3. QUANTUM CHROMODYNAMICS

- QCD Lagrangian
- Running Coupling
- Asymptotic Freedom
- α_s Measurements
- Deep Inelastic Scattering

$$\mathcal{L}_{QCD} = -\frac{1}{2}\,\mathrm{Tr}\,(\mathbf{G}^{\mu\nu}\,\mathbf{G}_{\mu\nu}) + \bar{\mathbf{q}}\,[i\,\gamma^{\mu}\,\mathbf{D}_{\mu} - m_q]\,\mathbf{q}$$

$$= -\frac{1}{4}\left(\partial^{\mu}G^{\nu}_{a} - \partial^{\nu}G^{\mu}_{a}\right)\left(\partial_{\mu}G^{a}_{\nu} - \partial_{\nu}G^{a}_{\mu}\right)$$

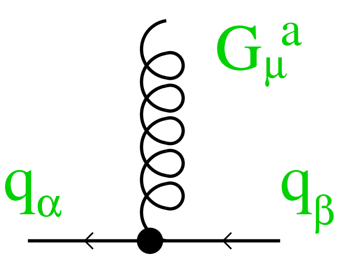
Kinetic

$$+ \sum_{q} \bar{q_{\alpha}}\;[i\,\gamma^{\mu}\,\partial_{\mu} - m_q]\;q_{\alpha}$$

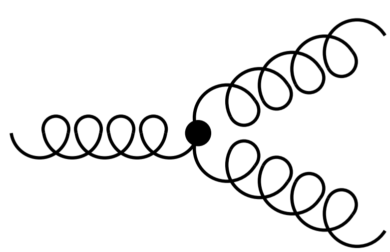
$$+ \frac{1}{2}\sum_{q} g_s \left[\bar{q_{\alpha}}\,(\lambda^{a})_{\alpha\beta}\,\gamma^{\mu}\,q_{\beta}\right] G^{a}_{\mu}$$

$$- \frac{1}{2}\,g_s\;f_{abc}\left(\partial_{\mu}G^{a}_{\nu} - \partial_{\nu}G^{a}_{\mu}\right) G^{\mu}_{b}\,G^{\nu}_{c}$$

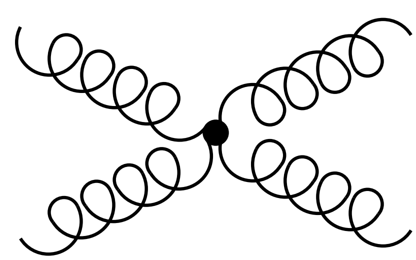
$$- \frac{1}{4}\,g_s^2\;f_{abc}\,f_{ade}\;G^{\mu}_{b}\,G^{\nu}_{c}\,G^{d}_{\mu}\,G^{e}_{\nu}$$



$g_s\,\frac{\lambda^{a}_{\alpha\beta}}{2}\,\gamma^{\mu}$



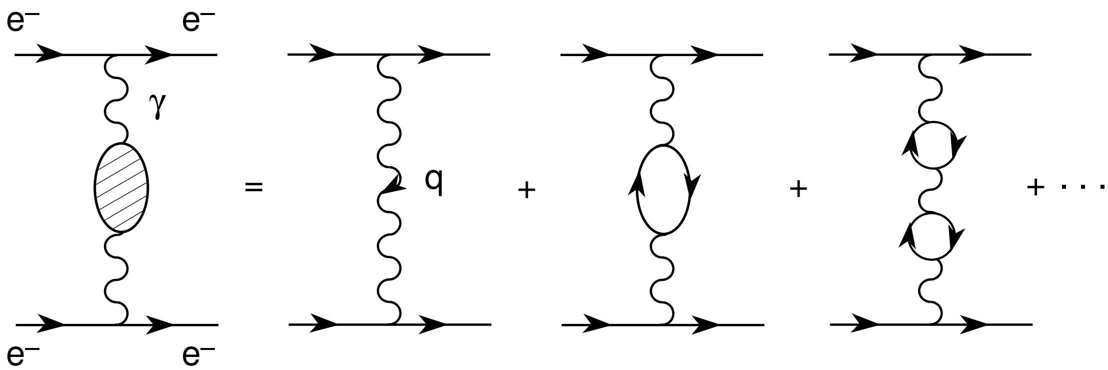
$g_s\;f_{abc}$



$g_s^2\,f_{abc}\,f_{ade}$

- **Gluon Self-interactions** G^3, G^4
- **Universal Coupling** g_s
- **No Colour Charges**

QUANTUM CORRECTIONS



$$T(Q^2) \sim \frac{\alpha}{Q^2} \left\{1 + \Pi(Q^2) + \Pi(Q^2)^2 + \cdots\right\} \sim \frac{\alpha(Q^2)}{Q^2}$$

Effective *(Running)* **Coupling:**

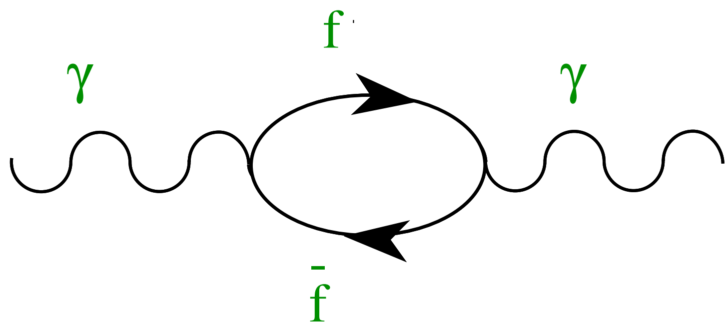
$$\alpha(Q^2) = \frac{\alpha}{1 - \Pi(Q^2)} \approx \frac{\alpha}{1 - \frac{\alpha}{3\pi} \log\left(\frac{Q^2}{m^2}\right)}$$

