5. Symmetry Breaking

- Scalar Potential
- Ground State Symmetry
- Higgs Mechanism
- The Higgs Boson
- Fermion Masses
- Fermion Generations
- Quark Mixing
- Lepton Mixing
- Standard Model Parameters

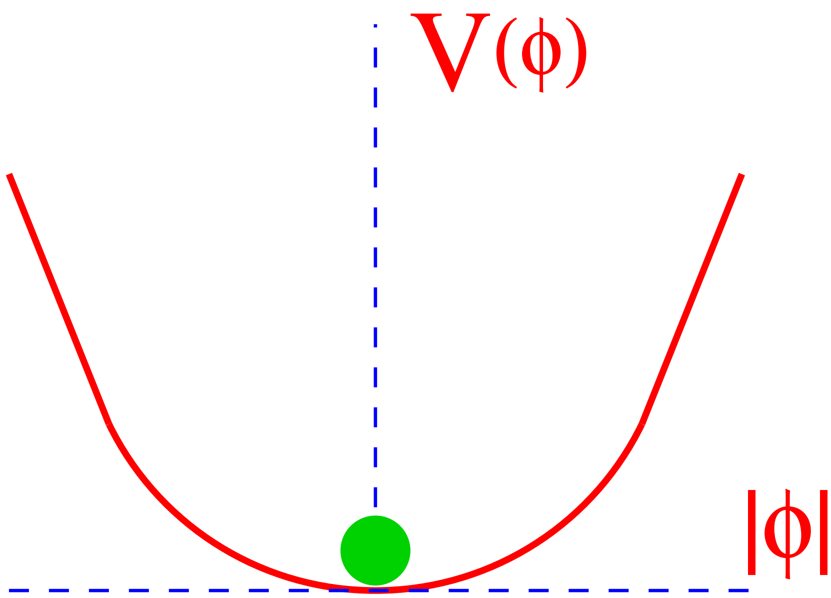
$$\mathcal{L}(\phi) = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi)$$

$$V(\phi) = \mu^2 \phi^\dagger \phi + h \left(\phi^\dagger \phi\right)^2$$

Phase Symmetry: $\phi(x) \longrightarrow e^{i\theta} \phi(x)$

$\mu^2 > 0$

$M_\phi = \mu$



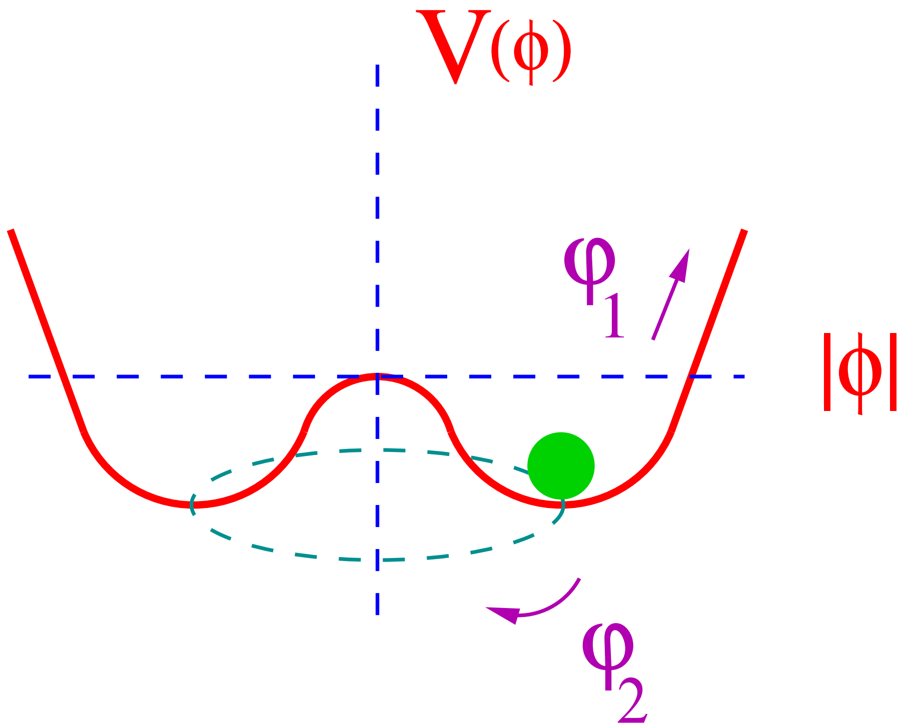
Trivial Minimum (Ground State / Vacuum):

$$\phi = \phi_0 = 0$$

$$V(\phi) = \mu^2\, \phi^\dagger \phi + h\, (\phi^\dagger \phi)$$

Phase Symmetry: $\phi(x) \longrightarrow e^{i\theta}\, \phi(x)$

$\mu^2 < 0$



Degenerate Minima (Ground State / Vacuum):

