4. Electroweak Unification

- Experimental Facts
- $SU(2) \otimes U(1)$ Gauge Theory
- Charged Current Interaction
- Neutral Current Interaction
- Gauge Self-Interactions

♠ Family Structure

$$\begin{bmatrix} \nu_l & q_u \\ l^- & q_d \end{bmatrix} \equiv \begin{cases} \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L , \quad (\nu_l)_R \quad , \quad l^-_R \\ \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L , \quad (q_u)_R \quad , \quad (q_d)_R \end{cases}$$

♠ Three Families

$$\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix} \quad , \quad \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix} \quad , \quad \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix}$$

♠ Charged Currents $W^\pm$

- Left-handed fermions only
- Flavour Changing: $\nu_l \Leftrightarrow l_l \quad , \quad q_u \Leftrightarrow q_d$

♠ Neutral Currents $\gamma \, , \, Z$

- Flavour Conserving

♠ Universality (Family–Independent Couplings)

♠ $(\nu_l)_R$?

$\mathbf{SU(2)_L \otimes U(1)_Y}$ GAUGE THEORY

Fields	$\psi_1(x)$	$\psi_2(x)$	$\psi_3(x)$
Quarks	$\begin{pmatrix} q_u \\ q_d \end{pmatrix}_L$	$(q_u)_R$	$(q_d)_R$
Leptons	$\begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L$	$(\nu_l)_R$	$(l^-)_R$

Free Lagrangian for Massless Fermions:

$$\mathcal{L}_0 \;=\; \sum_j \; i\; \bar{\psi}_j\, \gamma^\mu\, \partial_\mu \psi_j$$

$\mathbf{SU(2)_L \otimes U(1)_Y}$ **Flavour Symmetry:**

$$\psi_j \quad \longrightarrow \quad \exp\{i\,\vec{\tau}\,\vec{\alpha}/2\}\;\; \exp\left\{i\, y_j\, \beta\right\} \quad \psi_j$$

$$\bar{\psi}_j \quad \longrightarrow \quad \exp\{-i\,\vec{\tau}\,\vec{\alpha}/2\}\; \exp\left\{-i\, y_j\, \beta\right\} \quad \bar{\psi}_j$$

Gauge Principle: $\alpha \equiv \alpha(x)$, $\beta \equiv \beta(x)$

$$\mathbf{D}_\mu \, \psi_j(x) \equiv \left[\partial_\mu - i\, g\, \mathbf{W}_\mu(x) - i\, g'\, y_j\, B_\mu(x)\right] \psi_j(x)$$

$$\mathbf{D}_\mu \, \psi_j(x) \longrightarrow \mathbf{U}(x) \;\; \mathrm{exp}\left\{i\, y_j\, \beta(x)\right\} \;\; \mathbf{D}_\mu \, \psi_j(x)$$

$$B_\mu(x) \longrightarrow B_\mu(x) + \frac{1}{g'}\, \partial_\mu \beta(x)$$

$$\mathbf{W}_\mu(x) \longrightarrow \mathbf{U}(x)\, \mathbf{W}_\mu(x)\, \mathbf{U}^\dagger(x) + \frac{i}{g}\, \mathbf{U}(x)\, \partial_\mu \mathbf{U}^\dagger(x)$$

$$\mathbf{W}_\mu(x) \equiv \tfrac{\vec{\tau}}{2} \cdot \vec{W}_\mu(x) \quad ; \quad \mathbf{U}(x) \equiv \mathrm{exp}\left\{i\, \tfrac{\vec{\tau}}{2} \cdot \alpha(\vec{x})\right\}$$

$$W^i_\mu \longrightarrow W^i_\mu + \tfrac{1}{g}\, \partial_\mu \alpha^i - \epsilon^{ijk}\, \alpha^j\, W^k_\mu + O(\alpha^j \alpha^k)$$

4 Massless Gauge Bosons

$$W^\pm_\mu \quad , \quad W^3_\mu \quad , \quad B^0_\mu$$

CHARGED CURRENTS

$$\sum_j i\,\bar\psi_j\,\gamma^\mu\,\mathbf{D}_\mu\,\psi_j$$

$$\longrightarrow\; g\;\bar\psi_1\,\gamma^\mu\,\mathbf{W}_\mu\,\psi_1 \;+\; g'\,B_\mu\,\sum_j y_j\;\bar\psi_j\,\gamma^\mu\,\psi_j$$

$$\mathbf{W}_\mu \;\equiv\; \frac{\vec\tau}{2}\cdot\vec W_\mu \;=\; \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}\,W_\mu^\dagger \\ \sqrt{2}\,W_\mu & -W_\mu^3 \end{pmatrix}$$

$$W_\mu \;\equiv\; W_\mu^1 + i\,W_\mu^2$$

$$\mathcal{L}_{CC} \;=\; \frac{g}{2\sqrt{2}}\,W_\mu^\dagger\;\left[\bar q_u\,\gamma^\mu\,(1-\gamma_5)\,q_d + \bar\nu_l\,\gamma^\mu\,(1-\gamma_5)\,l\right] \;+\; \text{h.c.}$$