$\alpha(Q^2)$ Increases with $Q^2 \equiv -q^2$

➡ **Decreases at Large Distances**

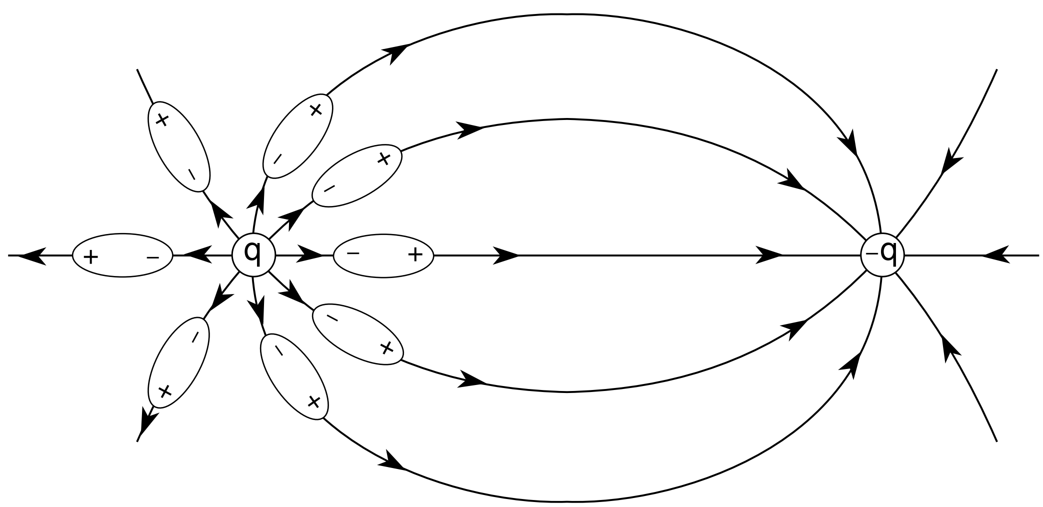
SCREENING

VACUUM POLARIZATION



The Photon Couples to *Virtual* $f\,\bar{f}$ *Pairs*

Vacuum ⟷ Polarized Dielectric Medium

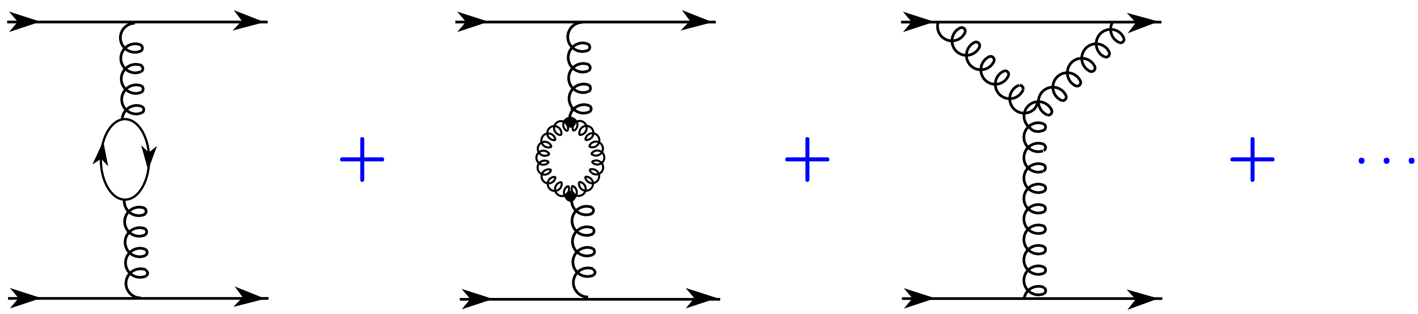


$$1/\alpha \;=\; 1/\alpha(m_e^2) \;=\; 137.03599976\;(50)$$

$$1/\alpha(M_Z^2) \;=\; 128.95 \pm 0.05$$

($l^- l^+$ and $q\,\bar{q}$ contributions included)

QCD RUNNING COUPLING



$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 - \beta_1 \frac{\alpha_s(Q_0^2)}{2\pi} \log\left(\frac{Q^2}{Q_0^2}\right)}$$