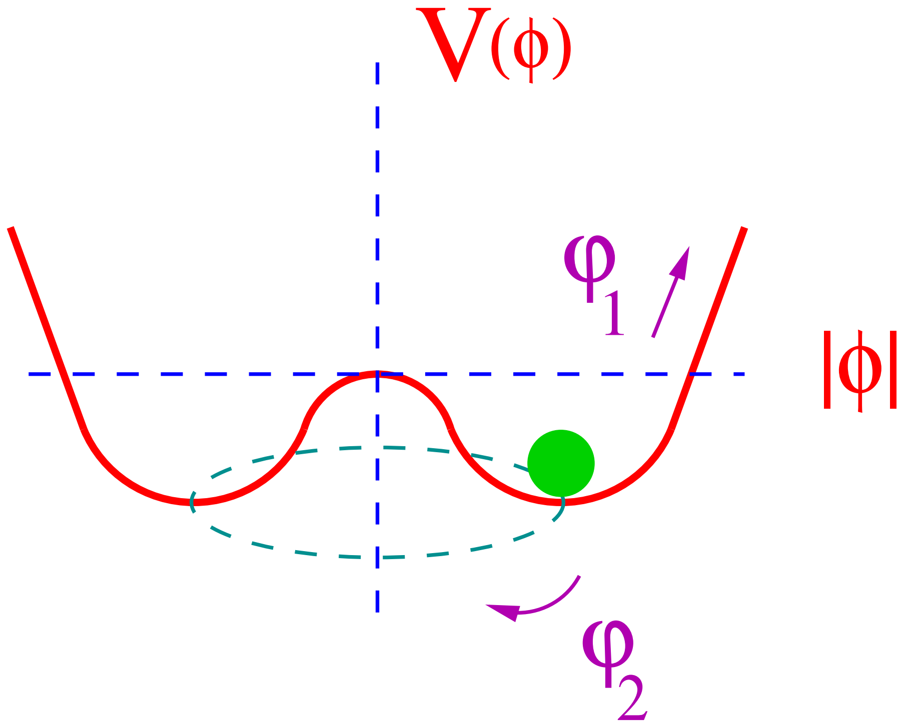
$$|\phi_0| = \sqrt{\frac{-\mu^2}{2\,h}} \equiv \frac{v}{\sqrt{2}} > 0 \qquad ; \qquad V(\phi_0) = -\frac{1}{4}\, h\, v^4$$

Spontaneous Symmetry Breaking:

$$\phi \equiv \frac{1}{\sqrt{2}}\, [v + \varphi_1(x)]\;\; e^{i\,\varphi_2(x)/v}$$

Vacuum Choice

$\mu^2 < 0$



$$\phi \equiv \frac{1}{\sqrt{2}}\left[v + \varphi_1(x)\right] \; e^{i\,\varphi_2(x)/v}$$

$$\mathcal{L}(\phi) = \frac{1}{2}\,\partial_\mu\varphi_1\,\partial^\mu\varphi_1 + \frac{1}{2}\left(1+\frac{\varphi_1}{v}\right)^2 \partial_\mu\varphi_2\,\partial^\mu\varphi_2 - V(\phi)$$

$$V(\phi) = V(\phi_0) + \frac{1}{2}\,M_{\varphi_1}^2\,\varphi_1^2 + h\,v\,\varphi_1^3 + \frac{1}{4}\,h\,\varphi_1^4$$

$$M_{\varphi_1}^2 = -2\,\mu^2 > 0 \qquad ; \qquad M_{\varphi_2}^2 = 0$$

1 Massless Goldstone Boson

ELECTROWEAK SSB

New Scalar Doublet

$$\phi(x) \equiv \begin{pmatrix} \phi^{(+)}(x) \\ \phi^{(0)}(x) \end{pmatrix} \qquad ; \qquad y_\phi = \frac{1}{2}$$

$$\mathcal{L}(\phi) = (\mathbf{D}_\mu \phi)^\dagger \, \mathbf{D}^\mu \phi - \mu^2 \, \phi^\dagger \phi - h \, \left(\phi^\dagger \phi\right)^2$$

$$\mathbf{D}^\mu \phi = \left[\partial^\mu - i \, g \, \mathbf{W}^\mu - i \, g' \, y_\phi \, B^\mu\right] \phi \quad ; \quad \mathbf{W}^\mu = \frac{\vec{\tau}}{2} \cdot \vec{W}^\mu$$

