QCD Phenomenology at High Energy

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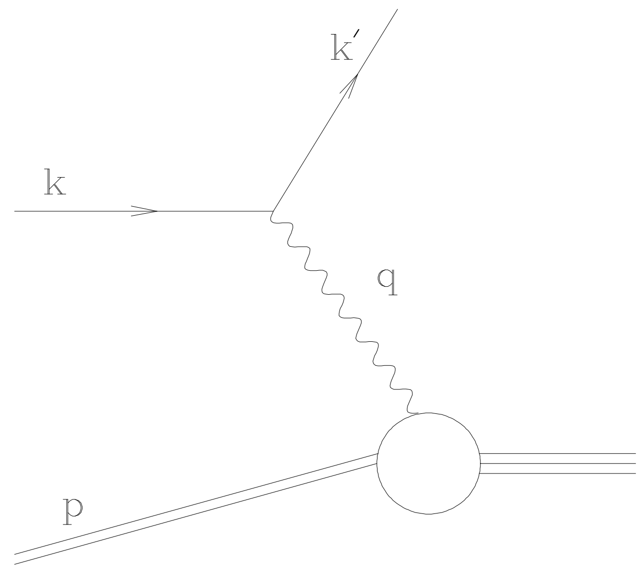
Lecture 5: DIS & hadron-hadron processes

- Deep inelastic scattering
  - Scaling violation
  - Small x
- Hadron-hadron processes
  - Lepton pair production
  - Jet production
  - Heavy quark production

Deep inelastic scattering

- Consider lepton-proton scattering via exchange of virtual photon:



- Standard variables are:

$$x = \frac{-q^2}{2p \cdot q} = \frac{Q^2}{2M(E-E')}$$
$$y = \frac{q \cdot p}{k \cdot p} = 1 - \frac{E'}{E}$$

where $Q^2 = -q^2 > 0$, $M^2 = p^2$ and energies refer to target rest frame.

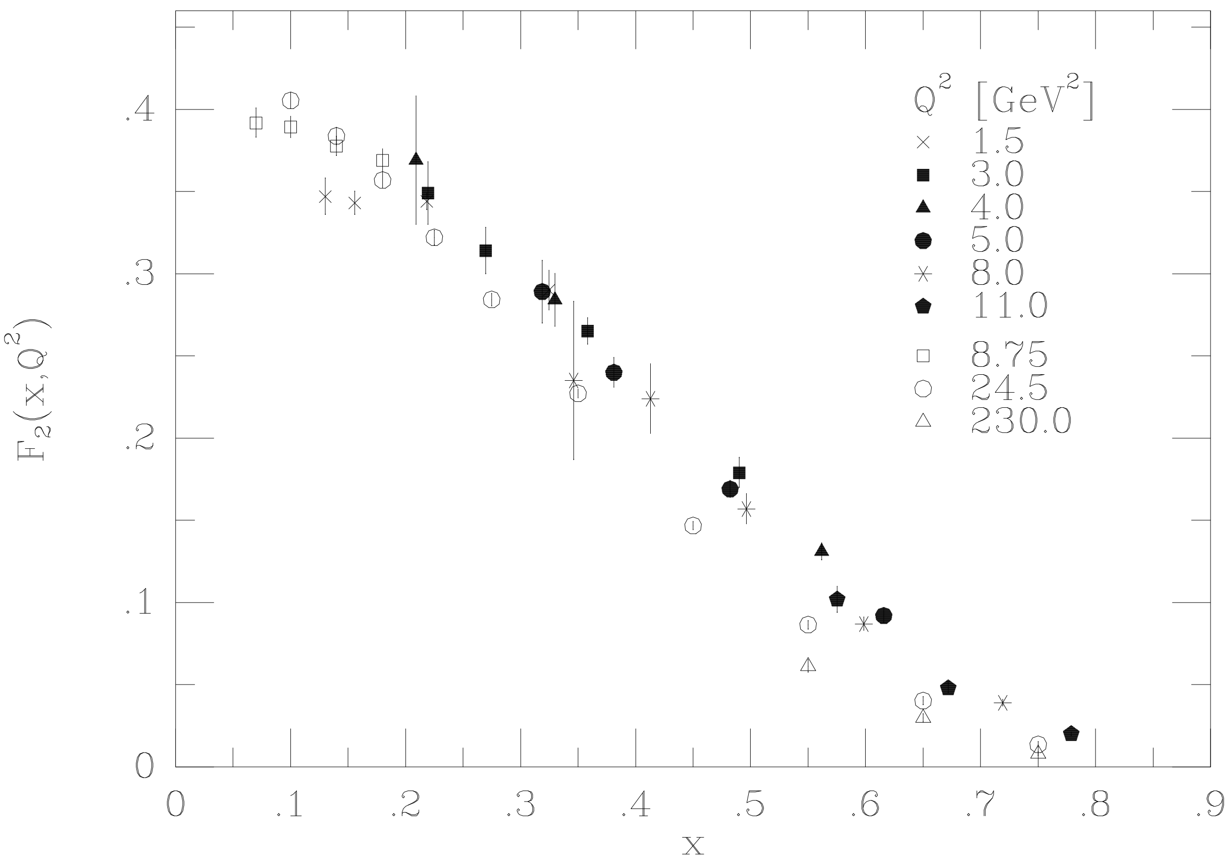
- Elastic scattering has $(p+q)^2 = M^2$, i.e. $x = 1$. Hence deep inelastic scattering (DIS) means $Q^2 \gg M^2$ and $x < 1$.

- Structure functions $F_i(x, Q^2)$ parametrise target structure as 'seen' by virtual photon. Defined in terms of cross section

$$\frac{d^2\sigma}{dxdy} = \frac{8\pi\alpha^2 ME}{Q^4}\left[\left(\frac{1+(1-y)^2}{2}\right)2xF_1 + (1-y)(F_2 - 2xF_1) - (M/2E)xyF_2\right].$$

- Bjorken limit is Q^2, $p \cdot q \to \infty$ with x fixed. In this limit structure functions obey approximate Bjorken scaling law, i.e. depend only on dimensionless variable x:

$$F_i(x, Q^2) \longrightarrow F_i(x).$$



- Figure shows F_2 structure function for proton target. Although Q^2 varies by two orders of magnitude, in first approximation data lie on universal curve.
- Bjorken scaling implies that virtual photon is scattered by *pointlike constituents* (partons) — otherwise structure functions would depend on ratio Q/Q_0, with $1/Q_0$ a length scale characterizing size of constituents.