$$\beta_1 = \frac{1}{3} N_F - \frac{11}{6} N_C$$

quarks gluons

$N_C = 3$, $N_F = 6$ ➡ $\beta_1 < 0$

$Q^2 > Q_0^2$ ➡ $\alpha_s(Q^2) < \alpha_s(Q_0^2)$

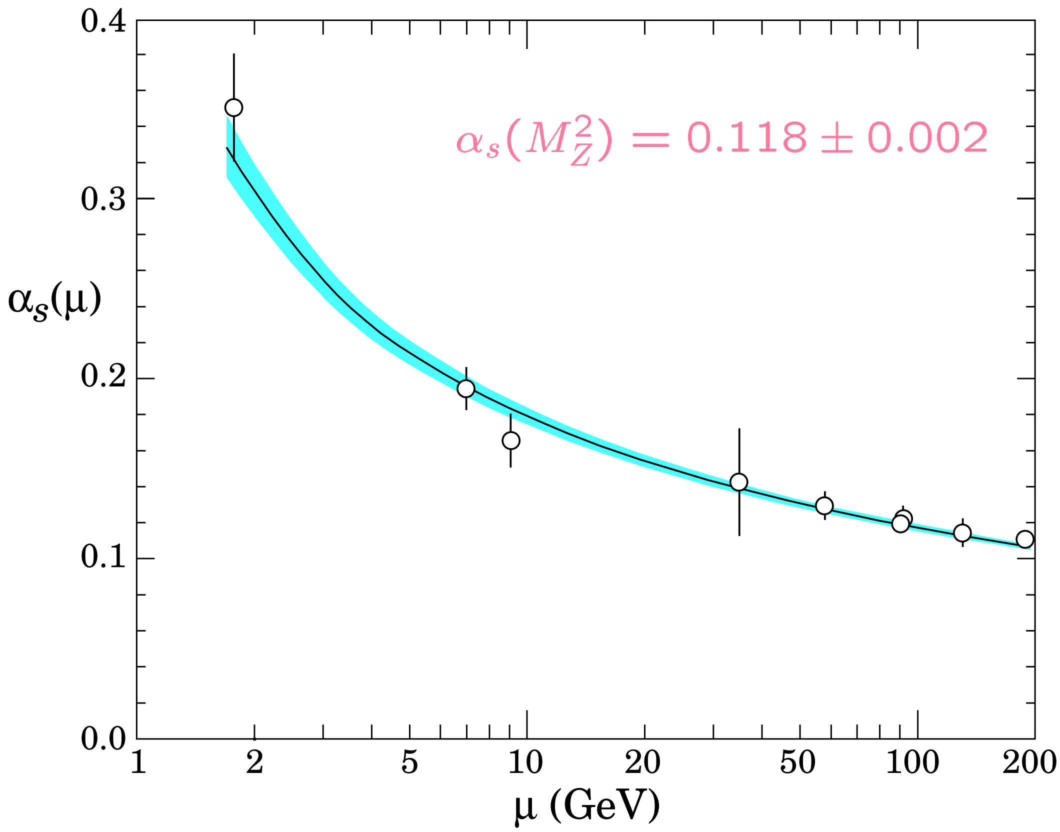
$\alpha_s(Q^2)$ **Decreases at Short Distances**

ANTI–SCREENING

$$\alpha_s(Q^2) \;=\; \frac{\alpha_s(Q_0^2)}{1 - \beta_1 \frac{\alpha_s(Q_0^2)}{2\pi} \log\left(\frac{Q^2}{Q_0^2}\right)}$$

$\beta_1 < 0$ ➡ $\lim_{Q^2 \to \infty} \alpha_s(Q^2) = 0$

ASYMPTOTIC FREEDOM

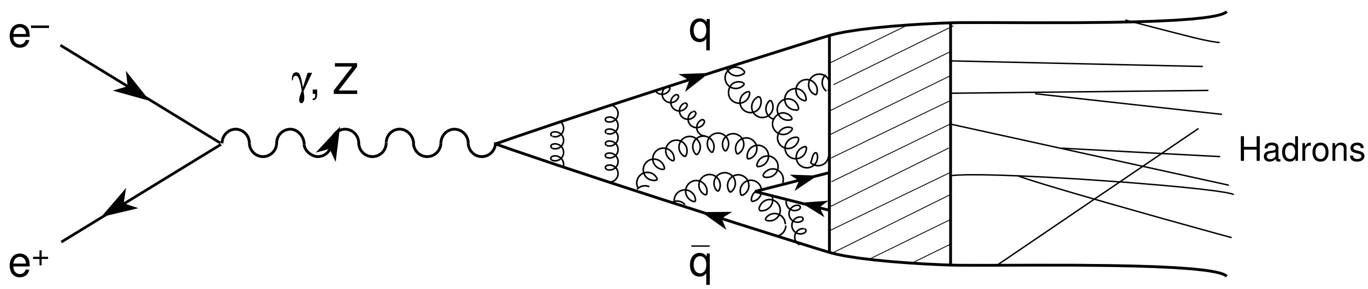


$\alpha_s(Q^2)$ **increases at low energies**

$\alpha_s \sim O(1)$ at 1 GeV Non-Perturbative Region

➡ **CONFINEMENT ?**

Confinement ⟷ Prob. Hadronization = 1



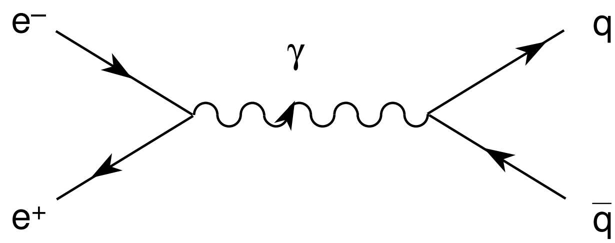
$$\sigma(e^+e^- \to \text{hadrons}) \; =$$

$$\sigma\left(e^+e^- \to q\,\bar{q} + q\,\bar{q}\,G + q\,\bar{q}\,G\,G + q\,\bar{q}\,q\,\bar{q} + \cdots\right)$$

