6. Electroweak Phenomenology

- Inputs
- $Z \to f\,\bar{f}$, $W \to f_1\,\bar{f}_2$
- Z Peak Asymmetries
- Sensitivity to Higher Scales
- Standard Model Fits: M_H
- $e^+e^- \to W^+W^-$, $e^+e^- \to Z\,Z$
- Higgs Search
- Quark Mixing
- CP Violation

INPUTS

$$G_F = (1.166\,39 \pm 0.000\,01) \times 10^{-5} \ \text{GeV}^{-2}$$

$$\alpha^{-1} = 137.035\,999\,76 \pm 0.000\,000\,50$$

$$M_Z = (91.187\,5 \pm 0.002\,1) \ \text{GeV}$$

$$\alpha^{-1}(M_Z^2) = 128.95 \pm 0.05$$

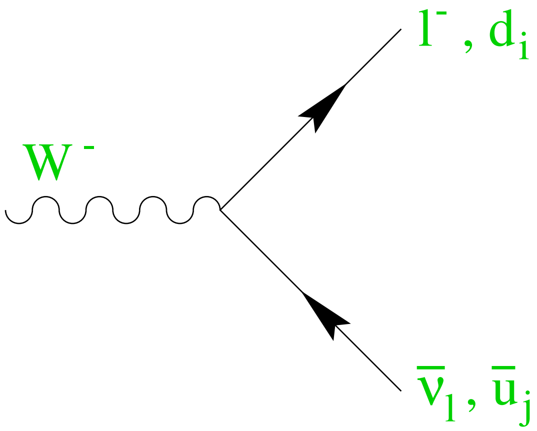
$$M_W^2 \sin^2\theta_W = \frac{\pi\,\alpha}{\sqrt{2}\,G_F}$$

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

$M_W = 80.94$ GeV [Exp: 80.451 ± 0.033]

(79.96)

$\sin^2\theta_W = 0.212$ (0.231)



$$\bar{u}_j = \bar{u}\ ,\ \bar{c}$$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$\mathrm{Br}(W^- \to l^- \bar{\nu}_l) \equiv \frac{\Gamma(W^- \to l^- \bar{\nu}_l)}{\Gamma(W^- \to \text{all})} = \frac{1}{3 + 2\,N_C} = 11.1\%$$

QCD: $$N_C \left\{1 + \frac{\alpha_s(M_Z)}{\pi}\right\} \approx 3.115$$

$\Rightarrow$ $$\mathrm{Br}(W^- \to l^- \bar{\nu}_l) \approx 10.8\%$$

Experiment:

$$\mathrm{Br}(W^- \to e^- \bar{\nu}_e) = (10.54 \pm 0.17)\%$$

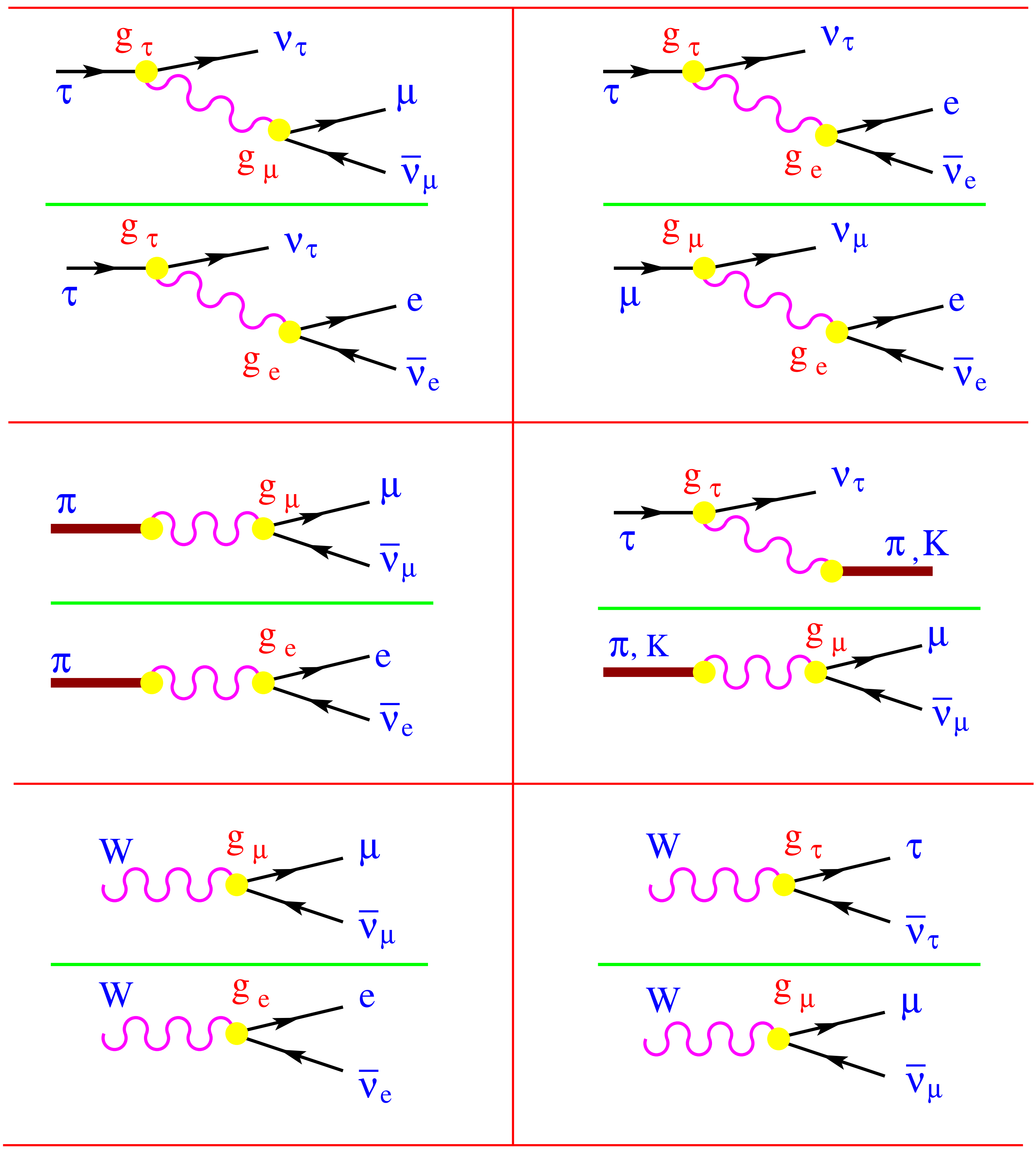
$$\mathrm{Br}(W^- \to \mu^- \bar{\nu}_\mu) = (10.54 \pm 0.16)\%$$

$$\mathrm{Br}(W^- \to \tau^- \bar{\nu}_\tau) = (11.09 \pm 0.22)\%$$

Universal $\mathrm{W}\,\mathrm{l}\,\bar{\nu}_\mathrm{l}$ Couplings

LEPTON UNIVERSALITY

$g_\mu \,/\, g_e$ $\qquad$ $g_\tau \,/\, g_\mu$



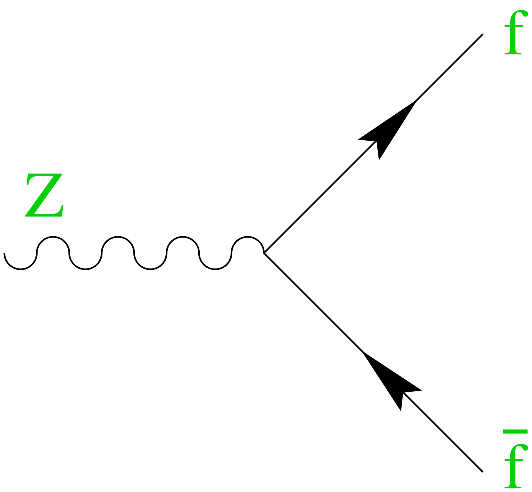
CHARGED–CURRENT UNIVERSALITY

	$\lvert g_\mu / g_e \rvert$
$B_{\tau\to\mu}/B_{\tau\to e}$	1.0006 ± 0.0021
$B_{\pi\to e}/B_{\pi\to\mu}$	1.0017 ± 0.0015
$B_{W\to\mu/e}$	1.000 ± 0.011

	$\lvert g_\tau / g_\mu \rvert$
$B_{\tau\to e}\ \tau_\mu/\tau_\tau$	0.9995 ± 0.0023
$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	1.005 ± 0.007
$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$	0.977 ± 0.016
$B_{W\to\tau/\mu}$	1.026 ± 0.014

	$\lvert g_\tau / g_e \rvert$
$B_{\tau\to\mu}\ \tau_\mu/\tau_\tau$	1.0001 ± 0.0023
$B_{W\to\tau/e}$	1.026 ± 0.014