- Parton model of DIS is formulated in a frame where target proton is moving very fast — *infinite momentum frame.*
  - Suppose that, in this frame, photon scatters from pointlike quark with fraction ξ of proton's momentum. Since $(\xi p + q)^2 = m_q^2 \ll Q^2$, we must have $\xi = Q^2/2p \cdot q = x$.
  - In terms of Mandelstam variables $\hat{s}, \hat{t}, \hat{u}$, spin-averaged matrix element squared for massless $eq \to eq$ scattering (related by crossing to $e^+e^- \to q\bar{q}$) is

$$\overline{\sum}|\mathcal{M}|^2 = 2e_q^2 e^4 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$$

    where $\overline{\sum}$ denotes average (sum) over initial (final) colours and spins.
  - In terms of DIS variables, $\hat{t} = -Q^2$, $\hat{u} = \hat{s}(y-1)$ and $\hat{s} = Q^2/xy$. Differential cross section is then

$$\frac{d^2\hat{\sigma}}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}[1 + (1-y)^2]\frac{1}{2}e_q^2 \delta(x - \xi).$$

  - From structure function definition (neglecting M)

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left\{[1 + (1-y)^2]F_1 + \frac{(1-y)}{x}(F_2 - 2xF_1)\right\}.$$

- Hence structure functions for scattering from parton with momentum fraction ξ is

$$\hat{F}_2 = x e_q^2 \delta(x-\xi) = 2x\hat{F}_1 \ .$$

- Suppose probability that quark q carries momentum fraction between ξ and $\xi + d\xi$ is $q(\xi)\, d\xi$. Then

$$\begin{aligned} F_2(x) &= \sum_q \int_0^1 d\xi \; q(\xi)\; x e_q^2 \delta(x-\xi) \\ &= \sum_q e_q^2 x q(x) = 2xF_1(x) \ . \end{aligned}$$

- Relationship $F_2 = 2xF_1$ (Callan-Gross relation) follows from spin-$\frac{1}{2}$ property of quarks ($F_1 = 0$ for spin-0).

- Proton consists of three valence quarks (uud), which carry its electric charge and baryon number, and infinite sea of light $q\bar{q}$ pairs.

- Probed at scale Q, sea contains all quark flavours with $m_q \ll Q$. Thus at $Q \sim 1$ GeV we expect

$$F_2^{em}(x) \simeq \frac{4}{9} x[u(x) + \bar{u}(x)] + \frac{1}{9} x[d(x) + \bar{d}(x) + s(x) + \bar{s}(x)]$$

where

$$\begin{aligned} u(x) &= u_V(x) + \bar{u}(x) \\ d(x) &= d_V(x) + \bar{d}(x) \\ s(x) &= \bar{s}(x) \end{aligned}$$

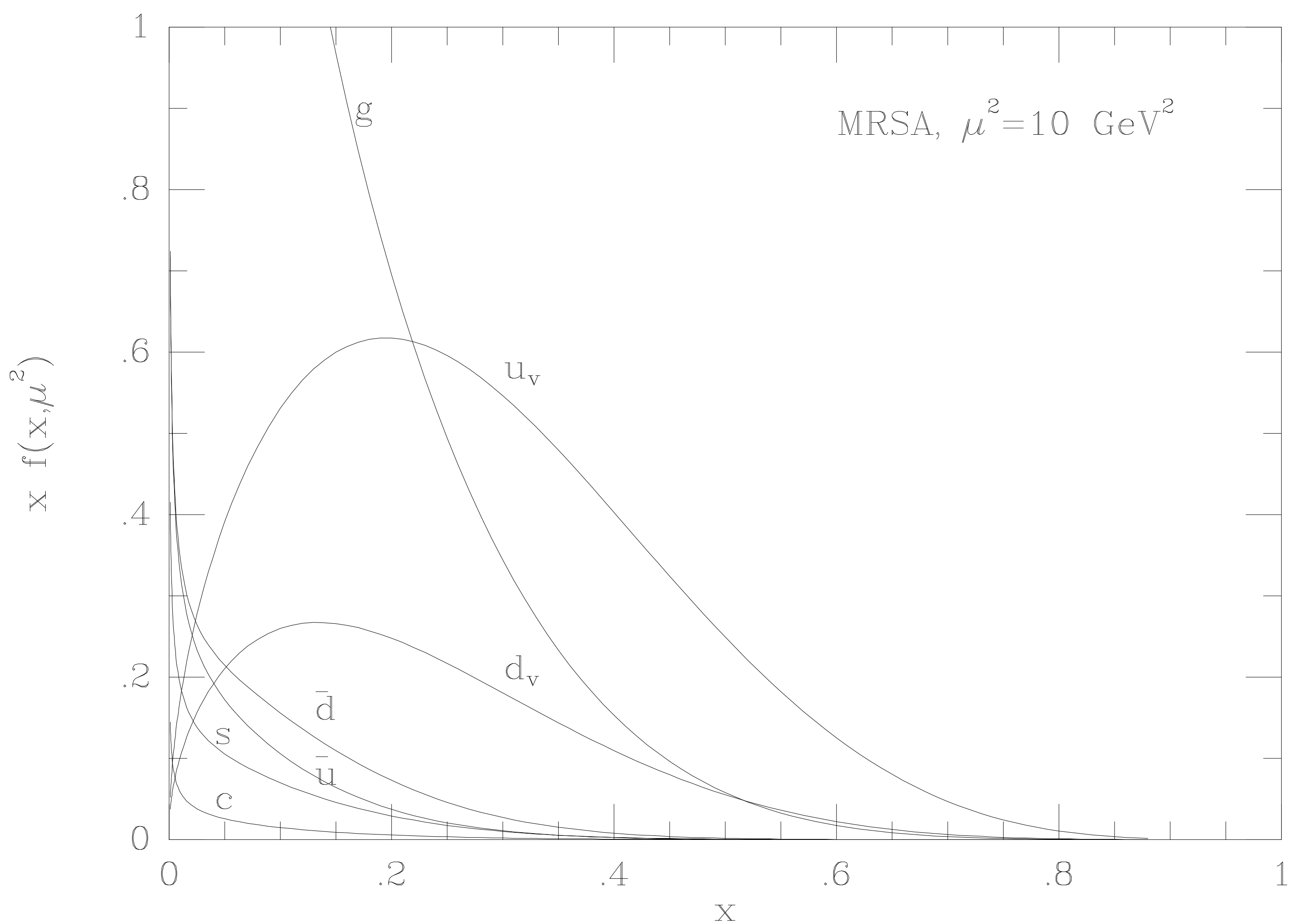
with sum rules

$$\int_0^1 dx\, u_V(x) = 2\,, \quad \int_0^1 dx\, d_V(x) = 1\,.$$

- Experimentally one finds

$$\sum_q \int_0^1 dx\, x[q(x) + \bar{q}(x)] \simeq 0.5\,.$$

Thus quarks only carry about 50% of proton's momentum. Rest is carried by *gluons*. Although not directly measured in DIS, gluons participate in other hard scattering processes such as large-p_T jet and prompt photon production.