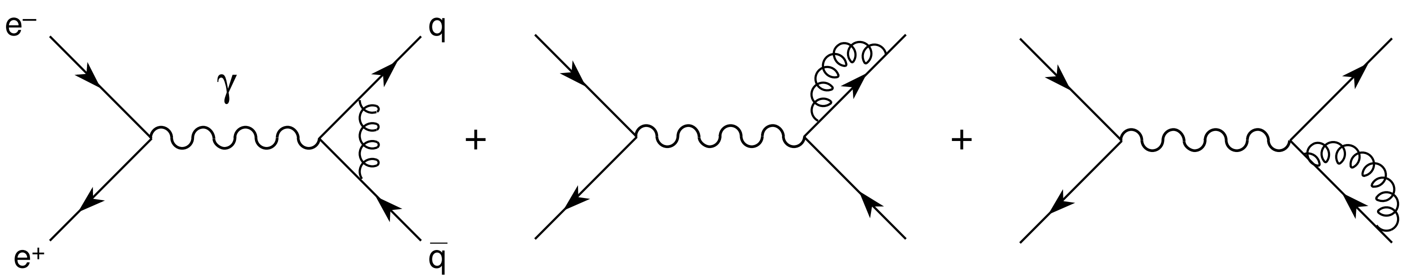
$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_q Q_q^2 \, N_C \quad \left\{1 + \frac{\alpha_s(s)}{\pi} + \cdots\right\}$$

$$R_Z \equiv \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to e^+e^-)} = R_Z^{EW} \, N_C \quad \left\{1 + \frac{\alpha_s(s)}{\pi} + \cdots\right\}$$

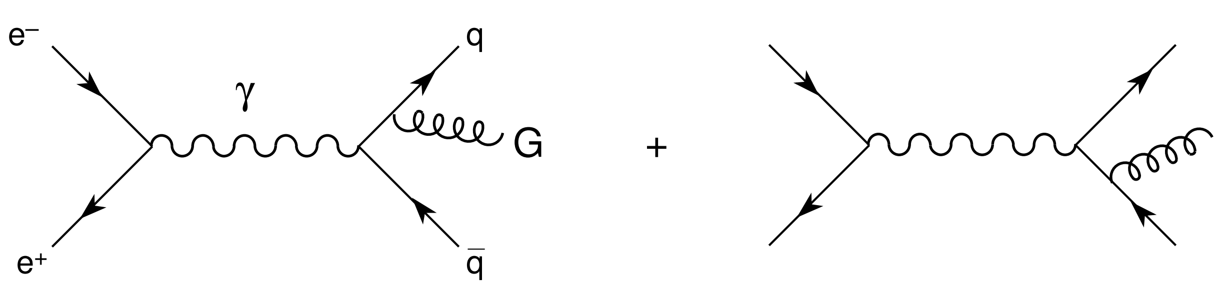
$$T\left(e^{+}e^{-} \to q\,\bar{q}\right) =$$



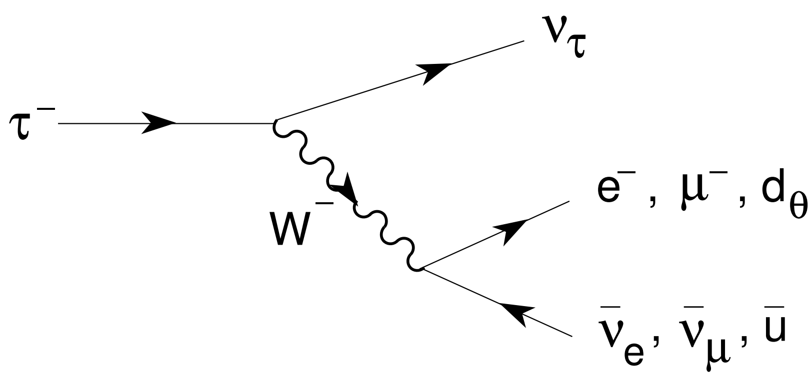
$$+$$



$$T\left(e^{+}e^{-} \to q\,\bar{q}\,G\right) =$$



$\tau^- \to \nu_\tau + \text{Hadrons}$



$d_\theta = \cos\theta_C \, d + \sin\theta_C \, s$

$$B_l \equiv \text{Br}\left(\tau^- \to \nu_\tau \, l^- \, \bar{\nu}_l\right) \approx \frac{1}{2+N_C} = \frac{1}{5} = 20\%$$

