QCD Phenomenology at High Energy

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CERN Academic Training Lectures, October 2003

Lecture 4: Jet fragmentation

- Fragmentation functions
  - Scaling violation
  - Small-x fragmentation
  - Average multiplicity
- Hadronization models
  - General ideas
  - Cluster & string models
- Particle yields and spectra
  - Identified particle yields & spectra
  - Quark & gluon jets

Fragmentation functions

- In addition to infrared-safe quantities like e^+e^- total cross section and shape variable distributions, there are factorizable quantities, in which infrared divergences of PT can be factorized out and replaced by experimentally measured factor that contains all long-distance effects.

- An example is fragmentation function $F_i^h(x,t)$, which gives distribution of momentum fraction x for hadrons of type h in jet initiated by parton of type i, produced in hard process at scale t.

- In e^+e^- annihilation, for example, hard process is $e^+e^- \to q\bar{q}$ at scale equal to c.m. energy squared s and distribution of $x = 2p_h/\sqrt{s}$ is (for $s \ll M_Z^2$)

$$\frac{d\sigma}{dx} = 3\sigma_0 \sum_q Q_q^2 \left\{ F_q^h(x,s) + F_{\bar{q}}^h(x,s) \right\}$$

where σ_0 is $e^+e^- \to \mu^+\mu^-$ cross section.

Scaling violation

- Fragmentation functions satisfy DGLAP evolution equation

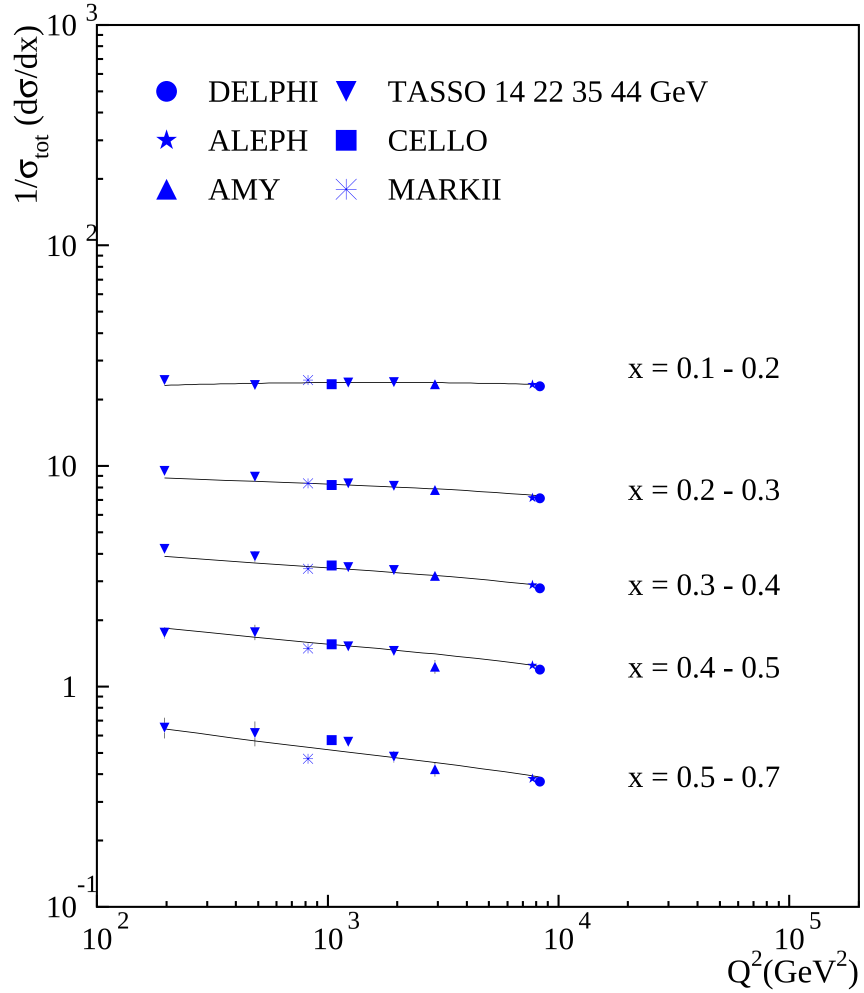
$$t\frac{\partial}{\partial t}F_i^h(x,t) = \sum_j \int_x^1 \frac{dz}{z}\frac{\alpha_{\rm S}}{2\pi}P_{ji}(z,\alpha_{\rm S})F_j^h(x/z,t)\,.$$

Splitting functions P_{ji} have perturbative expansions of the form

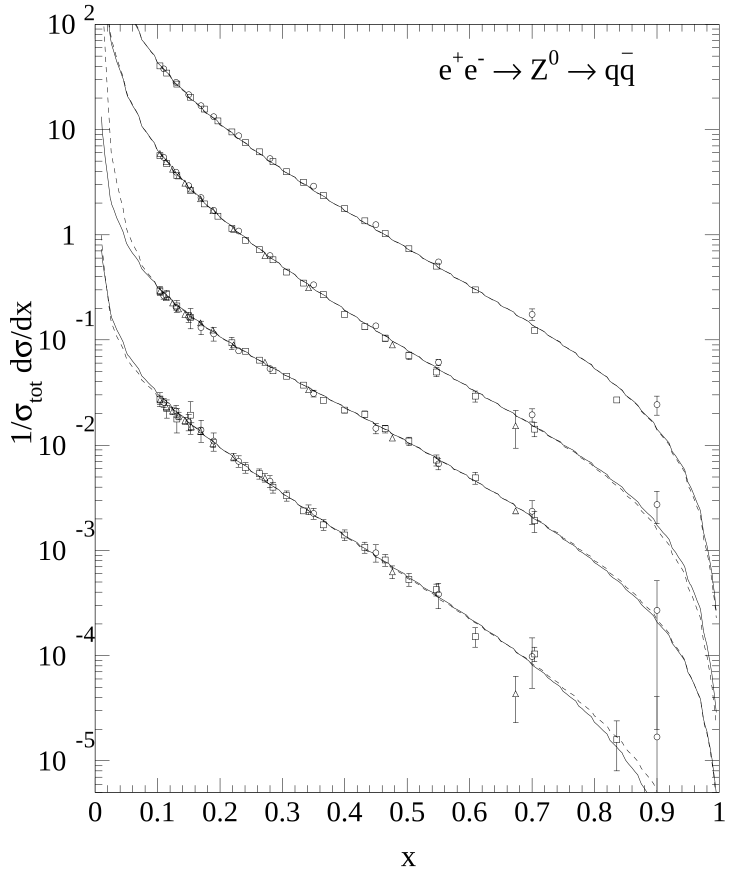
$$P_{ji}(z,\alpha_{\rm S}) = P_{ji}^{(0)}(z) + \frac{\alpha_{\rm S}}{2\pi}P_{ji}^{(1)}(z) + \cdots$$

Leading terms $P_{ji}^{(0)}(z)$ were given earlier. Notice that splitting function is P_{ji} rather than P_{ij} since F_j^h represents fragmentation of final parton j.

- Observed scaling violation can be used to measure α_S.



- Fitted fragmentation functions at $Q = M_Z$:



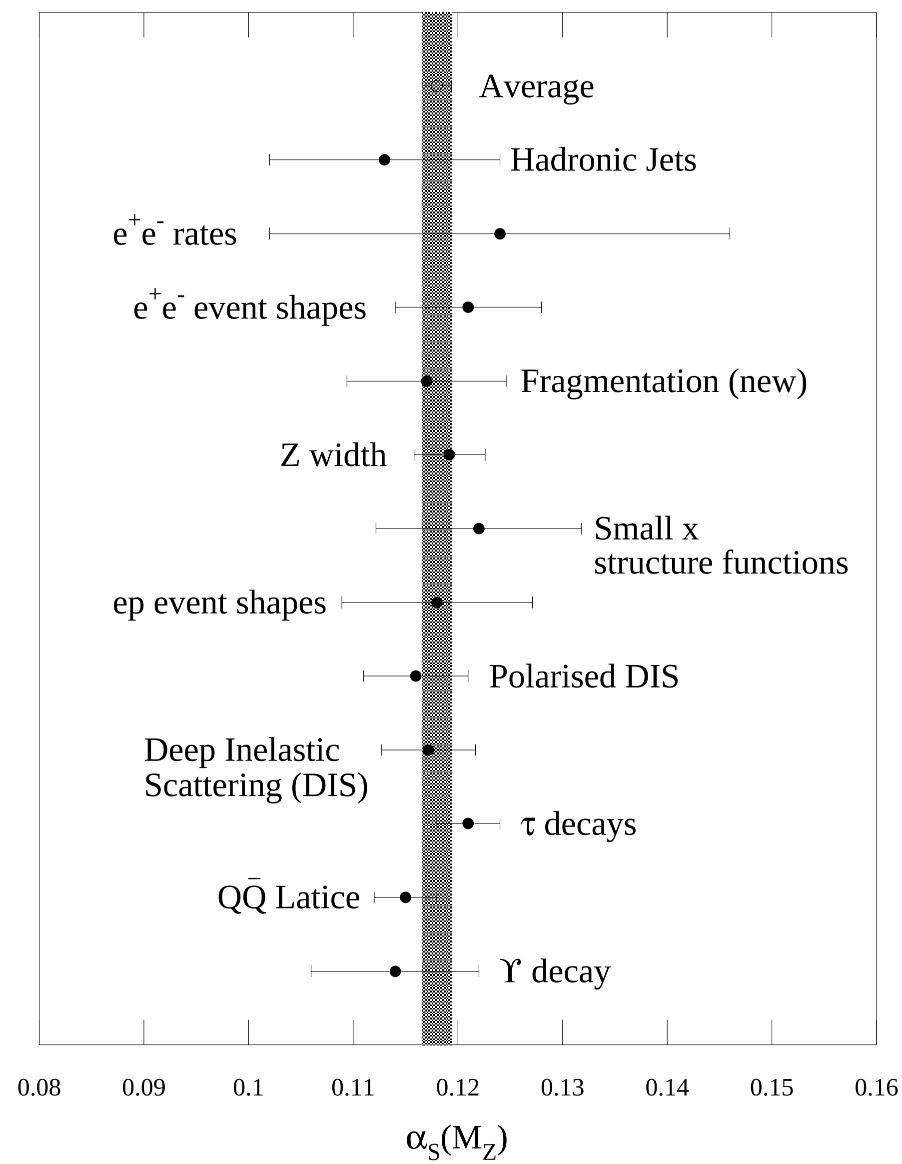
— charged

— $\pi^{\pm}$ $(\times 0.2)$

— $K^{\pm}$ $(\times 0.04)$

— $p/\bar{p}$ $(\times 0.008)$

- α_S from scaling violation is in good agreement with other measurements.



Small-x fragmentation

- Evolution of fragmentation functions at small x is sensitive to moments near $N = 1$. However, anomalous dimensions $\gamma_{gq}^{(0)}$, $\gamma_{gg}^{(0)}$ are not defined at $N = 1$: moment integrals for $N \leq 1$ are dominated by small z, where $P_{gi}(z)$ diverges due to soft gluon emission.

- At small z we must take into account coherence effects. Recall evolution variable becomes $\tilde{t} = E^2[1 - \cos\theta]$, with angular ordering condition $\tilde{t}' < z^2\tilde{t}$. Thus, redefining t as $\tilde{t}$, evolution equation in integrated form is

$$F_i(x,t) = F_i(x,t_0) + \sum_j \int_x^1 \frac{dz}{z} \int_{t_0}^{z^2 t} \frac{dt'}{t'} \frac{\alpha_S}{2\pi} P_{ji}(z) F_j(x/z, t')$$

or in differential form

$$t\frac{\partial}{\partial t} F_i(x,t) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_S}{2\pi} P_{ji}(z) F_j(x/z, z^2 t) \; .$$

- Only difference from DGLAP equation is z-dependent scale on the right-hand side — not important for most values of x but crucial at small x.

- For simplicity, consider first α_S fixed and neglect sum over j. Taking moments as usual,

$$t\frac{\partial}{\partial t}\tilde{F}(N,t) = \frac{\alpha_S}{2\pi}\int_x^1 dz\, z^{N-1}P(z)\tilde{F}(N,z^2t)\ .$$

  - Try solution of form $F(N,t) \propto t^{\gamma(N,\alpha_S)}$. Then anomalous dimension $\gamma(N,\alpha_S)$ must satisfy

$$\gamma(N,\alpha_S) = \frac{\alpha_S}{2\pi}\int_0^1 z^{N-1+2\gamma(N,\alpha_S)}P(z)\ .$$

  - For $N-1$ not small, we can neglect $2\gamma(N,\alpha_S)$ in exponent and obtain usual formula for anomalous dimension. For $N \simeq 1$, $z \to 0$ region dominates, where $P_{gg}(z) \simeq 2C_A/z$. Hence

$$\begin{aligned}\gamma_{gg}(N,\alpha_S) &= \frac{C_A\alpha_S}{\pi}\frac{1}{N-1+2\gamma_{gg}(N,\alpha_S)}\\ &= \frac{1}{4}\left[\sqrt{(N-1)^2+\frac{8C_A\alpha_S}{\pi}}-(N-1)\right]\\ &= \sqrt{\frac{C_A\alpha_S}{2\pi}}-\frac{1}{4}(N-1)+\frac{1}{32}\sqrt{\frac{2\pi}{C_A\alpha_S}}(N-1)^2+\cdots\end{aligned}$$

- To take account of running α_S, write

$$\tilde{F}(N,t) \sim \exp\left[\int^t \gamma_{gg}(N,\alpha_S)\frac{dt'}{t'}\right] ,$$

and note that $\gamma_{gg}(N,\alpha_S)$ should be $\gamma_{gg}(N,\alpha_S(t'))$. Use

$$\int^t \gamma_{gg}(N,\alpha_S(t'))\frac{dt'}{t'} = \int^{\alpha_S(t)} \frac{\gamma_{gg}(N,\alpha_S)}{\beta(\alpha_S)}\, d\alpha_S ,$$

where $\beta(\alpha_S) = -b\alpha_S^2 + \cdots$, to find

$$\tilde{F}(N,t) \sim \exp\left[\frac{1}{b}\sqrt{\frac{2C_A}{\pi\alpha_S}} - \frac{1}{4b\alpha_S}(N-1) + \frac{1}{48b}\sqrt{\frac{2\pi}{C_A\alpha_S^3}}(N-1)^2 + \cdots\right]_{\alpha_S=\alpha_S(t)} .$$

- In e^+e^- annihilation, scale $t \sim s$ and behaviour of $\tilde{F}(N,s)$ near $N=1$ determines form of small-x fragmentation functions. Keeping terms up to $(N-1)^2$ in exponent gives Gaussian function of N which transforms into Gaussian function of $\xi \equiv \ln(1/x)$:

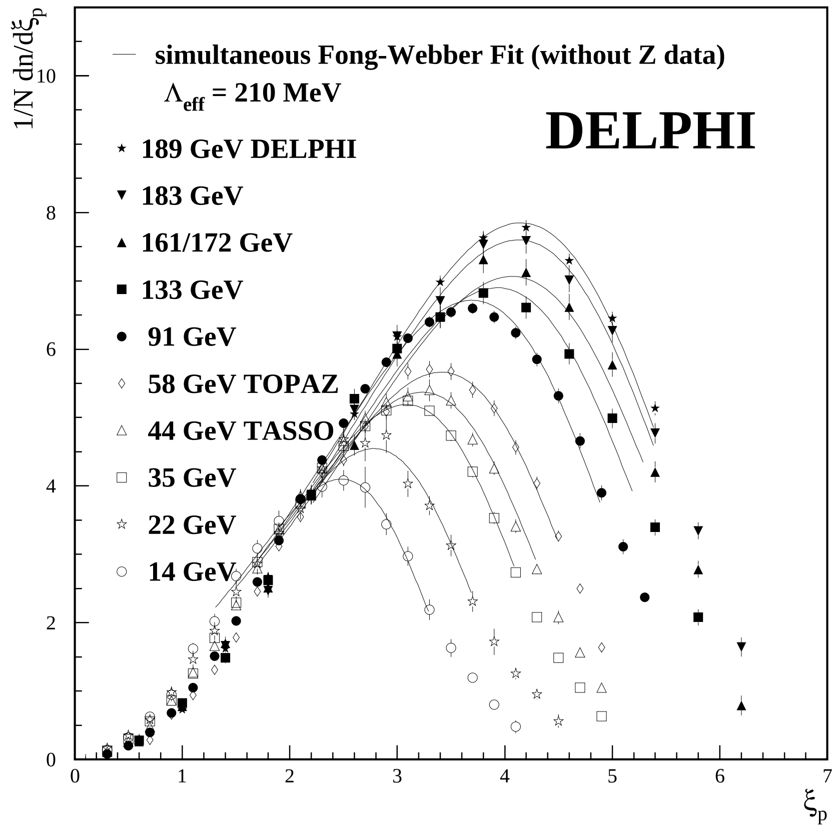
$$xF(x,s) \propto \exp\left[-\frac{1}{2\sigma^2}(\xi-\xi_p)^2\right] ,$$