- Figure shows typical set of parton distributions extracted from fits to DIS data, at $Q^2 = 10$ GeV^2.

Scaling violation

- Bjorken scaling is not exact. Structure functions decrease at large x and grow at small x with increasing Q^2. This is due to Q^2 dependence of parton distributions, considered earlier. In present notation, they satisfy DGLAP evolution equations of form

$$t\frac{\partial}{\partial t}q(x,t) = \frac{\alpha_S(t)}{2\pi}\int_x^1 \frac{dz}{z}P(z)q\left(\frac{x}{z},t\right) \equiv \frac{\alpha_S(t)}{2\pi}P \otimes q$$

where P is $q \to qg$ splitting function.

- Taking into account other types of parton branching that can occur in addition to $q \to qg$, we obtain coupled evolution equations

$$\begin{aligned} t\frac{\partial q_i}{\partial t} &= \frac{\alpha_S(t)}{2\pi}\left[P_{qq} \otimes q_i + P_{qg} \otimes g\right] \\ t\frac{\partial \bar{q}_i}{\partial t} &= \frac{\alpha_S(t)}{2\pi}\left[P_{qq} \otimes \bar{q}_i + P_{qg} \otimes g\right] \\ t\frac{\partial g}{\partial t} &= \frac{\alpha_S(t)}{2\pi}\left[P_{gq} \otimes \sum(q_i + \bar{q}_i) + P_{gg} \otimes g\right] . \end{aligned}$$

- Lowest-order splitting functions were derived in Lecture 2. More generally they are power series in α_S, same for jet fragmentation (timelike branching) and deep inelastic scattering (spacelike branching) in leading order, but differing in higher orders. Consequently, behaviour of structure functions at small x is different from that derived in Lecture 4 for jet fragmentation functions.
- For the present, we concentrate on larger x values ($x \gtrsim 0.01$), where PT expansion converges better.
- Recall solution of evolution equations for flavour non-singlet combinations V, e.g. $V = q_i - \bar{q}_i$ or $q_i - q_j$. Mixing with gluons drops out and

$$t\frac{\partial}{\partial t}V(x,t) = \frac{\alpha_S(t)}{2\pi}P_{qq} \otimes V\ .$$

Taking moments (Mellin transform)

$$\tilde{V}(N,t) = \int_0^1 dx\, x^{N-1}\, V(x,t)\ ,$$

we find

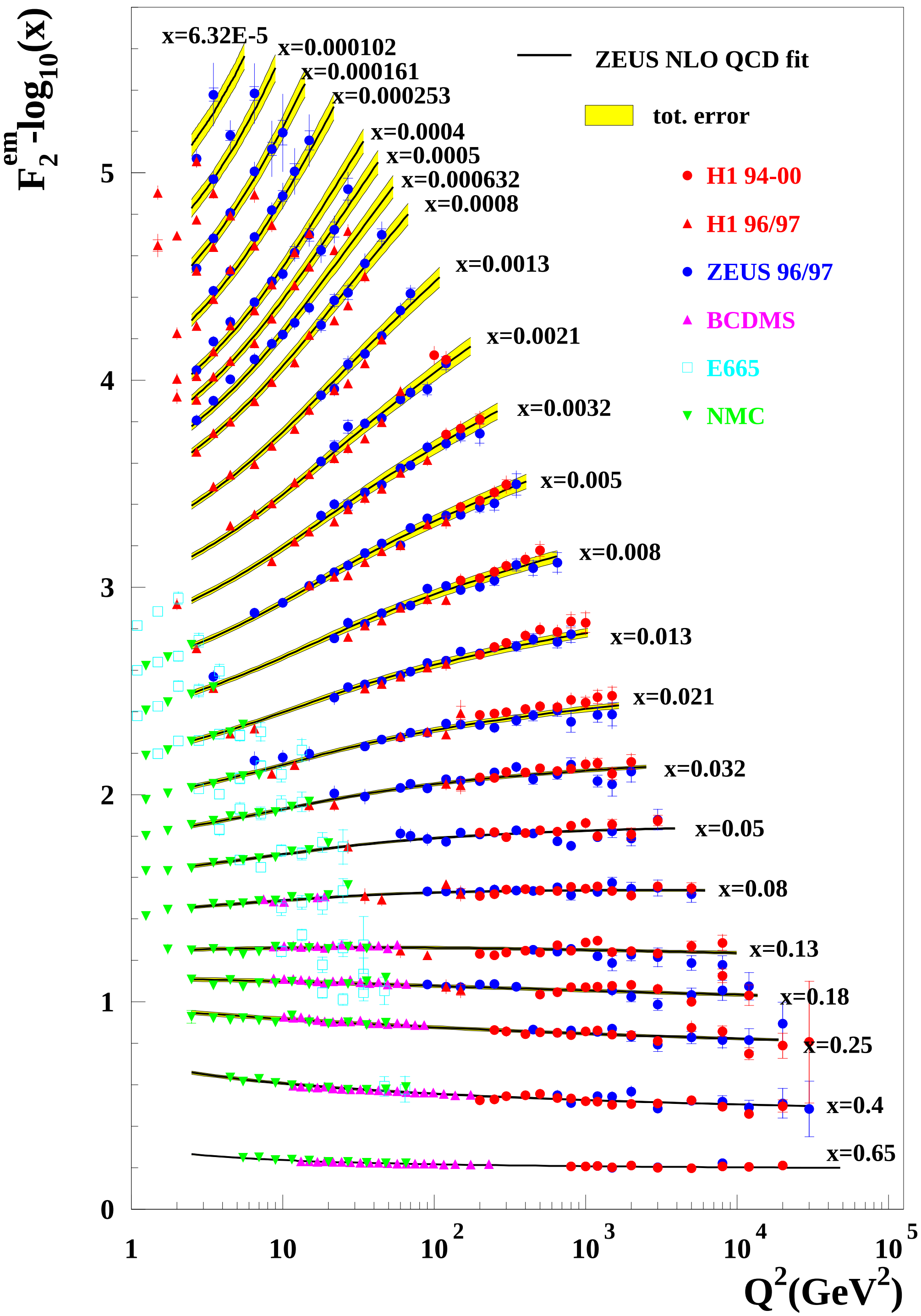
$$t\frac{\partial}{\partial t}\tilde{V}(N,t) = \frac{\alpha_S(t)}{2\pi}\gamma_{qq}^{(0)}(N)\,\tilde{V}(N,t)$$

where $\gamma_{qq}^{(0)}(N)$ is Mellin transform of $P_{qq}^{(0)}$. Solution is

$$\tilde{V}(N,t) = \tilde{V}(N,t_0)\left(\frac{\alpha_S(t_0)}{\alpha_S(t)}\right)^{d_{qq}(N)}$$

where $d_{qq}(N) = \gamma_{qq}^{(0)}(N)/2\pi b$.