$Z \to l^+ l^-, \ \nu_l \bar{\nu}_l$



$$\Gamma(Z \to l\,\bar{l}) \ \propto \ \left(|v_l|^2 + |a_l|^2\right)$$

$$\frac{\Gamma_{\text{inv}}}{\Gamma_{ll}} \equiv \frac{\Gamma(Z \to \ \text{invisible})}{\Gamma(Z \to l^+ l^-)} = N_\nu \ \frac{\Gamma(Z \to \ \nu_l \bar{\nu}_l)}{\Gamma(Z \to l^+ l^-)}$$

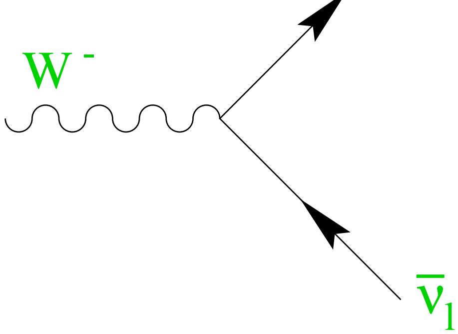
$$= \ N_\nu \ \frac{2}{(1 - 4\sin^2\theta_W)^2 + 1} = 1.955 \ N_\nu$$

(1.989)

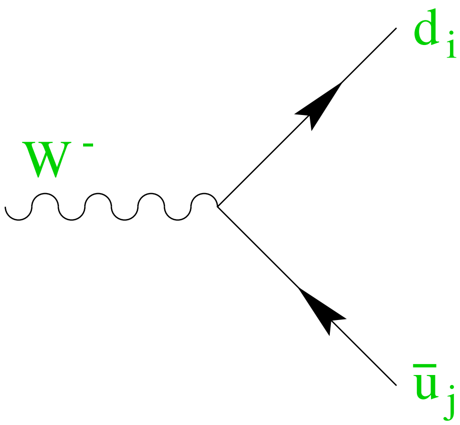
Experiment: $\frac{\Gamma_{\text{inv}}}{\Gamma_{ll}} = 5.942 \pm 0.016$

➡ $N_\nu = 3.04$ (2.99)

$N_\nu = 2.9841 \pm 0.0083$

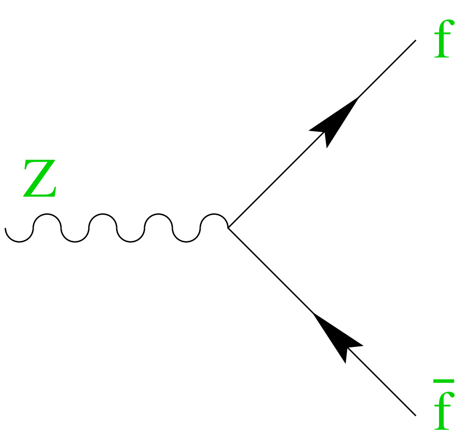


$$\Gamma = \frac{G_F M_W^3}{6\pi\sqrt{2}}$$



$$W^- \to d' \bar{u} \, , \; s' \bar{c}$$

$$\Gamma = \frac{G_F M_W^3}{6\pi\sqrt{2}} \; \left|\mathbf{V}_{ij}\right|^2 N_C$$



$$Z \to l^- l^+ \, , \; \nu_i \bar{\nu}_i \, , \; q \bar{q}$$

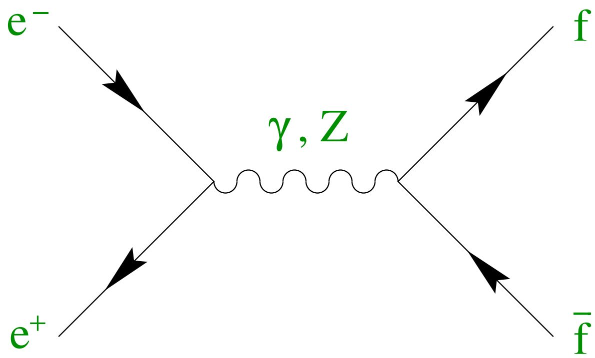
(q=u,d,s,c,b)

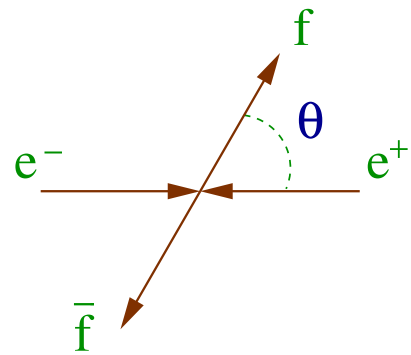
$$\Gamma = \frac{G_F M_Z^3}{6\pi\sqrt{2}} \left(|v_f|^2 + |a_f|^2\right) N_f$$

$$N_l = 1 \quad , \quad N_q = N_C$$

➡	$\Gamma_W = 2.09$ GeV ,	$\Gamma_Z = 2.48$ GeV
Exp:	2.12 (5)	2.495 (2)

$e^+e^- \to \gamma, Z \to f\bar{f}$





$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8\,s}\; N_f \;\Big\{ A\,\left(1+\cos^2\theta\right) + B\,\cos\theta - h_f\left[C\,\left(1+\cos^2\theta\right) + D\,\cos\theta\right]\Big\}$$

$$N_l = 1 \quad , \quad N_q = N_C\,\left\{1 + \frac{\alpha_s(M_Z^2)}{\pi} + \ldots\right\}$$

$$A = 1 + 2\,v_e\,v_f\,\mathrm{Re}(\chi) + \left(v_e^2 + a_e^2\right)\left(v_f^2 + a_f^2\right)|\chi|^2$$

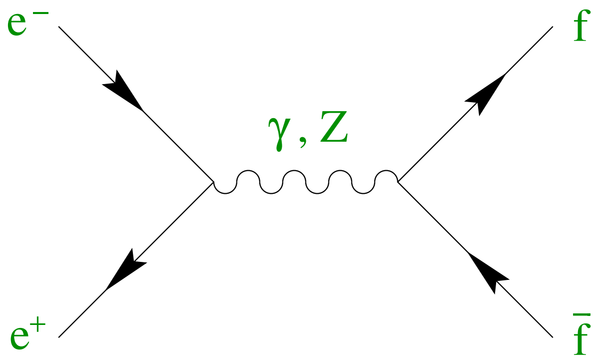
$$B = 4\,a_e\,a_f\,\mathrm{Re}(\chi) + 8\,v_e\,a_e\,v_f\,a_f\,|\chi|^2$$

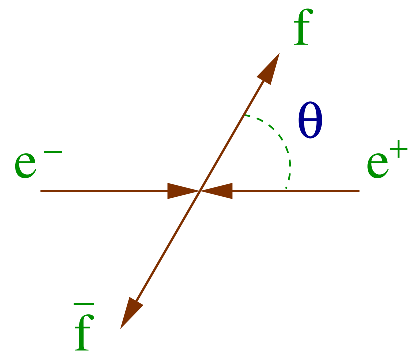
$$C = 2\,v_e\,a_f\,\mathrm{Re}(\chi) + 2\,\left(v_e^2 + a_e^2\right)v_f\,a_f\,|\chi|^2$$

$$D = 4\,a_e\,v_f\,\mathrm{Re}(\chi) + 4\,v_e\,a_e\,\left(v_f^2 + a_f^2\right)|\chi|^2$$

$$h_f = \pm 1 \quad , \quad \chi = \frac{G_F\,M_Z^2}{2\sqrt{2}\,\pi\,\alpha}\;\frac{s}{s - M_Z^2 + i\,s\,\Gamma_Z/M_Z}$$

$e^+e^- \to \gamma, Z \to f\bar{f}$





$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8\,s}\; N_f \;\Big\{A\,\left(1+\cos^2\theta\right) + B\,\cos\theta$$
$$- h_f\left[C\,\left(1+\cos^2\theta\right) + D\,\cos\theta\right]\Big\}$$

$$\sigma = \frac{4\,\pi\,\alpha^2}{3\,s}\; N_f\; A$$

$$\mathcal{A}_{\mathrm{FB}}(s) \equiv \frac{N_F - N_B}{N_F + N_B} = \frac{3}{8}\,\frac{B}{A}$$

$$\mathcal{A}_{\mathrm{Pol}}(s) \equiv \frac{\sigma^{(h_f=+1)} - \sigma^{(h_f=-1)}}{\sigma^{(h_f=+1)} + \sigma^{(h_f=-1)}} = -\frac{C}{A}$$

$$\mathcal{A}_{\mathrm{FB}}^{\mathrm{Pol}}(s) \equiv \frac{N_F^{(+1)} - N_F^{(-1)} - N_B^{(+1)} + N_B^{(-1)}}{N_F^{(+1)} + N_F^{(-1)} + N_B^{(+1)} + N_B^{(-1)}} = -\frac{3}{8}\,\frac{D}{A}$$