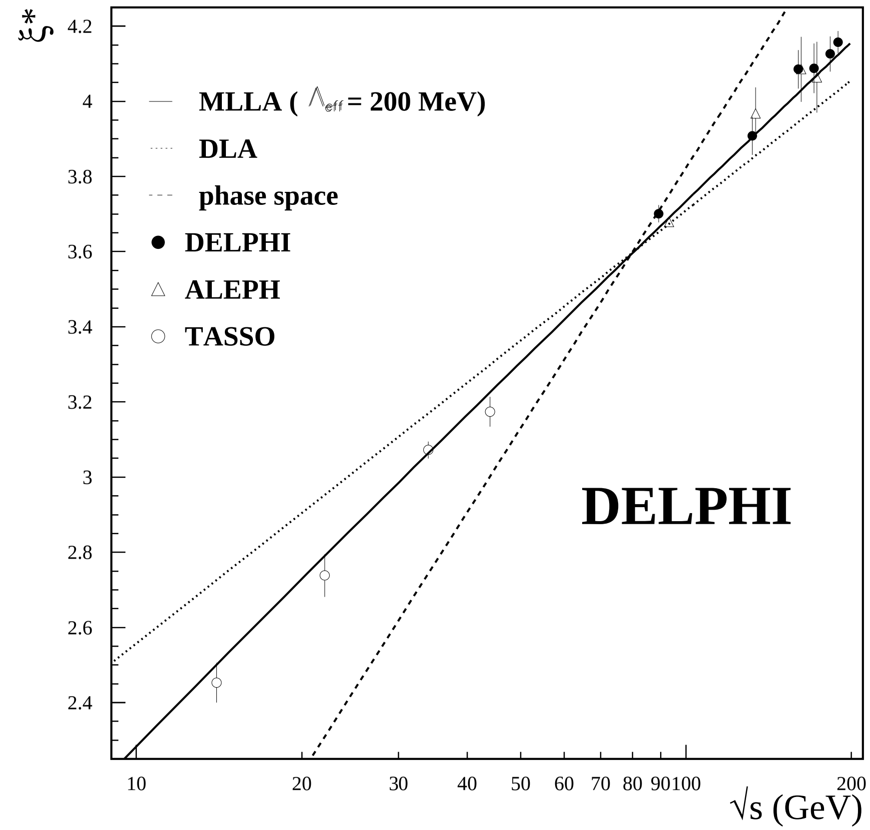
- Peak position

$$\xi_p = \frac{1}{4b\alpha_{\mathrm{S}}(s)} \sim \frac{1}{4}\ln s$$

- Width of distribution

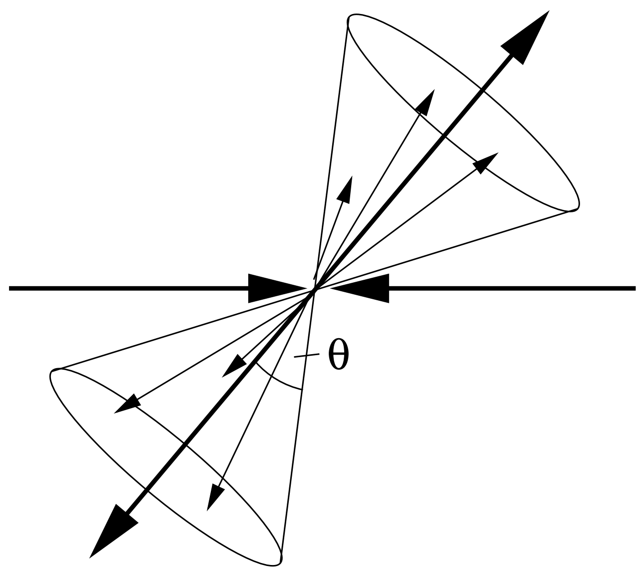
$$\sigma = \left(\frac{1}{24b}\sqrt{\frac{2\pi}{C_A\alpha_{\mathrm{S}}^3(s)}}\right)^{\frac{1}{2}} \propto (\ln s)^{\frac{3}{4}} \ .$$