Quark / Lepton Universality

Left-Handed Interaction

NEUTRAL CURRENTS

$$\mathcal{L}_{NC} = \sum_j \bar{\psi}_j \gamma^\mu \left[g \frac{\tau_3}{2} W^3_\mu + g' y_j B_\mu \right] \psi_j$$

Massless Fields ➡ Arbitrary Combination

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

A_μ has the **QED** Interaction **IF**

$$g \sin\theta_W = g' \cos\theta_W = e \quad ; \quad \mathbf{Y} = \mathbf{Q} - \mathbf{T}_3$$

$$\left(y_1 = Q_u - \tfrac{1}{2} = Q_d + \tfrac{1}{2} \quad , \quad y_2 = Q_u \quad , \quad y_3 = Q_d \right)$$

⬇ **Electroweak Unification**

$$\mathcal{L}_{NC} = e A_\mu \sum_j \bar{\psi}_j \gamma^\mu \mathbf{Q}_j \psi_j + \mathcal{L}^Z_{NC}$$

$$\mathbf{Q}_1 = \begin{pmatrix} Q_u & 0 \\ 0 & Q_d \end{pmatrix} \quad ; \quad Q_2 = Q_u \quad ; \quad Q_3 = Q_d$$

$$\mathcal{L}_{NC}^{Z} = \frac{e}{\sin\theta_W \cos\theta_W} \; Z_\mu \; \sum_j \bar{\psi}_j \gamma^\mu \left[\frac{\tau_3}{2} - \sin^2\theta_W \, \mathbf{Q}_j\right] \psi_j$$

$$= \frac{e}{2\sin\theta_W \cos\theta_W} \; Z_\mu \; \sum_f \; \bar{f} \gamma^\mu \left[v_f - a_f \gamma_5\right] f$$

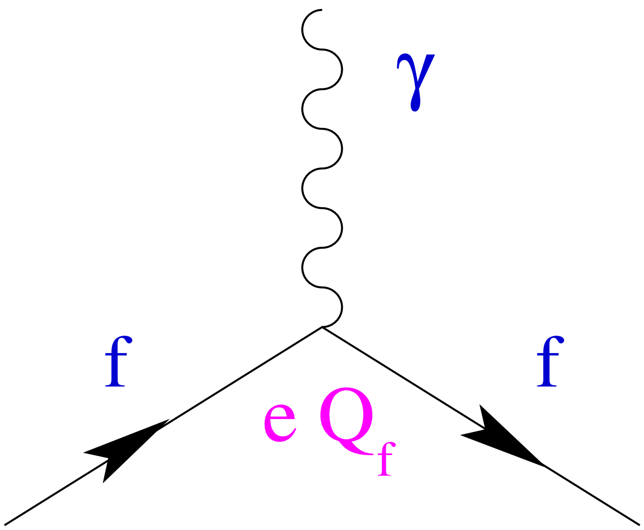
	$2\,v_f$	$2\,a_f$
q_u	$1 - \frac{8}{3}\sin^2\theta_W$	1
q_d	$-1 + \frac{4}{3}\sin^2\theta_W$	-1
ν_l	1	1
l^-	$-1 + 4\sin^2\theta_W$	-1

