form a complicated web with ($10^8$?) components (with in general different dimensions, say 100) :



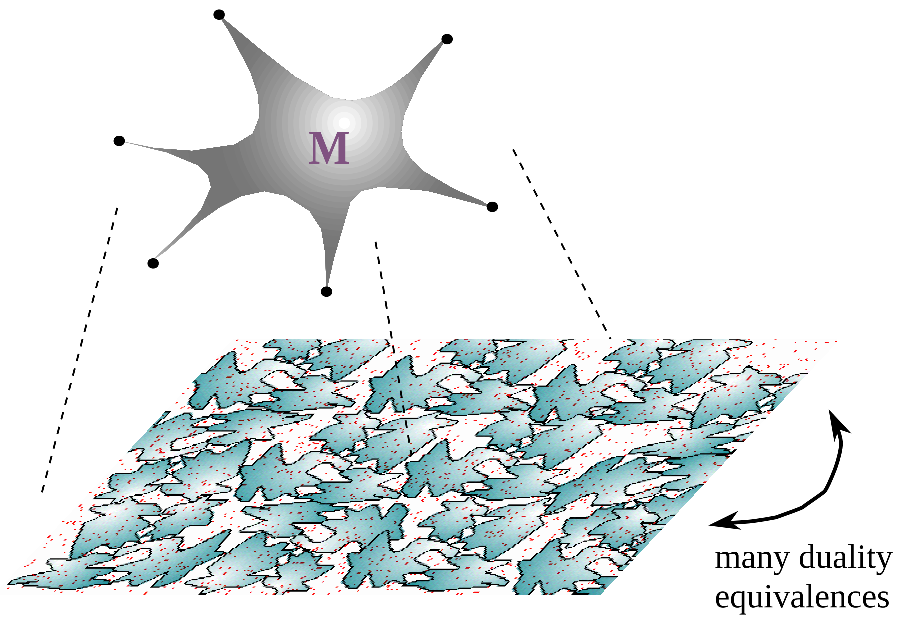

Connected via singular transitions:
massless states open up new branches

Different kinds of singularities give rise to many kinds of known, as well as novel physical phenomena in D=4....



## N=1 Supersymmetric Theories ?

- We are beginning to investigate N=0,1 SUSY strings in D=4, which it is a problem of enormous complexity .............




- Can we still hope for a single unique vacuum state? Nobody knows....

Despite all complexity: it seems that what crystallizes here is just **one single** theory, with many many facets.....

It may be that this is just the space of all possible consistent quantum theories that include gravity.........