Cosmology and the origin of structure

Rocky **I**: The universe observed

Rocky **II**: The growth of cosmological structure

Rocky **III**: Inflation and the origin of perturbations

Rocky **IV**: Dark matter and dark energy

Academic Training Lectures
Rocky Kolb
Fermilab, University of Chicago, & CERN

Rocky I: The universe observed

- **Cosmological parameters:**

$H_0 \quad \rightarrow$ Hubble's constant

$q_0 \quad \rightarrow$ deceleration parameter

$\Omega_i \quad \rightarrow$ the cosmic food chain

$$\left(\Omega_0 \;,\; \Omega_M \;,\; \Omega_B \;,\; \Omega_\Lambda \;,\; \Omega_\gamma \;,\; \Omega_\nu \;,\; \ldots\ldots \right)$$

$t_0 \quad \rightarrow$ age of the universe

$T_0 \quad \rightarrow$ temperature of the universe

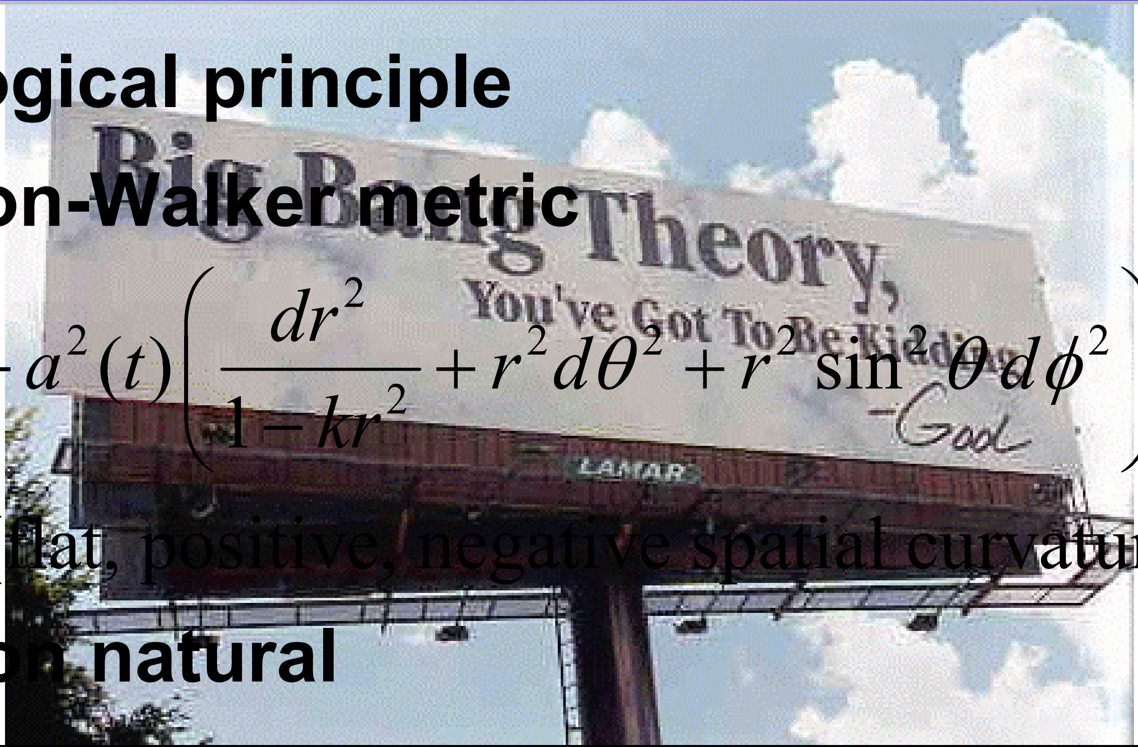
- **Power spectrum of large-scale structure:**

$P(k)$

- **Anisotropy of CMB:**

C_l

Theoretical developments (1917-1927)

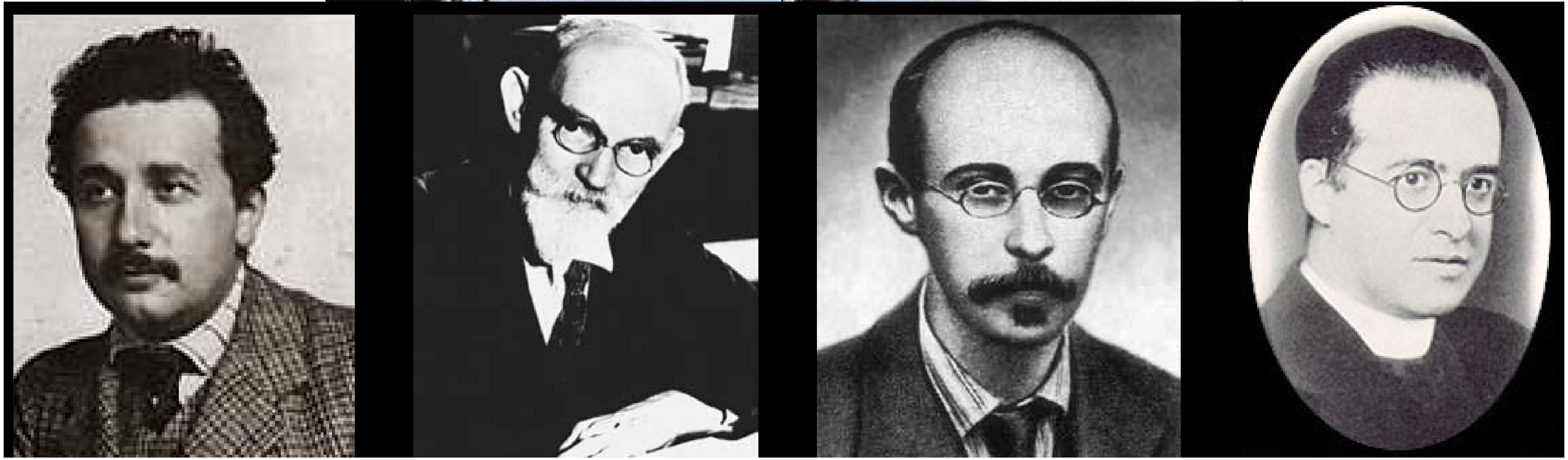

Big Bang Theory,
You've Got To Be Kidding.
-God
LAMAR

- **Cosmological principle**
- **Robertson-Walker metric**

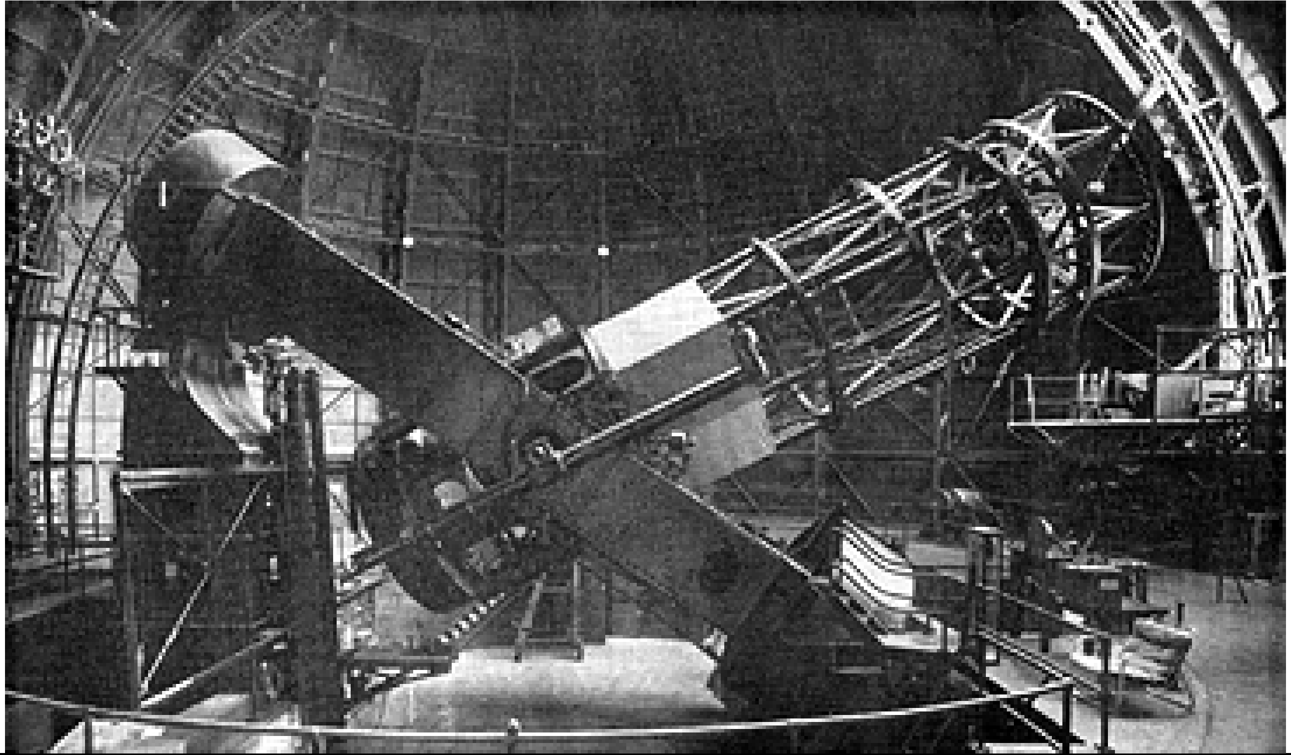
$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2\right)$$

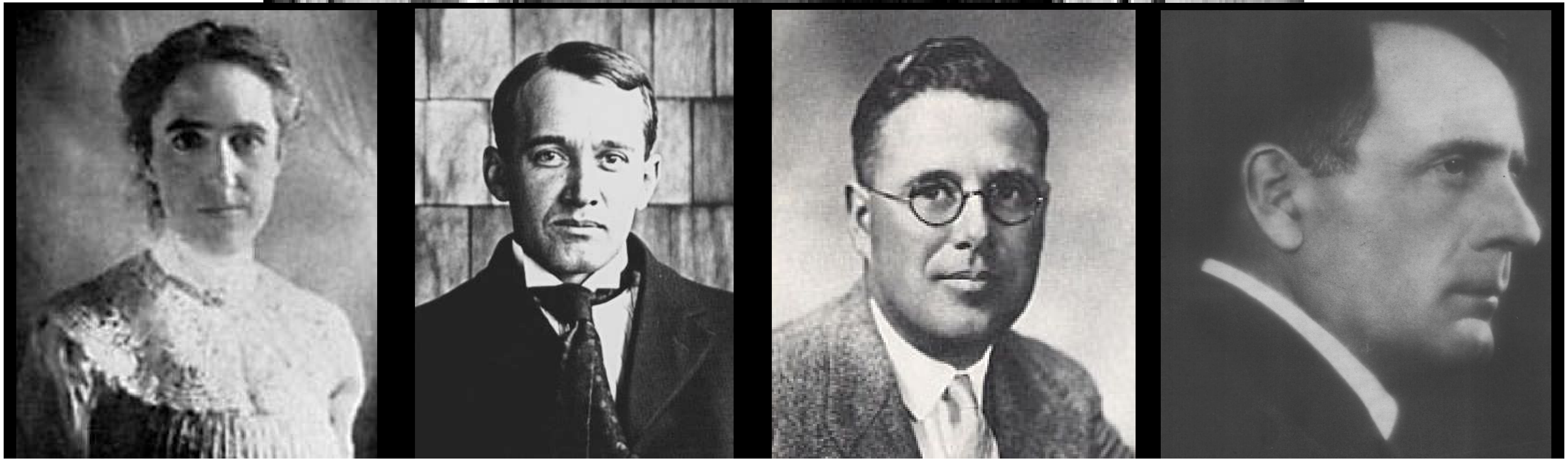
$$k = 0, \pm 1 \quad \text{(flat, positive, negative spatial curvature)}$$

- **Expansion natural**



Observational developments (1912-1929)





“… physically, he is a splendid specimen…”
“… magnificent physique ….”
“… manly …”
“… loveable character …”

INTERCOLLEGIATE
1909
CHAMPIONS

University of Chicago 1909 National Champions

Observational developments (1912-1929)

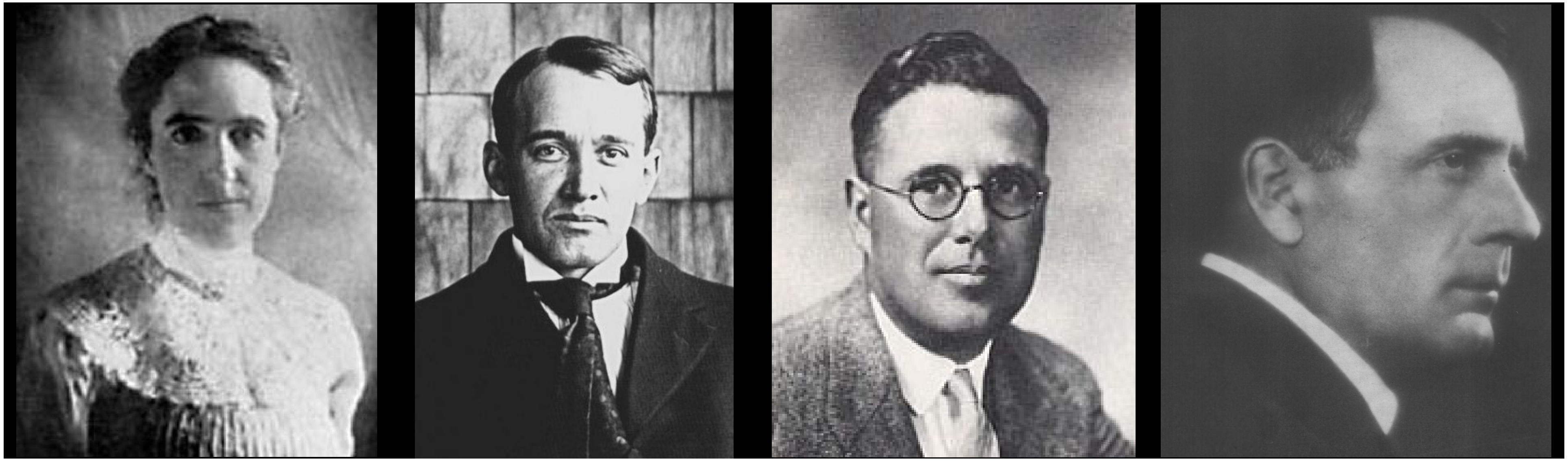
- **Cosmological principle (*extragalactic* astronomy)**
- **Universe expands**