- Now $d_{qq}(1) = 0$ and $d_{qq}(N) < 0$ for $N \geq 2$. Thus as t increases V *decreases* at large x and *increases* at small x. Physically, this is due to increase in the phase space for gluon emission by quarks as t increases, leading to loss of momentum. This is clearly visible in data.



F_2^{em}-log$_{10}$(x)
Q^2(GeV^2)
ZEUS NLO QCD fit
tot. error
H1 94-00
H1 96/97
ZEUS 96/97
BCDMS
E665
NMC
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65

- For flavour-singlet combination, define $\Sigma = \sum_i (q_i + \bar{q}_i)$. Then we obtain

$$\begin{aligned} t\frac{\partial \Sigma}{\partial t} &= \frac{\alpha_S(t)}{2\pi}\left[P_{qq} \otimes \Sigma + 2N_f P_{qg} \otimes g\right] \\ t\frac{\partial g}{\partial t} &= \frac{\alpha_S(t)}{2\pi}\left[P_{gq} \otimes \Sigma + P_{gg} \otimes g\right] . \end{aligned}$$

- Thus flavour-singlet quark distribution Σ mixes with gluon distribution g: evolution equation for moments has matrix form

$$t\frac{\partial}{\partial t}\begin{pmatrix} \tilde{\Sigma} \\ \tilde{g} \end{pmatrix} = \begin{pmatrix} \gamma_{qq} & 2N_f\gamma_{qg} \\ \gamma_{gq} & \gamma_{gg} \end{pmatrix}\begin{pmatrix} \tilde{\Sigma} \\ \tilde{g} \end{pmatrix}$$

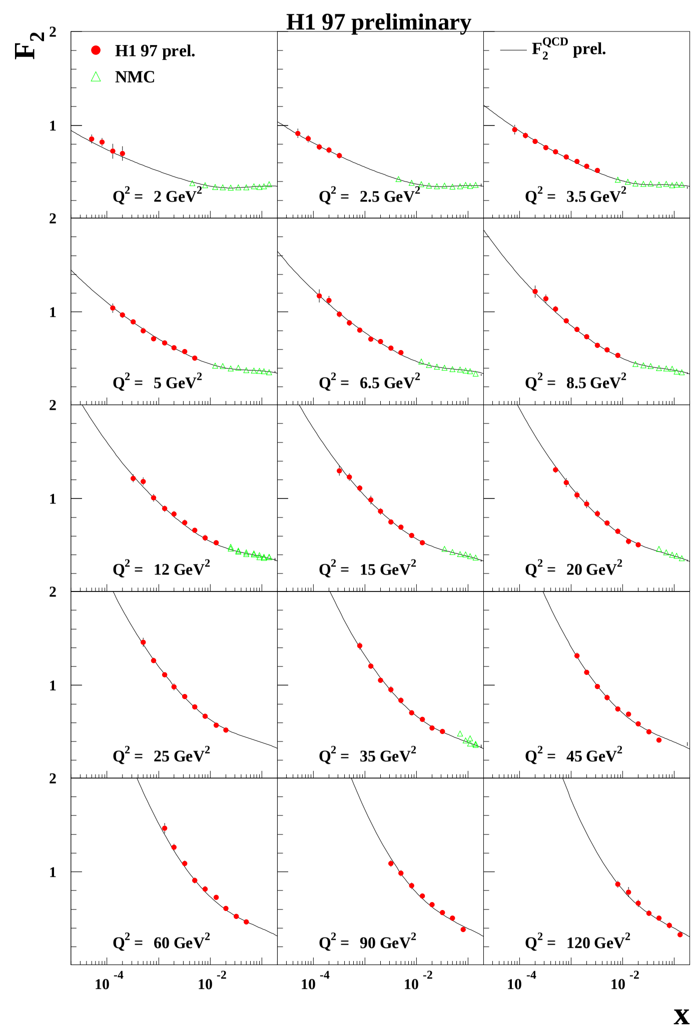
- Singlet anomalous dimension matrix has two real eigenvalues $\gamma_\pm$ given by

$$\gamma_\pm = \tfrac{1}{2}\left[\gamma_{gg} + \gamma_{qq} \pm \sqrt{(\gamma_{gg} - \gamma_{qq})^2 + 8N_f\gamma_{gq}\gamma_{qg}}\right] .$$

- Expressing $\tilde{\Sigma}$ and $\tilde{g}$ as linear combinations of eigenvectors $\tilde{\Sigma}_+$ and $\tilde{\Sigma}_-$, we find they evolve as superpositions of terms of above form with $\gamma_\pm$ in place of γ_{qq}.

Small x

- At small x, corresponding to $N \to 1$, we find $\gamma_+ \to \gamma_{gg} \to \infty$, $\gamma_- \to \gamma_{qq} \to 0$. Therefore structure functions grow rapidly at small x.



- Higher-order corrections also become large in this region:

$$
\begin{aligned}
\gamma_{qq}^{(1)}(N) &\to \frac{40C_F N_f T_R}{9(N-1)} \\
\gamma_{qg}^{(1)}(N) &\to \frac{40C_A T_R}{9(N-1)} \\
\gamma_{gq}^{(1)}(N) &\to \frac{9C_F C_A - 40C_F N_f T_R}{9(N-1)} \\
\gamma_{gg}^{(1)}(N) &\to \frac{(12C_F - 46C_A) N_f T_R}{9(N-1)} \,.
\end{aligned}
$$

- Thus we find

$$
\begin{aligned}
\gamma_+ &\to \frac{2C_A}{N-1}\frac{\alpha_S}{2\pi}\left[1 + \frac{(26C_F - 23C_A)N_f}{18C_A}\frac{\alpha_S}{2\pi} + \ldots\right] \\
&= \frac{2C_A}{N-1}\frac{\alpha_S}{2\pi}\left[1 - 0.64N_f\frac{\alpha_S}{2\pi} + \ldots\right]
\end{aligned}
$$

where neglected terms are either non-singular at $N = 1$ or higher-order in α_S. Thus NLO corrections is relatively small.