$\mathrm{SU(2)_L \otimes U(1)_Y}$ Symmetry

Degenerate Vacuum States: $\mu^2 < 0 \; , \; h > 0$

$$|\langle 0 | \phi_0 | 0 \rangle| = \sqrt{\frac{-\mu^2}{2\,h}} \equiv \frac{v}{\sqrt{2}}$$

Spontaneous Symmetry Breaking:

$$\phi(x) = \exp\left\{i \, \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x)\right\} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

HIGGS MECHANISM

$$\phi(x) = \exp\left\{i\,\frac{\vec{\tau}}{2}\cdot\vec{\theta}(x)\right\}\quad\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H(x)\end{pmatrix}$$

$SU(2)_L$ Invariance ➡ $\vec{\theta}(x)$ Unphysical

Unitary Gauge: $\vec{\theta}(x) = 0$

$$(\mathbf{D}_\mu\,\phi)^\dagger\,\mathbf{D}^\mu\phi \quad\longrightarrow\quad \frac{1}{2}\,\partial_\mu H\,\partial^\mu H$$

$$+\ \frac{g^2}{4}\,(v+H)^2\left\{W_\mu^\dagger\,W^\mu + \frac{1}{2\cos^2\theta_W}\,Z_\mu\,Z^\mu\right\}$$

➡ $$M_Z\ \cos\theta_W = M_W = \tfrac{1}{2}\,v\,g$$

Massive Gauge Bosons

Bosonic Degrees of Freedom

Massless $W^{\pm}\,,\ Z$

3×2 helicities $=$ **6**

$+$

3 Goldstones $\vec{\theta}$

$\downarrow$ SSB

Massive $W^{\pm}\,,\ Z$

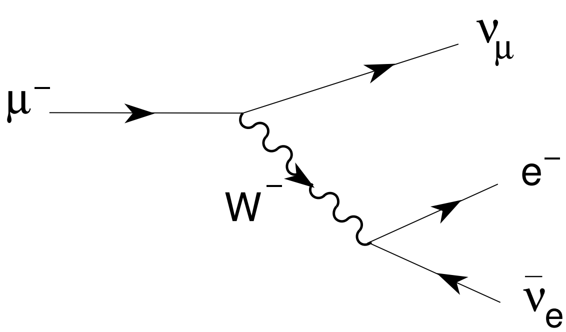
3×3 helicities $=$ **9**

SAME PHYSICS

$$M_Z \cos\theta_W = M_W = \frac{1}{2} v\, g$$

$$M_Z = 91.187\,5 \text{ GeV} \quad > \quad M_W = 80.451 \text{ GeV}$$

$$\Rightarrow \qquad \sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.222$$



$$\frac{g^2}{M_W^2 - q^2} \approx \frac{g^2}{M_W^2} \equiv 4\sqrt{2}\, G_F$$

$$\frac{1}{\tau_\mu} \equiv \Gamma = \frac{G_F^2\, m_\mu^5}{192\,\pi^3}$$

$$G_F = 1.166\,39 \times 10^{-5} \text{ GeV}^{-2}$$

$$g = \frac{e}{\sin\theta_W} \quad , \quad M_W$$

$$\Rightarrow \qquad \sin^2\theta_W = 0.215$$

$$v = \left(\sqrt{2}\, G_F\right)^{-1/2} = 246 \text{ GeV}$$

THE HIGGS BOSON

$$\mathcal{L}_S = \frac{h\,v^4}{4} + \mathcal{L}_H + \mathcal{L}_{HG^2}$$

$$\mathcal{L}_H = \tfrac{1}{2}\,\partial_\mu H\,\partial^\mu H - \tfrac{1}{2}\,M_H^2\,H^2 - \frac{M_H^2}{2\,v}\,H^3 - \frac{M_H^2}{8\,v^2}\,H^4$$

$$\begin{aligned}\mathcal{L}_{HG^2} &= M_W^2\,W_\mu^\dagger W^\mu \left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\} \\ &+ \frac{1}{2}M_Z^2\,Z_\mu Z^\mu \left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\}\end{aligned}$$