$$v_R = H_0 d_L$$

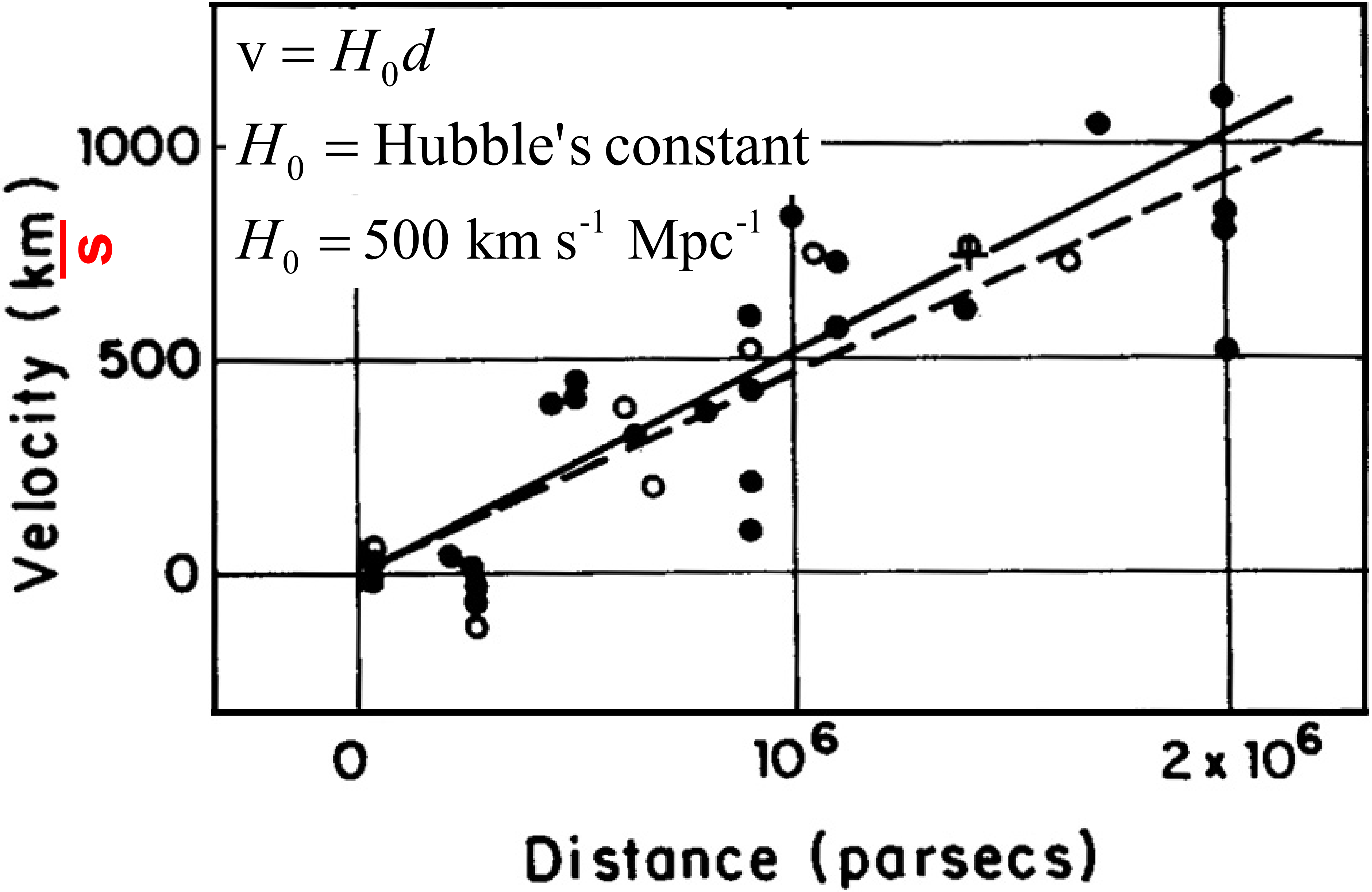
$$v_R = \text{ recessional velocity } = cz = c\,\Delta\lambda/\lambda$$

$$H_0 = \text{ Hubble's constant} = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$d_L = \text{ Luminosity distance}$$



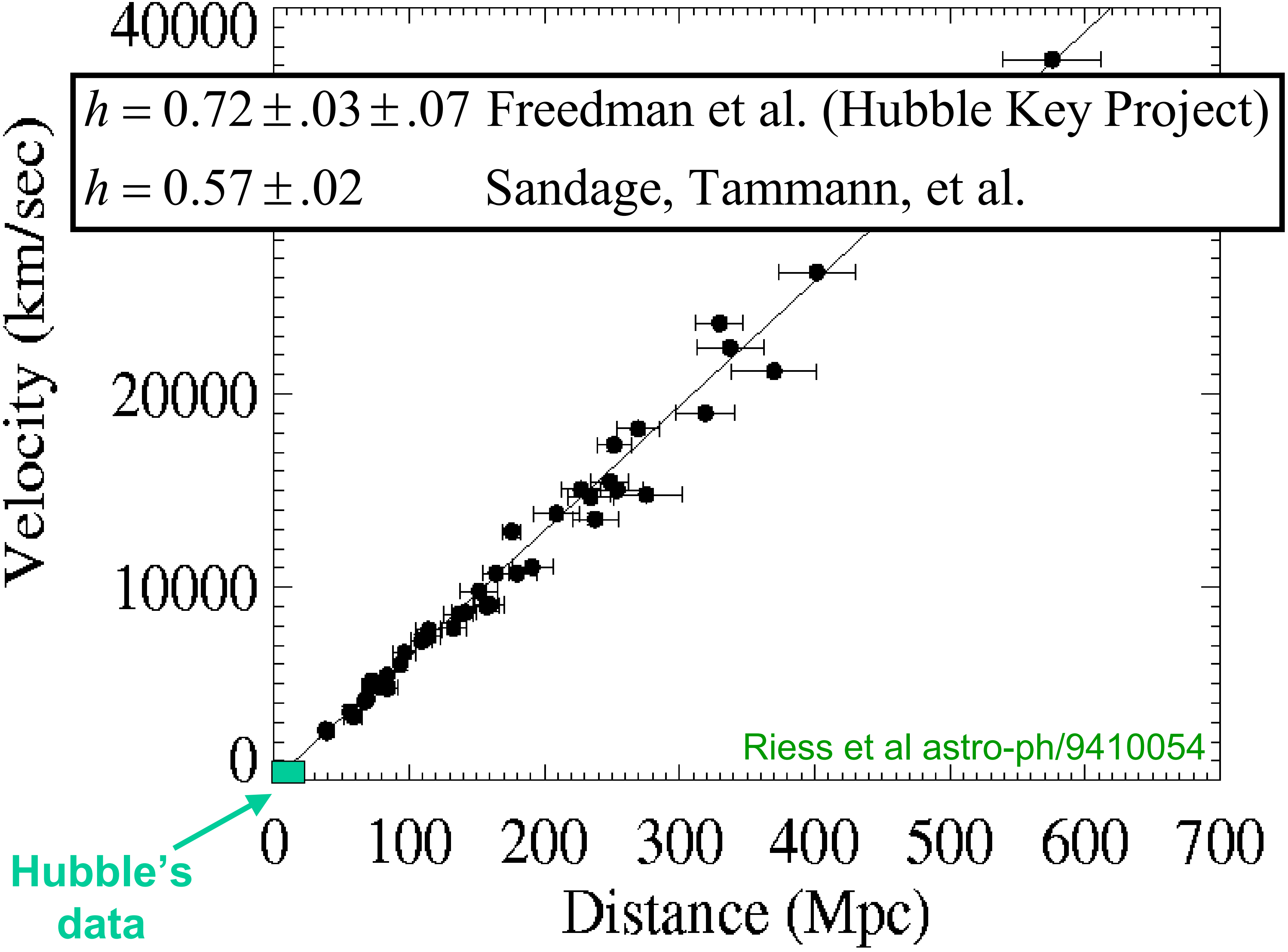
Hubble’s Discovery Paper - 1929



$\mathrm{v} = H_0 d$

H_0 = Hubble's constant

$H_0 = 500 \text{ km s}^{-1} \text{ Mpc}^{-1}$



$h = 0.72 \pm .03 \pm .07$ Freedman et al. (Hubble Key Project)

$h = 0.57 \pm .02$ Sandage, Tammann, et al.

Riess et al astro-ph/9410054

Hubble's data

Distance-luminosity relation

$F = \dfrac{L}{4\pi d_L^2}$ defines luminosity distance - "know" L, measure F

Conservation of energy: flux redshifted: $(1+z)^2 = \left[a(t_0)/a(t_1)\right]^2$

redshift of energy × redshift of time interval: $(1+z)^2$

$F = \dfrac{L}{A}$ $A =$ area of ${}^2S$ centered on source at time of detection, t_0

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \Rightarrow A = 4\pi a^2(t_0) r^2$$

$$F = \frac{L}{4\pi a^2(t_0) r^2 (1+z)^2} \qquad \Rightarrow \qquad \boxed{d_L = a(t_0)\, r\, (1+z)}$$

light from comoving coordinate r reaches us now redshifted by an amount $(1+z)$

Field equations

$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \qquad H \equiv \frac{\dot{a}}{a}$ expansion rate

$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right) \qquad q \equiv -\frac{\ddot{a}}{a}\frac{1}{H^2}$ deceleration parameter

$$\rho = \rho_M + \rho_R + \rho_\Lambda + \rho_w + \ldots\ldots$$

whatever: $p_w = w\rho_w \qquad w = p/\rho \qquad \rho_w \propto \left(1+z\right)^{3(1+w)}$

matter: $p_M = 0 \qquad w = 0 \qquad \rho_M \propto \left(1+z\right)^3$

radiation: $p_R = \rho_R/3 \qquad w = 1/3 \qquad \rho_R \propto \left(1+z\right)^4$

vacuum: $p_\Lambda = -\rho_\Lambda \qquad w = -1 \qquad \rho_\Lambda \propto \left(1+z\right)^0$

Distance-luminosity relation

$$\boxed{d_L = a(t_0)\, r\,(1+z)}$$

$$\boxed{ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)}$$

light travels on geodesics

$$ds^2 = 0$$

$$\int \frac{dr}{\sqrt{1-kr^2}} = \int \frac{dt}{a(t)} = \int \frac{da}{H(a)a^2}$$

$$H^2 = H_0^2\left[(1-\Omega_0)(1+z)^2 + \Omega_M(1+z)^3 + \Omega_w(1+z)^{3(1+w)} + \ldots\right]$$

$$\Omega_i = \rho_i \Big/ \left(3H_0^2/8\pi G\right)$$

$$\Omega_0 = \Omega_M + \Omega_\Lambda + \Omega_R + \Omega_w + \ldots \qquad (1-\Omega_0) \propto k$$

$$\int_0^r \frac{dr'}{\sqrt{1-kr'^2}} = \int_0^z \frac{a^{-1}(t_0)H_0^{-1} \quad dz'}{\sqrt{(1-\Omega_0)(1+z')^2 + \Omega_M(1+z')^3 + \Omega_w(1+z')^{3(1+w)} + \ldots}}$$

Distance-luminosity relation

$$\boxed{d_L = a(t_0)\, r\, (1+z)}$$

$$\int_0^r \frac{dr'}{\sqrt{1-kr'^2}} = \int_0^z \frac{a^{-1}(t_0) H_0^{-1} \quad dz'}{\sqrt{(1-\Omega_0)(1+z')^2 + \Omega_M (1+z')^3 + \Omega_w (1+z')^{3(1+w)} + \ldots}}$$

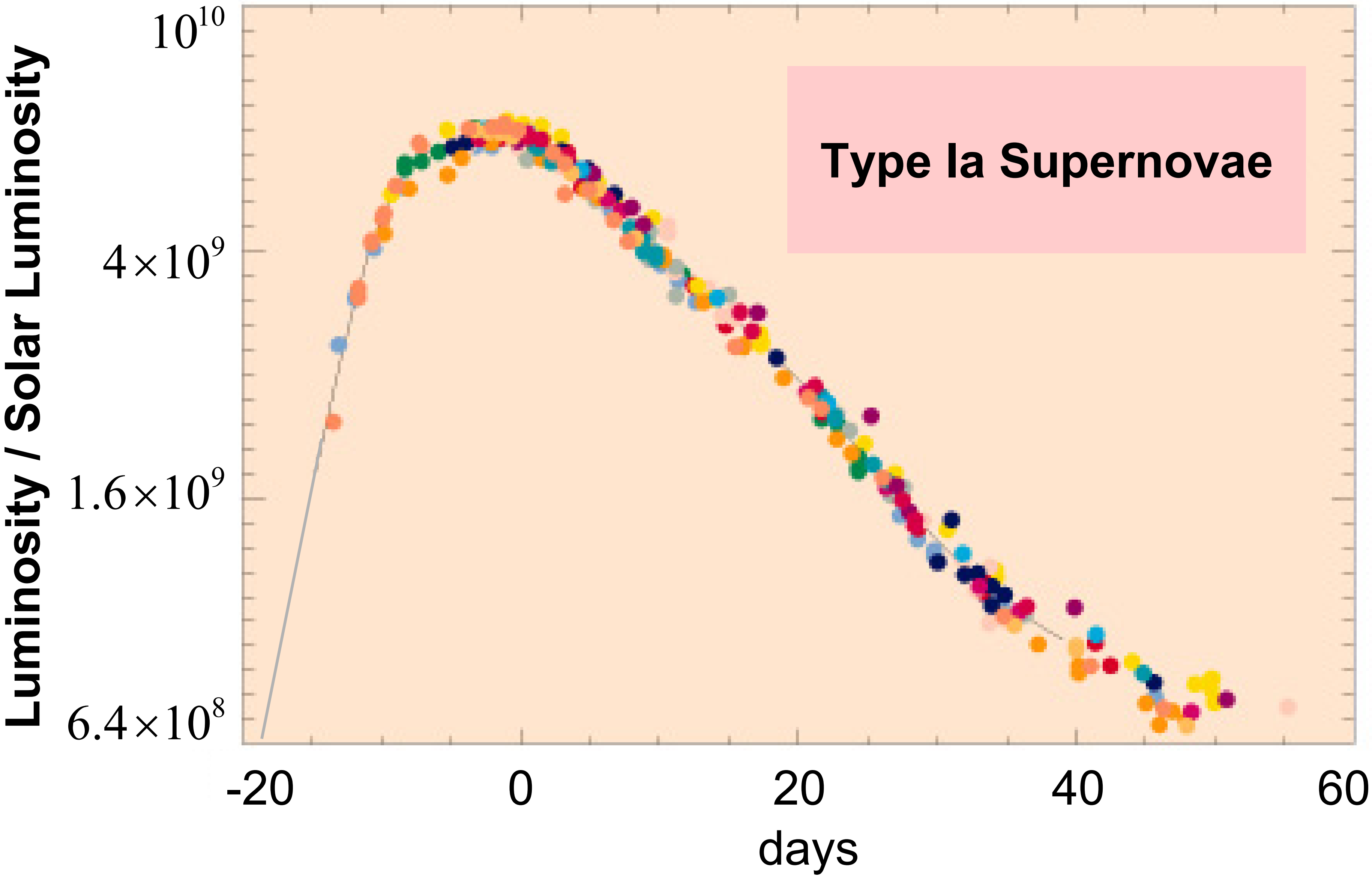
Example: matter + lambda ⟶ $\Omega_0 = \Omega_M + \Omega_\Lambda$

Program:

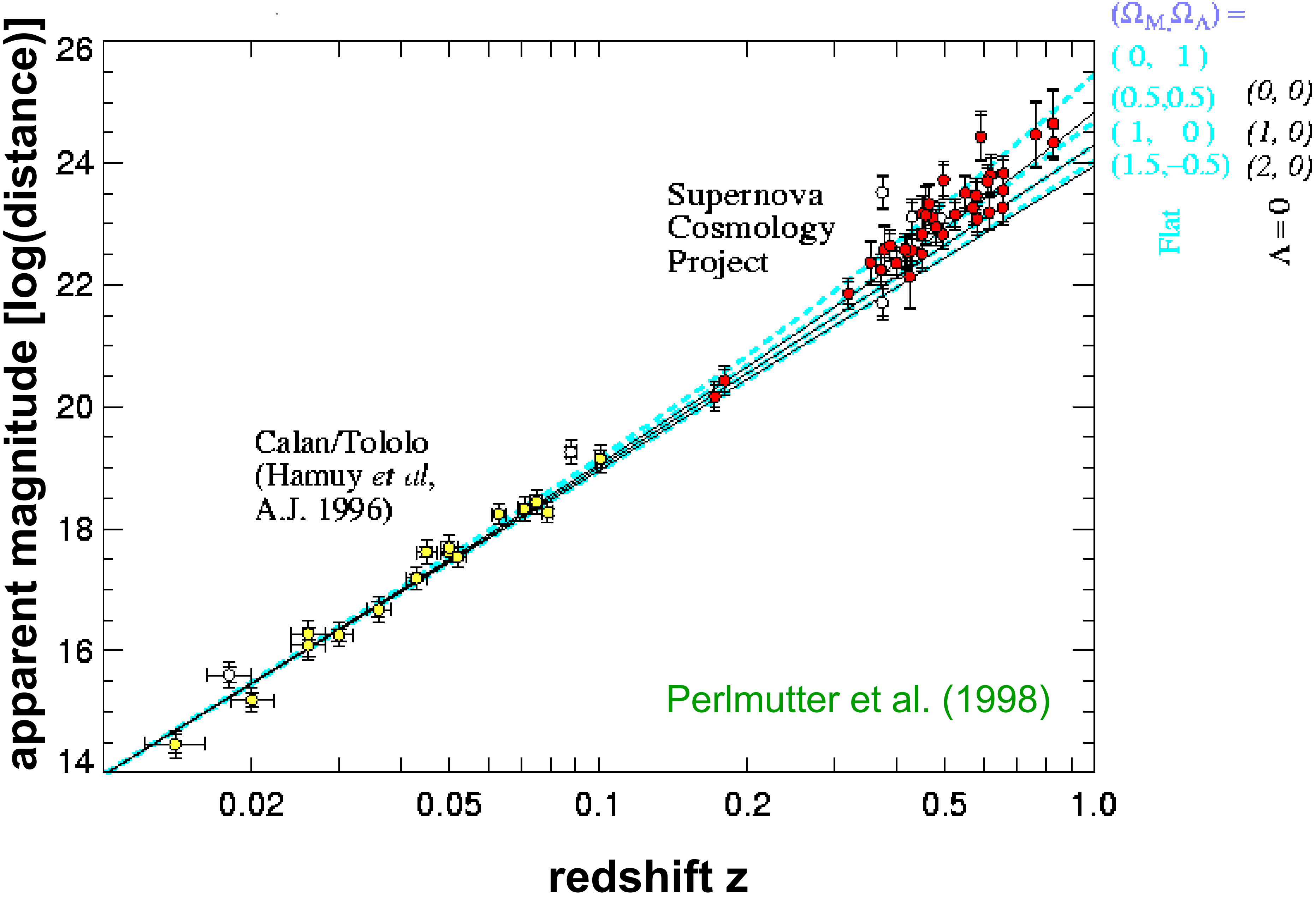
- **measure** $d_L(z)$ **via** $d_L^2 = L/4\pi F$
- **input** Ω_i **and calculate** $a(t_0)r$

$$\boxed{\begin{array}{c} H_0 d_L = z + O(z^2) \\ \Omega_i \hookleftarrow \end{array}}$$

Type Ia supernova are standard candles

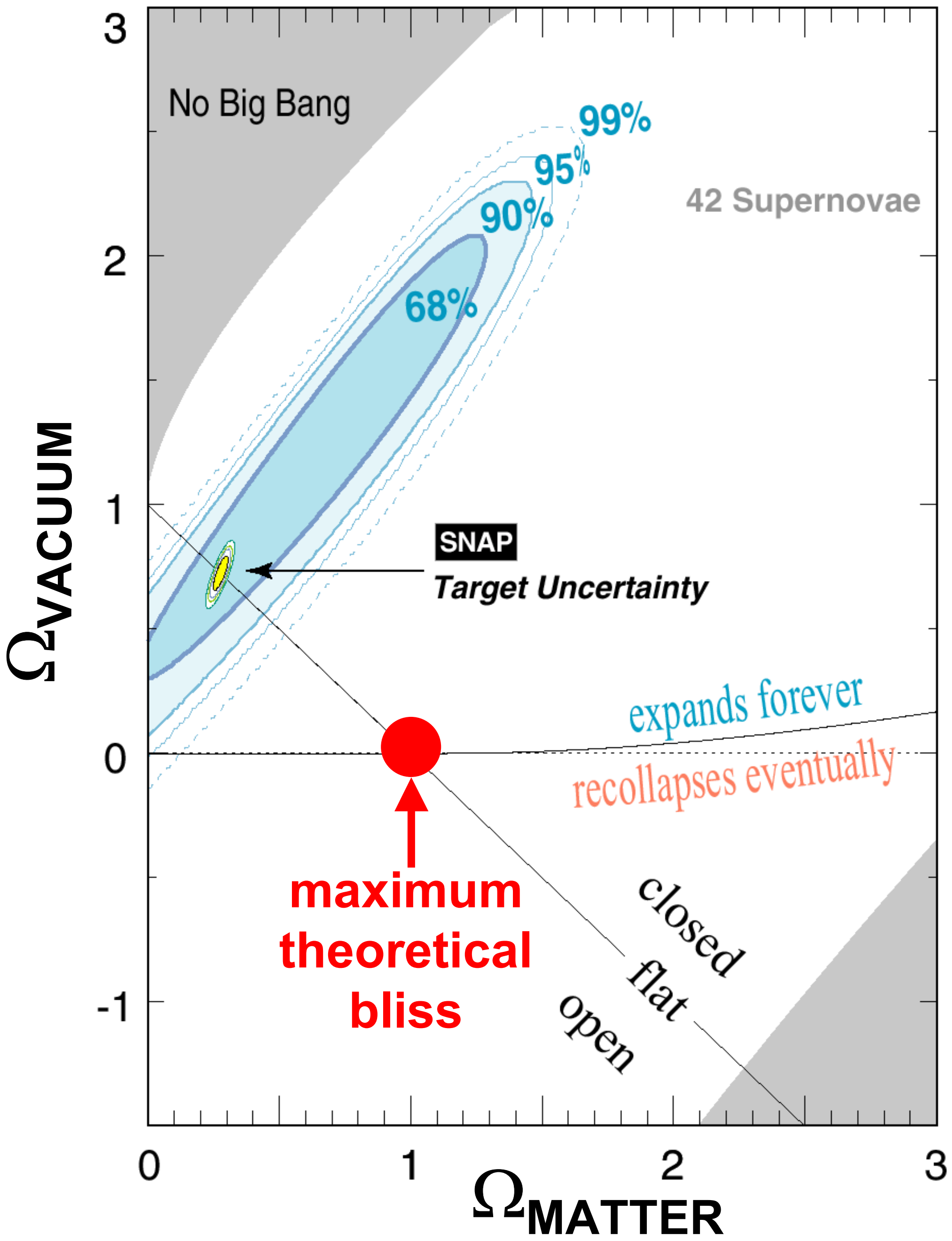


Type Ia supernova Hubble diagram



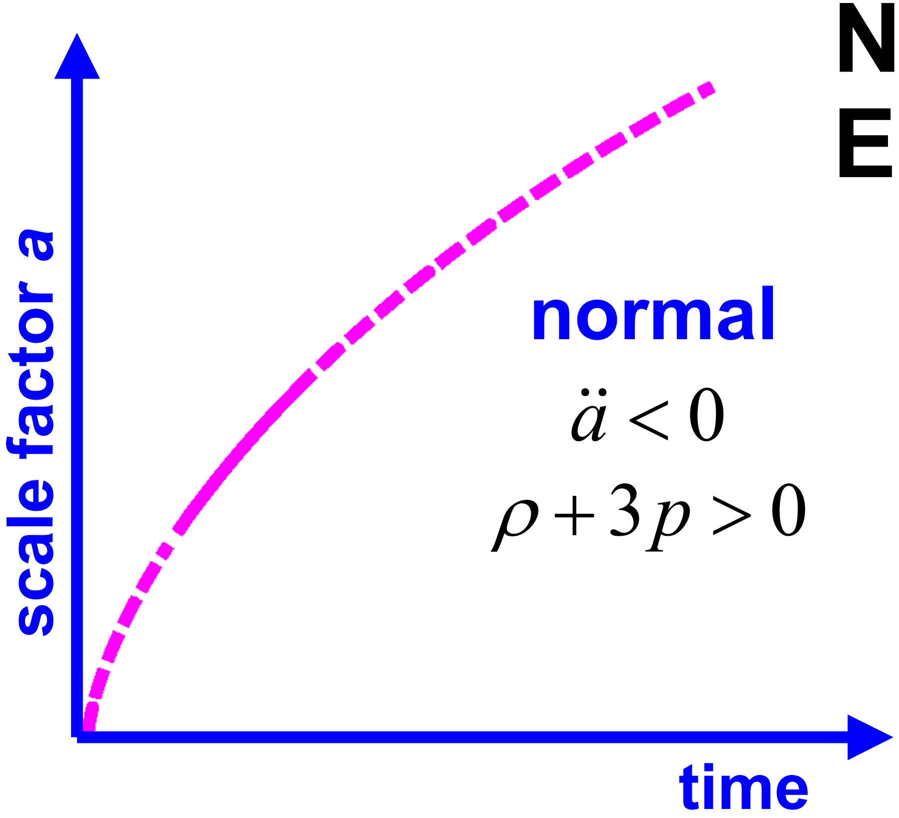
Perlmutter et al. (1998)

Supernova Cosmology Project
Perlmutter *et al.* (1998)



maximum theoretical bliss

cosmological constant,
…some changing non-zero vacuum energy,
… or some unknown systematic effect(s)