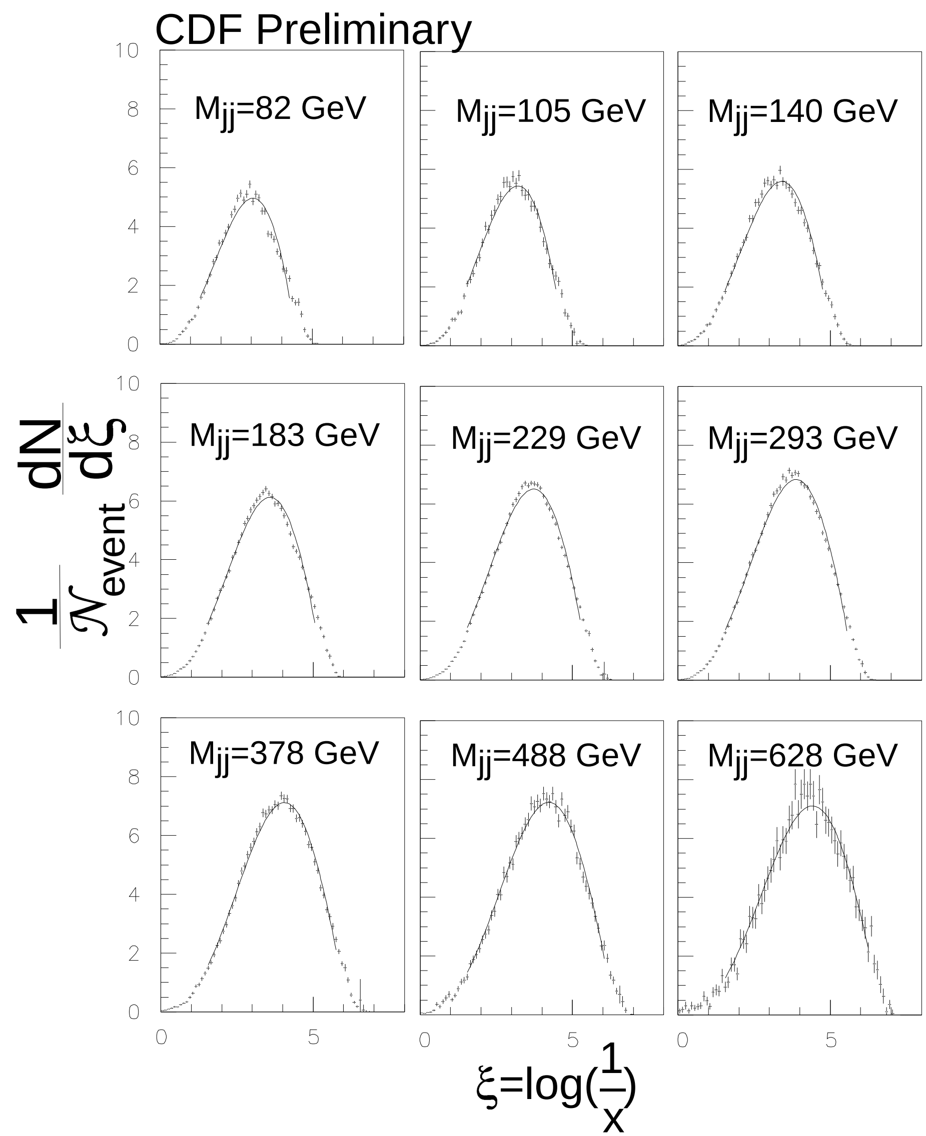


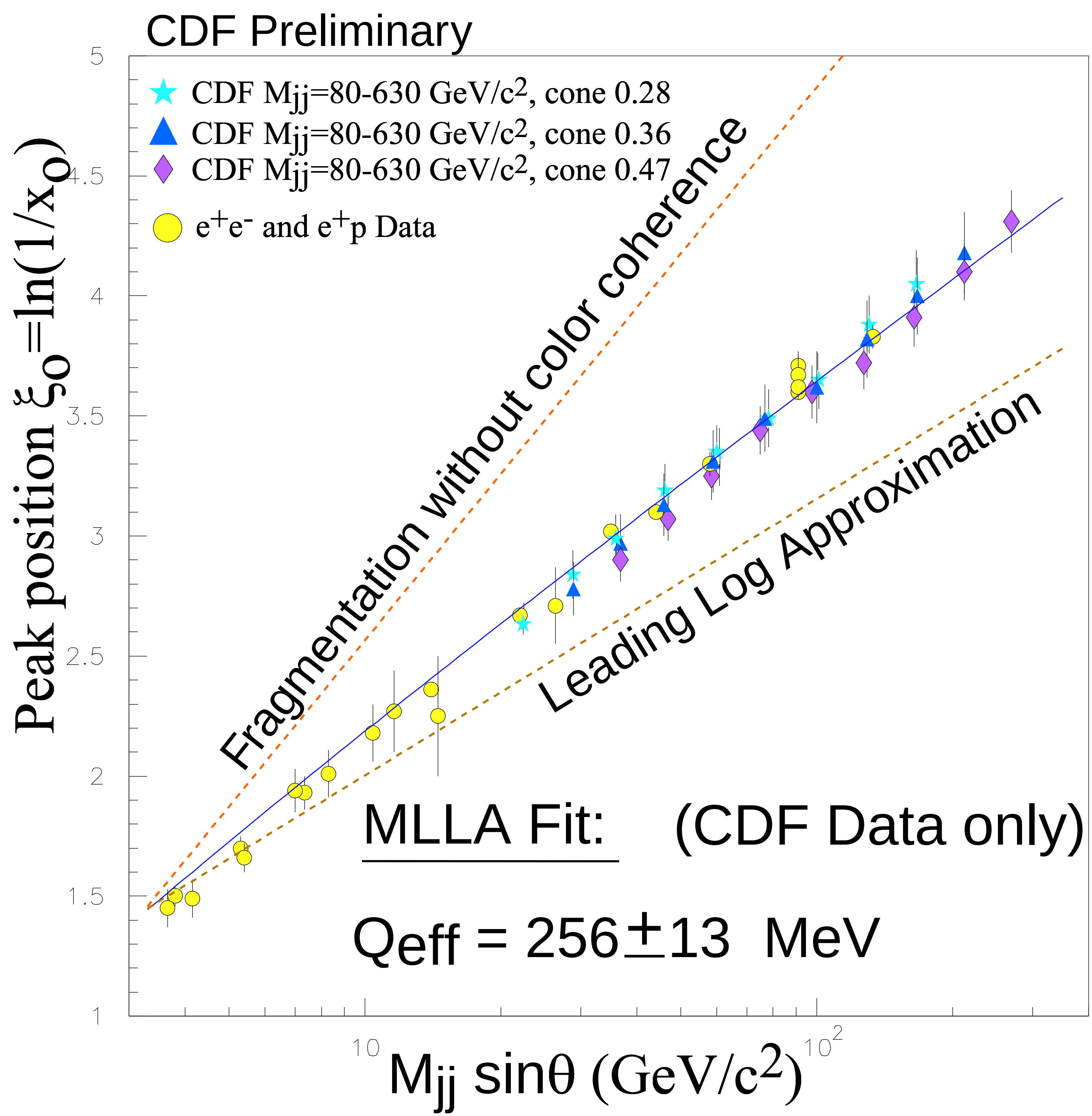


- Results agree well with observed form and energy dependence.
- Energy-dependence of the peak position ξ_p tests suppression of hadron production at small x due to soft gluon coherence. Decrease at very small x is expected on kinematical grounds, but this would occur at particle energies proportional to their masses, i.e. at $x \propto m/\sqrt{s}$, giving $\xi_p \sim \frac{1}{2}\ln s$. Thus purely kinematic suppression would give ξ_p increasing twice as fast.

- In $p\bar{p} \to$ dijets, $\sqrt{s}$ is replaced by $M_{JJ} \sin\theta$ where M_{JJ} is dijet mass and θ is jet cone angle.





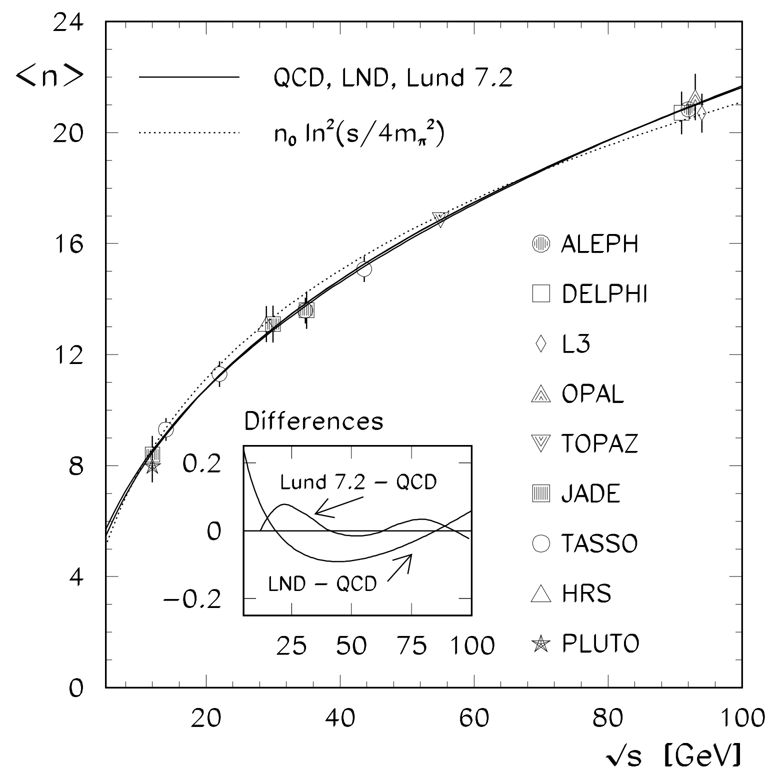

CDF Preliminary
CDF Mjj=80-630 GeV/c2, cone 0.28
CDF Mjj=80-630 GeV/c2, cone 0.36
CDF Mjj=80-630 GeV/c2, cone 0.47
e+e- and e+p Data
Fragmentation without color coherence
Leading Log Approximation
MLLA Fit: (CDF Data only)
Qeff = 256±13 MeV
Peak position ξ0=ln(1/x0)
Mjj sinθ (GeV/c2)
1
1.5
2
2.5
3
3.5
4
4.5
5
10
10^2

Average multiplicity

- Mean number of hadrons is $N = 1$ moment of fragmentation function:

$$\langle n(s)\rangle = \int_0^1 dx\, F(x,s) = \tilde{F}(1,s) \sim \exp \frac{1}{b}\sqrt{\frac{2C_A}{\pi\alpha_S(s)}} \sim \exp\sqrt{\frac{2C_A}{\pi b}\ln\left(\frac{s}{\Lambda^2}\right)}$$

(solid curve) in good agreement with data.