Z Peak ($s = M_Z^2$)

$$\sigma = \frac{12\,\pi}{M_Z^2}\;\frac{\Gamma_e\,\Gamma_f}{\Gamma_Z^2} \qquad ; \qquad \Gamma_f \equiv \Gamma(Z \to f\,\bar{f})$$

$$\mathcal{A}_{\rm FB}(s) = \frac{3}{4}\;\mathcal{P}_e\,\mathcal{P}_f$$

$$\mathcal{A}_{\rm Pol}(s) = \mathcal{P}_f \qquad ; \qquad \mathcal{A}_{\rm FB}^{\rm Pol}(s) = \frac{3}{4}\;\mathcal{P}_e$$

$$\mathcal{A}_{\rm LR}(s) \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\,\mathcal{P}_e \quad ; \quad \mathcal{A}_{\rm FB}^{\rm LR}(s) = -\frac{3}{4}\;\mathcal{P}_f$$

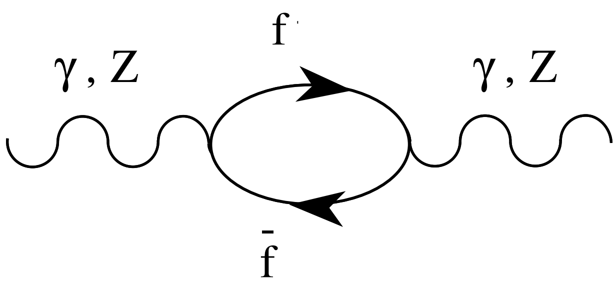
$$\mathcal{P}_f \equiv -\,A_f = \frac{-\,2\,v_f\,a_f}{|v_f|^2 + |a_f|^2}$$

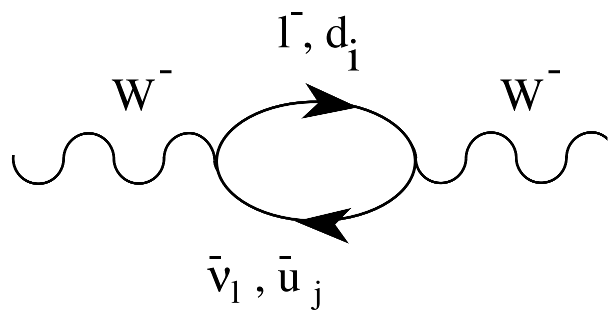
Final Polarization Only Available for $\mathbf{f} = \tau$

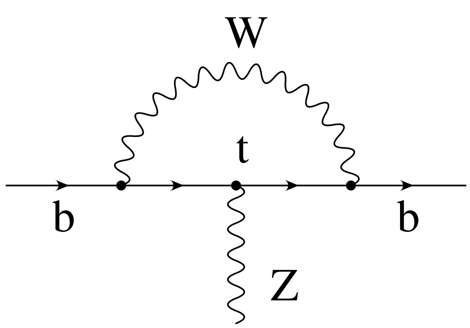
$\mathbf{A_l}$ **Sensitive to Higher Order Corrections**

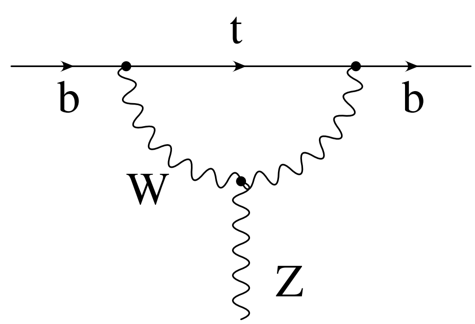
$$|v_l| = \frac{1}{2}\;\left|-1 + 4\,\sin^2\theta\right| \ll 1$$