$$B_e = (17.81 \pm 0.06)\% \qquad ; \qquad B_\mu = (17.33 \pm 0.06)\%$$

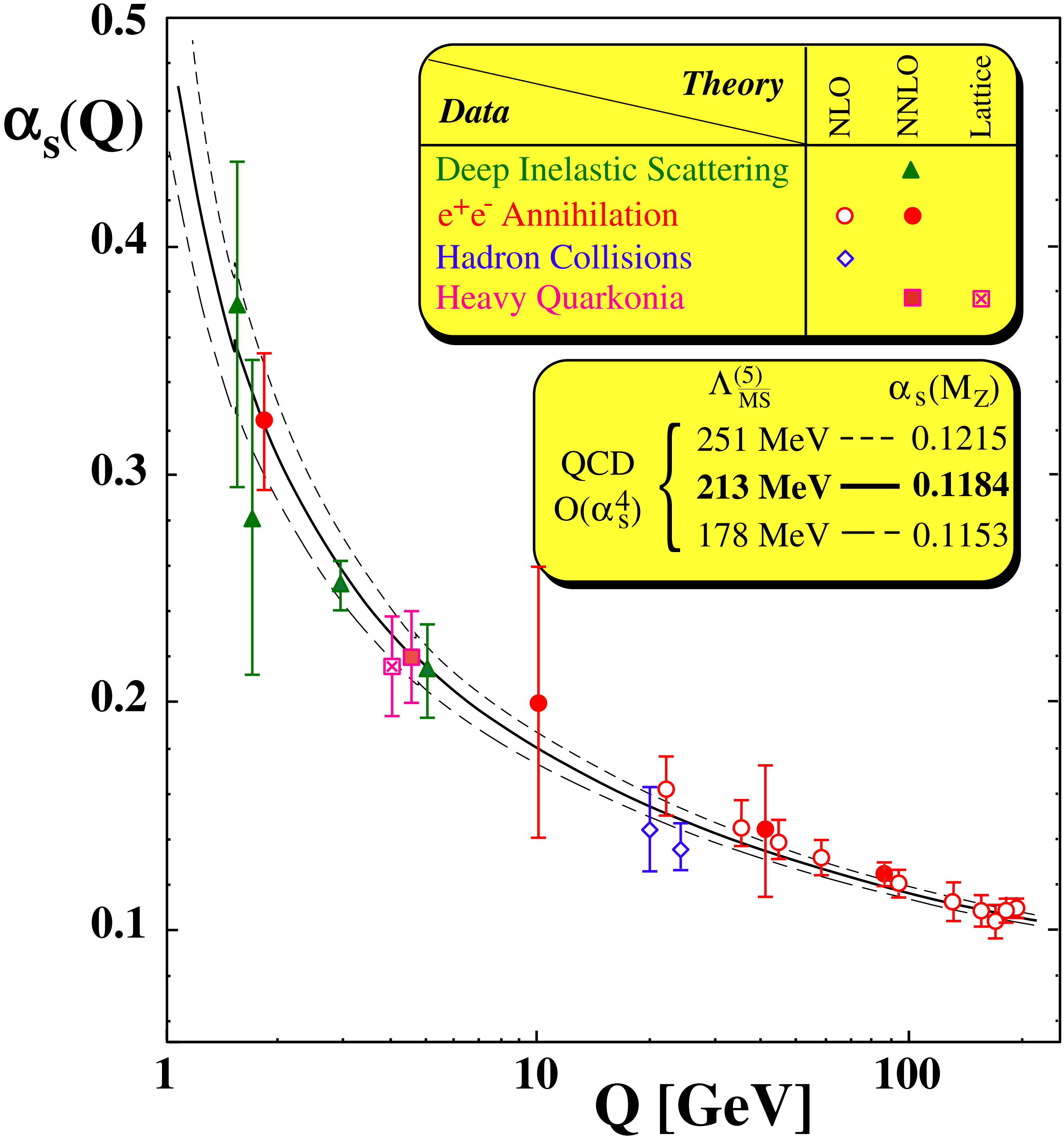
QCD :

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{Hadrons})}{\Gamma(\tau^- \to \nu_\tau \, e^- \, \bar{\nu}_e)} = N_C \left\{ 1 + \frac{\alpha_s(m_\tau^2)}{\pi} + \cdots \right\}$$

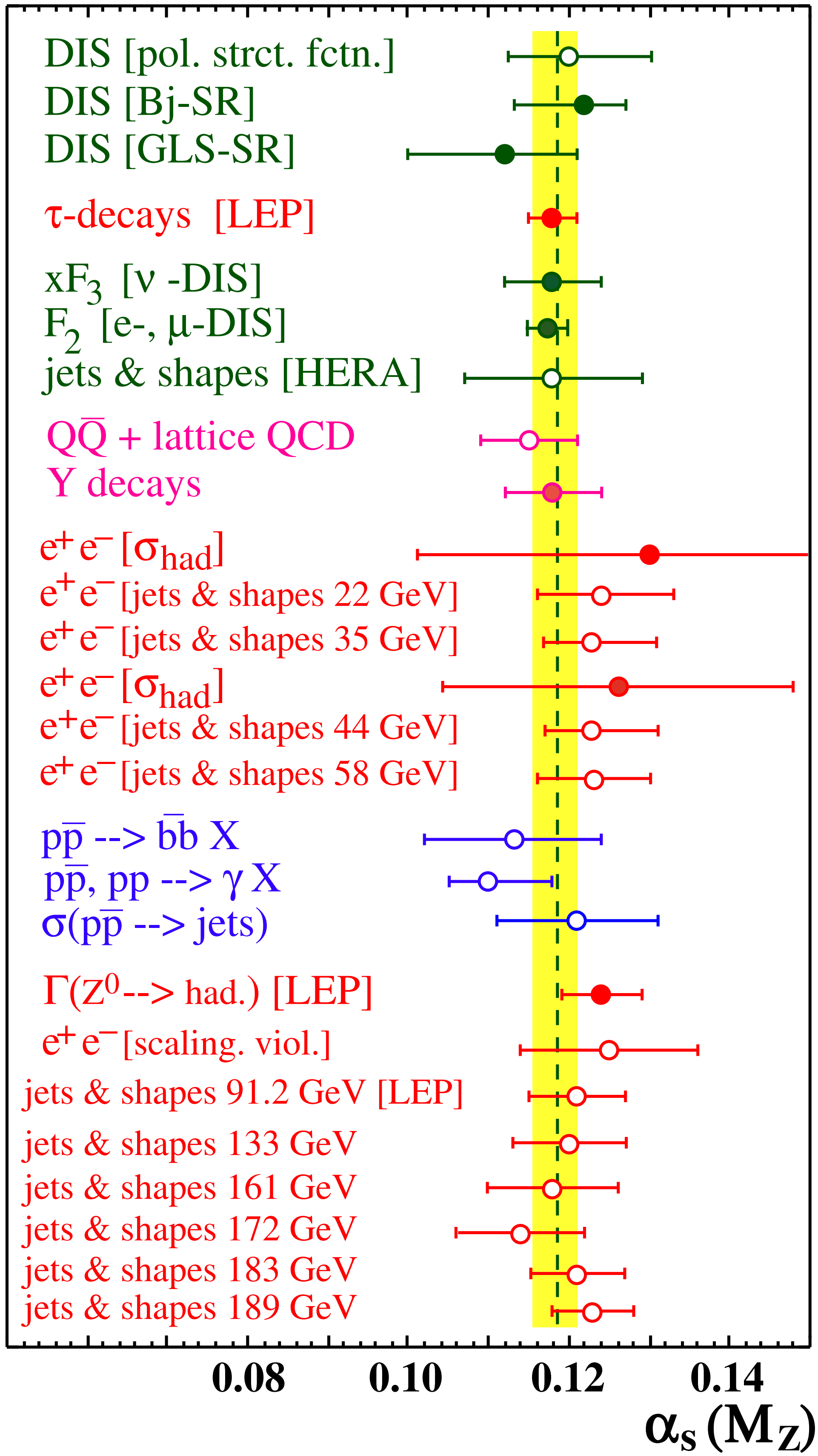
$$R_\tau = 3.484 \pm 0.024 \quad \Rightarrow \quad \alpha_s(m_\tau^2) = 0.345 \pm 0.020$$