1 Scalar Particle $\mathbf{H^0}$ to be Discovered

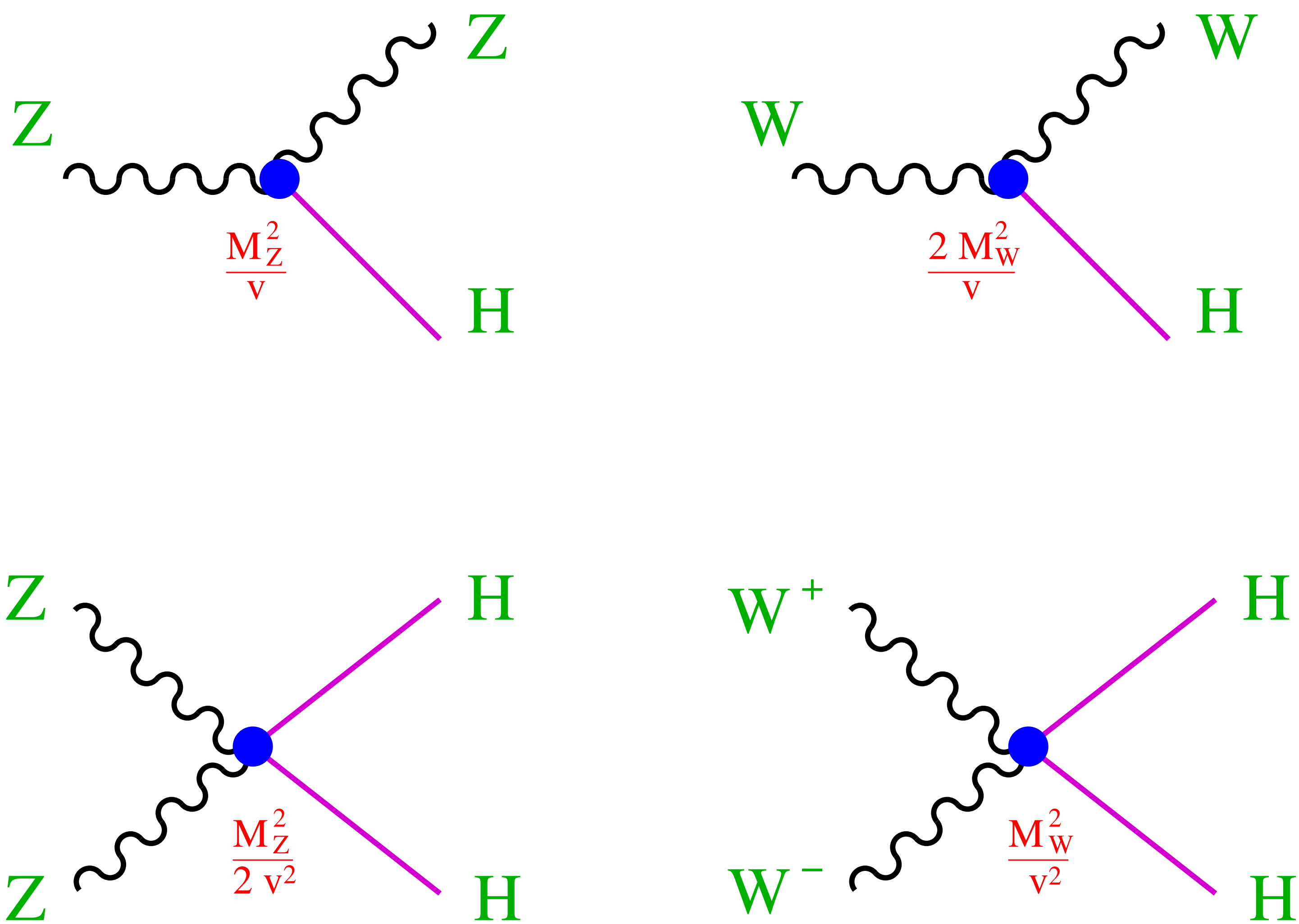
$M_H = \sqrt{-2\,\mu^2} = \sqrt{2\,h}\,v$ **Free Parameter**

LEP: (95% CL)

$$114 \text{ GeV} < M_H < 196 \text{ GeV}$$

(Direct) (Indirect)