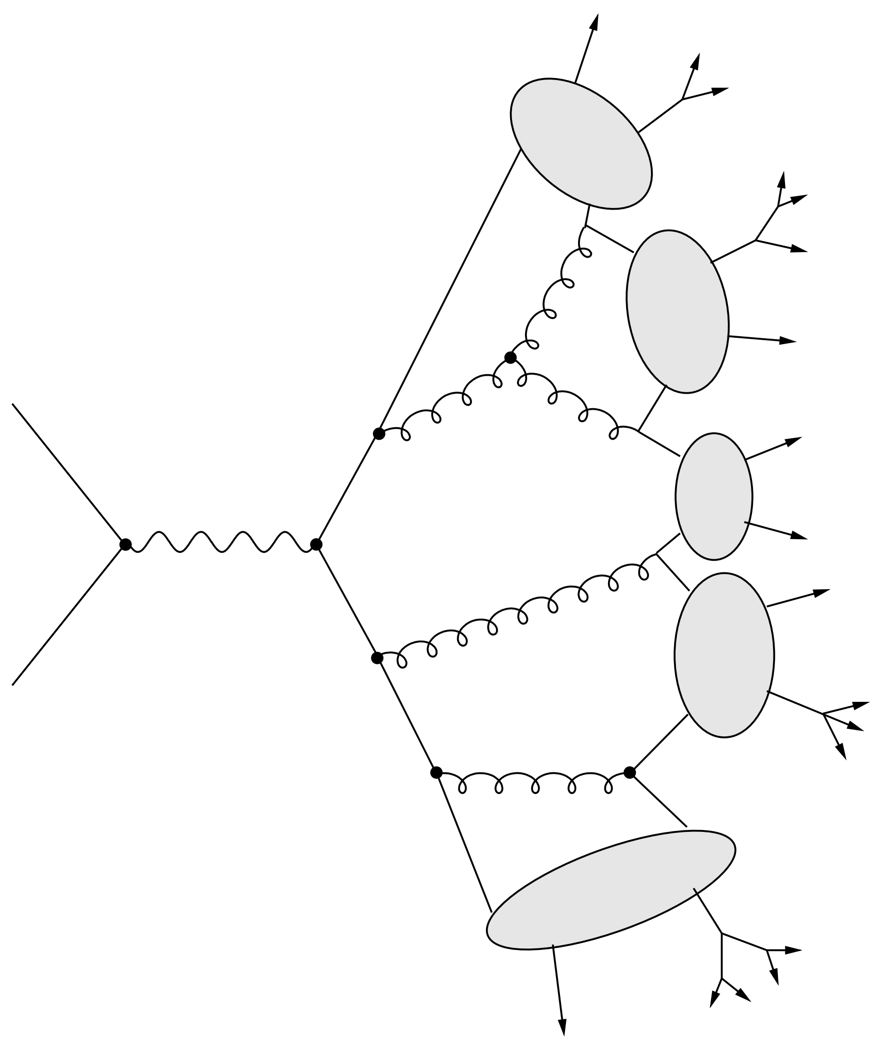
Hadronization Models

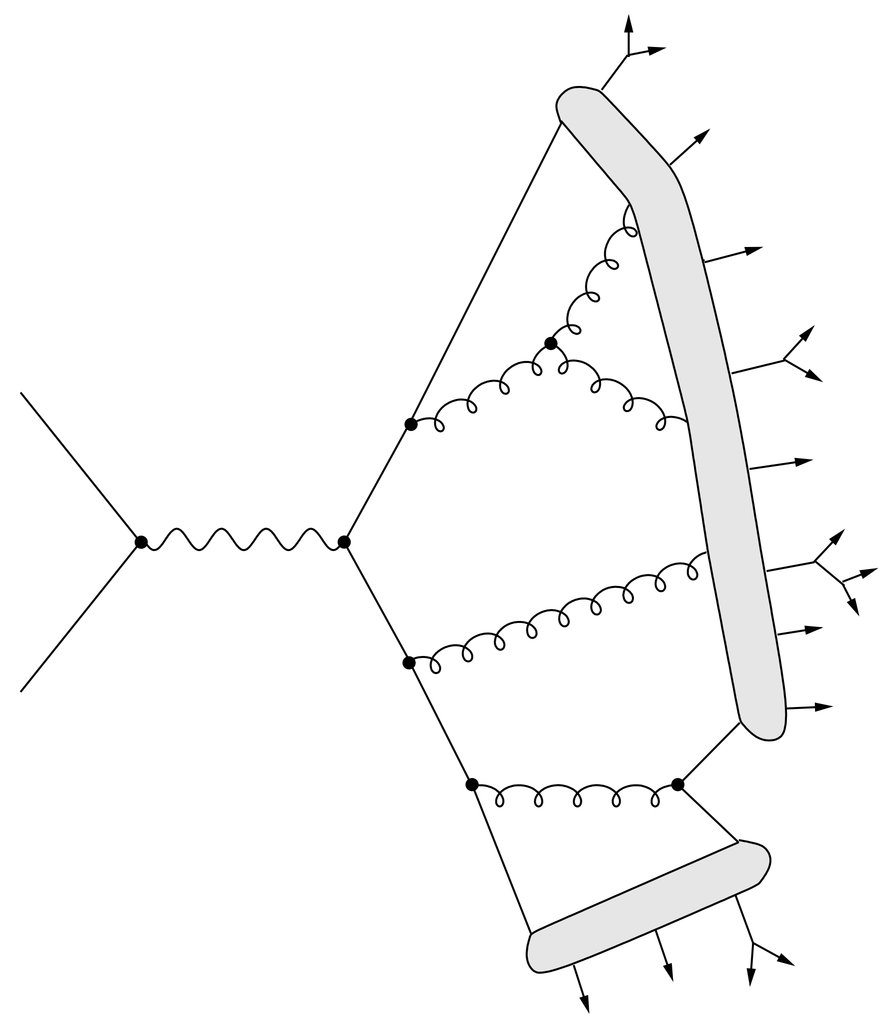
General ideas

- Local parton-hadron duality
  - Hadronization is long-distance process, involving small momentum transfers. Hence hadron-level flow of energy-momentum, flavour should follow parton level.
  - Results on spectra and multiplicities support this.
- Universal low-scale $\alpha_{\rm S}$
  - PT works well down to very low scales, $Q \sim 1$ GeV.
  - Assume $\alpha_{\rm S}(Q)$ defined (non-perturbatively) for all Q.
  - Good description of heavy quark spectra, event shapes.

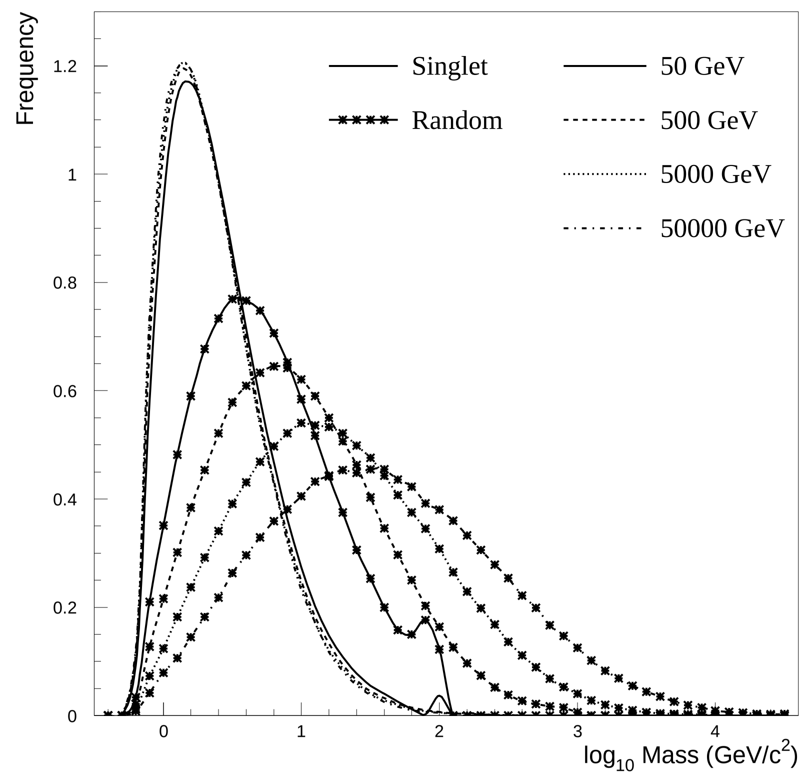
Specific models

- General ideas do not describe hadron formation. Main current models are cluster and string.



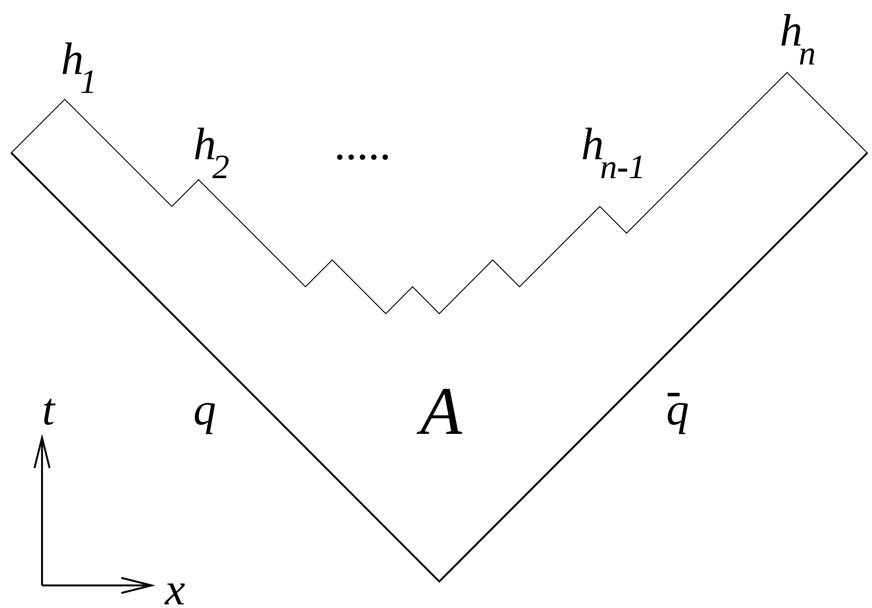


- Cluster (HERWIG)
  - Non-perturbative $g \to q\bar{q}$ splitting after parton shower.
  - Colour singlet $q\bar{q}$ clusters have lower mass due to preconfinement property of parton shower.



  - Clusters decay according to 2-hadron density of states.
  - Few parameters: natural p_T and heavy particle suppression
  - Problems with massive clusters, baryons, heavy quarks

- String (JETSET)
  - Uses string dynamics: colour string stretched between initial $q\bar{q}$ breaks up into hadrons via $q\bar{q}$ pair production.
  - String gives linear confinement potential, area law for matrix elements.
  - Gluons produced in shower give ‘kinks’ on string.



$$|M(q\bar{q} \to h_1 \cdots h_n)|^2 \propto e^{-bA}$$

  - Extra parameters for p_T and heavy particle suppression.
  - Some problems with baryons.

- String (UCLA)
  - Takes area law more seriously (mass suppression).
  - Extra parameters for p_T.
  - Some problems with baryons.