Higher Order Corrections



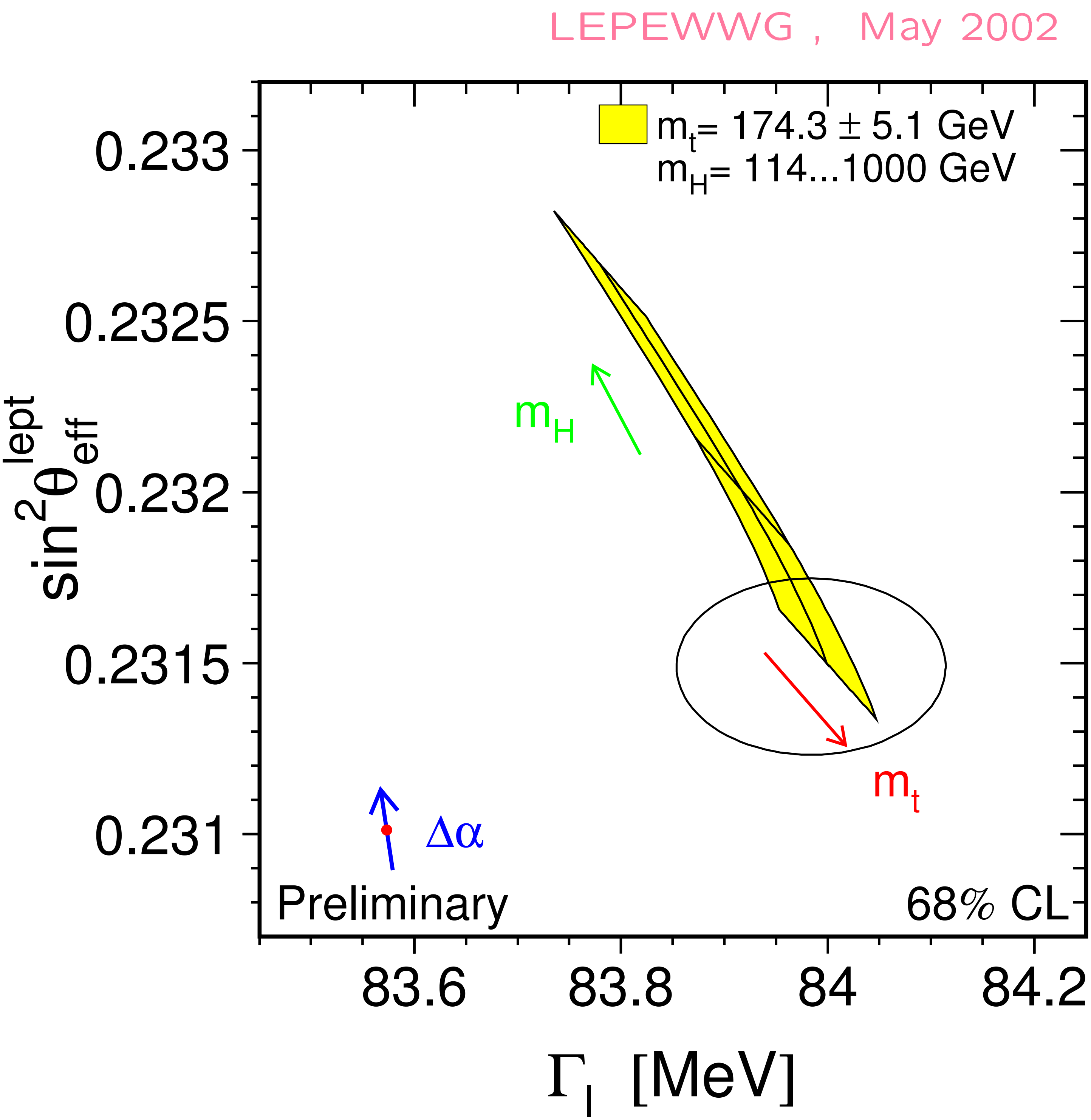






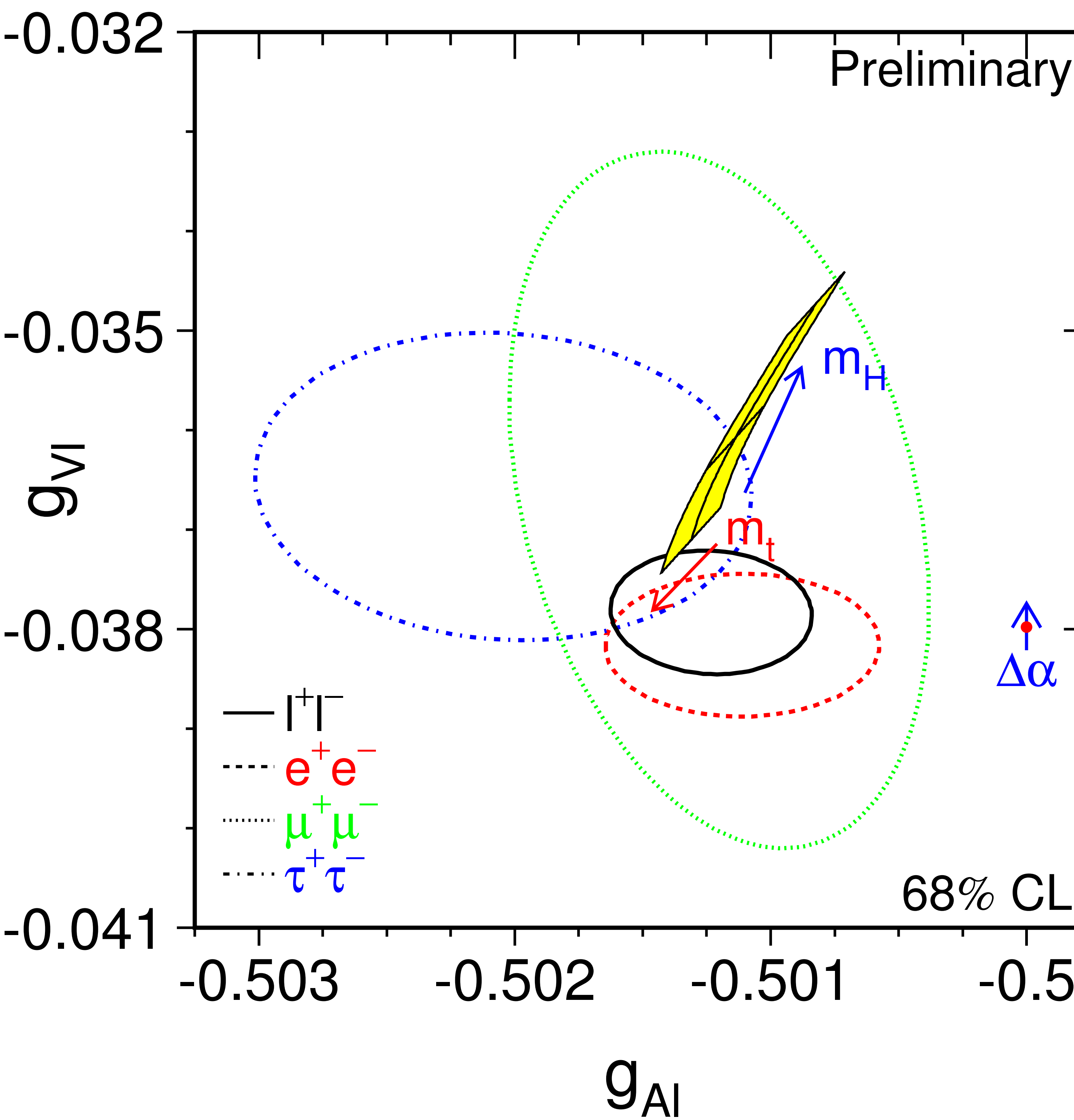
Sensitive to Heavier Particles

TOP QUARK , HIGGS



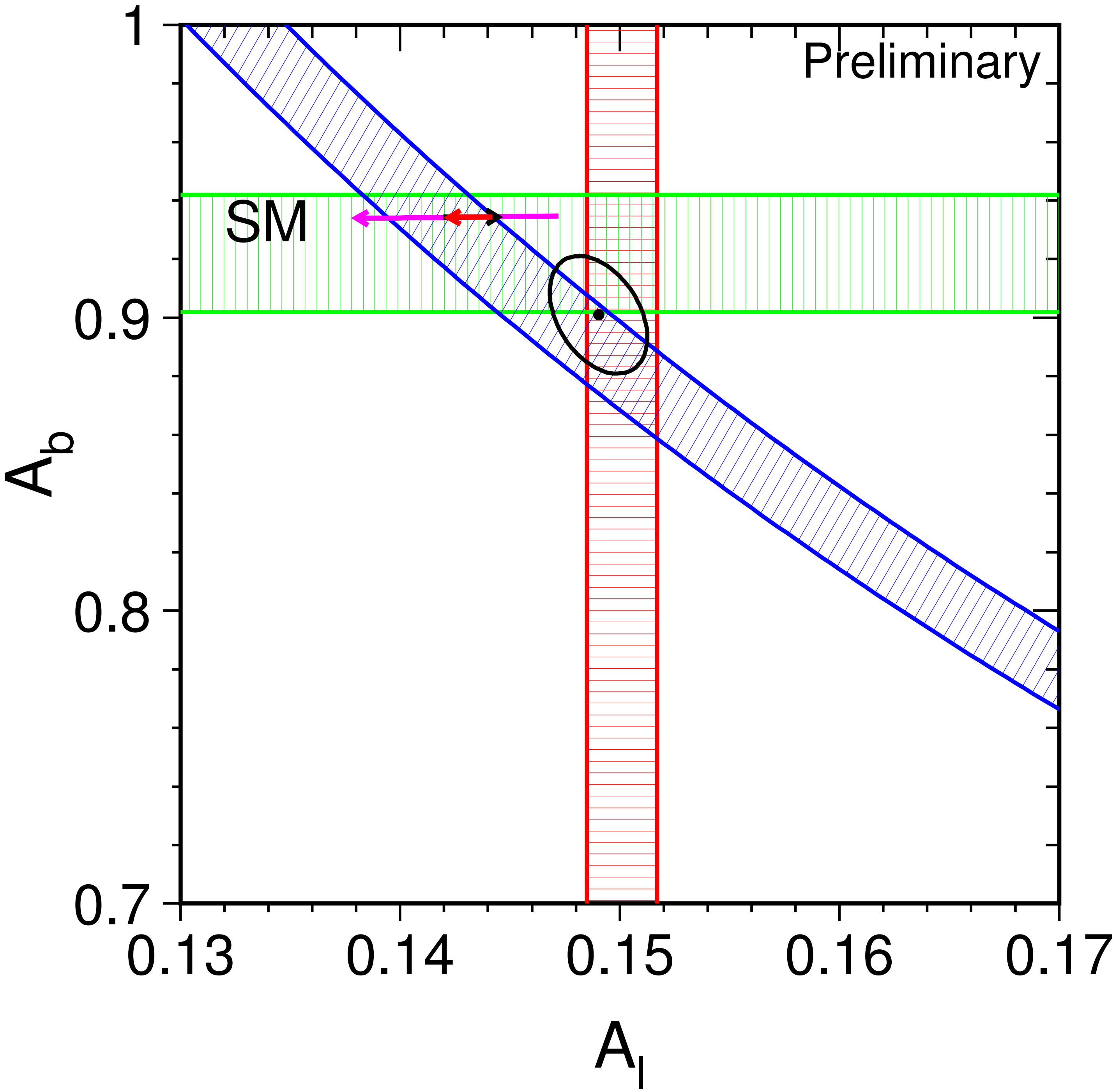
$\alpha(M_Z^2)^{-1} = 128.95 \pm 0.05$

Evidence of Electroweak Corrections



$m_t = (174.3 \pm 5.1)$ GeV ; $M_H = \left(300\,^{+700}_{-186}\right)$ GeV

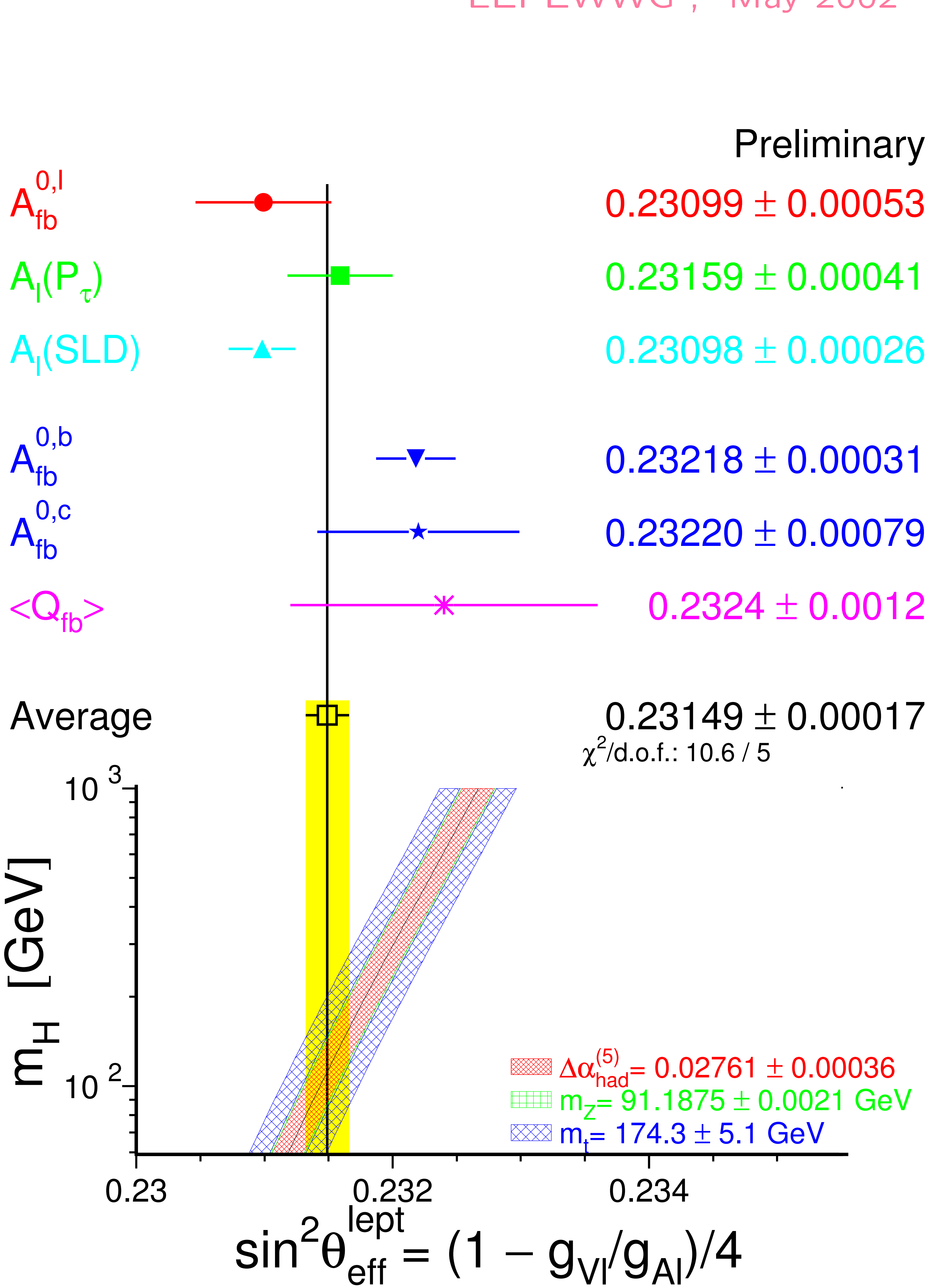
Low Values of M_H Preferred

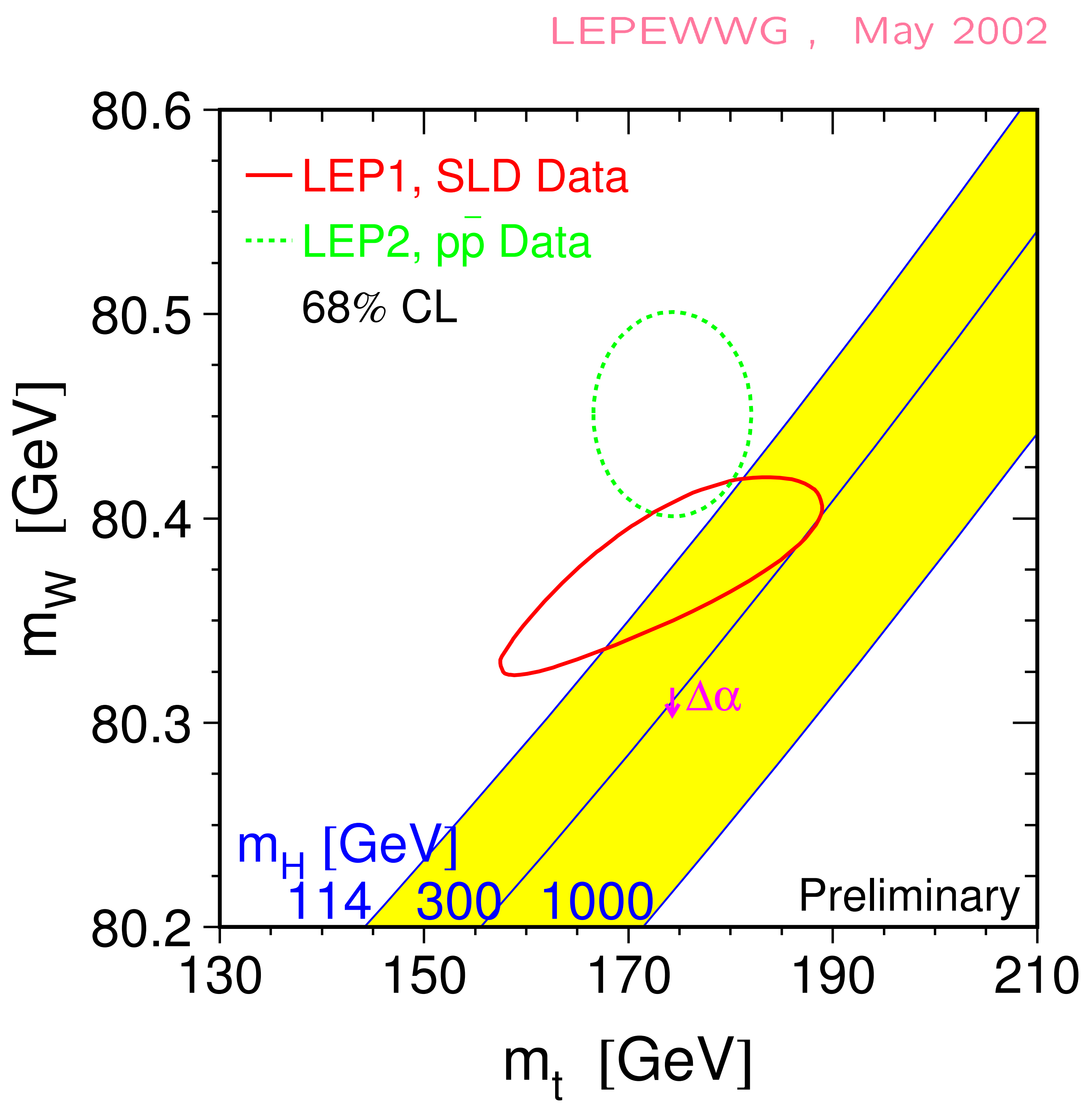