Higgs Couplings $\propto$ Masses



$$v = \left(\sqrt{2}\,G_F\right)^{-1/2} = 246 \text{ GeV}$$

FERMION MASSES

Scalar–Fermion Couplings allowed by Gauge Symmetry

$$\mathcal{L}_Y = (\bar{q}_u, \bar{q}_d)_L \left[c^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} (q_u)_R \right]$$

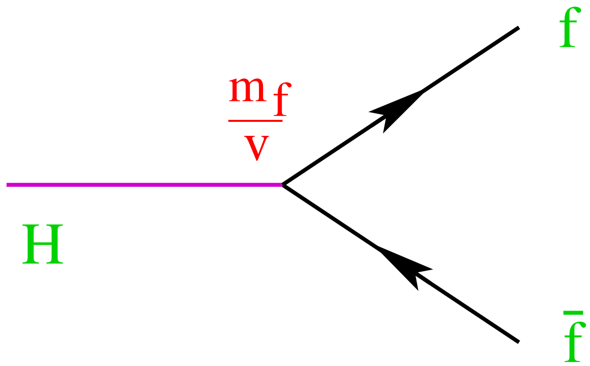
$$+ \; (\bar{\nu}_l, \bar{l})_L \; c^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l_R \quad + \quad \text{h.c.}$$

⬇ SSB

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) \left\{ m_{q_d} \; \bar{q}_d \, q_d + m_{q_u} \; \bar{q}_u \, q_u + m_l \; \bar{l} \, l \right\}$$

Fermion Masses are New Free Parameters

$$\left[m_{q_d} \, , \, m_{q_u} \, , \, m_l \right] = - \left[c^{(d)} \, , \, c^{(u)} \, , \, c^{(l)} \right] \; \frac{v}{\sqrt{2}}$$



Couplings Fixed

$$g_{Hf\bar{f}} = \frac{m_f}{v}$$

$\mathbf{N_G = 3}$ **Identical Copies** **WHY ?**

$$\begin{array}{l} Q=0 \\ Q=-1 \end{array} \qquad \begin{pmatrix} \nu'_j & | & u'_j \\ l'_j & | & d'_j \end{pmatrix} \qquad \begin{array}{l} Q=+\frac{2}{3} \\ Q=-\frac{1}{3} \end{array} \qquad (j=1,\cdots,N_G)$$

Masses are the only difference

$$\mathcal{L}_Y = \sum_{jk} \left\{ \left(\bar{u}'_j, \bar{d}'_j\right)_L \left[c^{(d)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c^{(u)}_{jk} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] + \left(\bar{\nu}'_j, \bar{l}'_j\right)_L c^{(l)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l'_{kR} \right\} + \text{h.c.}$$

$\Downarrow$ SSB

$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right) \left\{ \bar{\mathbf{d}}'_L \, \mathbf{M}'_d \, \mathbf{d}'_R + \bar{\mathbf{u}}'_L \, \mathbf{M}'_u \, \mathbf{u}'_R + \bar{\mathbf{l}}'_L \, \mathbf{M}'_l \, \mathbf{l}'_R + \text{h.c.} \right\}$$

Arbitrary Non-Diagonal Complex Mass Matrices

$$\left[\mathbf{M}'_d, \mathbf{M}'_u, \mathbf{M}'_l\right]_{ij} = -\left[c^{(d)}_{ij}, c^{(u)}_{ij}, c^{(l)}_{ij}\right] \frac{v}{\sqrt{2}}$$

$$\mathbf{M}'_d = \mathbf{H}_d\,\mathbf{U}_d = \mathbf{S}_d^\dagger\,\mathcal{M}_d\,\mathbf{S}_d\,\mathbf{U}_d \qquad \mathbf{H}_f = \mathbf{H}_f^\dagger$$

$$\mathbf{M}'_u = \mathbf{H}_u\,\mathbf{U}_u = \mathbf{S}_u^\dagger\,\mathcal{M}_u\,\mathbf{S}_u\,\mathbf{U}_u \qquad \mathbf{U}_f\,\mathbf{U}_f^\dagger = 1$$

$$\mathbf{M}'_l = \mathbf{H}_l\,\mathbf{U}_l = \mathbf{S}_l^\dagger\,\mathcal{M}_l\,\mathbf{S}_l\,\mathbf{U}_l \qquad \mathbf{S}_f\,\mathbf{S}_f^\dagger = 1$$