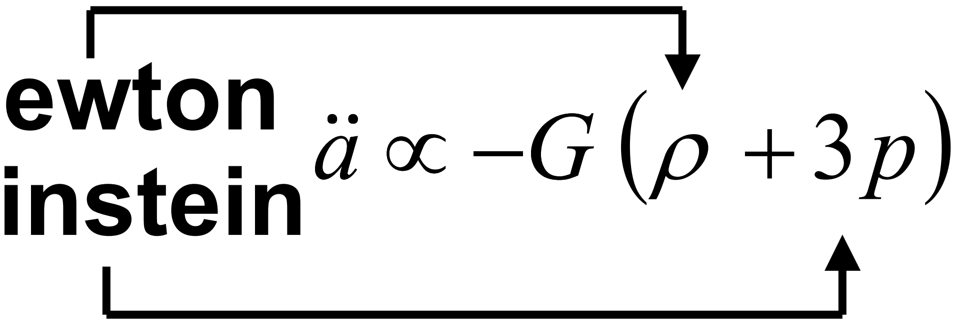
normal

$$\ddot{a} < 0$$

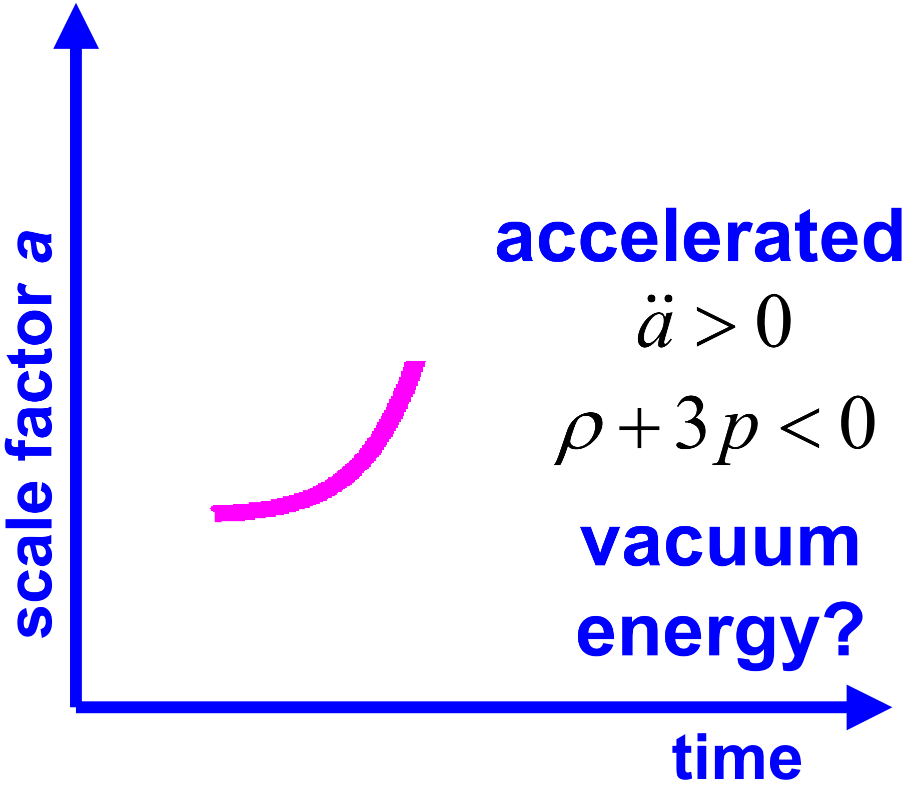
$$\rho + 3p > 0$$

Newton

Einstein



$$\ddot{a} \propto -G\left(\rho + 3p\right)$$



accelerated

$$\ddot{a} > 0$$

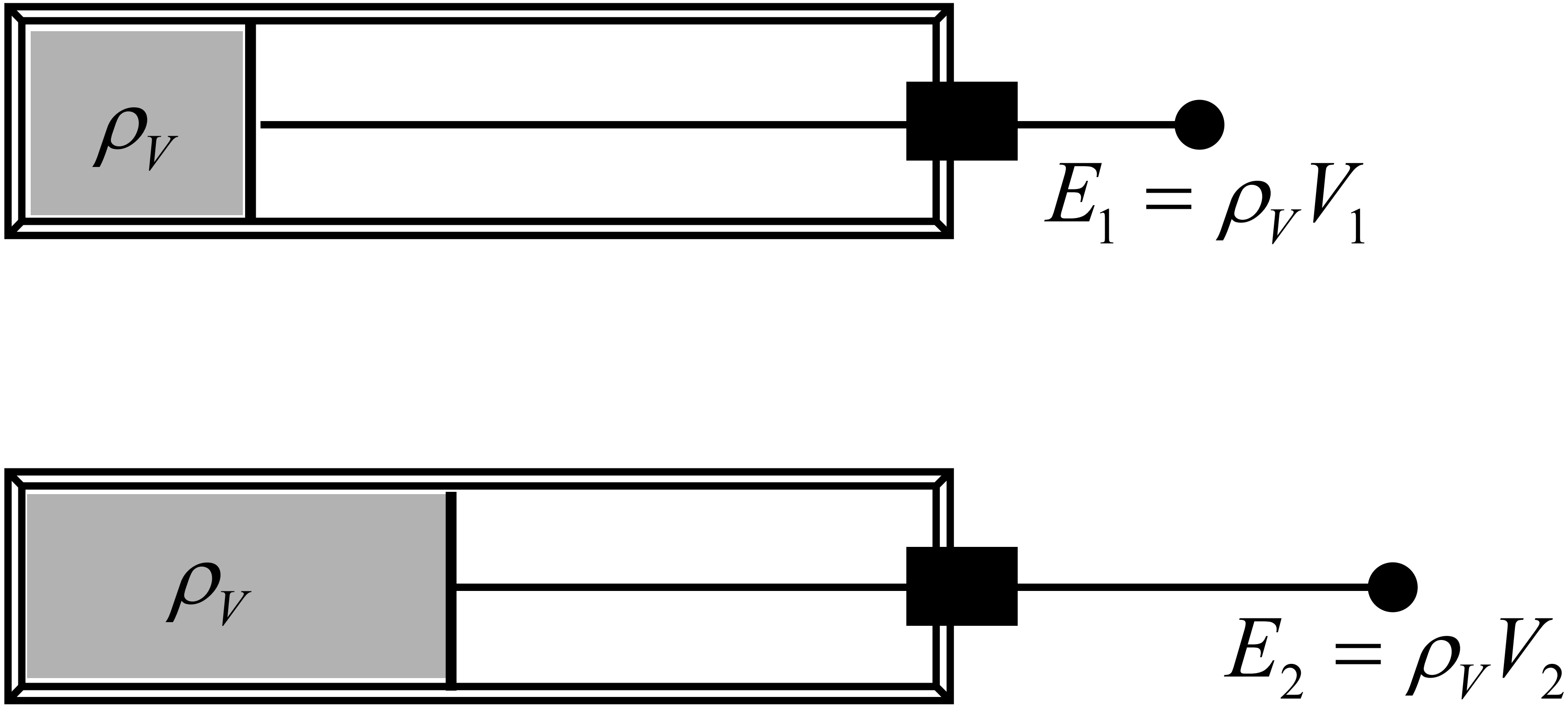
$$\rho + 3p < 0$$

vacuum energy?

The accelerating universe?

- **Normal matter slows the expansion of the universe (deceleration). Gravity is attractive.**
- **Negative pressure would push apart space.**
- **"Vacuum energy" (the mass-energy density of space) is positive, but its pressure is *negative.***

Vacuum pressure



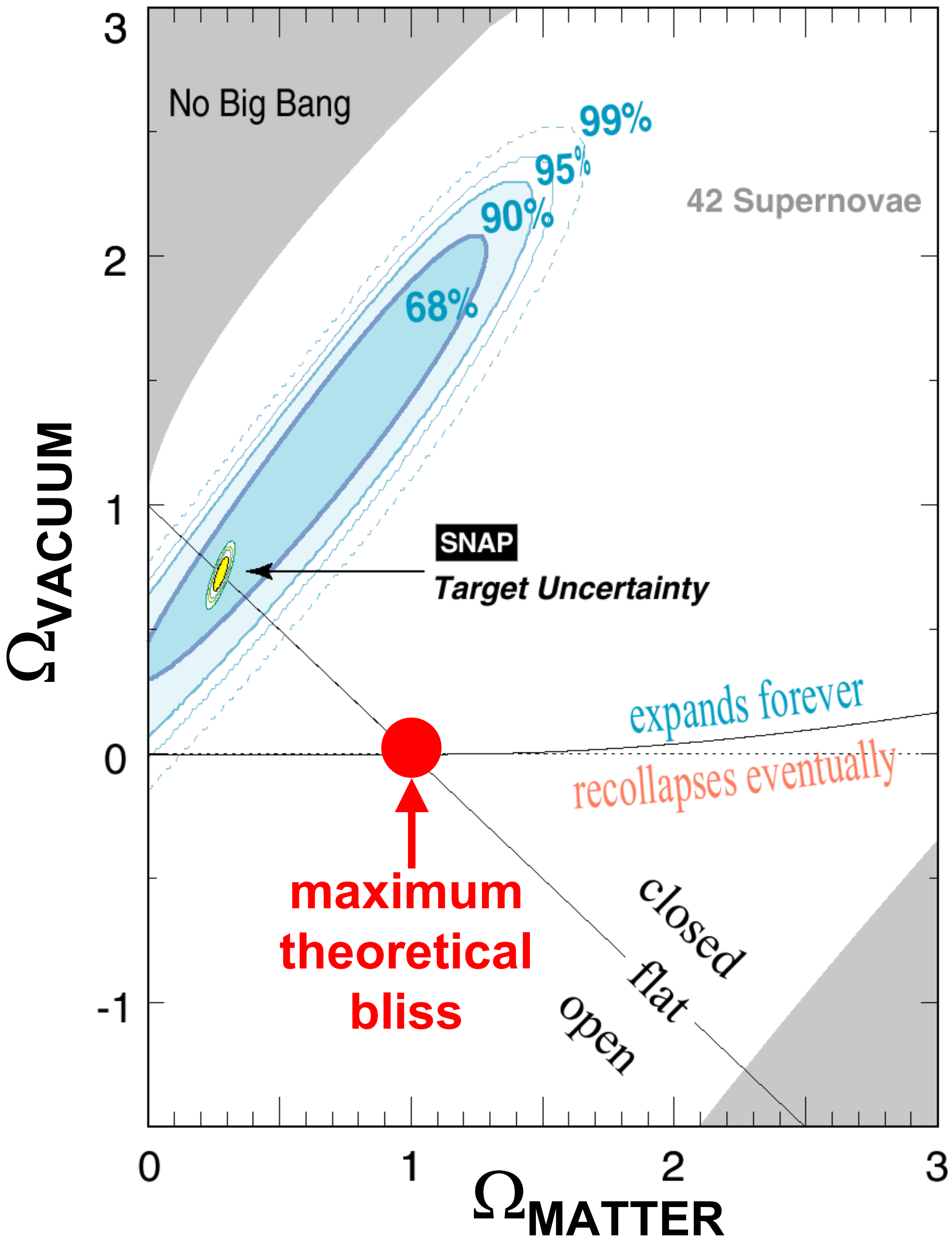
$$E_1 = \rho_V V_1$$

$$E_2 = \rho_V V_2$$

$$E_2 > E_1$$

had to *pull* piston
"negative pressure"

Supernova Cosmology Project
Perlmutter *et al.* (1998)



maximum theoretical bliss

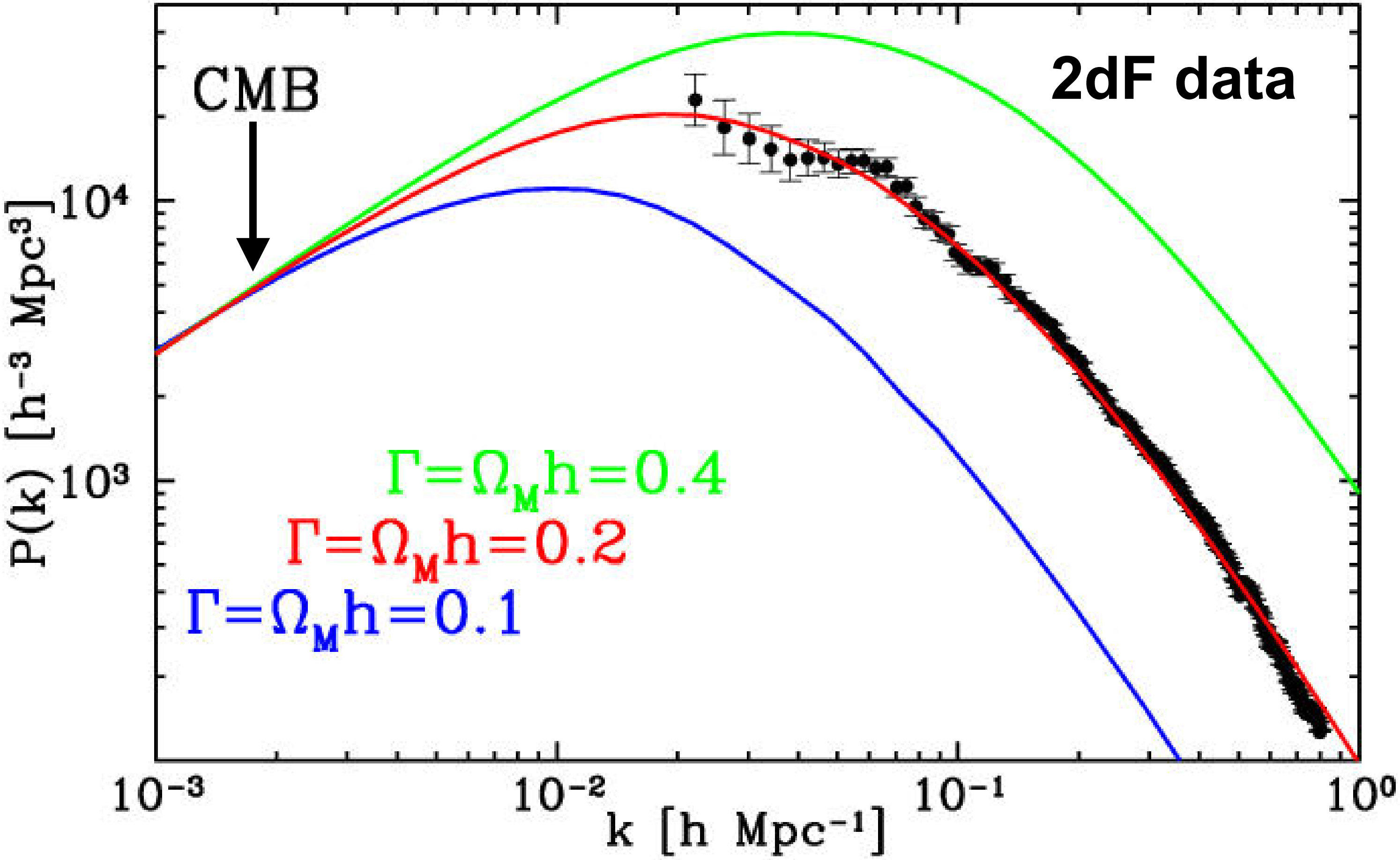
cosmological constant, …some changing non-zero vacuum energy,
… or some unknown systematic effect(s)

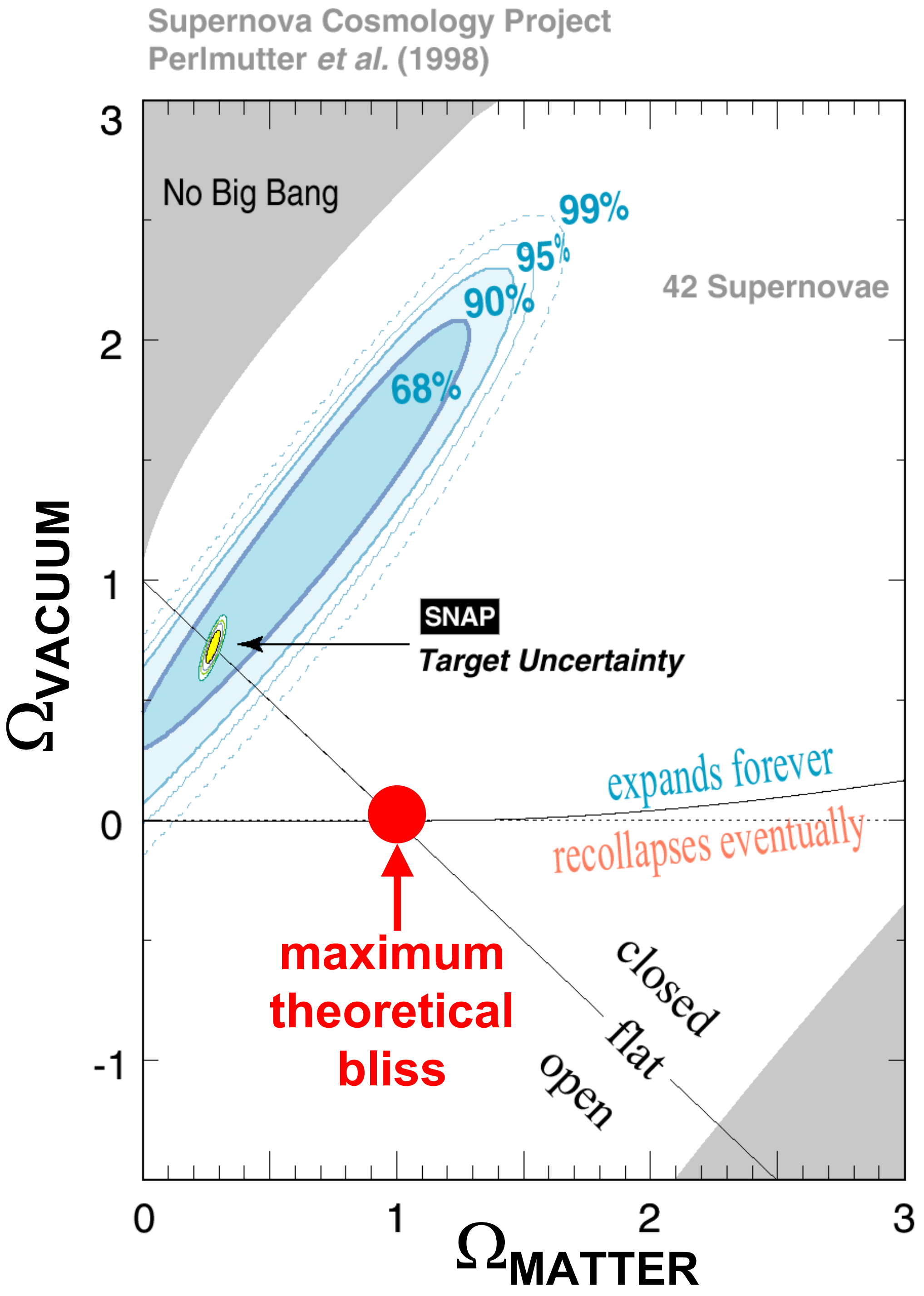
The case for Λ:

1) **acceleration**
2) **large-scale structure**

$$\Gamma \; □ \; \Omega_M h \; □ \; 0.25 \pm 0.05$$

$\Omega_M h$	Ω_M	h
0.25	1	0.25
0.25	0.35	0.70





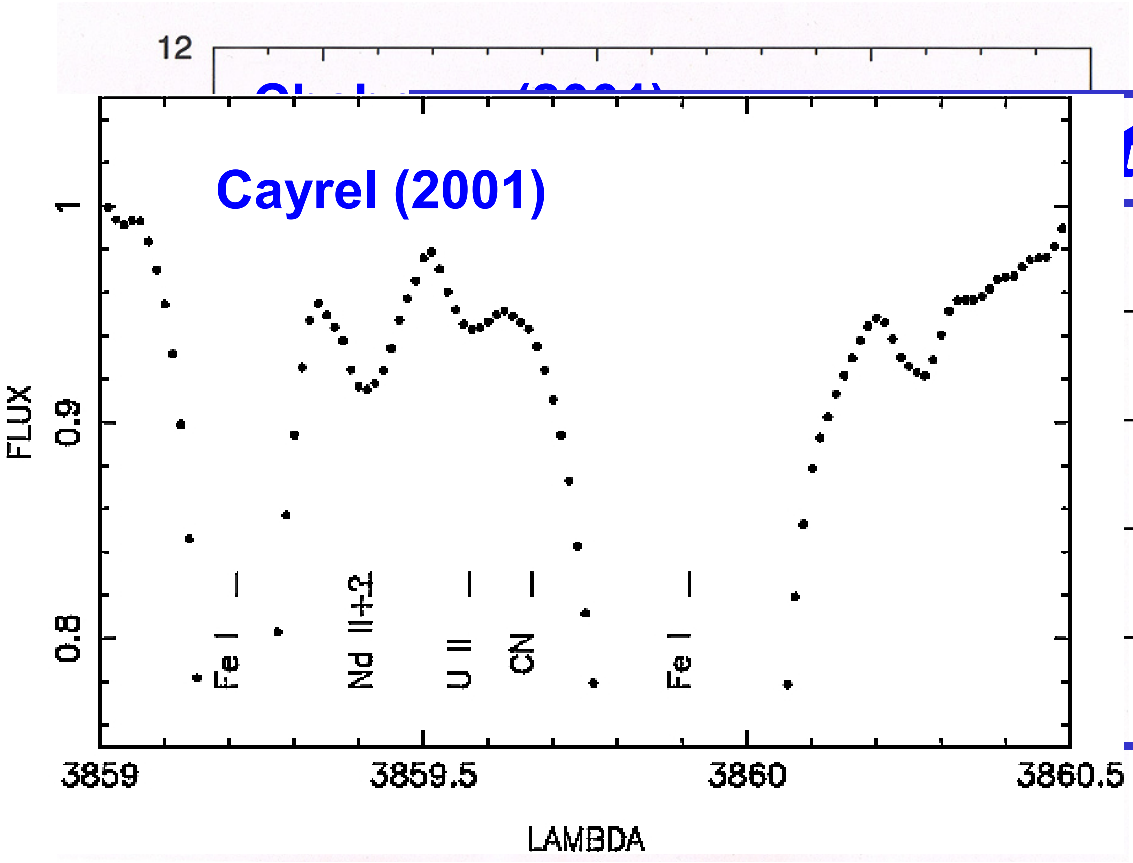
cosmological constant,
…some changing non-zero vacuum energy,
… or some unknown systematic effect(s)

The case for Λ:

1) **acceleration**
2) **large-scale structure**
3) **age of the universe**

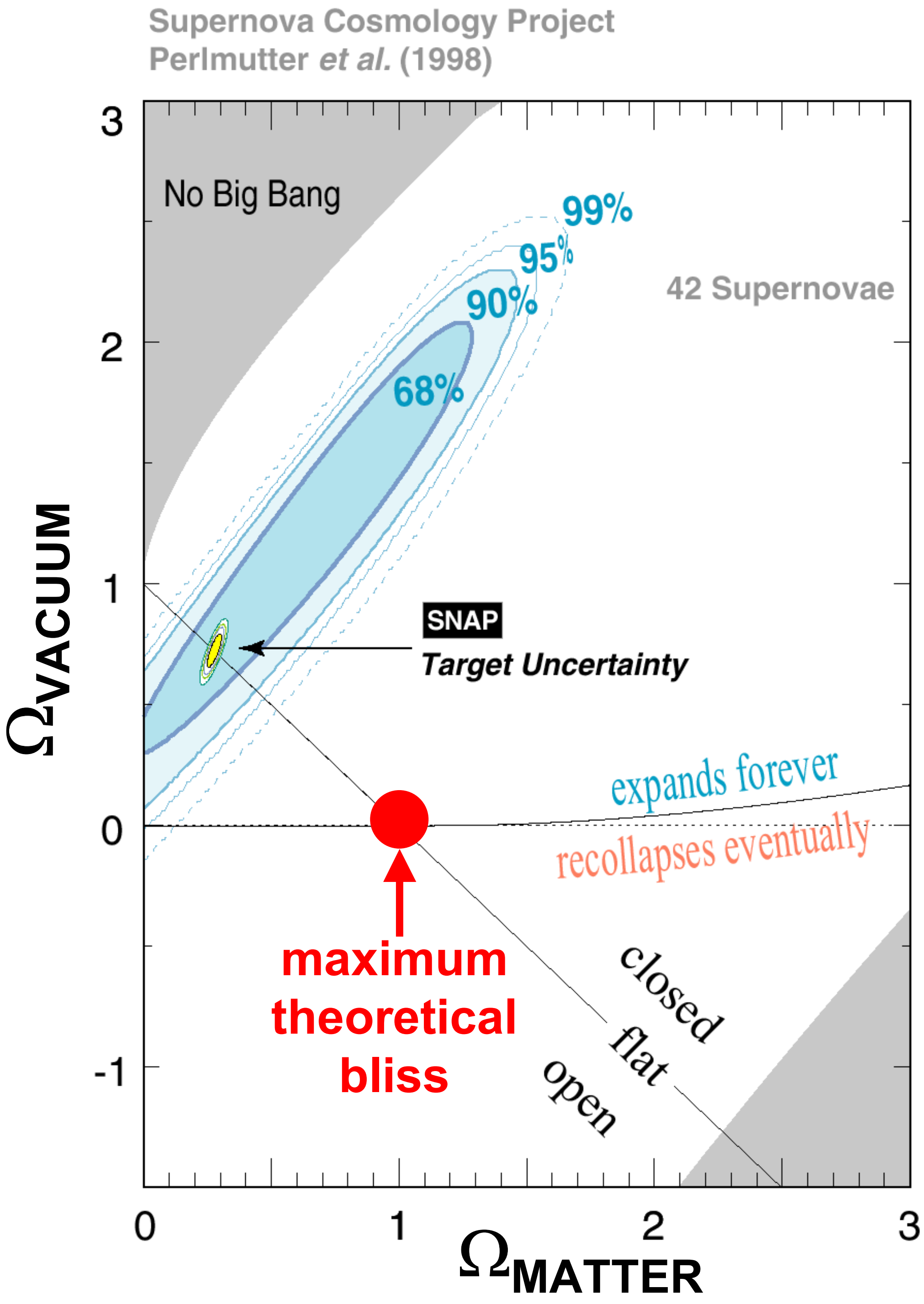
t_0 : age of the universe

- white dwarf cooling
- nucleocosmochronology 12.6 ± 3 Gyr
- globular cluster evolution 13.5 ± 2 Gyr



t_0 (Gyr)
9.3
12
14
13.5
15

First measurement of stellar uranium



Supernova Cosmology Project
Perlmutter *et al.* (1998)

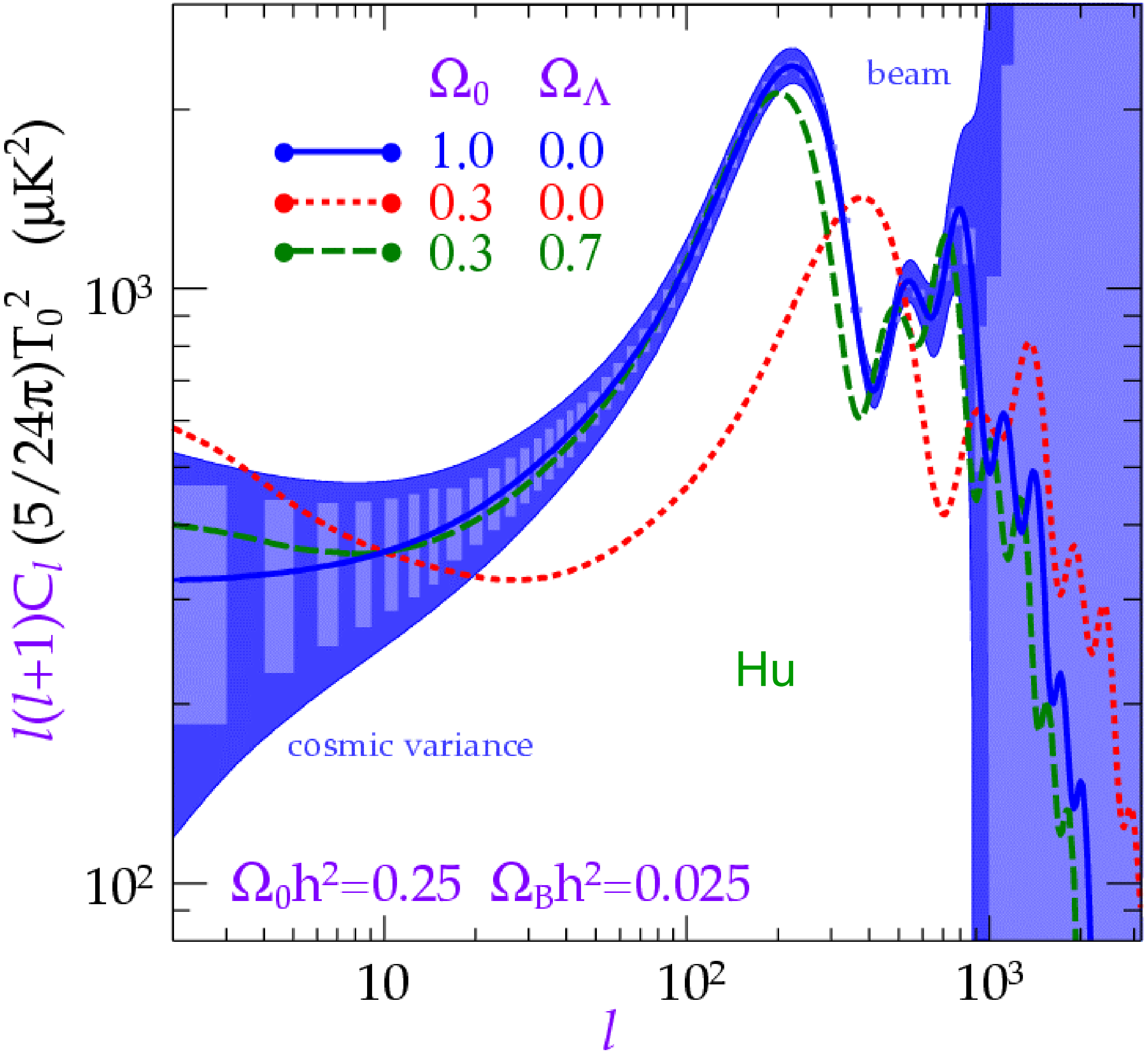
cosmological constant, …some changing non-zero vacuum energy, … or some unknown systematic effect(s)

The case for Λ:

1) acceleration
2) large-scale structure
3) age of the universe
4) flatness

$\Omega_0 = 1$
$\Omega_M = 0.3$
$1 - 0.3 = 0.7$

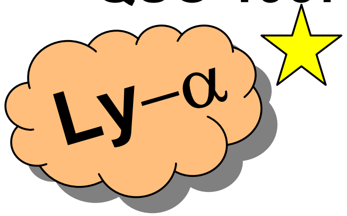
Flat universe

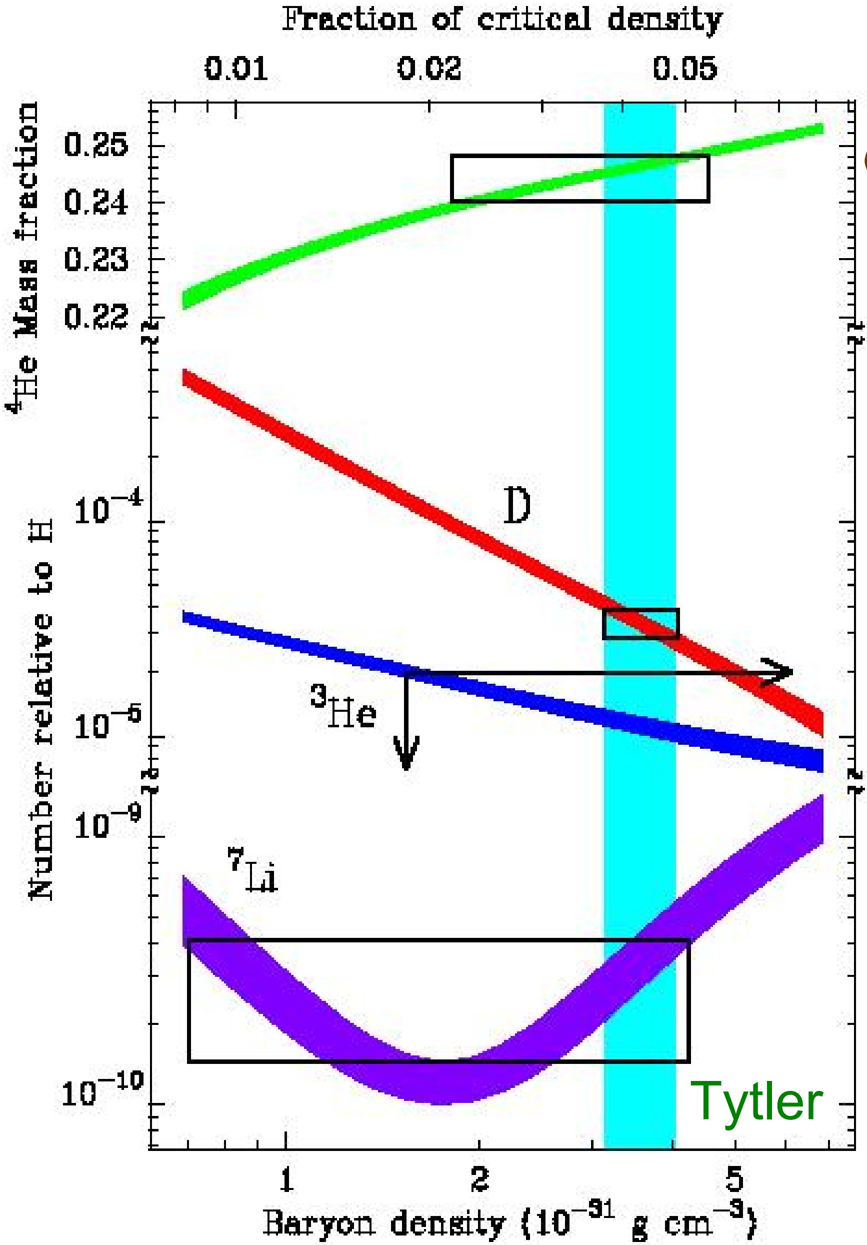


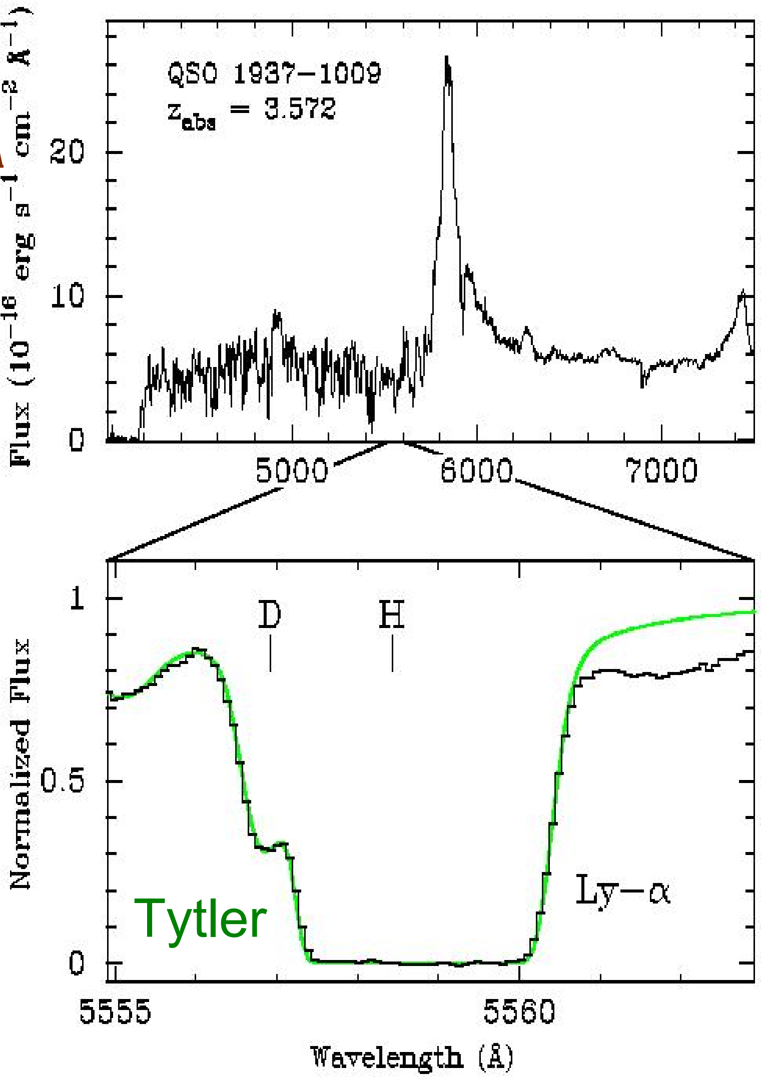
$$\Omega_0 = 1.03 \pm 0.06$$

$\Omega_B h^2 \sim 0.02$

QSO 1937-1009







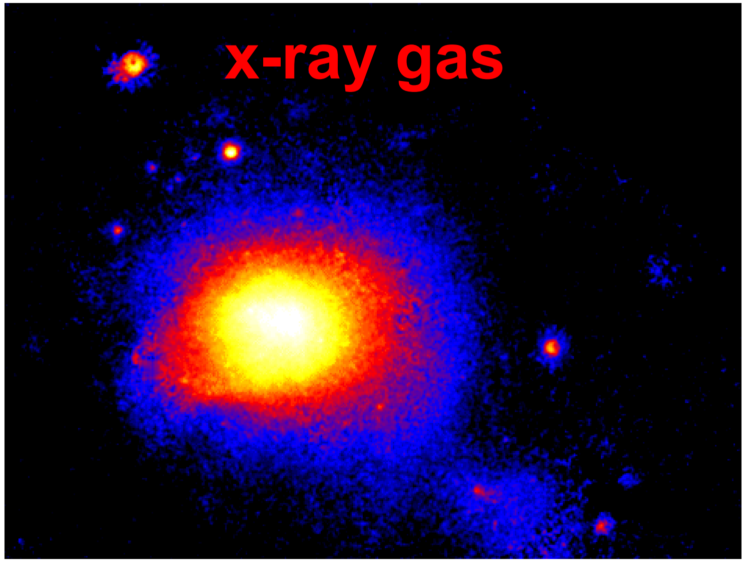
$\Omega_M \sim 0.3$

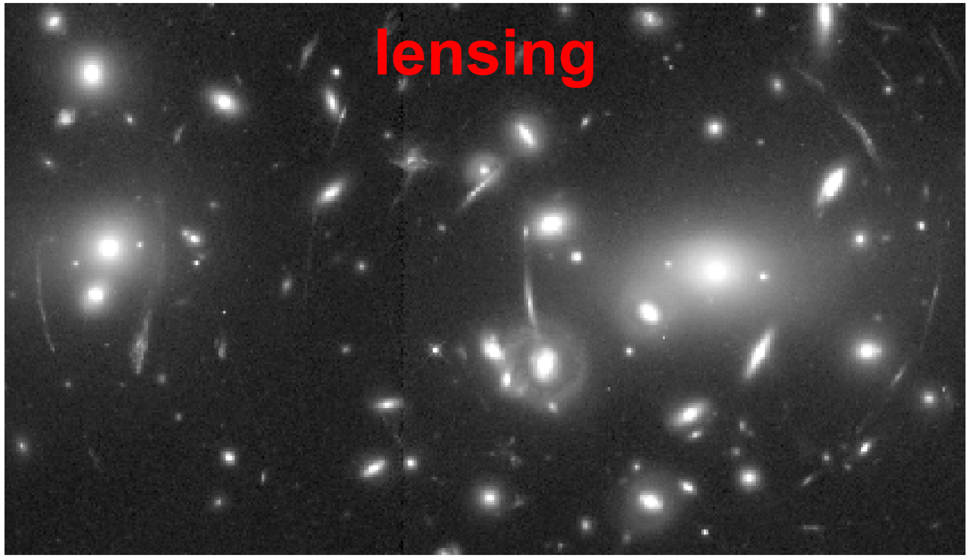
galaxy kinematics

$$\Omega_0 = 1$$

$$\Omega_M = 0.3$$

$$1 - 0.3 = 0.7$$





Cosmo-illogical constant

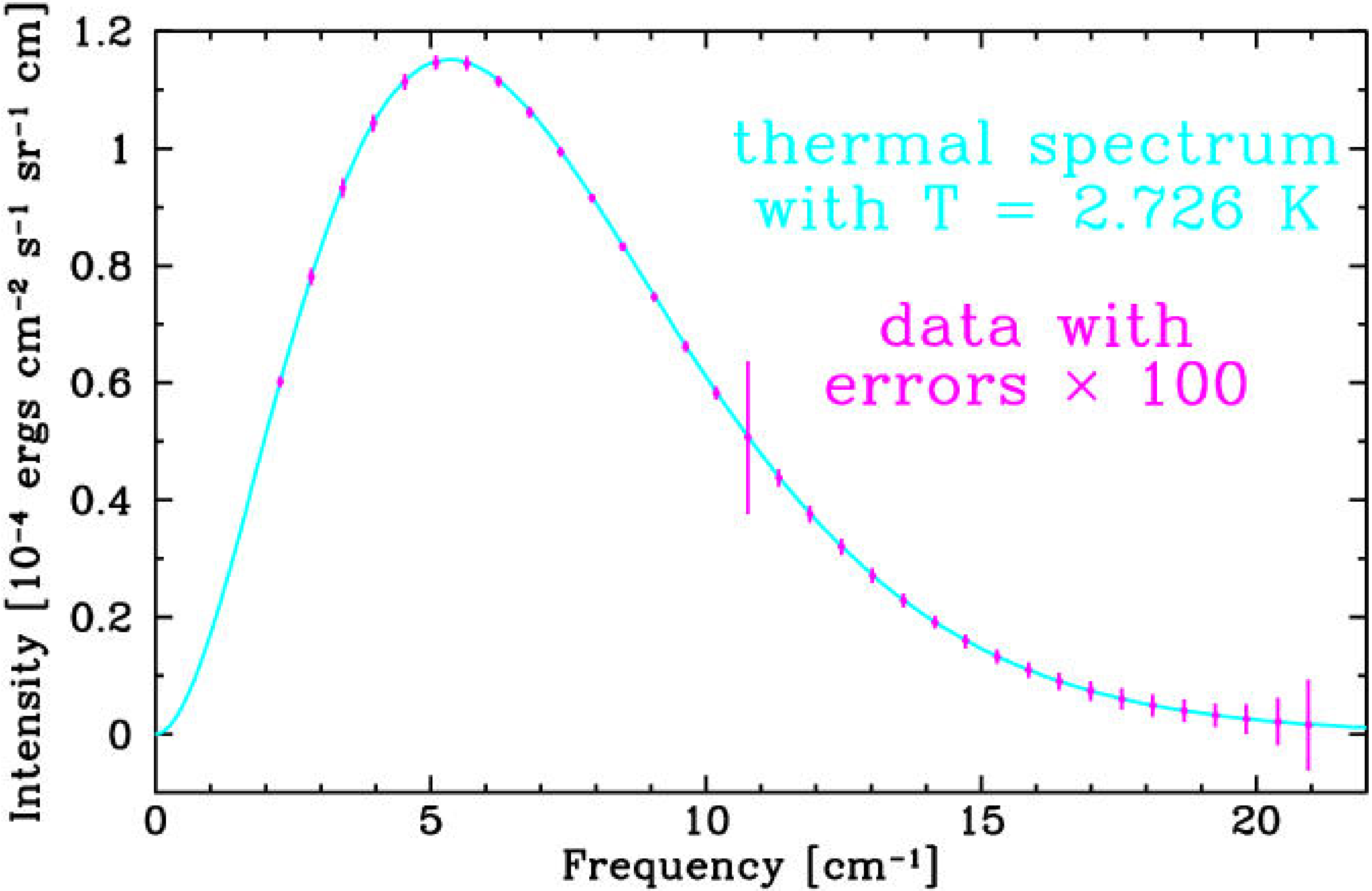
Mass density of space:

$$\boxed{\rho \Box 10^{-30}\,\mathrm{g\ cm}^{-3}}$$

Who ordered that?

The unbearable lightness of nothing!

Temperature of the universe



The cosmic food chain (Ω_i)

Radiation: 0.02%

Visible matter: 0.1%

Neutrinos: 0.1%

Dark neutrons & protons: 9.78%

Dark matter: 30%

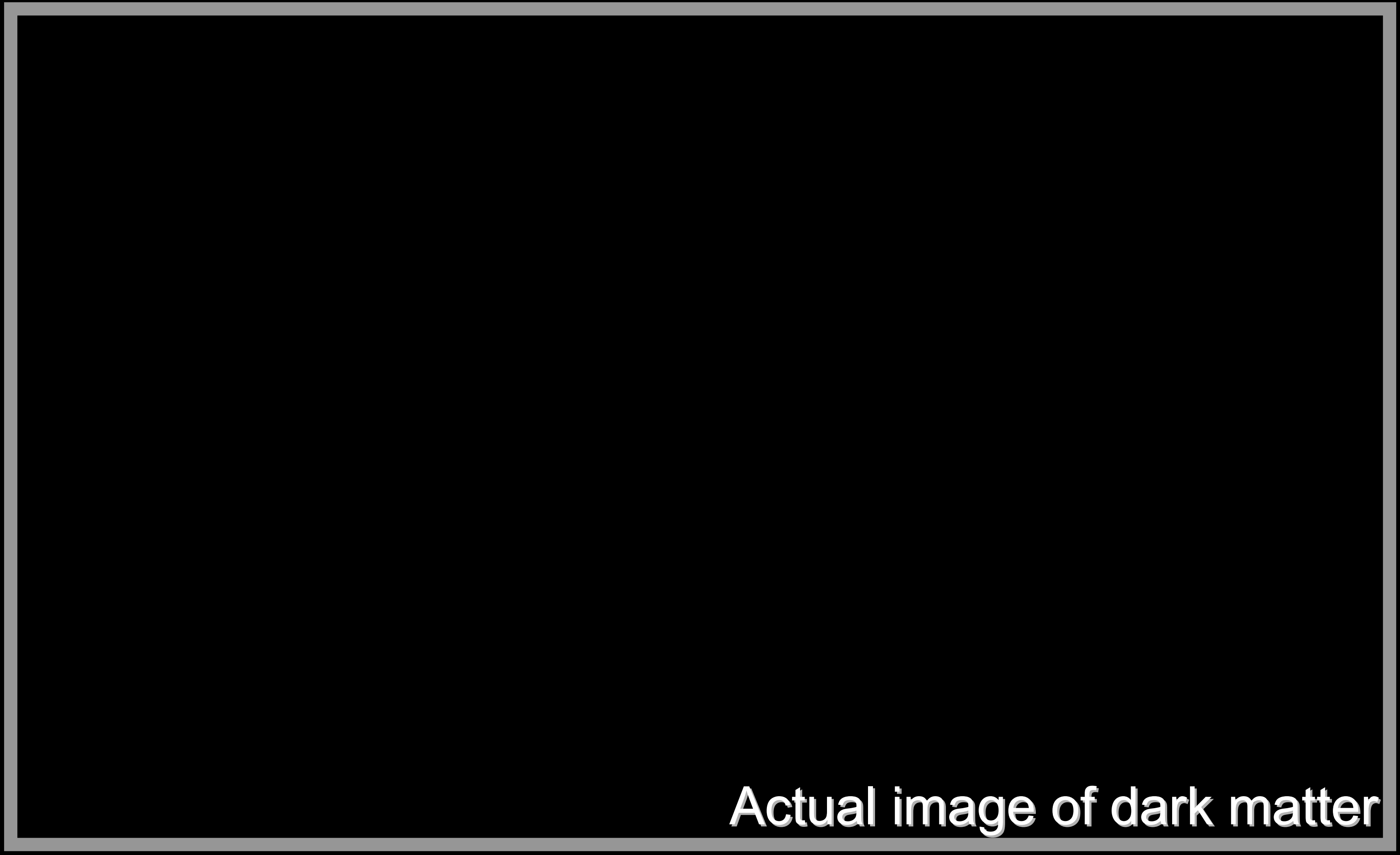
Dark energy: 60%

Observer's view of the universe

NGC 6070

lumpy (inhomogeneous and anisotropic)
full of stars, galaxies, clusters, ….