- In general one finds (BFKL) that for small x ($N \to 1$)

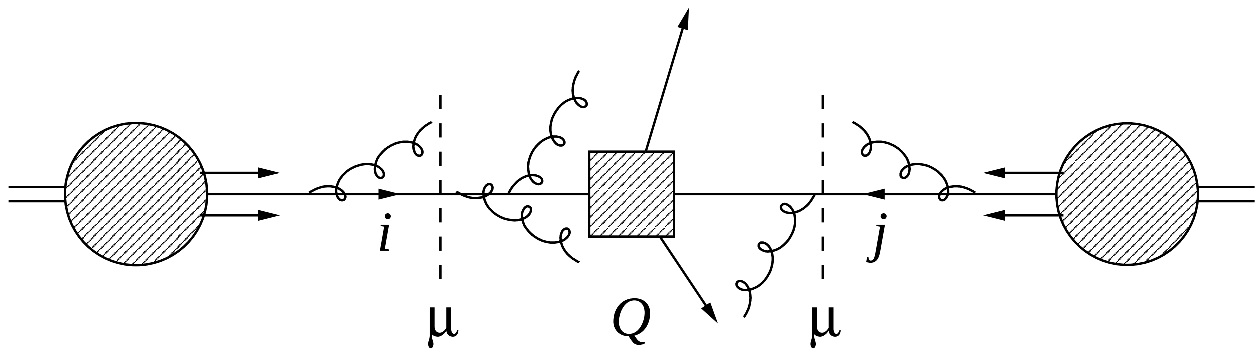
$$\gamma_+ \to \sum_{n=1}^{\infty} \sum_{m=0}^{n} \frac{\gamma^{(n,m)}}{(N-1)^m} \left(\frac{\alpha_S}{2\pi}\right)^n$$

where it happens that $\gamma^{(2,2)}$ (and $\gamma^{(3,3)}$) are zero. This is much less singular than the timelike (jet fragmentation) case, where we saw that $m \leq 2n-1$ and $\gamma^{(2,3)}$ and $\gamma^{(3,5)}$ are not zero.

  - This is probably why significant deviations from NLO QCD have not yet been seen in DIS at small x, whereas they are obvious in jet fragmentation.
  - Crucial difference is coherence (angular ordering), which we saw suppresses soft gluon emission in low-x fragmentation, but does not suppress low-x spacelike branching in DIS.

Hadron-hadron processes

- In hard hadron-hadron scattering, constituent partons from each incoming hadron interact at short distance (large momentum transfer Q^2).



- For hadron momenta P_1, P_2 ($S = 2P_1 \cdot P_2$), form of cross section is

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 D_i(x_1, \mu^2) D_j(x_2, \mu^2) \hat{\sigma}_{ij}(\hat{s} = x_1 x_2 S, \alpha_S(\mu^2), Q^2/\mu^2)$$

where μ^2 is factorization scale and $\hat{\sigma}_{ij}$ is subprocess cross section for parton types i, j.

❖ Notice that factorization scale is in principle arbitrary: affects only what we call part of subprocess or part of initial-state evolution (parton shower).

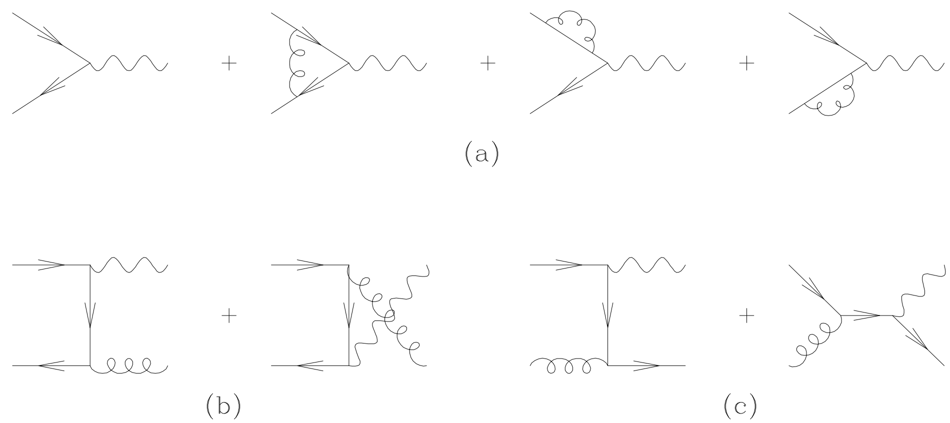
❖ Unlike e^+e^- or ep, we may have interaction between spectator partons, leading to *soft underlying event* and/or *multiple hard scattering*.

Lepton pair production

- Inverse of $e^+e^- \to q\bar{q}$ is Drell-Yan process. At $\mathcal{O}(\alpha_S^0)$, mass distribution of lepton pair is given by

$$\frac{d\hat{\sigma}}{dM^2}(q\bar{q} \to \gamma^* \to l^+l^-) = \frac{4\pi\alpha^2}{\hat{s}}\frac{1}{3}Q_q^2\,\delta(M^2 - \hat{s})$$