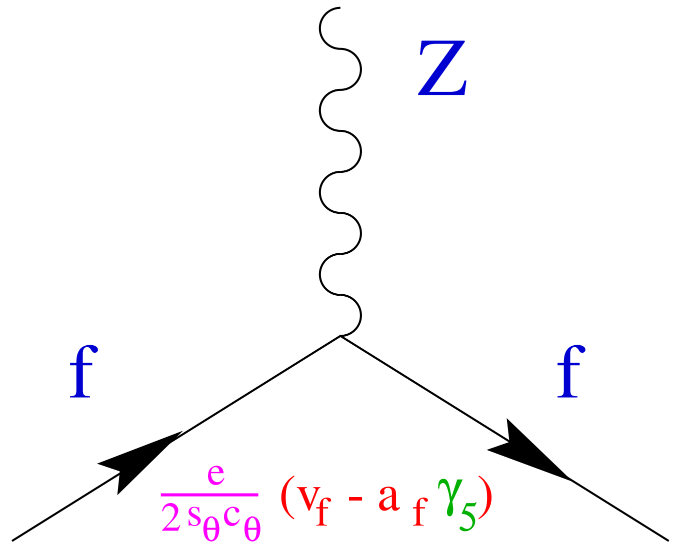
♠ **IF ν_{R} do exist :**

$y(\nu_R) = Q_\nu = 0$ ➡ No ν_R Interactions

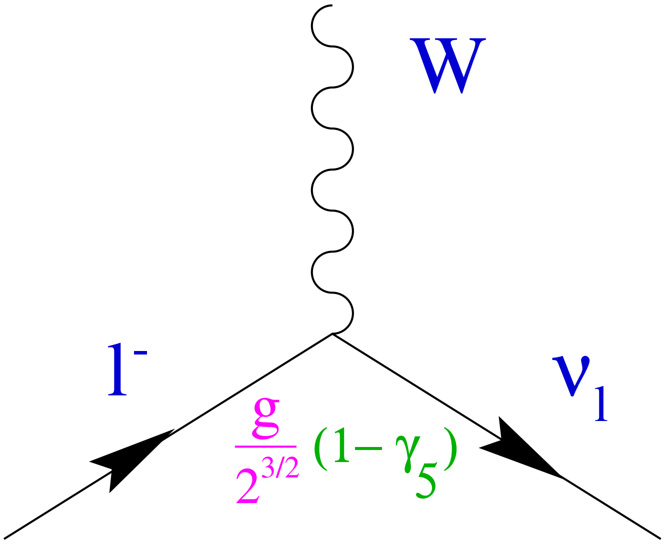
Sterile Neutrinos

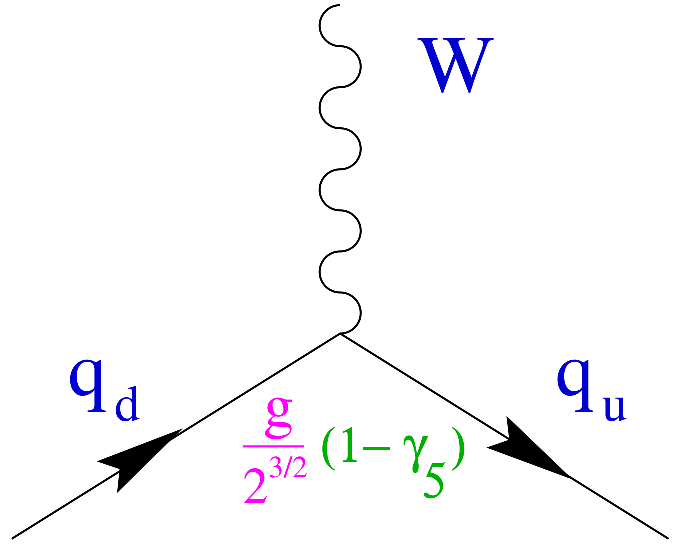
NEUTRAL CURRENTS





CHARGED CURRENTS





$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\, \epsilon^{ijk}\, W^j_\mu\, W^k_\nu$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \longrightarrow B_{\mu\nu}$$

$$\mathcal{L}_K = -\frac{1}{4} B_{\mu\nu}\, B^{\mu\nu} - \frac{1}{2} \mathrm{Tr}\,(\mathbf{W}_{\mu\nu}\, \mathbf{W}^{\mu\nu})$$

$$= -\frac{1}{4} B_{\mu\nu}\, B^{\mu\nu} - \frac{1}{4} \vec{W}_{\mu\nu}\, \vec{W}_{\mu\nu}$$

$$= \mathcal{L}_3 + \mathcal{L}_4$$

$$\mathcal{L}_3 = -i\, e\, \cot\theta_W\, \left\{ (\partial^\mu W^\nu - \partial^\nu W^\mu)\, W^\dagger_\mu\, Z_\nu - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}\right) W_\mu\, Z_\nu + W_\mu\, W^\dagger_\nu\, (\partial^\mu Z^\nu - \partial^\nu Z^\mu) \right\}$$

$$-i\, e\, \left\{ (\partial^\mu W^\nu - \partial^\nu W^\mu)\, W^\dagger_\mu\, A_\nu - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}\right) W_\mu\, A_\nu + W_\mu\, W^\dagger_\nu\, (\partial^\mu A^\nu - \partial^\nu A^\mu) \right\}$$