Theorist’s view of the universe



Actual image of dark matter

smooth (homogeneous and isotropic)
full of dark matter (and dark energy)

Power spectrum

- Assume there is an average density $\bar{\rho}$
- Expand density contrast $\delta(\vec{x})$ in Fourier modes

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}} = \int \delta_{\vec{k}} \exp\left(-i\vec{k} \cdot \vec{x}\right) d^3k$$

- Autocorrelation function defines power spectrum

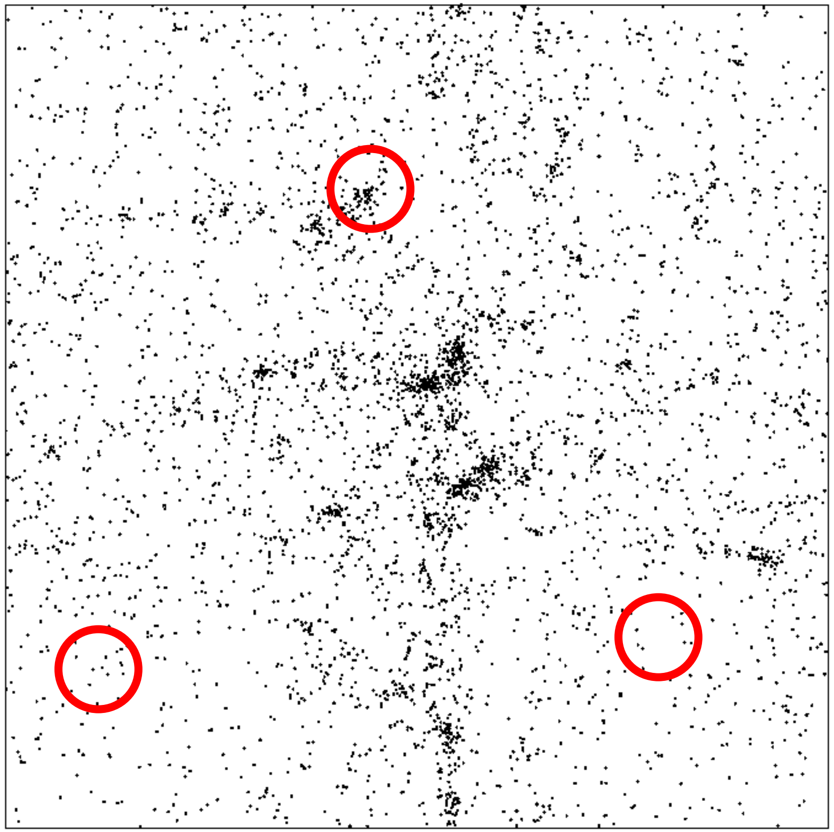
$$\left\langle \frac{\delta\rho(\vec{x})}{\rho} \right\rangle^2 = \langle \delta(\vec{x})\delta(\vec{x}) \rangle = \int_0^\infty \frac{dk}{k} \frac{k^3 \left|\delta_{\vec{k}}^2\right|}{2\pi^2}$$

$$\Delta^2(k) \equiv \frac{k^3 \left|\delta_{\vec{k}}^2\right|}{2\pi^2} \qquad P(k) \equiv \left|\delta_{\vec{k}}^2\right|$$

Power spectrum

- Power spectrum related to *rms* fluctuations

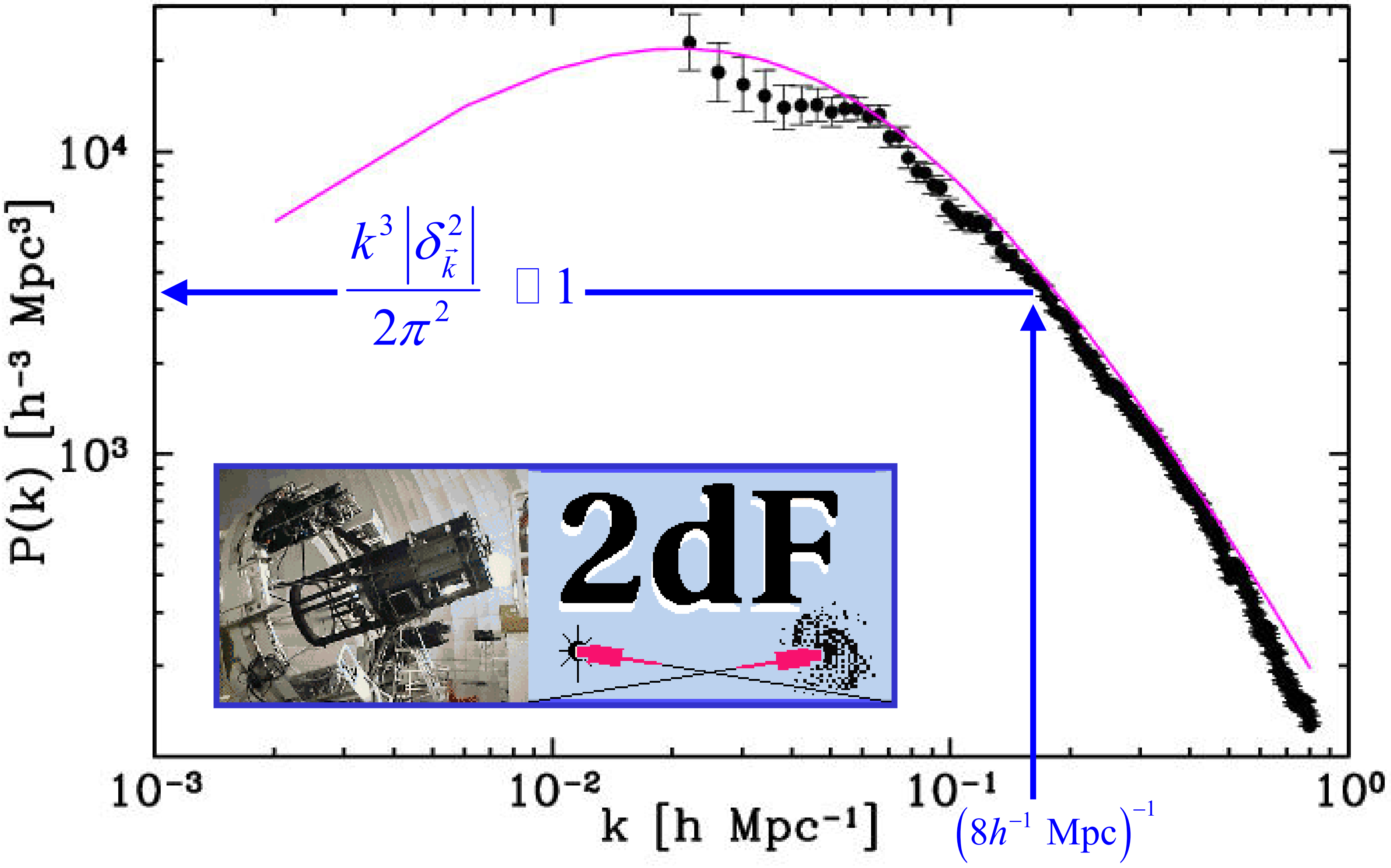
$$\left\langle \frac{\delta\rho(\vec{x})}{\rho} \right\rangle_R \approx \Delta(k = R^{-1})$$



○ sphere of radius *R*

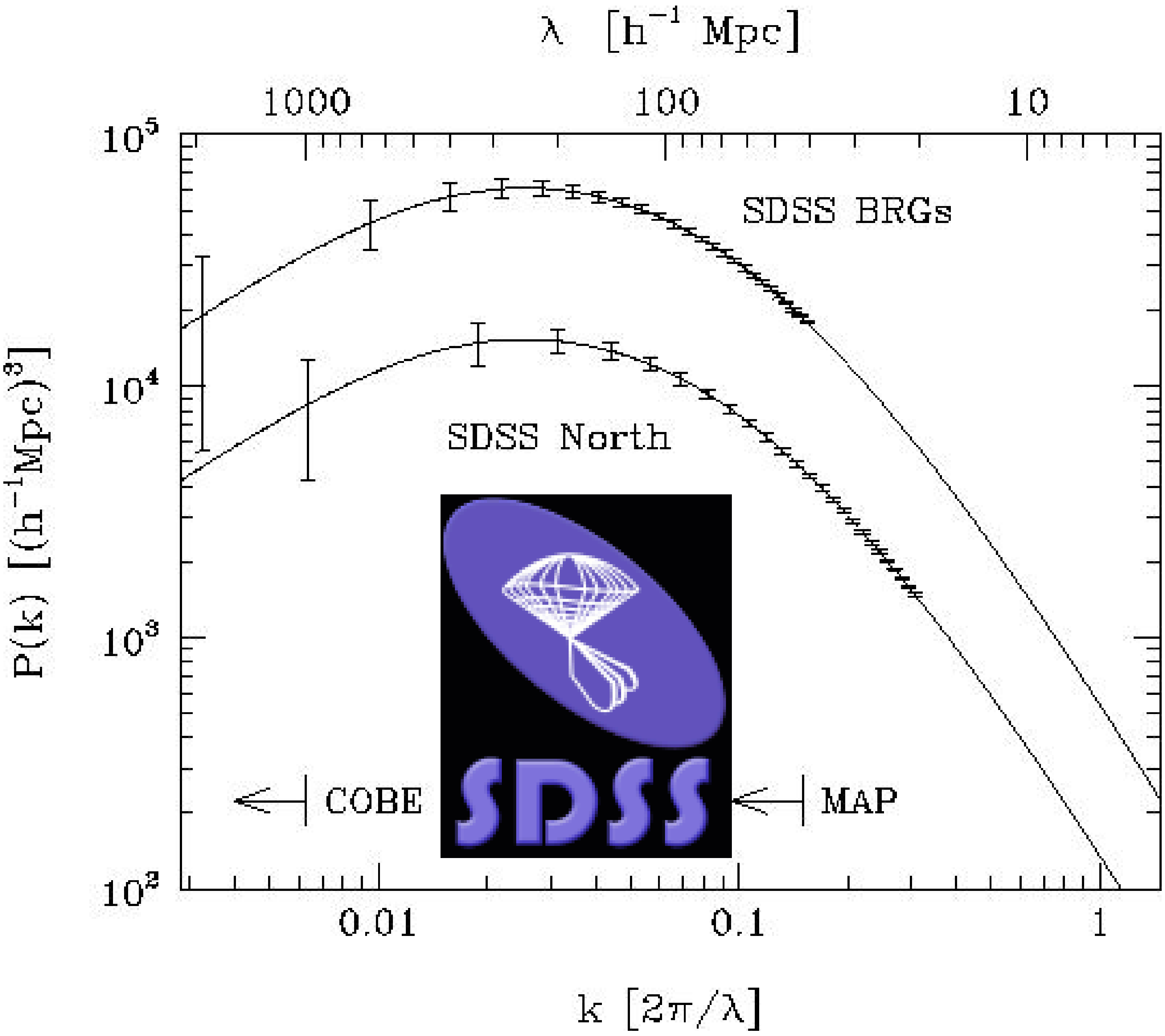
trial	N	variance
1	N_1	$N_1 - \bar{N}$
2	N_2	$N_2 - \bar{N}$
3	N_3	$N_3 - \bar{N}$
⋮	⋮	⋮
	$\bar{N}$	$\sqrt{\sum (N_i - \bar{N})^2}$

Power spectrum



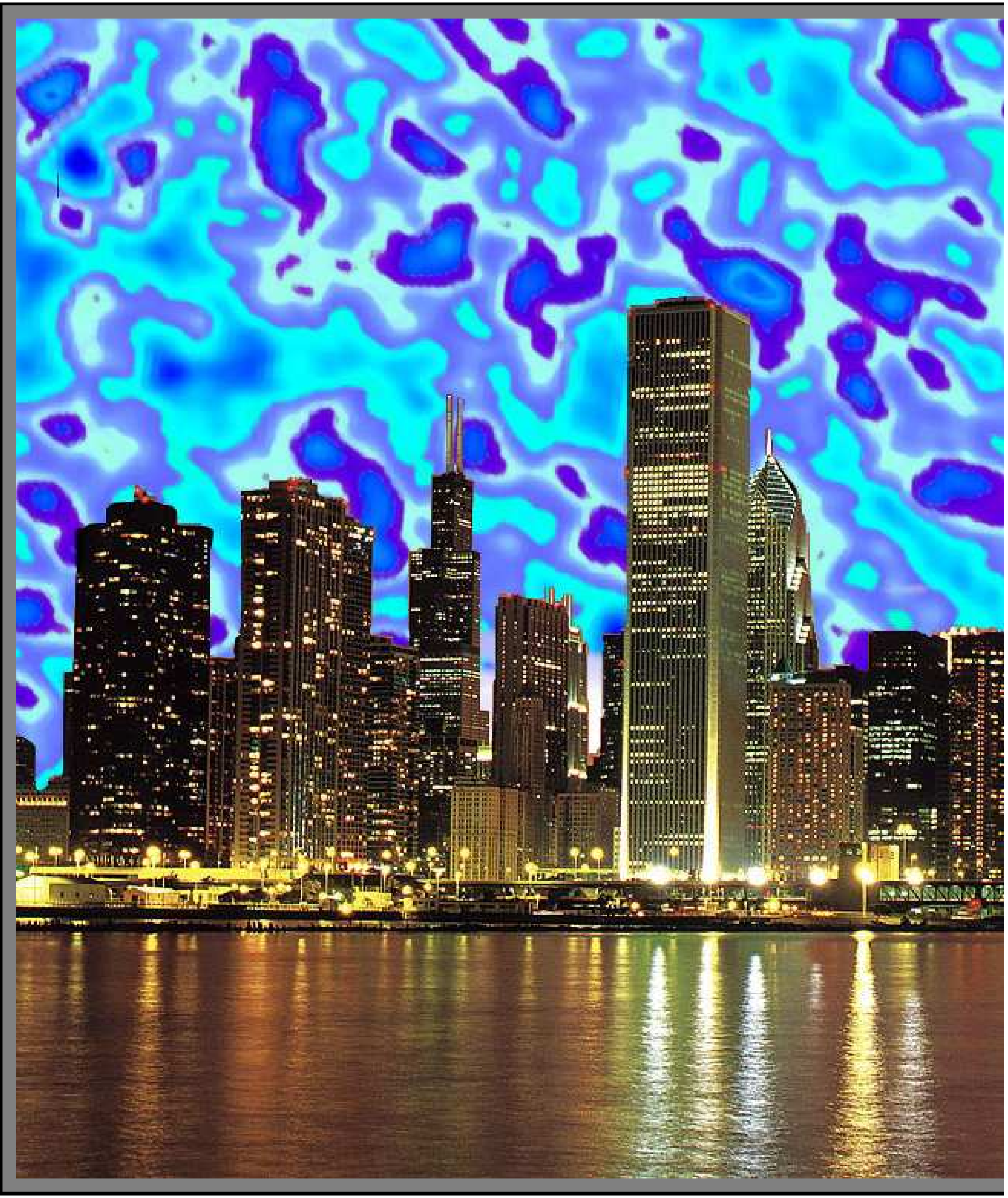
$$\frac{k^3\left|\delta_{\vec{k}}^2\right|}{2\pi^2} \;\square\; 1$$

$\left(8h^{-1}\ \mathrm{Mpc}\right)^{-1}$


λ [h^{-1} Mpc]
1000
100
10
10^5
10^4
10^3
10^2
P(k) [(h^{-1}Mpc)^3]
SDSS BRGs
SDSS North
COBE
MAP
SDSS
0.01
0.1
1
k [2π/λ]