$m_t = (174.3 \pm 5.1)$ GeV ; $M_H = \left(300^{+700}_{-186}\right)$ GeV

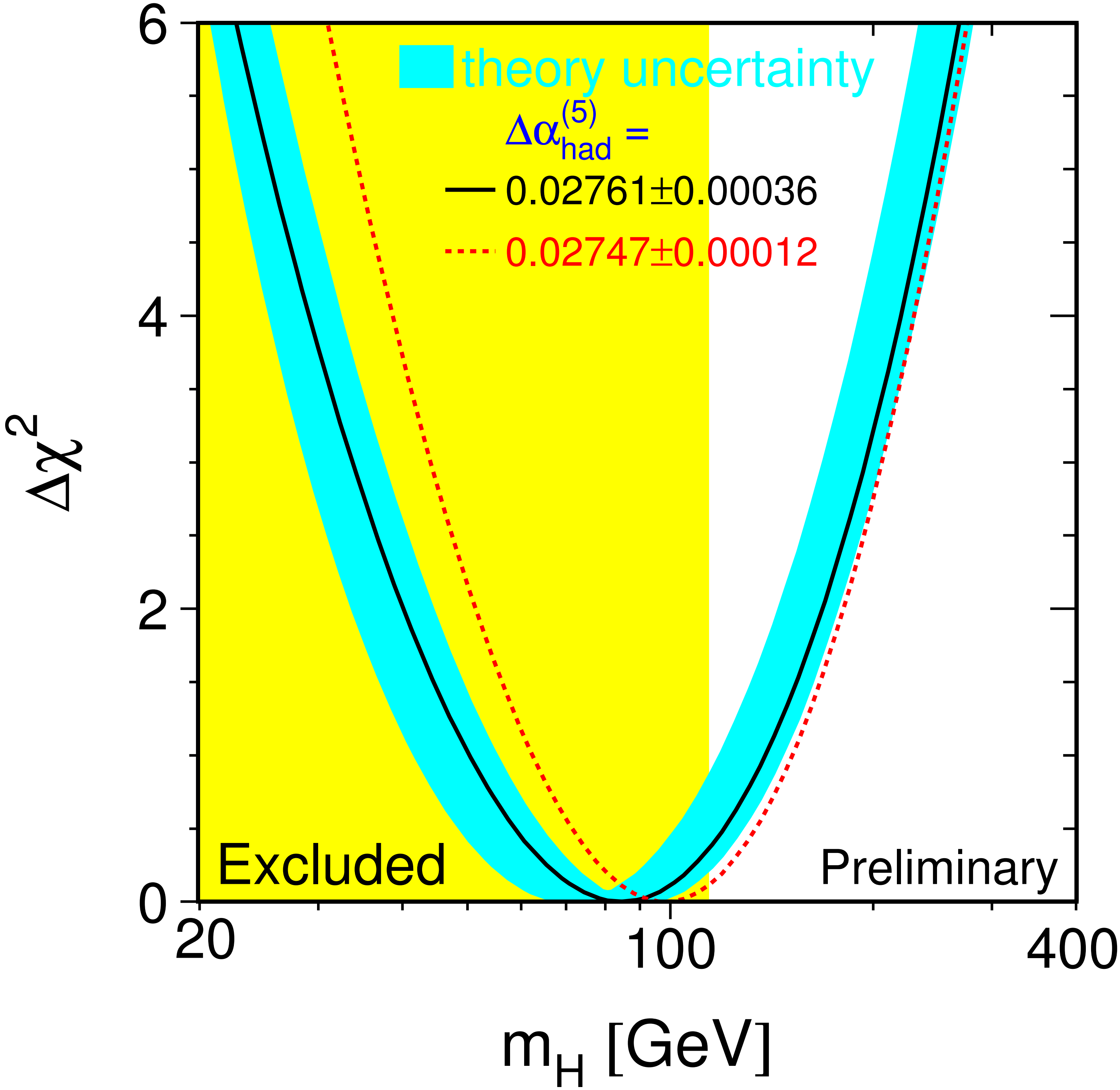
Heavy Quarks Favour High $\mathbf{M_H}$

Leptons Favour Low $\mathbf{M_H}$


LEP EWWG , May 2002
Preliminary
$A_{fb}^{0,l}$ 0.23099 ± 0.00053
$A_l(P_\tau)$ 0.23159 ± 0.00041
A_l(SLD) 0.23098 ± 0.00026
$A_{fb}^{0,b}$ 0.23218 ± 0.00031
$A_{fb}^{0,c}$ 0.23220 ± 0.00079
<Q_{fb}> 0.2324 ± 0.0012
Average 0.23149 ± 0.00017
χ^2/d.o.f.: 10.6 / 5
10^3
10^2
m_H [GeV]
$\Delta\alpha_{had}^{(5)}$= 0.02761 ± 0.00036