Meson yields in Z^0 decay

Particle	Multiplicity	HERWIG 5.9*	JETSET 7.4	UCLA 7.4	Expts
Charged	20.96(18)	20.95	20.95	20.88	ADLMO
$\pi^{\pm}$	17.06(24)	17.41	16.95	17.04	ADO
π^0	9.43(38)	9.97	9.59	9.61	ADLO
η	0.99(4)	1.02	1.00	0.78	ALO
$\rho(770)^0$	1.24(10)	1.18	1.50	1.17	AD
$\omega(782)$	1.09(09)	1.17	1.35	1.01	ALO
$\eta'(958)$	0.159(26)	0.097	0.155	0.121	ALO
$f_0(980)$	0.155(8)	0.111	$\sim$ 0.1	—	ADO
$a_0(980)^{\pm}$	0.14(6)	0.162	—	—	O
$\phi(1020)$	0.097(7)	0.104	0.194	0.132	ADO
$f_2(1270)$	0.188(14)	0.186	$\sim$ 0.2	—	ADO
$f_2'(1525)$	0.012(6)	0.021	—	—	D
$K^{\pm}$	2.26(6)	2.16	2.30	2.24	ADO
K^0	2.074(14)	1.98	2.07	2.06	ADLO
$K^*(892)^{\pm}$	0.718(44)	0.670	1.10	0.779	ADO
$K^*(892)^0$	0.759(32)	0.676	1.10	0.760	ADO
$K_2^*(1430)^0$	0.084(40)	0.111	—	—	DO
$D^{\pm}$	0.187(14)	0.161	0.174	0.196	ADO
D^0	0.462(26)	0.506	0.490	0.497	ADO
$D^*(2010)^{\pm}$	0.181(10)	0.151	0.242	0.227	ADO
$D_s^{\pm}$	0.131(20)	0.115	0.129	0.130	O
B^*	0.28(3)	0.201	0.260	0.254	D
$B_{u,d}^{**}$	0.118(24)	0.013	—	—	D
J/ψ	0.0054(4)	0.0018	0.0050	0.0050	ADLO
$\psi(3685)$	0.0023(5)	0.0009	0.0019	0.0019	DO
χ_{c1}	0.0086(27)	0.0001	—	—	DL

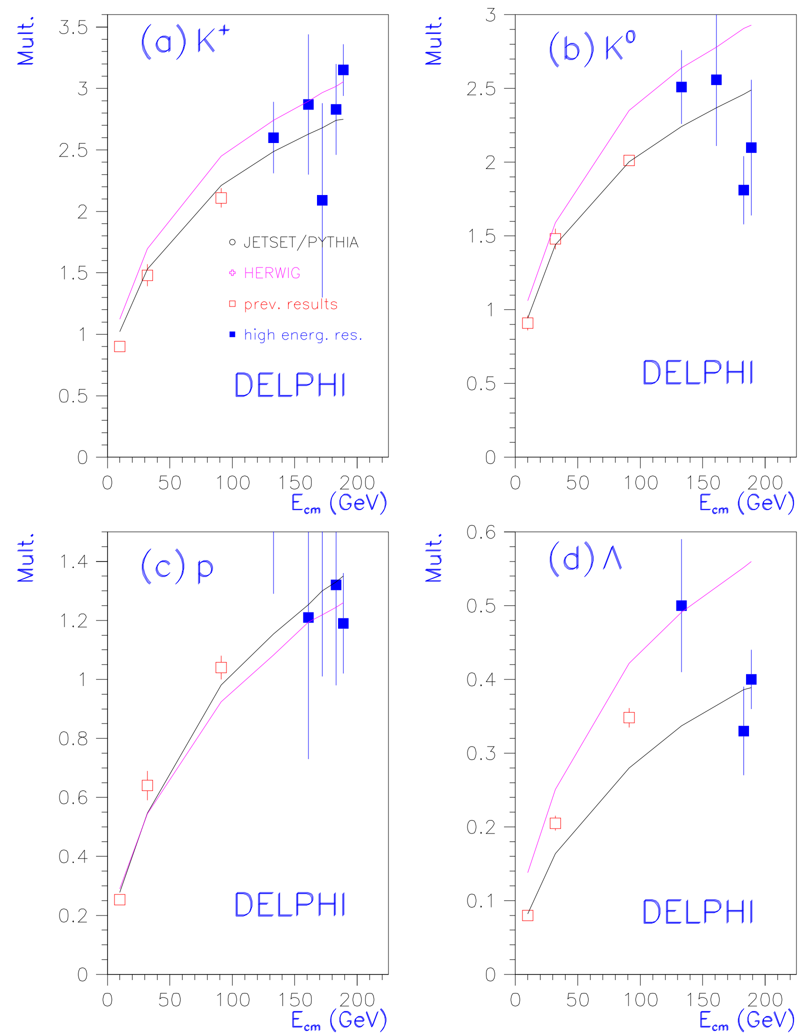
Baryon yields in Z^0 decay

Particle	Multiplicity	HERWIG 5.9*	JETSET 7.4	UCLA 7.4	Expts
p	1.04(4)	0.863	1.19	1.09	ADO
Δ^{++}	0.079(15)	0.156	0.189	0.139	D
	0.22(6)	0.156	0.189	0.139	O
Λ	0.399(8)	0.387	0.385	0.382	ADLO
$\Lambda(1520)$	0.0229(25)	—	—	—	DO
$\Sigma^{\pm}$	0.174(16)	0.154	0.140	0.118	DO
Σ^0	0.074(9)	0.068	0.073	0.074	ADO
$\Sigma^{\star\pm}$	0.0474(44)	0.111	0.074	0.074	ADO
Ξ^-	0.0265(9)	0.0493	0.0271	0.0220	ADO
$\Xi(1530)^0$	0.0058(10)	0.0205	0.0053	0.0081	ADO
Ω^-	0.0012(2)	0.0056	0.00072	0.0011	ADO
Λ_c^+	0.078(17)	0.0123	0.059	0.026	O

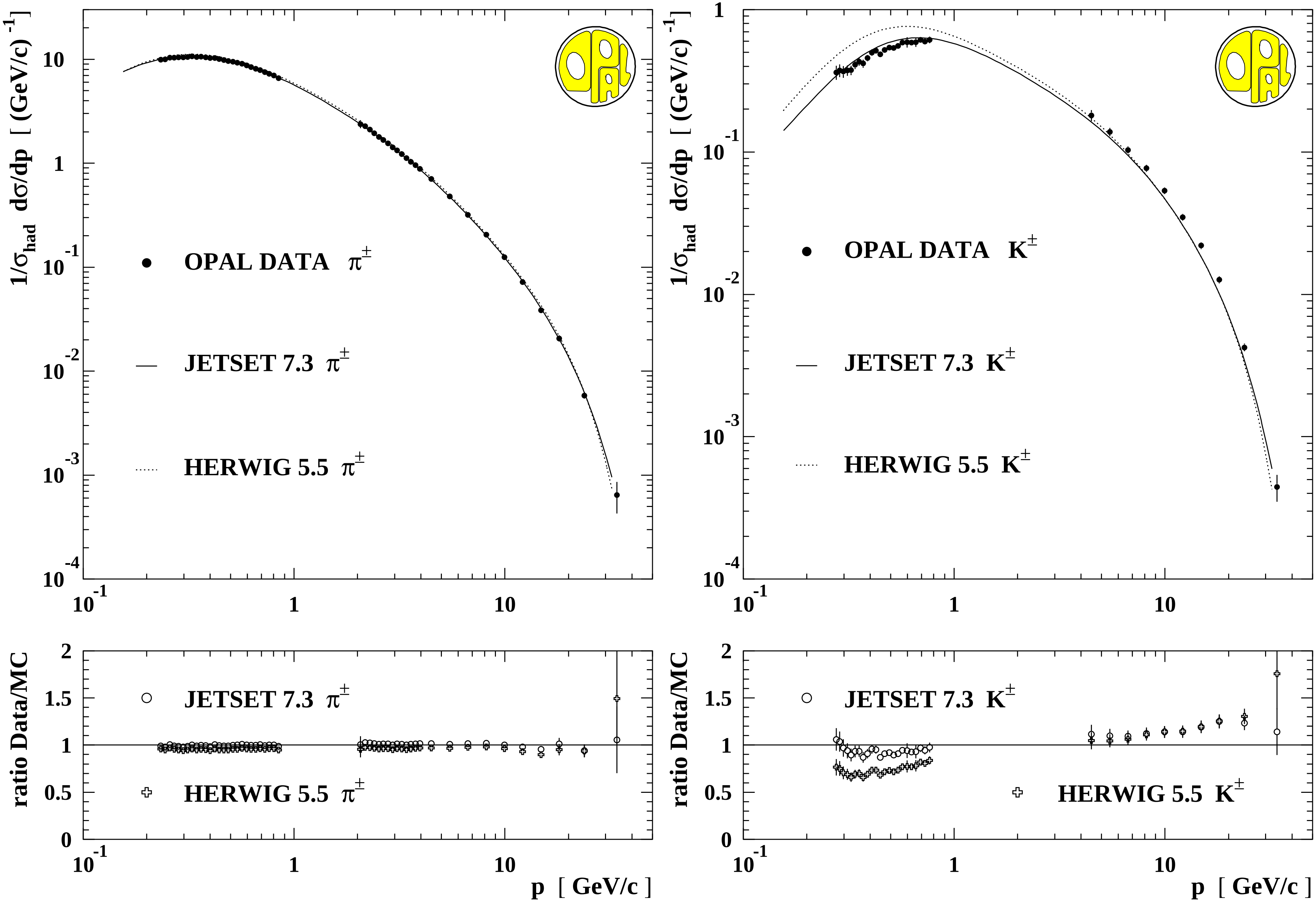
* ALEPH tuning with strangeness suppression 0.8 (G. Rudolph)