$$\alpha_s(m_\tau^2) \quad > \quad \alpha_s(M_Z^2) \quad = 0.119$$

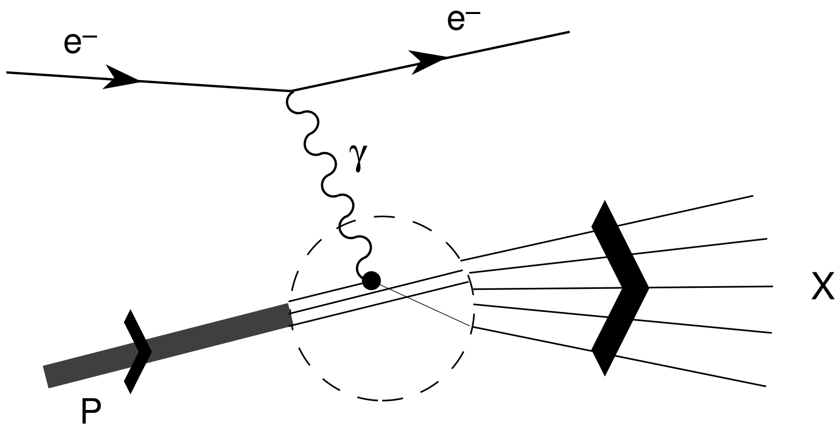
MEASUREMENTS OF $\alpha_s(Q)$ (S. Bethke)



MEASUREMENTS OF $\alpha_s(M_Z)$ (S. Bethke)



DEEP INELASTIC SCATTERING



$$\nu \equiv \frac{(P\cdot q)}{M_p} = E_e - E_e'$$
$$= \frac{Q^2 + P_X^2 - M_p^2}{2M_p}$$

$$Q^2 \equiv -q^2$$

$$\frac{d\sigma}{dQ^2\, d\nu} = \frac{\pi\alpha^2 \cos^2\left(\frac{\theta}{2}\right)}{4E_e^2 \sin^4\left(\frac{\theta}{2}\right) E_e E_e'} \left\{ W_2 + 2\, W_1 \tan^2\left(\frac{\theta}{2}\right) \right\}$$

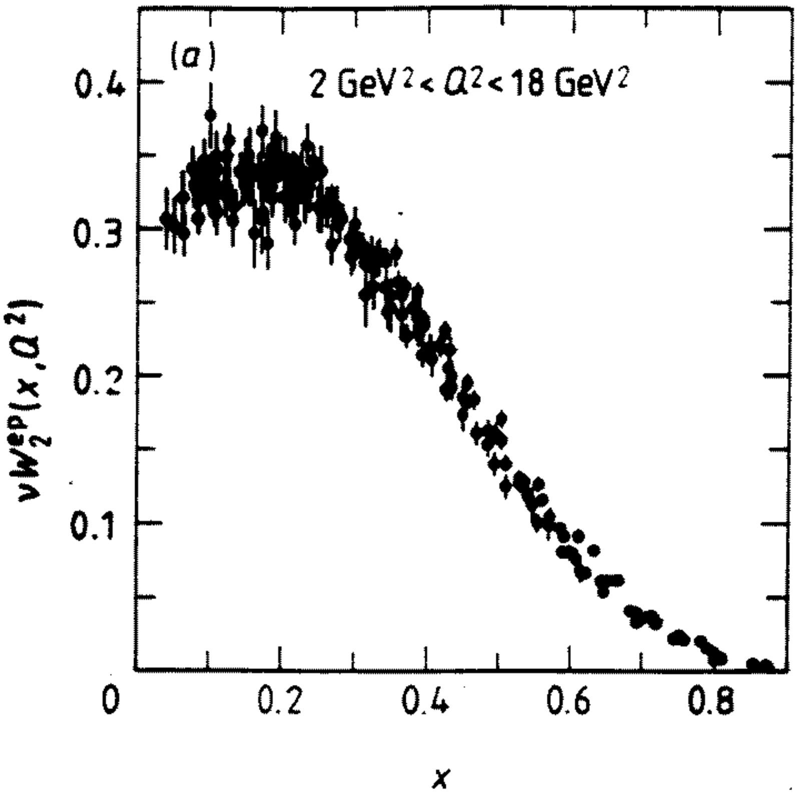
Free Parton Model: $p_i^\mu = \xi_i\, P^\mu \quad ; \quad m_i = \xi_i\, M_p$

$$W_1(Q^2,\nu) = \sum_i \int_0^1 d\xi_i\ f_i(\xi_i)\ \frac{e_i^2 Q^2}{4m_i^2}\ \delta\left(\nu - \frac{Q^2}{2m_i}\right) = \frac{1}{M_p} F_1(x)$$
$$W_2(Q^2,\nu) = \sum_i \int_0^1 d\xi_i\ f_i(\xi_i)\ e_i^2\ \delta\left(\nu - \frac{Q^2}{2m_i}\right) = \frac{1}{\nu} F_2(x)$$