m_Z= 91.1875 ± 0.0021 GeV
m_t= 174.3 ± 5.1 GeV
0.23
0.232
0.234
$\sin^2\theta_{eff}^{lept} = (1 - g_{Vl}/g_{Al})/4$



LEPEWWG , May 2002
LEP1, SLD Data
LEP2, p$\bar{p}$ Data
68% CL
$\Delta\alpha$
m_H [GeV]
114
300
1000
Preliminary
80.6
80.5
80.4
80.3
80.2
130
150
170
190
210
m_W [GeV]
m_t [GeV]

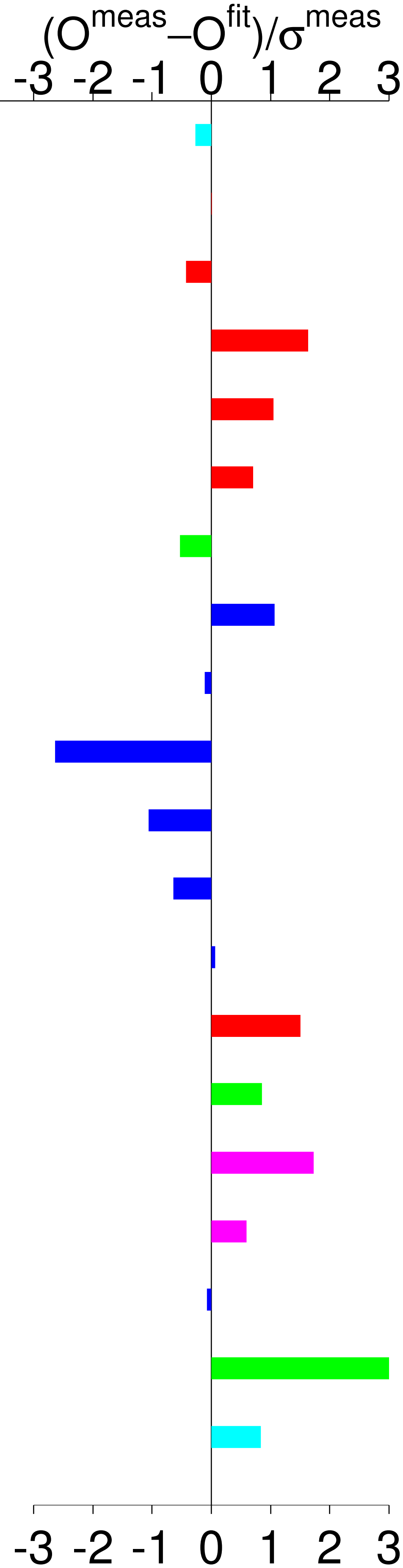


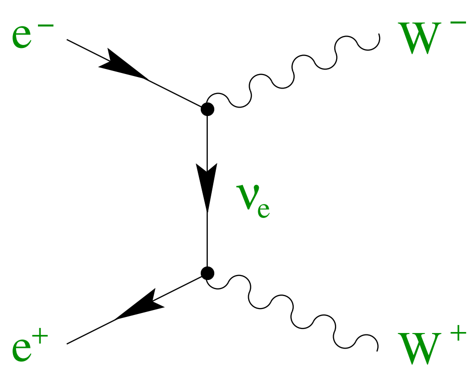
$$114 \text{ GeV} < M_H < 196 \text{ GeV}$$

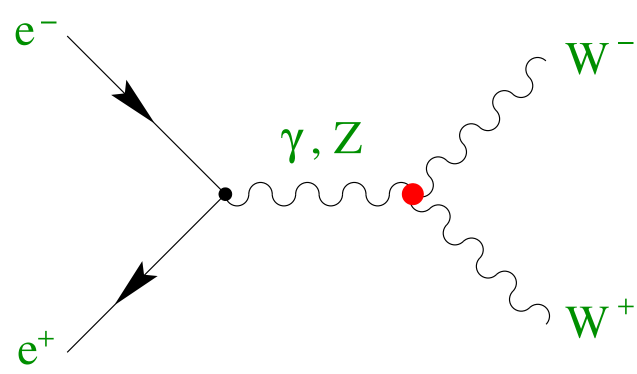
(95% CL)