Observer's view of the universe

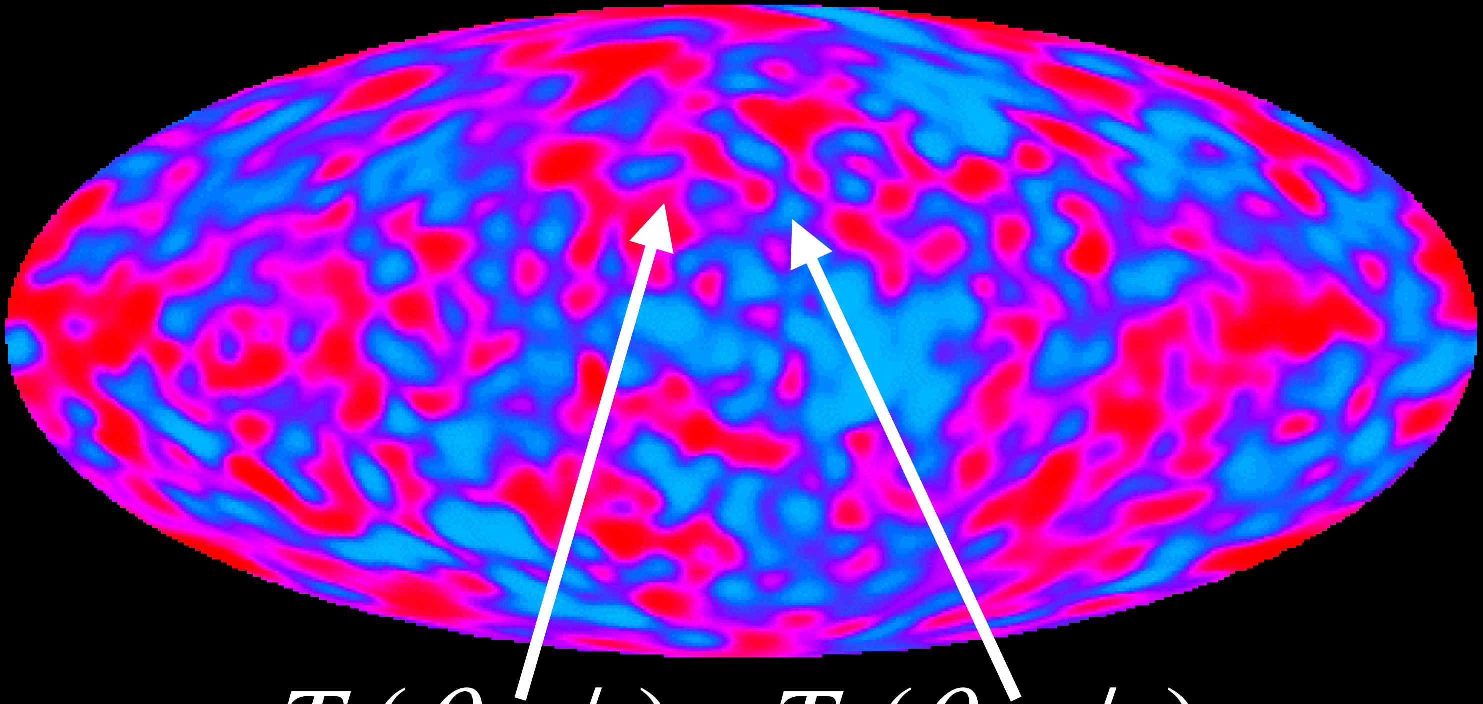
(fluctuations)



Theorist's view of the universe

(isotropic)

Angular correlation function, C_l

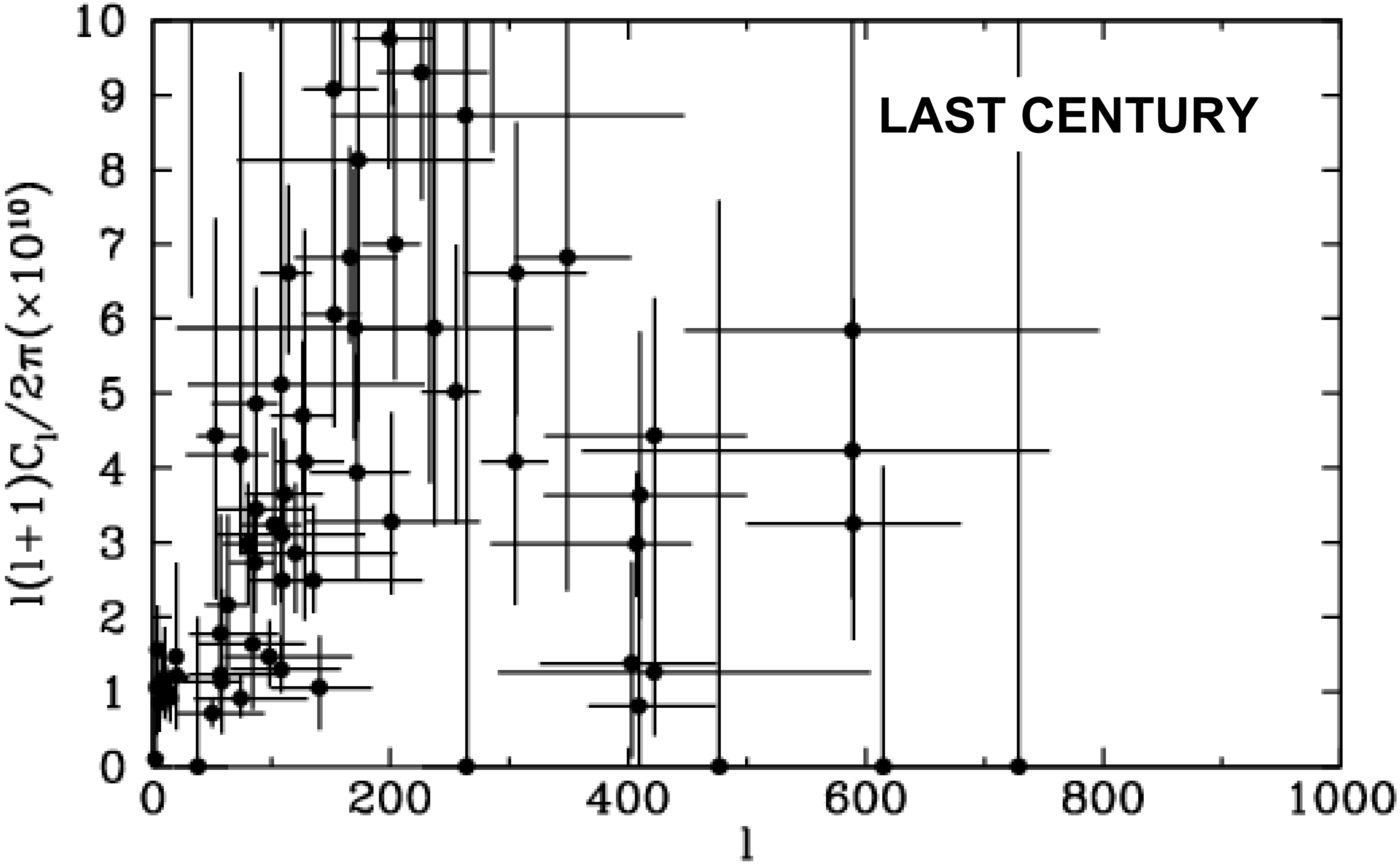


$$T_1(\theta_1,\phi_1) \quad T_2(\theta_2,\phi_2)$$

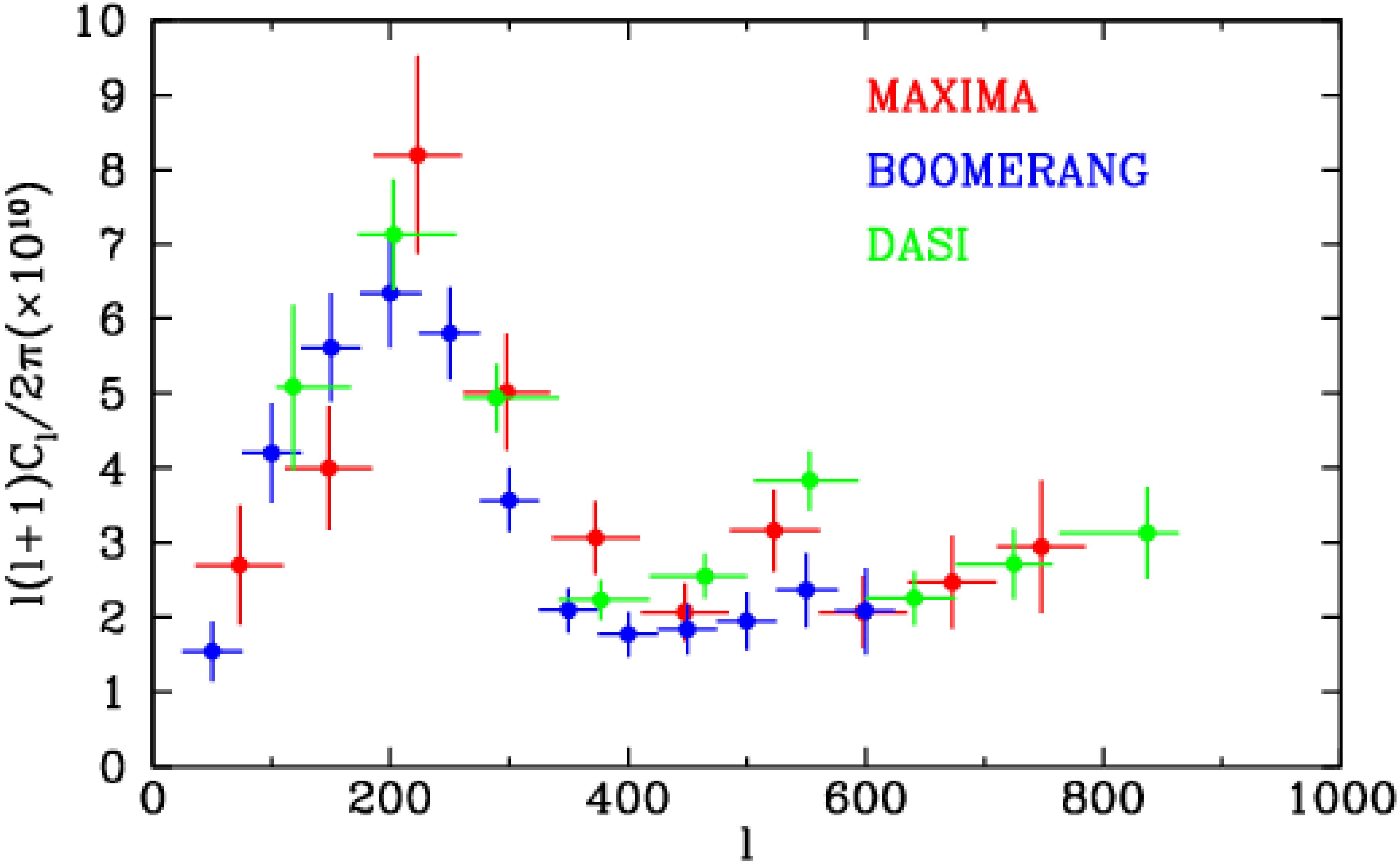
$$\langle T_1 T_2 \rangle = \sum a_{lm} Y_{lm}(\theta,\phi)$$

$$\left\langle \left| a_{lm} \right|^2 \right\rangle^{1/2} \equiv C_l$$

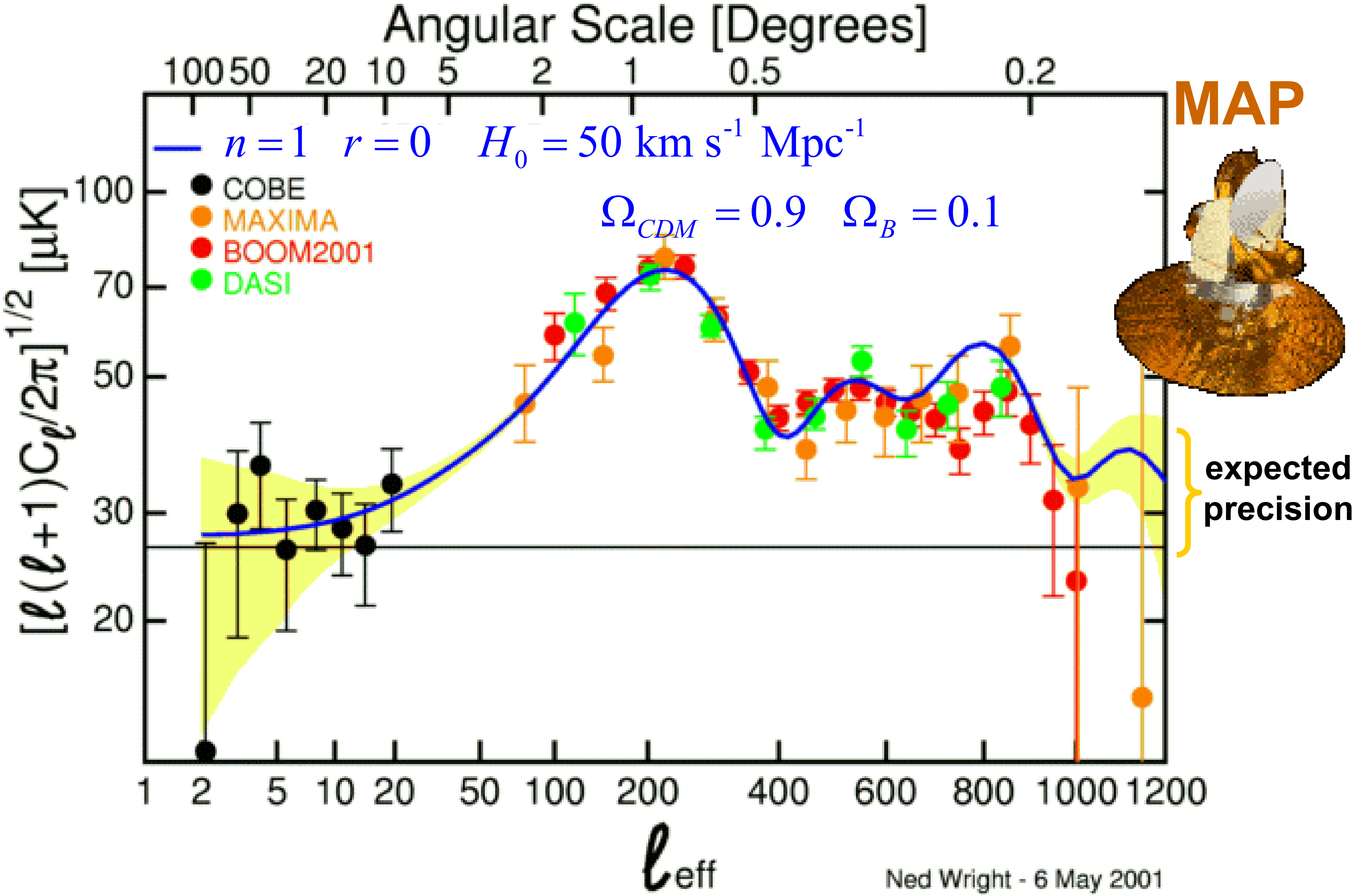
Angular power spectrum



Angular power spectrum



Angular power spectrum



$n = 1 \quad r = 0 \quad H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$

$\Omega_{CDM} = 0.9 \quad \Omega_B = 0.1$

MAP

expected precision

Rocky I: The universe observed

- **Cosmological parameters:**

$H_0 \quad \rightarrow$ Hubble's constant

$q_0 \quad \rightarrow$ deceleration parameter

$\Omega_i \quad \rightarrow$ the cosmic food chain

$$\left(\Omega_0 \ , \ \Omega_M \ , \ \Omega_B \ , \ \Omega_\Lambda \ , \ \Omega_\gamma \ , \ \Omega_\nu \ , ... \right)$$

$t_0 \quad \rightarrow$ age of the universe

$T_0 \quad \rightarrow$ temperature of the universe

- **Power spectrum of large-scale structure:**

$P(k)$

- **Anisotropy of CMB:**

C_l

Cosmology and the origin of structure

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