SCALING: $x \equiv \frac{Q^2}{2M_p\,\nu}$

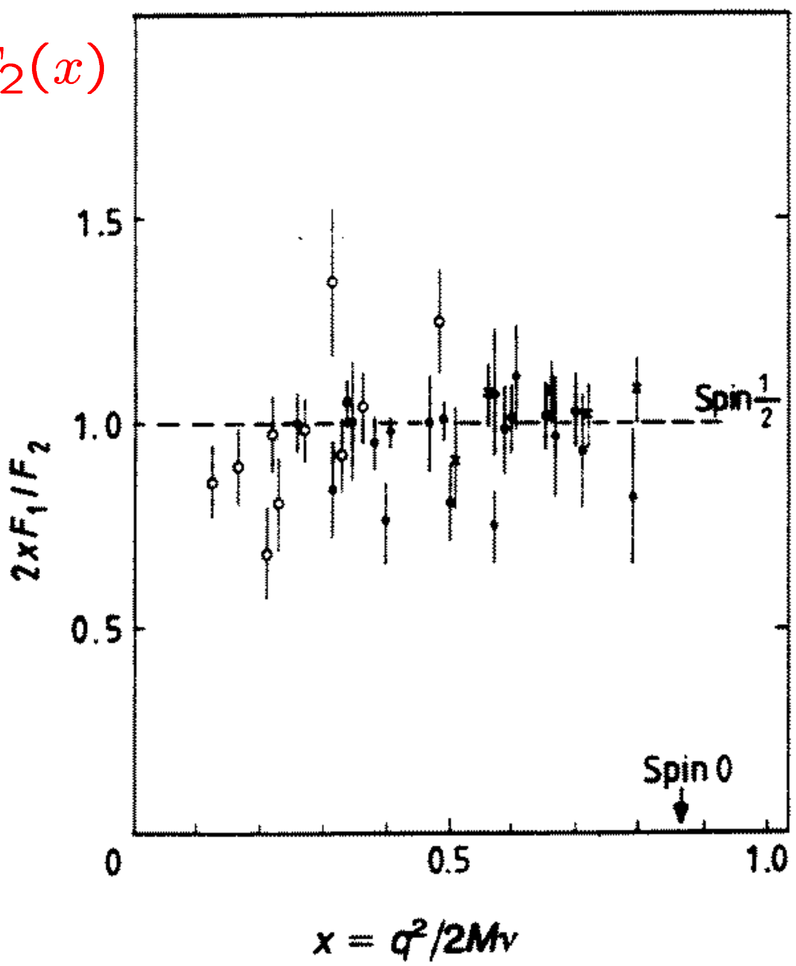
$$F_2(x) = 2\,x\ F_1(x) = x \textstyle\sum_i e_i^2\, f_i(x)$$