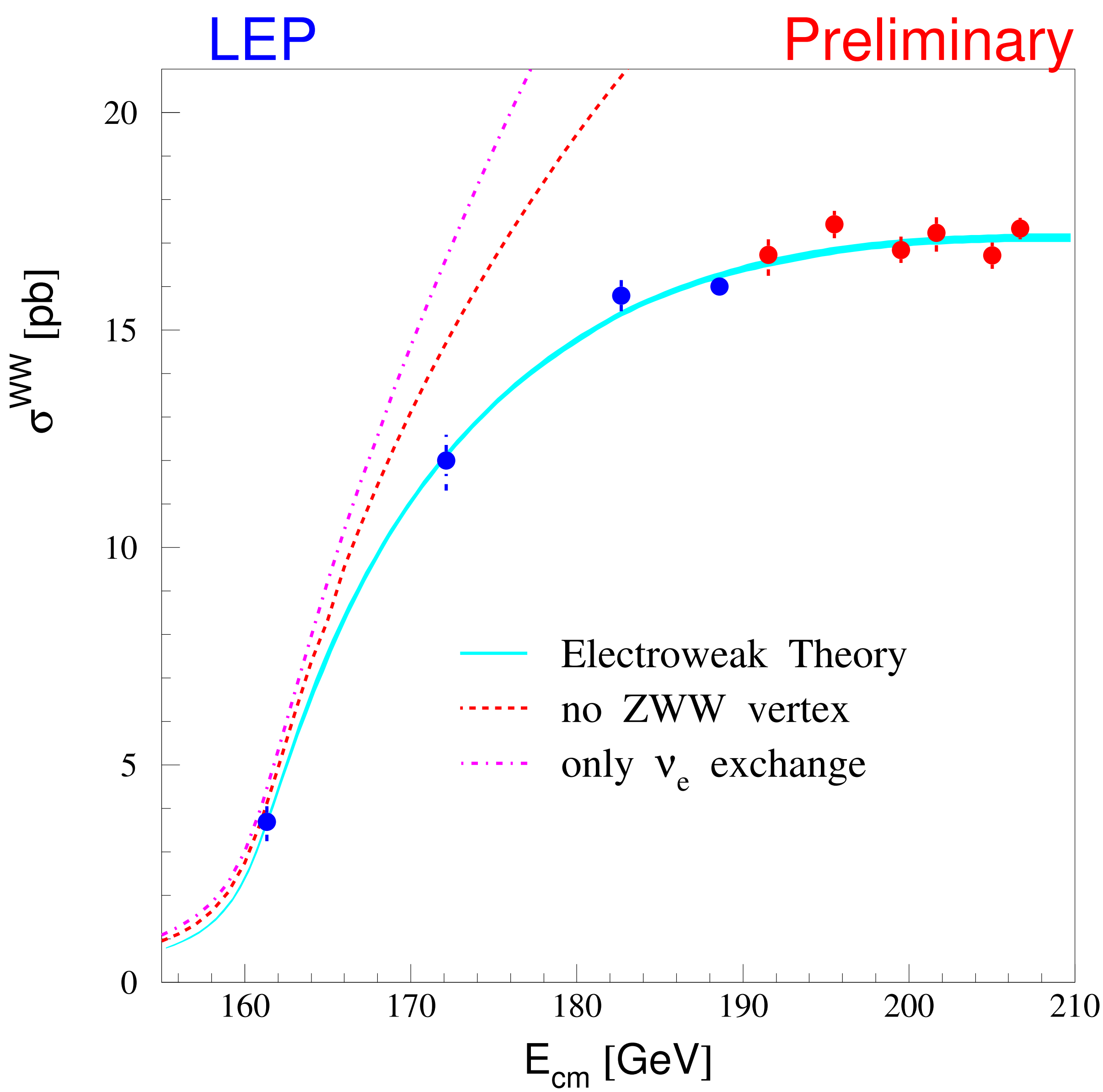
Winter 2002

	Measurement	Pull
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02761 ± 0.00036	-.27
m_Z [GeV]	91.1875 ± 0.0021	.01
Γ_Z [GeV]	2.4952 ± 0.0023	-.42
σ_{had}^{0} [nb]	41.540 ± 0.037	1.63
R_l	20.767 ± 0.025	1.05
$A_{fb}^{0,l}$	0.01714 ± 0.00095	.70
$A_l(P_\tau)$	0.1465 ± 0.0033	-.53
R_b	0.21646 ± 0.00065	1.06
R_c	0.1719 ± 0.0031	-.11
$A_{fb}^{0,b}$	0.0994 ± 0.0017	-2.64
$A_{fb}^{0,c}$	0.0707 ± 0.0034	-1.05
A_b	0.922 ± 0.020	-.64
A_c	0.670 ± 0.026	.06
A_l(SLD)	0.1513 ± 0.0021	1.50
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	.86
m_W [GeV]	80.451 ± 0.033	1.73
Γ_W [GeV]	2.134 ± 0.069	.59
m_t [GeV]	174.3 ± 5.1	-.08
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	3.00
Q_W(Cs)	-72.39 ± 0.59	.84