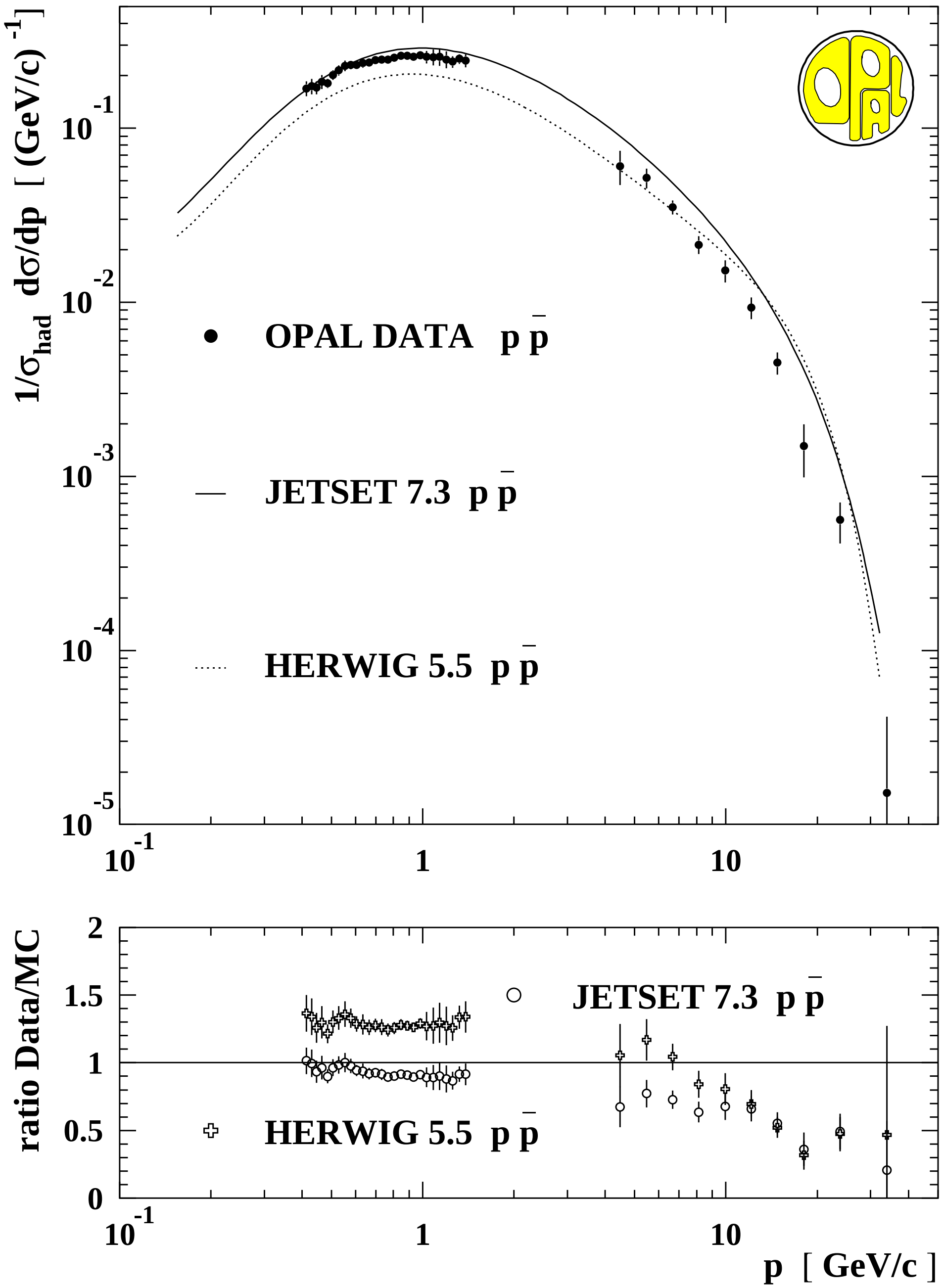
Yields at other energies

- Models in broad agreement with data at other energies.



Identified particle spectra





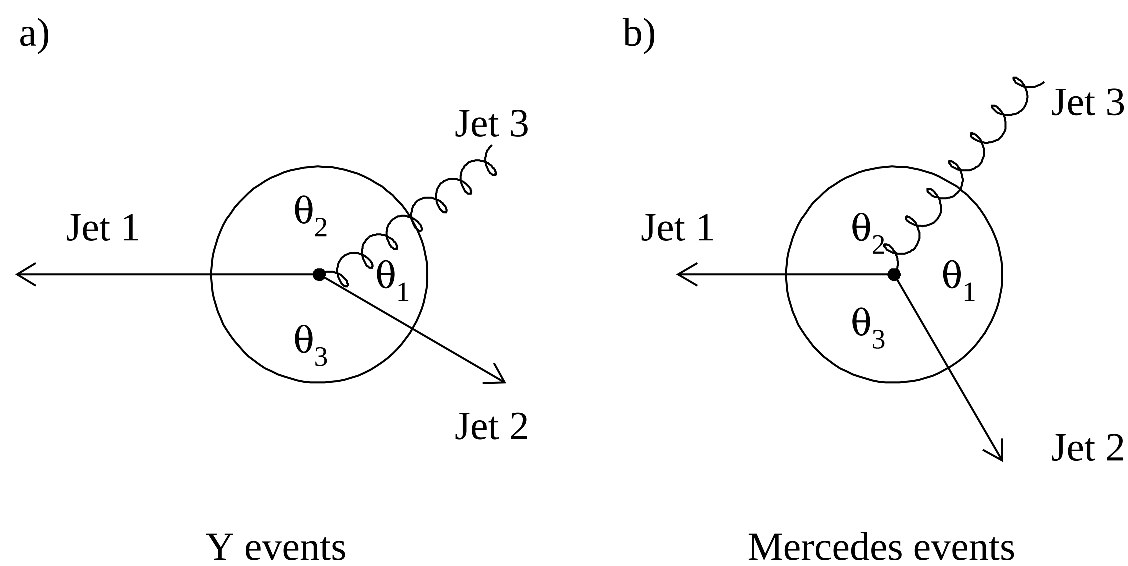
OPAL data on

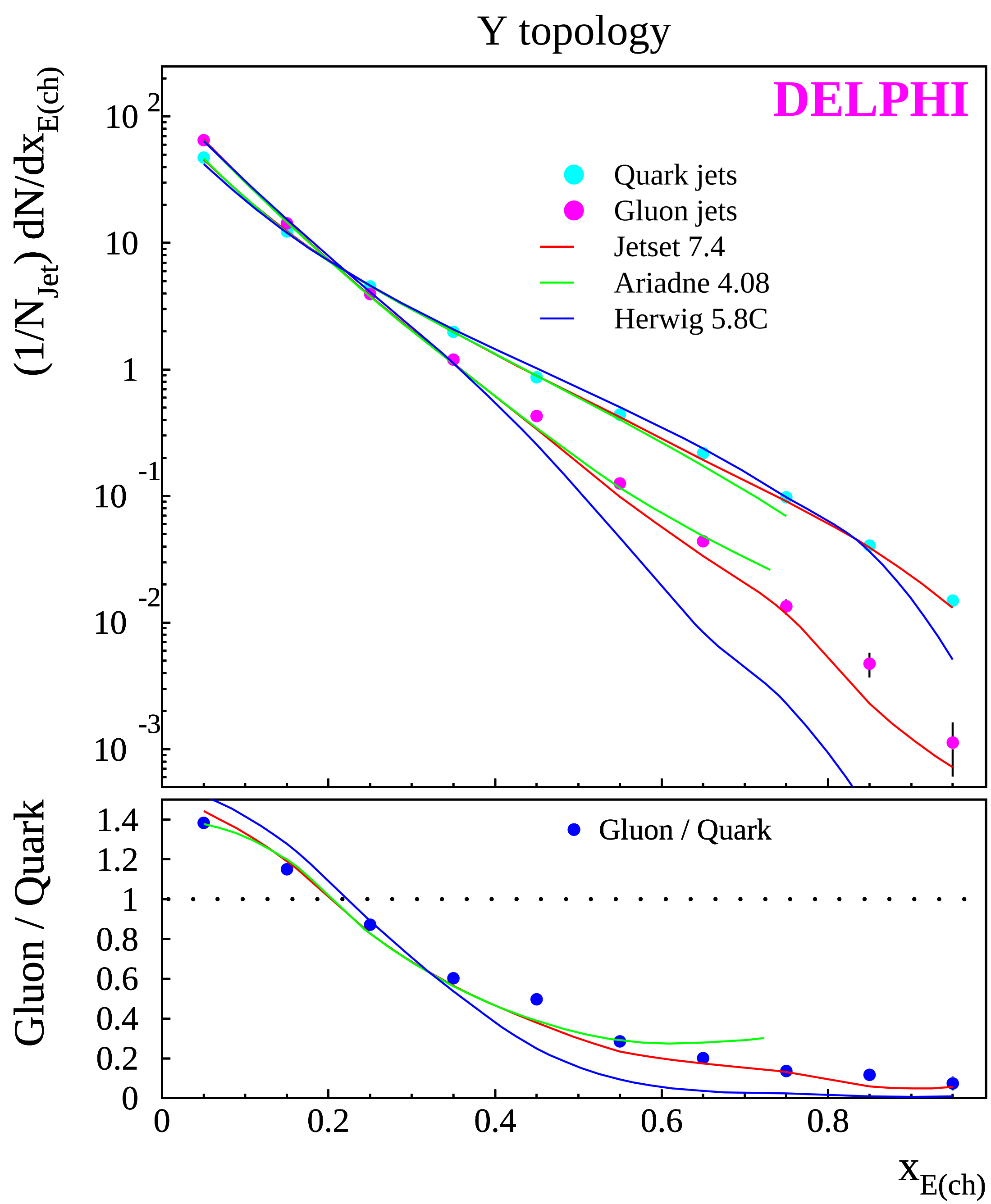
$Z^0 \to \pi^{\pm}, K^{\pm}, p/\bar{p}$.