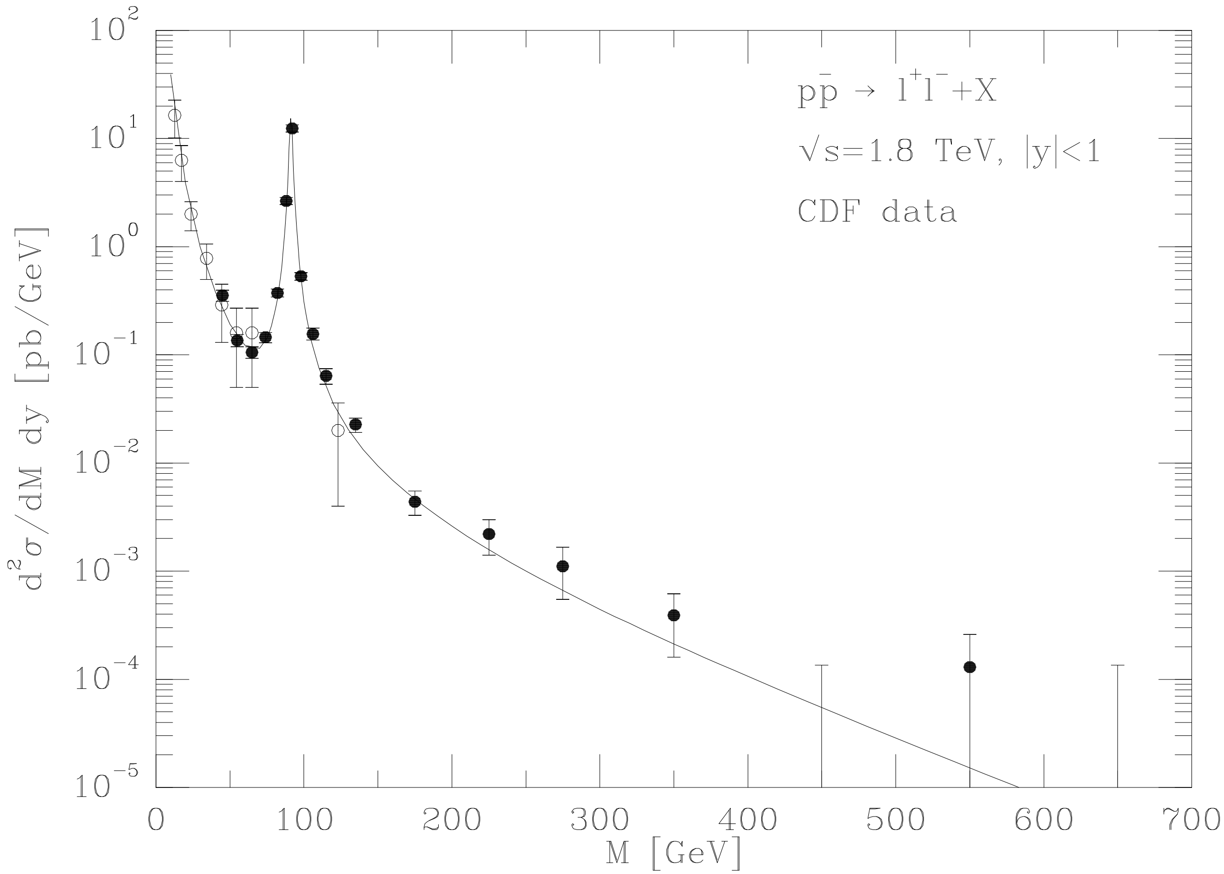
  - ❖ Factor of 1/3 instead of 3 because of *average* over colours of incoming q.



  - ❖ In higher orders *vertex corrections* (a) have $M^2 = \hat{s}$, *gluon emission* (b) and *QCD Compton* (c) diagrams give $M^2 < \hat{s}$.

- Rapidity of lepton pair in overall c.m. frame is ($p^\mu = p_1^\mu + p_2^\mu$)

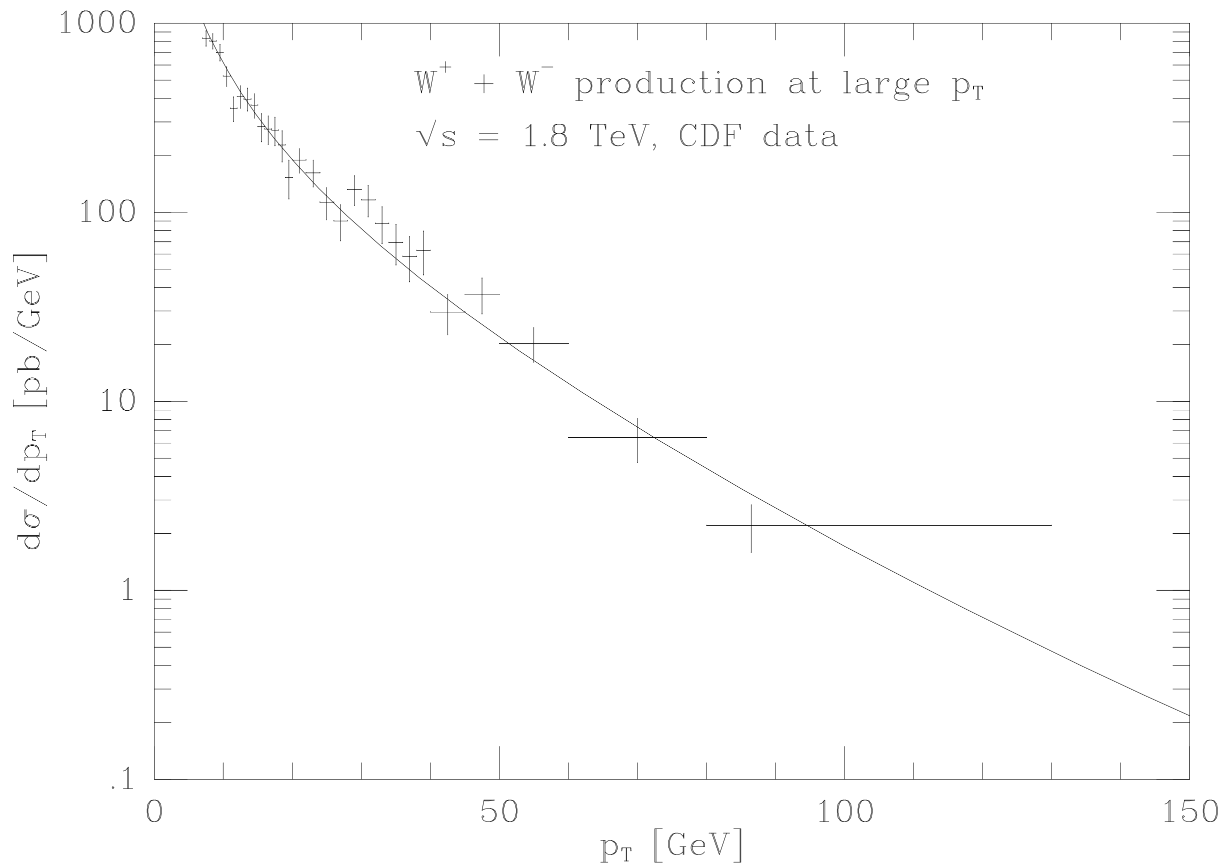
$$y \equiv \frac{1}{2}\ln\left(\frac{p^0 + p^3}{p^0 - p^3}\right) = \frac{1}{2}\ln\left(\frac{x_1}{x_2}\right)$$



- $W^\pm$ bosn production is similar, except sensitive to different parton distributions, e.g.

$$u\bar{d} \to W^+ \to l^+\nu_l$$

- Transverse momentum of lepton pair, p_T, measures net transverse momentum of emitted partons plus any *intrinsic* p_T:

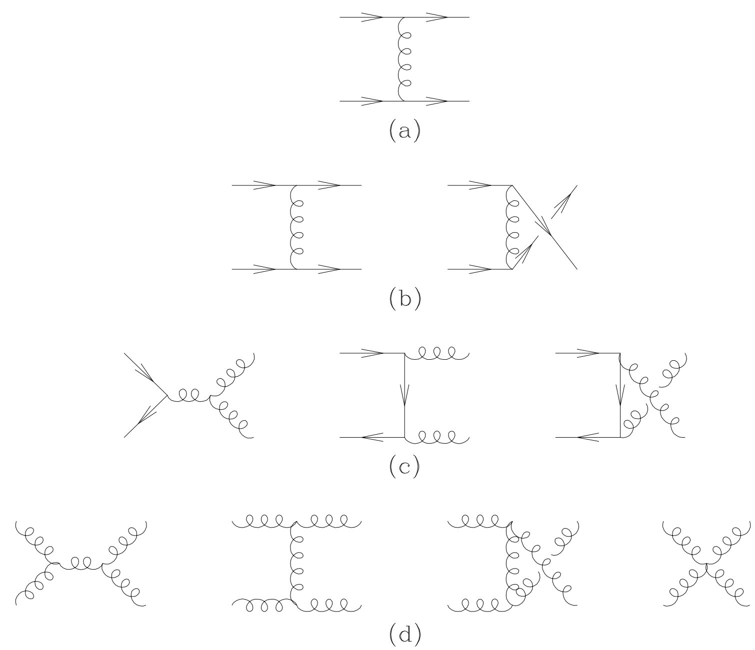


Jet production

- Lowest-order subprocess for purely hadronic jet production is $2 \to 2$ scattering $p_1 + p_2 \to p_3 + p_4$

$$\frac{E_3 E_4 d^6\hat{\sigma}}{d^3\boldsymbol{p}_3 d^3\boldsymbol{p}_4} = \frac{1}{32\pi^2\hat{s}}\overline{\sum}|\mathcal{M}|^2\,\delta^4(p_1+p_2-p_3-p_4)\,.$$

- Many processes even at $\mathcal{O}(\alpha_S^2)$:



- For single-jet inclusive cross section, integrate over one outgoing momentum:

$$\frac{E d^3\hat\sigma}{d^3\boldsymbol{p}} = \frac{d^3\hat\sigma}{d^2\boldsymbol{p}_T dy} \longrightarrow \frac{1}{2\pi E_T}\frac{d^3\hat\sigma}{dE_T\, d\eta} = \frac{1}{16\pi^2\hat s}\overline{\sum}|\mathcal{M}|^2\,\delta(\hat s+\hat t+\hat u)$$

where (neglecting jet mass)

$$E_T \equiv E\sin\theta = |\boldsymbol{p}_T|\,, \quad \eta \equiv -\ln\tan(\theta/2) \;=\; y\,.$$

- Jets in hadron-hadron usually defined using cone algorithm: combine all hadrons h with

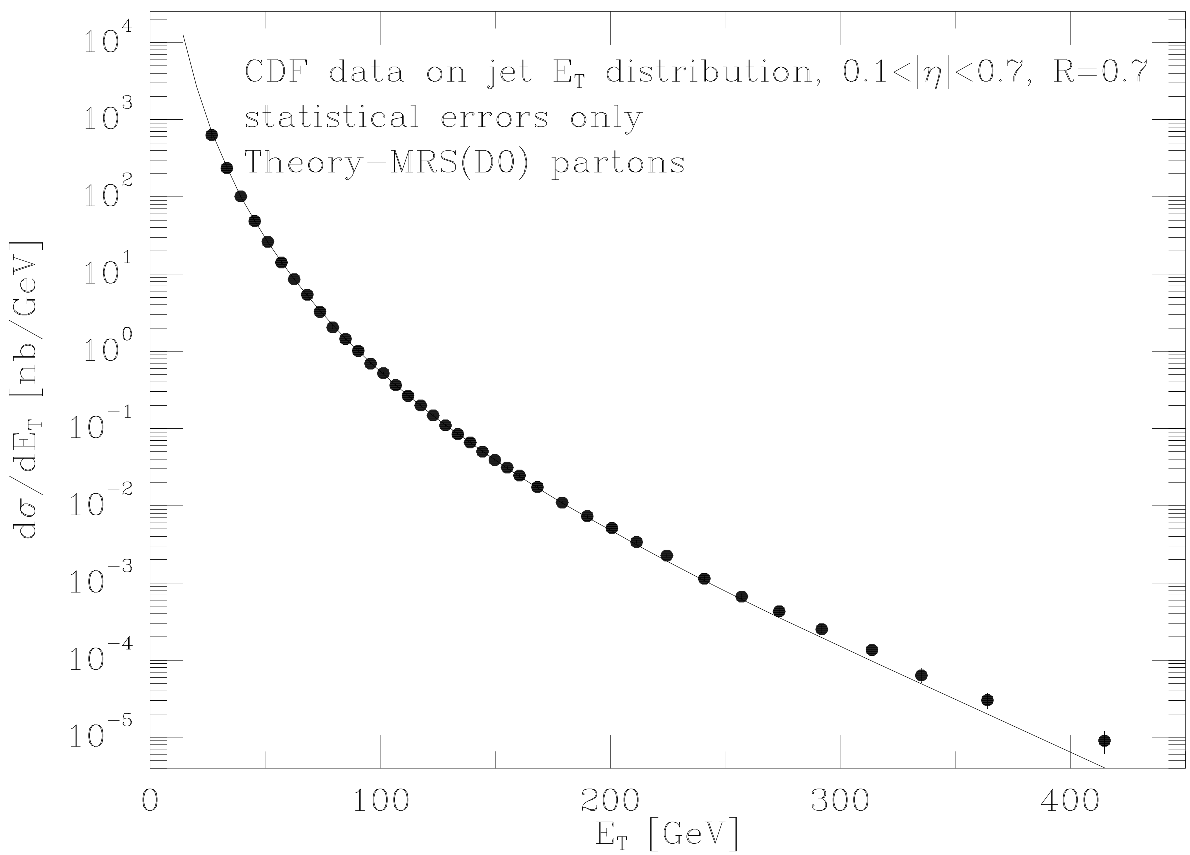
$$\Delta R_{hJ} \equiv \sqrt{(\eta_h-\eta_J)^2+(\phi_h-\phi_J)^2} < R$$

where η_J, ϕ_J refer to jet axis, chosen to maximize jet E_T, and $R \sim 0.7$ is cone size.