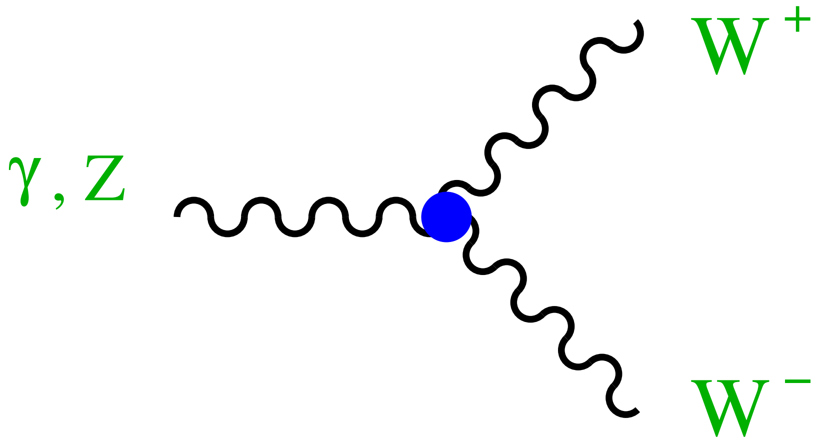
$$\mathcal{L}_4 = -\frac{e^2}{2\sin^2\theta_W} \left\{ \left(W^\dagger_\mu W^\mu\right)^2 - W^\dagger_\mu\, W^{\mu\dagger}\, W_\nu\, W^\nu \right\}$$

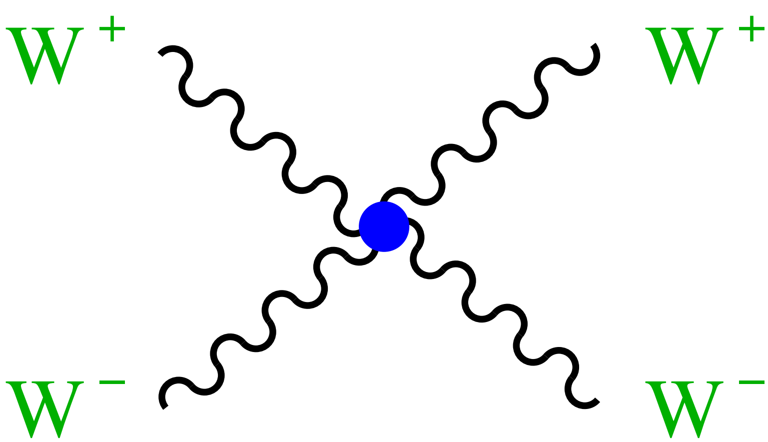
$$- e^2 \cot^2\theta_W\, \left\{ W^\dagger_\mu\, W^\mu\, Z_\nu\, Z^\nu - W^\dagger_\mu\, Z^\mu\, W_\nu\, Z^\nu \right\}$$

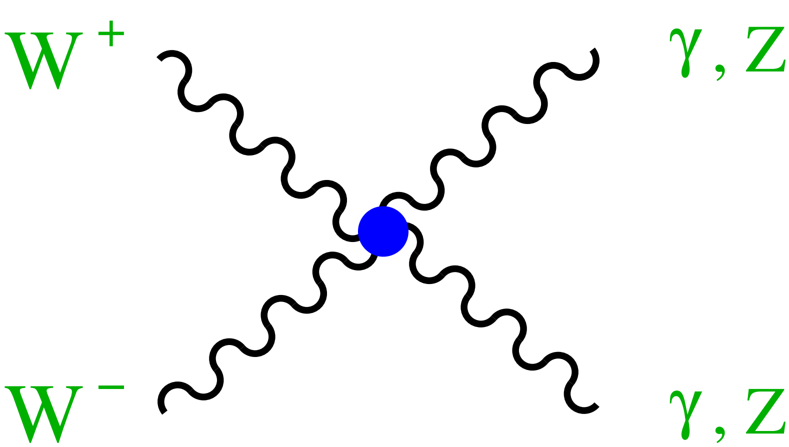
$$- e^2 \cot\theta_W\, \left\{ 2\, W^\dagger_\mu\, W^\mu\, Z_\nu\, A^\nu - W^\dagger_\mu\, Z^\mu\, W_\nu\, A^\nu - W^\dagger_\mu\, A^\mu\, W_\nu\, Z^\nu \right\}$$

$$- e^2\, \left\{ W^\dagger_\mu\, W^\mu\, A_\nu\, A^\nu - W^\dagger_\mu\, A^\mu\, W_\nu\, A^\nu \right\}$$

GAUGE SELF-INTERACTIONS







Gauge Symmetry

→ $m_\gamma = 0$ Good

$M_W = M_Z = 0$ Bad!

$M_W = 80.45$ GeV

$M_Z = 91.19$ GeV

Moreover

$$\mathcal{L}_{m_f} \equiv -m_f \; \bar{f}\, f = -m_f \left(\bar{f}_L\, f_R + \bar{f}_R\, f_L \right)$$

Also Forbidden by Gauge Symmetry

→ $m_f = 0$ $\forall\, f$

All Particles Massless