Logarithmic plots

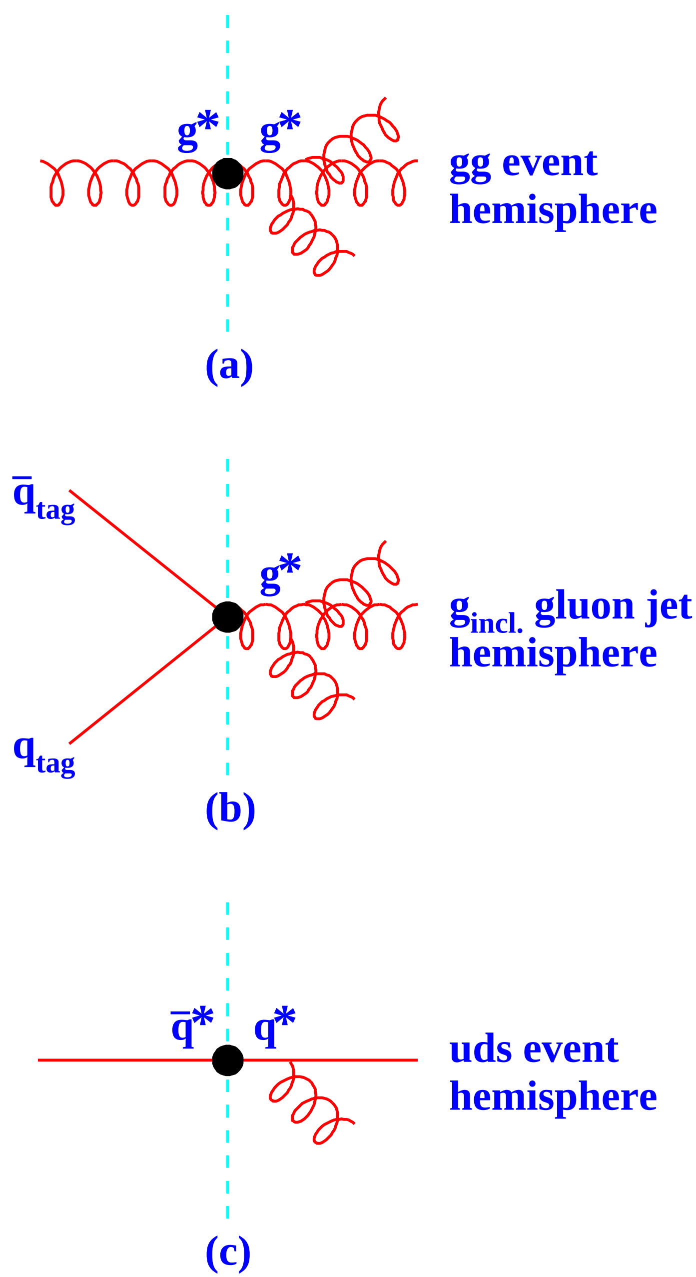
Quark and gluon jets

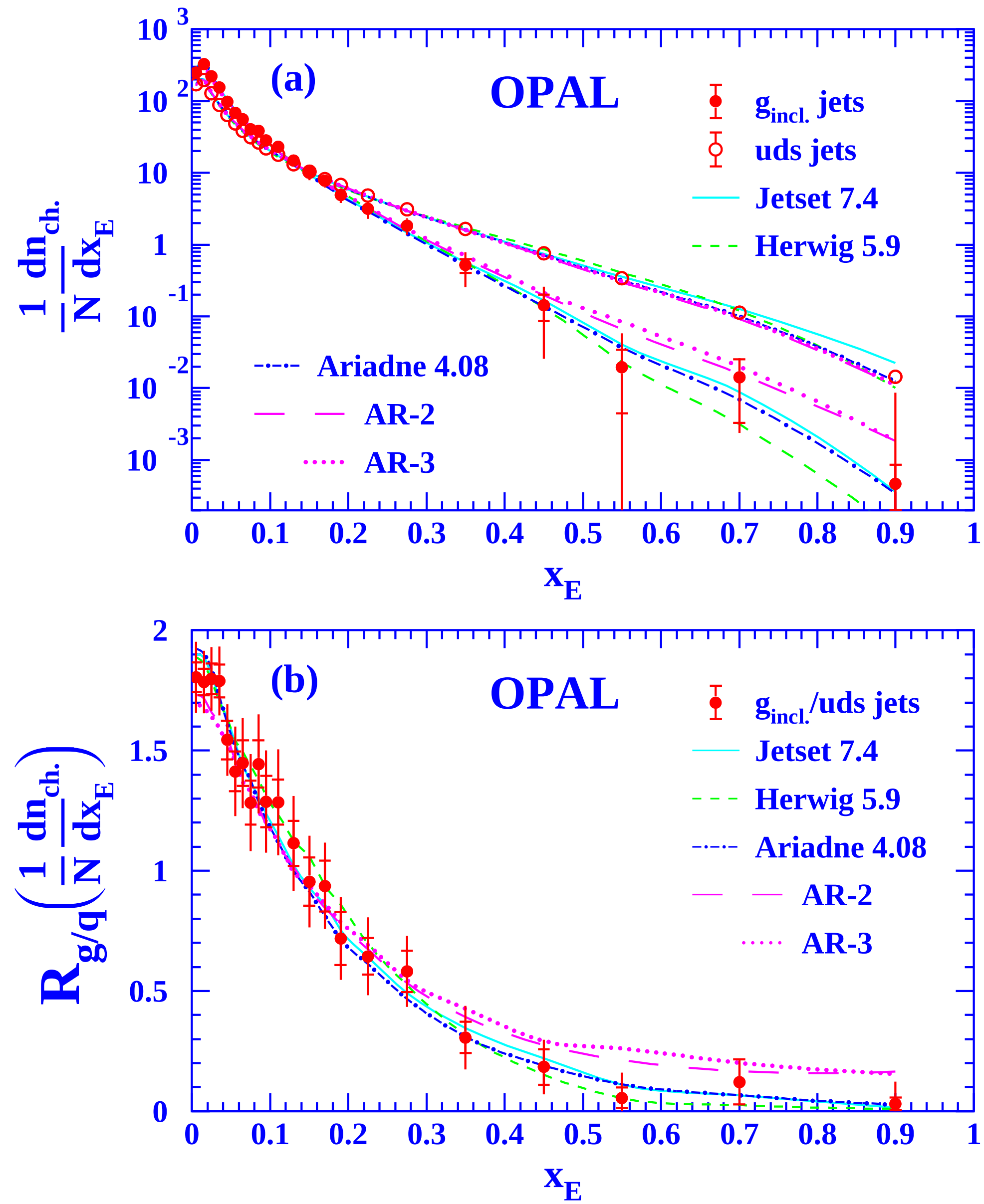
- DELPHI select gluon jets by anti-tagging heavy quark jets in ‘Y’ and ‘Mercedes’ three-jet events




Y topology
DELPHI
(1/N_Jet) dN/dx_E(ch)
Quark jets
Gluon jets
Jetset 7.4
Ariadne 4.08
Herwig 5.8C
Gluon / Quark
x_E(ch)

- OPAL select gluon jets recoiling against tagged b-jets in same hemisphere. Monte Carlo studies indicate that such jets should be similar to those emitted by a point source of gluon pairs (e.g. a $^1S_0$ $Q\bar{Q}$ state).





- Gluon jets have softer fragmentation functions than light quark jets, and higher charged multiplicity.

Summary of Lecture 4

- Fragmentation functions show expected scaling violation.
- Small-x fragmentation shows coherence effects:
  - Peak in $\ln(1/x)$ moves slowly ($\sim \frac{1}{4}\ln s$)
  - Multiplicity increases slowly ($\sim \exp[c\sqrt{\ln s}]$).
- Cluster and string hadronization models give good overall description of data.
- Gluon jets have softer fragmentation, higher multiplicity than quark jets.