$F_2(x)$



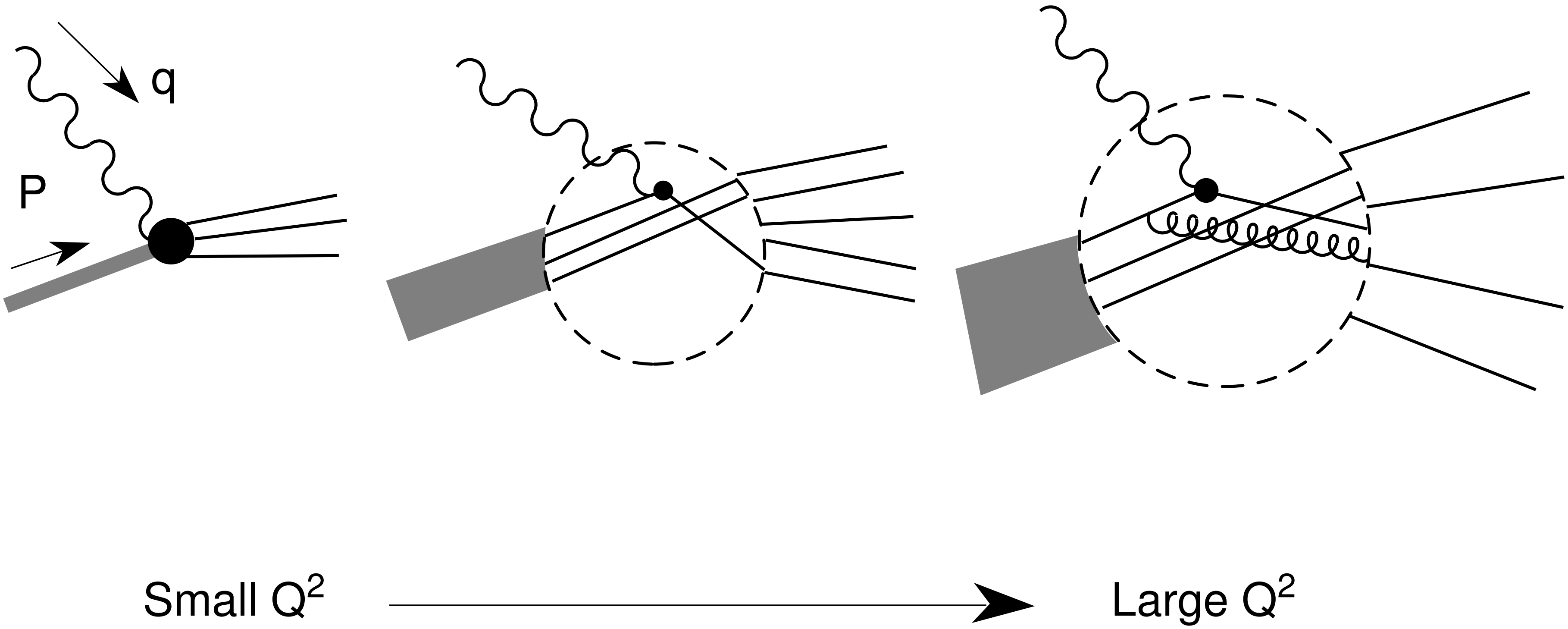
$\alpha_s \approx 0$

$2\,x\,F_1(x)/F_2(x)$

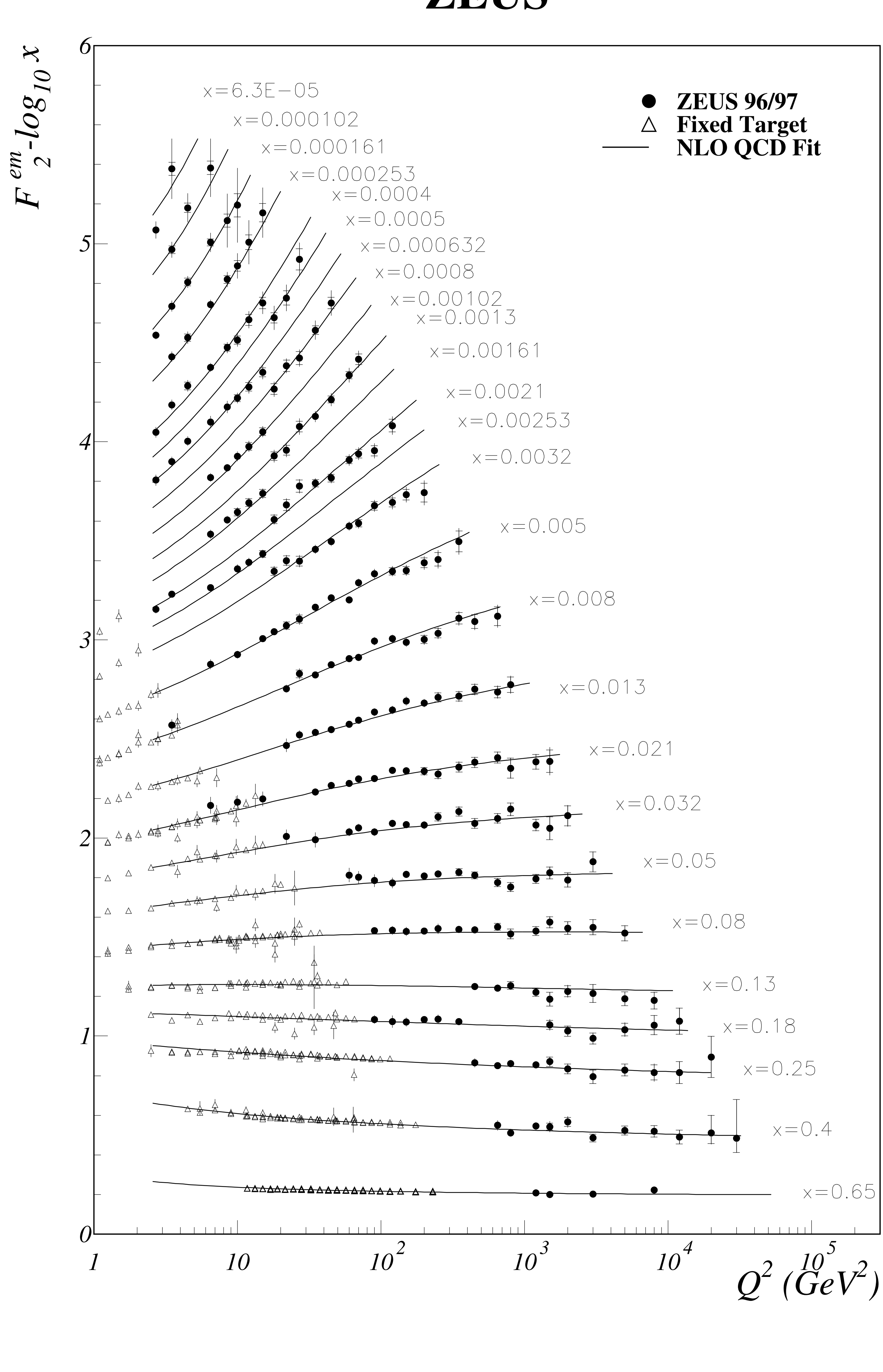


$J = \frac{1}{2}$

PROTON STRUCTURE WITHIN QCD



$O(\alpha_s)$ **Scaling Violations** → $F_i(x, Q^2)$



ZEUS
ZEUS 96/97
Fixed Target
NLO QCD Fit
F_2^{em}-$log_{10}x$
x=6.3E-05
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.00102
x=0.0013
x=0.00161
x=0.0021
x=0.00253
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
0
1
2
3
4
5
6
1
10
10^2
10^3
10^4
10^5
Q^2 (GeV^2)