$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right)\left\{\bar{\mathbf{d}}\,\mathcal{M}_d\,\mathbf{d} + \bar{\mathbf{u}}\,\mathcal{M}_u\,\mathbf{u} + \bar{\mathbf{l}}\,\mathcal{M}_l\,\mathbf{l}\right\}$$

$$\mathcal{M}_d = \mathrm{diag}\,(m_d, m_s, m_b) \quad , \quad \mathcal{M}_u = \mathrm{diag}\,(m_u, m_c, m_t)$$

$$\mathcal{M}_l = \mathrm{diag}\,(m_e, m_\mu, m_\tau)$$

Mass Eigenstates $\neq$ Weak Eigenstates

$$\mathbf{d}_L \equiv \mathbf{S}_d\,\mathbf{d}'_L \quad , \quad \mathbf{u}_L \equiv \mathbf{S}_u\,\mathbf{u}'_L \quad , \quad \mathbf{l}_L \equiv \mathbf{S}_l\,\mathbf{l}'_L$$

$$\mathbf{d}_R \equiv \mathbf{S}_d\,\mathbf{U}_d\,\mathbf{d}'_R \quad , \quad \mathbf{u}_R \equiv \mathbf{S}_u\,\mathbf{U}_u\,\mathbf{u}'_R \quad , \quad \mathbf{l}_R \equiv \mathbf{S}_l\,\mathbf{U}_l\,\mathbf{l}'_R$$

$$\bar{\mathbf{f}}'_L\,\mathbf{f}'_L = \bar{\mathbf{f}}_L\,\mathbf{f}_L \;,\; \bar{\mathbf{f}}'_R\,\mathbf{f}'_R = \bar{\mathbf{f}}_R\,\mathbf{f}_R \quad \longrightarrow \quad \mathcal{L}'_{NC} = \mathcal{L}_{NC}$$

$$\bar{\mathbf{u}}'_L\,\mathbf{d}'_L = \bar{\mathbf{u}}_L\,\mathbf{V}\,\mathbf{d}_L \quad \longrightarrow \quad \mathcal{L}'_{CC} \neq \mathcal{L}_{CC}$$

$$\mathbf{V} \equiv \mathbf{S}_u\,\mathbf{S}_d^\dagger$$

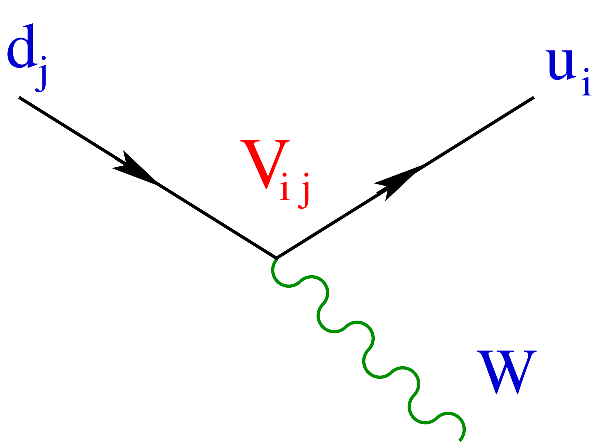
QUARK MIXING

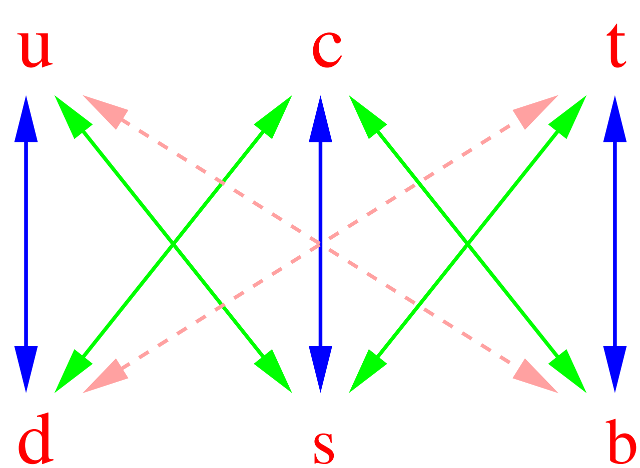
$$\mathcal{L}_{NC}^{Z} = \frac{e}{2\sin\theta_W\cos\theta_W} \; Z_\mu \sum_f \bar{f}\,\gamma^\mu \left[v_f - a_f\,\gamma_5\right] f$$

