A closer look to the goldstino

The goldstino field can be easily identified. The full fermion mass matrix of a generic supersymmetric theory is

$$M_f = \begin{pmatrix} 0 & \sqrt{2} g_a (T^a \langle\phi\rangle)_i \\ \sqrt{2} g_a (\langle\phi^*\rangle T^a)_j & \langle W_{ij}\rangle \end{pmatrix}$$

There is always one combination of fermion fields that corresponds to zero mass:

$$\tilde{G} = \begin{pmatrix} \langle D^a\rangle/\sqrt{2} \\ \langle F_i\rangle \end{pmatrix}$$

is an eigenvector of M_f with zero eigenvalue. This is the goldstino.

The first component of $M_f\tilde{G}$ is

$$\sqrt{2}\langle g_a(T^a\phi)_i F_i\rangle = -\sqrt{2}\langle g_a(T^a\phi)_i W_i^*\rangle = -\sqrt{2}\langle \frac{\partial W^*}{\partial \phi_i^*}\delta_{\rm gauge}\phi_i^*\rangle = 0$$

by gauge-invariance of the superpotential.

The second component is given by

$$\langle g_a(\phi^* T^a)_j D^a + W_{ij}F_i\rangle$$

The scalar potential is

$$V = F_i F_i^* + \frac{1}{2}D^a D^a = W_i W_i^* + \frac{1}{2}g_a^2(\phi^* T^a\phi)^2$$

It follows that

$$\begin{aligned}\langle\frac{\partial V}{\partial \phi_k}\rangle &= \langle W_i^* W_{ik} + g_a^2(\phi^* T^a\phi)(\phi^* T^a)_k\rangle \\ &= -\langle F_i W_{ik} + g_a D^a(\phi^* T^a)_k\rangle = 0\end{aligned}$$

Let us consider the case of local supersymmetry: $\eta = \eta(x)$. Then

$$(\delta_{\eta_1}\delta_{\eta_2} - \delta_{\eta_2}\delta_{\eta_1})X = -(\eta_1^\dagger\,\bar{\sigma}^\mu\,\eta_2 - \eta_2^\dagger\,\bar{\sigma}^\mu\,\eta_1)\,i\partial_\mu X$$

is a local translation, i.e. a general coordinate transformation.

Gravitation is included in a natural way.

Local supersymmetry is called supergravity.

In this case, the so-called super-Higgs effect takes place: the spin-3/2 partner of the graviton, the gravitino, acquires a mass.

The missing spin degrees of freedom are provided by the goldstino.

By dimensional analysis,

$$m_{3/2} \sim \frac{\langle F \rangle}{M_P}$$

Clearly, the gravitino mass has very different values in gravity-mediated and gauge mediated scenarios:

- **gravity-mediated: the gravitino mass**

$$m_{3/2} \sim \frac{\langle F \rangle}{M_P} \sim m_{\rm soft}$$

  and its interactions are gravitational. Almost no interest for phenomenology at colliders.

- **gauge-mediated:**

$$m_{3/2} \sim \frac{\langle F \rangle}{M_P} \sim \frac{10^5}{10^{19}}\,{\rm GeV}$$

  if $M \sim \langle F \rangle \ll M_P$. Its goldstino components may have non-negligible couplings, and may therefore play a role in LHC physics.

Gravity-mediated models: some details

The supergravity lagrangian is non-renormalizable (as expected). Assume there is a hidden chiral superfeld X whose auxiliary component F_X gets a non-zero vev. The lagrangian includes

$$\begin{aligned}\mathcal{L} &= -\frac{1}{M_P}F_X\sum_a\frac{1}{2}f_a\lambda^a\lambda^a + \text{h.c.}\\ &\quad +\frac{1}{M_P^2}F_XF_X^*k_{ij}\phi_i^*\phi_j\\ &\quad -\frac{1}{M_P}F_X\left(\frac{1}{6}y'_{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}\mu'_{ij}\phi_i\phi_j + \text{h.c.}\right)\end{aligned}$$

When $F_X \to \langle F_X\rangle$, this gives precisely the soft terms we need.

In a *minimal* version of supergravity, there are considerable simplifications:

$$f_a = f \quad k_{ij} = k\delta_{ij} \quad y' = \alpha y \quad \mu' = \beta\mu$$

This leads to

$$M_1 = M_2 = M_3 = m_{1/2}$$
$$m_Q^2 = m_U^2 = m_D^2 = m_L^2 = m_E^2 = m_{H_u}^2 = m_{H_d}^2 = m_0^2$$
$$A_u = A_0 y_u \quad A_d = A_0 y_d \quad A_e = A_0 y_e$$
$$B = B_0 \mu$$

Neutral flavour changing effects automatically suppressed.

Gauge-mediated models: some details

In the simplest version, the model contains four messenger supermultiplet:

$$q \sim \left(\mathbf{3}, \mathbf{1}, -\frac{2}{3}\right) \quad \bar{q} \sim \left(\bar{\mathbf{3}}, \mathbf{1}, +\frac{2}{3}\right) \quad \ell \sim (\mathbf{1}, \mathbf{2}, +1) \quad \bar{\ell} \sim (\mathbf{1}, \mathbf{2}, -1)$$

and a chiral multiplet S, whose scalar and auxiliary components take a vev (e.g. by the O'Raifeartaigh mechanism). They interact through a superpotential

$$W_{\text{messengers}} = y_2 S \ell \bar{\ell} + y_3 S q \bar{q}$$

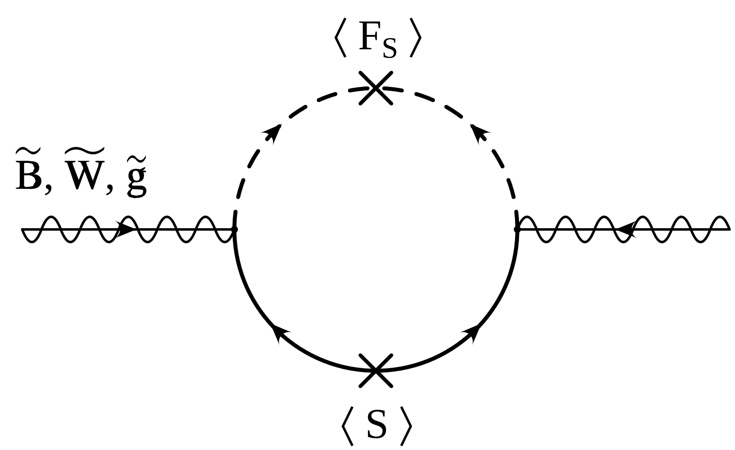
which becomes effectively

$$W_{\text{messengers}} = y_2 \langle S \rangle \ell \bar{\ell} + y_3 \langle S \rangle q \bar{q}$$

Gauginos get masses

$$M_a = \frac{\alpha_a}{4\pi} \frac{\langle F_S \rangle}{\langle S \rangle}$$

through one-loop diagrams:



Soft scalar masses and trilinear couplings arise at two-loops. One gets

$$A_u = A_d = A_e = 0$$

at the messenger scale.

General features of soft masses in gauge-mediated models:

- **larger for strongly-interacting particles**
- **determined by gauge properties only**

The second properties leads to a degeneracy in squarks and slepton masses, that suppresses FCNC effects.

Summary

- **Supersymmetry provides a solution to the naturalness problem without spoiling the good features of the Standard Model.**
- **It has a number of welcome side-effects:**
  - **a better context for grand unification**
  - **a natural candidate for cold dark matter.**
- **A large portion of the parameter space for supersymmetric theories has already been explored.**
- **Supersymmetry may be relevant in different forms (superstrings).**