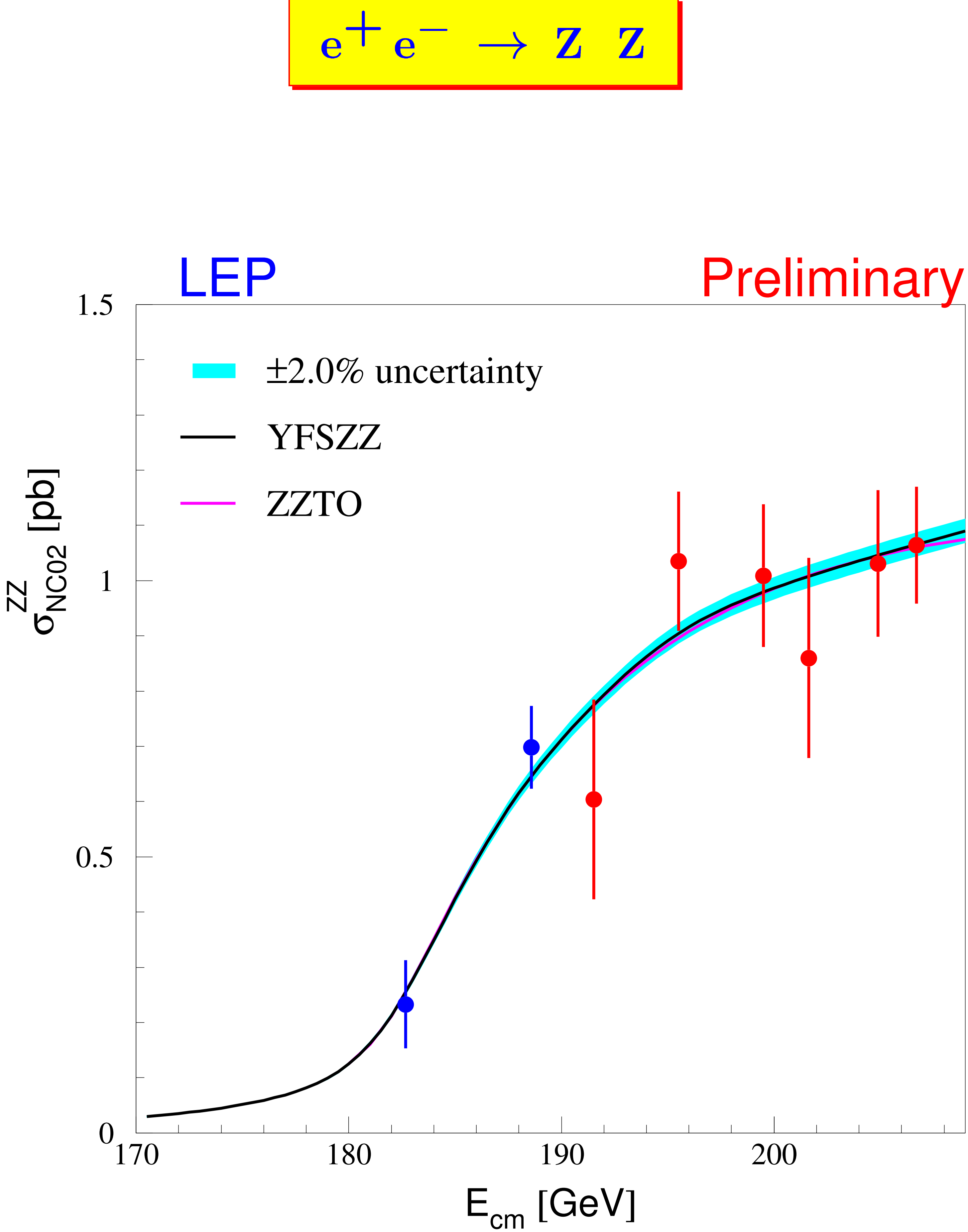






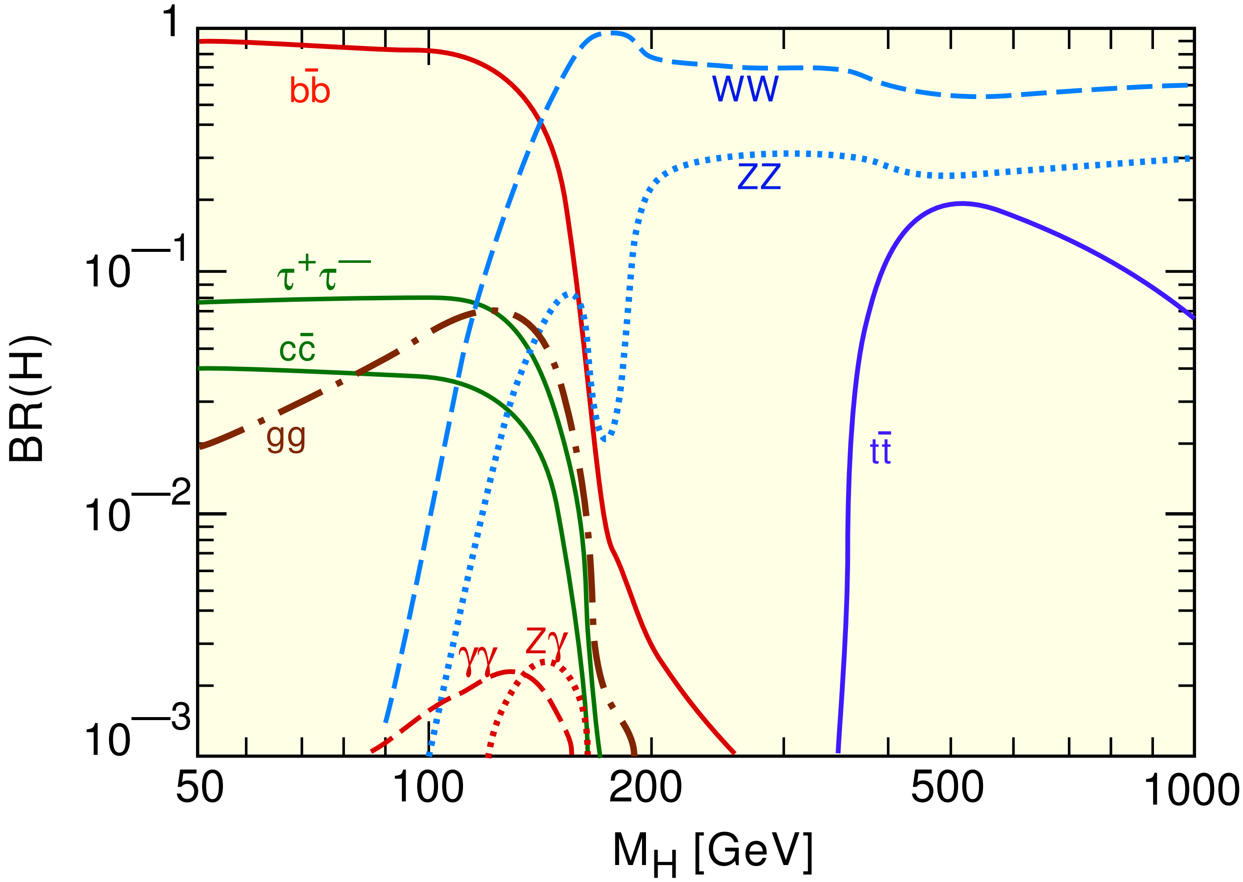


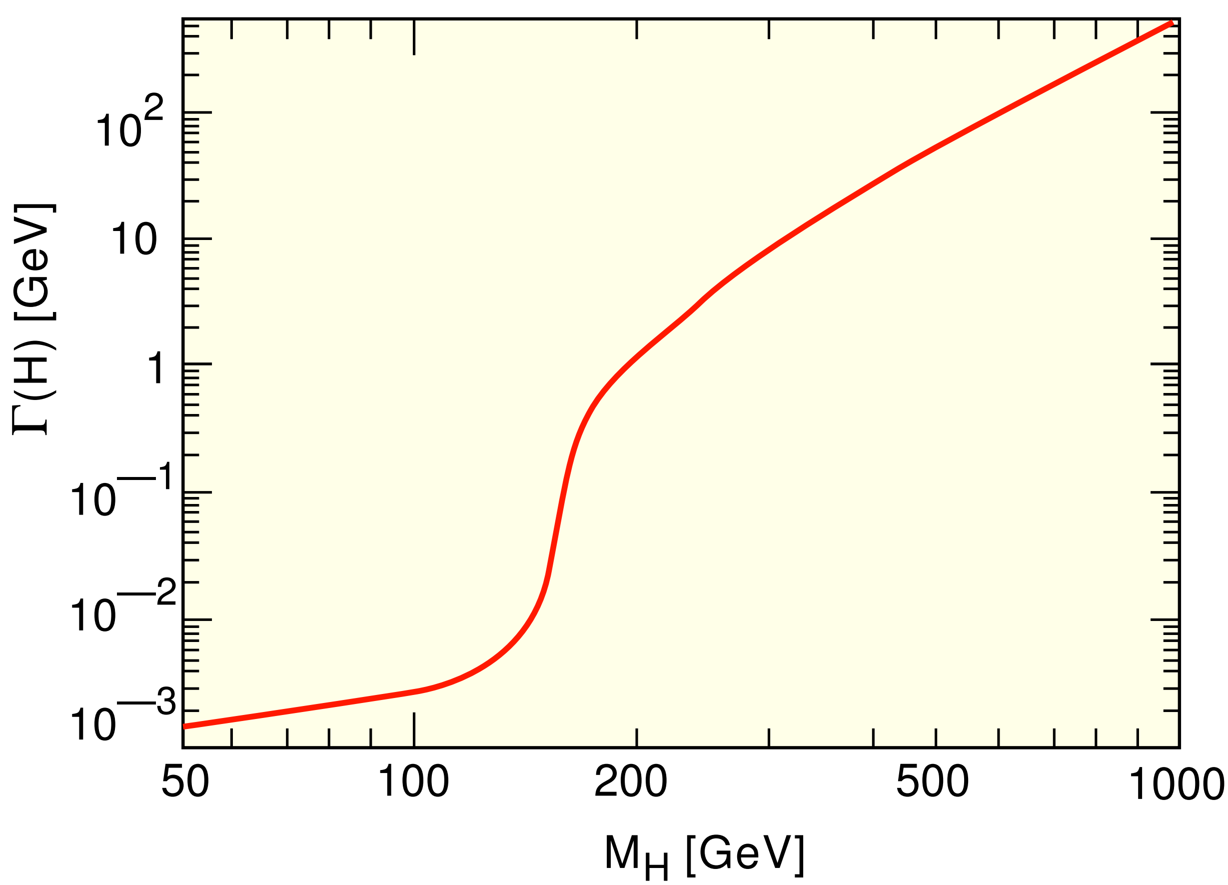
Evidence of Gauge Self-Interactions



e+ e− → Z Z
LEP
Preliminary
±2.0% uncertainty
YFSZZ
ZZTO
σ^ZZ_NC02 [pb]
1.5
1
0.5
0
170
180
190
200
E_cm [GeV]

Branching ratios and total decay width





(D. Denegri)

V_{ij} DETERMINATIONS

CKM entry	Value	Source
$\|V_{ud}\|$	0.9740 ± 0.0010	Nuclear β decay
	0.9725 ± 0.0013	$n \to p\, e^-\, \bar{\nu}_e$
	0.9734 ± 0.0008	
$\|V_{us}\|$	0.2196 ± 0.0026	K_{e3}
	0.2176 ± 0.0026	Hyperon decays
$\|V_{cd}\|$	0.224 ± 0.016	$\nu\, d \to c\, X$
$\|V_{cs}\|$	1.04 ± 0.16	$D \to \bar{K}\, e^+\, \nu_e$
	0.97 ± 0.11	$W^+ \to c\, \bar{s}$
$\|V_{cb}\|$	0.0421 ± 0.0022	$B \to D^*\, l\, \bar{\nu}_l$
	0.0404 ± 0.0011	$b \to c\, l\, \bar{\nu}_l$
	0.0412 ± 0.0020	
$\|V_{ub}\|$	0.0033 ± 0.0006	$B \to \rho\, l\, \bar{\nu}_l$
	0.0041 ± 0.0006	$b \to u\, l\, \bar{\nu}_l$
	0.0036 ± 0.0007	
$\|V_{tb}\| / \sqrt{\sum_q \|V_{tq}\|^2}$	$0.97\,^{+0.16}_{-0.12}$	$t \to b\, W / q\, W$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9957 \pm 0.0019$$

$$\sum_j \left(|V_{uj}|^2 + |V_{cj}|^2 \right) = 2.039 \pm 0.025 \qquad \text{(LEP)}$$

$\mathcal{C}$ / $\mathcal{P}$: Violated maximally in weak interactions

$\mathcal{CP}$: Symmetry of nearly all observed phenomena

Slight ($\sim$ 0.2%) $\not{\mathcal{CP}}$ in K^0 decays (1964)

Huge Matter — Antimatter Asymmetry in our Universe ➡ Baryogenesis

$\mathcal{CPT}$ Theorem: $\not{\mathcal{CP}}$ ⟺ $\not{\mathcal{T}}$

Thus, $\not{\mathcal{CP}}$ requires:

- Complex Phases
- Interferences

Standard Model Mechanism of $\not{CP}$

Complex phases in Yukawa couplings only:

$$\mathcal{L}_Y = \sum_{jk} \left\{ (\bar{u}'_j, \bar{d}'_j)_L \left[c^{(d)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c^{(u)}_{jk} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] \right\}$$

↓ SSB $\quad [\, \langle \phi^{(0)} \rangle = v/\sqrt{2} \,]$

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) \frac{v}{\sqrt{2}} \left\{ \bar{d}'_{jL}\, c^{(d)}_{jk}\, d'_{kR} + \bar{u}'_{jL}\, c^{(u)}_{jk}\, u'_{kR} + \text{h.c.} \right\}$$

↓ $c^{(q)}_{jk}$ diagonalization

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) \left\{ \bar{d}_{jL}\, m_{d_j}\, d'_{jR} + \bar{u}_{jL}\, m_{u_j}\, u_{jR} + \text{h.c.} \right\}$$

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} W^\dagger_\mu \sum_{ij} \bar{u}_i \gamma^\mu (1 - \gamma_5) V_{ij}\, d_j + \text{h.c.}$$

The CKM matrix $\mathbf{V_{ij}}$ is the only source of $\not{CP}$

Unitarity: $V \cdot V^\dagger = V^\dagger \cdot V = 1$

$\mathbf{N_f = 2}$ **:** 1 angle, 0 phases

$$V = \begin{bmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{bmatrix} \longrightarrow \text{No } \not{\mathcal{CP}}$$

$\mathbf{N_f = 3}$ **:** (CKM) 3 angles, 1 phase

$$\begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & c_{23}\, c_{13} \end{bmatrix}$$

$$\approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\,\lambda^3\,(\rho - i\,\eta) \\ -\lambda & 1-\lambda^2/2 & A\,\lambda^2 \\ A\,\lambda^3\,(1-\rho-i\,\eta) & -A\,\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4)$$

$c_{ij} \equiv \cos\theta_{ij}$; $s_{ij} \equiv \sin\theta_{ij}$ $(i\,,\,j = 1\,2\,3)$

$\lambda \approx \sin\theta_c \approx 0.223$; $A \approx 0.83$; $\sqrt{\rho^2+\eta^2} \approx 0.40$

$$\delta_{13} \neq 0 \quad (\eta \neq 0) \longrightarrow \not{\mathcal{CP}}$$

THE STANDARD THEORY OF FUNDAMENTAL INTERACTIONS

$$\mathbf{SU(3)_C \otimes SU(2)_L \otimes U(1)_Y}$$

Electroweak + Strong Forces

- Gauge Symmetry ➡ Dynamics
- 3 Gauge Parameters: $\alpha_s(M_Z^2)$, α , θ_W
- All Known Experimental Facts Explained
- Problem with **Mass Scales / Mixings**:

  - 15 Additional Parameters
  - Why 3 Families ?
  - Why Left $\neq$ Right ?
  - Why $m_t > M_Z$?
  - Does the Higgs Exist ?
  - Flavour Mixing
  - $\mathcal{CP}$ Violation
  - Neutrino Masses / Oscillations