Flavour Conserving Neutral Currents

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} W_\mu^\dagger \left[\sum_{ij} \bar{u}_i\,\gamma^\mu\,(1-\gamma_5)\,\mathbf{V}_{ij}\,d_j + \sum_l \bar{\nu}_l\,\gamma^\mu\,(1-\gamma_5)\,l \right] + \text{h.c.}$$

Flavour Changing Charged Currents





QUARK MIXING MATRIX

♠ Unitary $N_G \times N_G$ Matrix: N_G^2 parameters

♠ $2\,N_G - 1$ arbitrary phases:

$$\left. \begin{array}{l} u_i \;\to\; e^{i\phi_i}\, u_i \\ d_j \;\to\; e^{i\theta_j}\, d_j \end{array} \right\} \quad \longrightarrow \quad \mathbf{V}_{ij} \;\to\; e^{i(\theta_j - \phi_i)}\, \mathbf{V}_{ij}$$

$$\Downarrow$$

$\mathbf{V}_{ij}$ Physical Parameters

$\frac{1}{2}\, N_G\, (N_G - 1)$	moduli
$\frac{1}{2}\, (N_G - 1)\, (N_G - 2)$	phases

Unitarity: $V \cdot V^\dagger = V^\dagger \cdot V = 1$

$\mathbf{N_f = 2}$ **:** 1 angle, 0 phases

$$V = \begin{bmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{bmatrix} \longrightarrow \text{No } \not{\mathcal{CP}}$$

$\mathbf{N_f = 3}$ **:** (CKM) 3 angles, 1 phase

$$\begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & c_{23}\, c_{13} \end{bmatrix}$$

$$\approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\,\lambda^3\,(\rho - i\,\eta) \\ -\lambda & 1-\lambda^2/2 & A\,\lambda^2 \\ A\,\lambda^3\,(1-\rho-i\,\eta) & -A\,\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4)$$

$$c_{ij} \equiv \cos\theta_{ij} \quad ; \quad s_{ij} \equiv \sin\theta_{ij} \qquad (i\,,\, j = 1\,2\,3)$$

$$\lambda \approx \sin\theta_c \approx 0.223 \quad ; \quad A \approx 0.83 \quad ; \quad \sqrt{\rho^2+\eta^2} \approx 0.40$$

$$\delta_{13} \neq 0 \quad (\eta \neq 0) \longrightarrow \not{\mathcal{CP}}$$

$$\mathcal{L}_{CC}^{(l)} = \frac{g}{2\sqrt{2}} W_\mu^\dagger \sum_{i,j} \bar{\nu}_i \gamma^\mu (1-\gamma_5) \mathbf{V}_{ij}^{(l)} l_j + \text{h.c.}$$

♠ **IF** $\mathbf{m_{\nu_i} = 0}$, $\bar{\nu}_{l_j} \equiv \bar{\nu}_i \mathbf{V}_{ij}^{(l)}$

➡ $$\mathcal{L}_{CC}^{(l)} = \frac{g}{2\sqrt{2}} W_\mu^\dagger \sum_l \bar{\nu}_l \gamma^\mu (1-\gamma_5) l + \text{h.c.}$$

Separate Lepton Number Conservation

(True in Minimal SM without ν_R)

♠ **IF** $\mathbf{\nu_R^i}$ **exist and** $\mathbf{m_{\nu_i} \neq 0}$

$\not{L}_e$, $\not{L}_\mu$, $\not{L}_\tau$ ($L_e + L_\mu + L_\tau$ Conserved)

BUT

$B(\mu \to e\gamma) < 1.2 \times 10^{-11}$ (90% CL)

$B(\tau \to \mu\gamma) < 1.1 \times 10^{-6}$ (90% CL)

♠ **QCD:** $\alpha_s(M_Z)$ **1**

♠ **EW Gauge / Scalar Sector:** **4**

$$g, g', \mu^2, h \quad \Leftrightarrow \quad \alpha, \theta_W, M_W, M_H \quad \Leftrightarrow \quad \alpha, G_F, M_Z, M_H$$

♠ **Yukawa Sector:** **13**

$$m_e \; , \; m_\mu \; , \; m_\tau$$

$$m_d \; , \; m_s \; , \; m_b$$

$$m_u \; , \; m_c \; , \; m_t$$

$$\theta_1 \; , \; \theta_2 \; , \; \theta_3 \; , \; \delta$$

➡ **18 Free Parameters**

(+ Neutrino Masses / Mixings ?)

TOO MANY !