- Use η rather than θ for invariance under longitudinal boosts: $x_1 \to a x_1$, $x_2 \to x_2/a$ gives

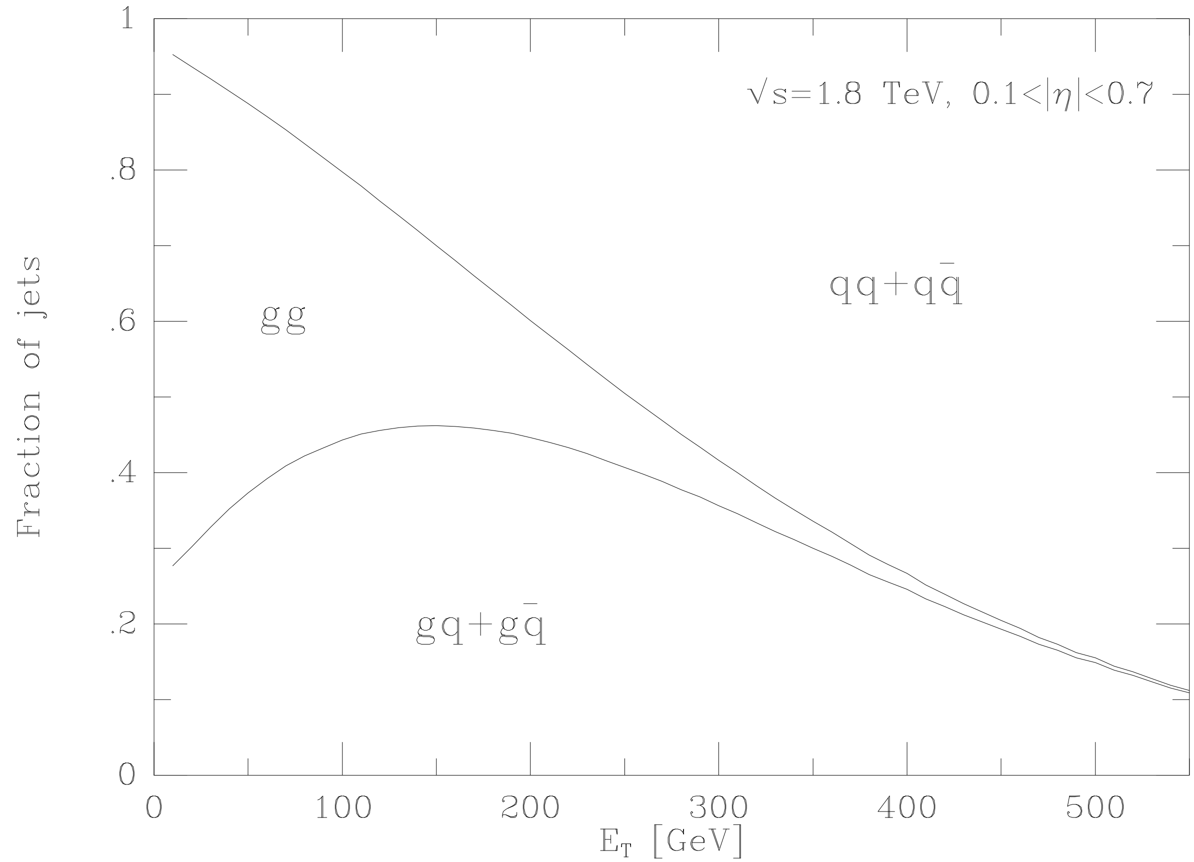
$$\eta_{h,J} \to \eta_{h,J} + \ln a$$

so $\eta_h - \eta_J$ is invariant.



- Slight excess at large E_T caused excitement, but can be reduced/removed by adjusting gluon distribution.

- Contribution of different parton combinations ij determined by subprocess cross sections and parton distributions.

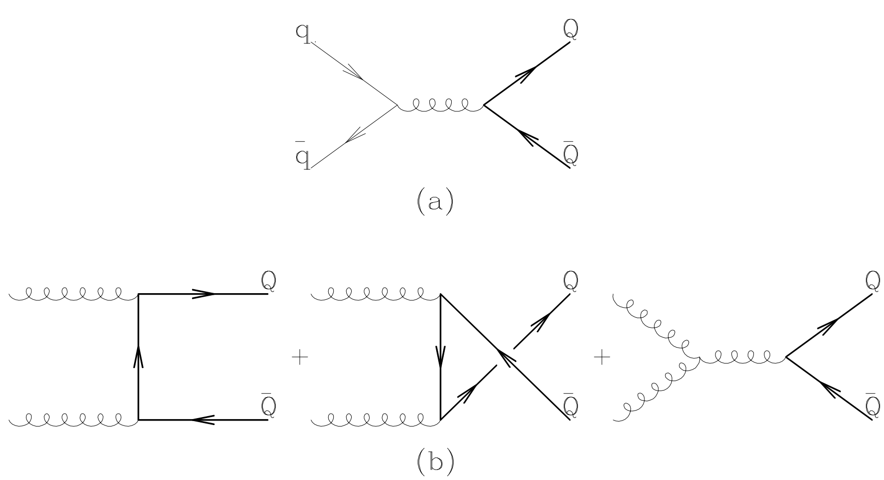


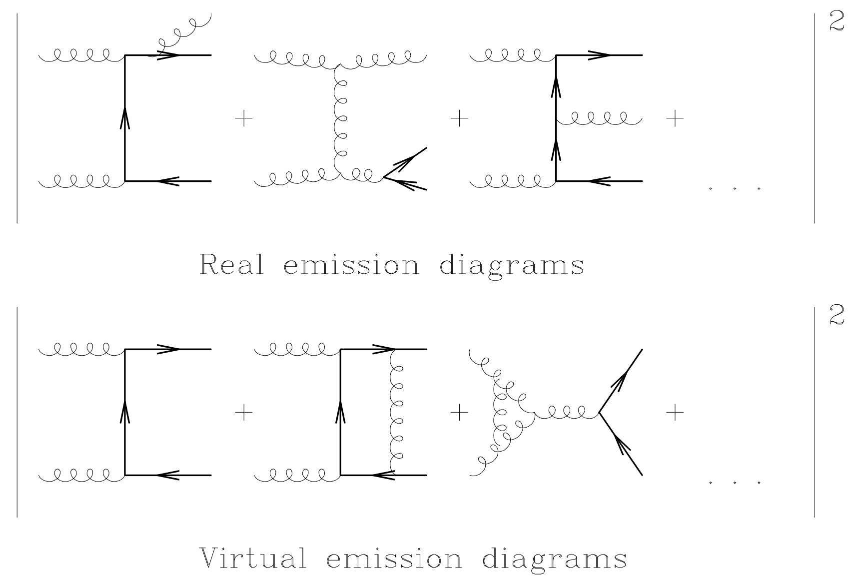
- Quarks dominate at large E_T since this selects large $x_{1,2}$:

$$\hat{s} = x_1 x_2 S > 4E_T^2$$

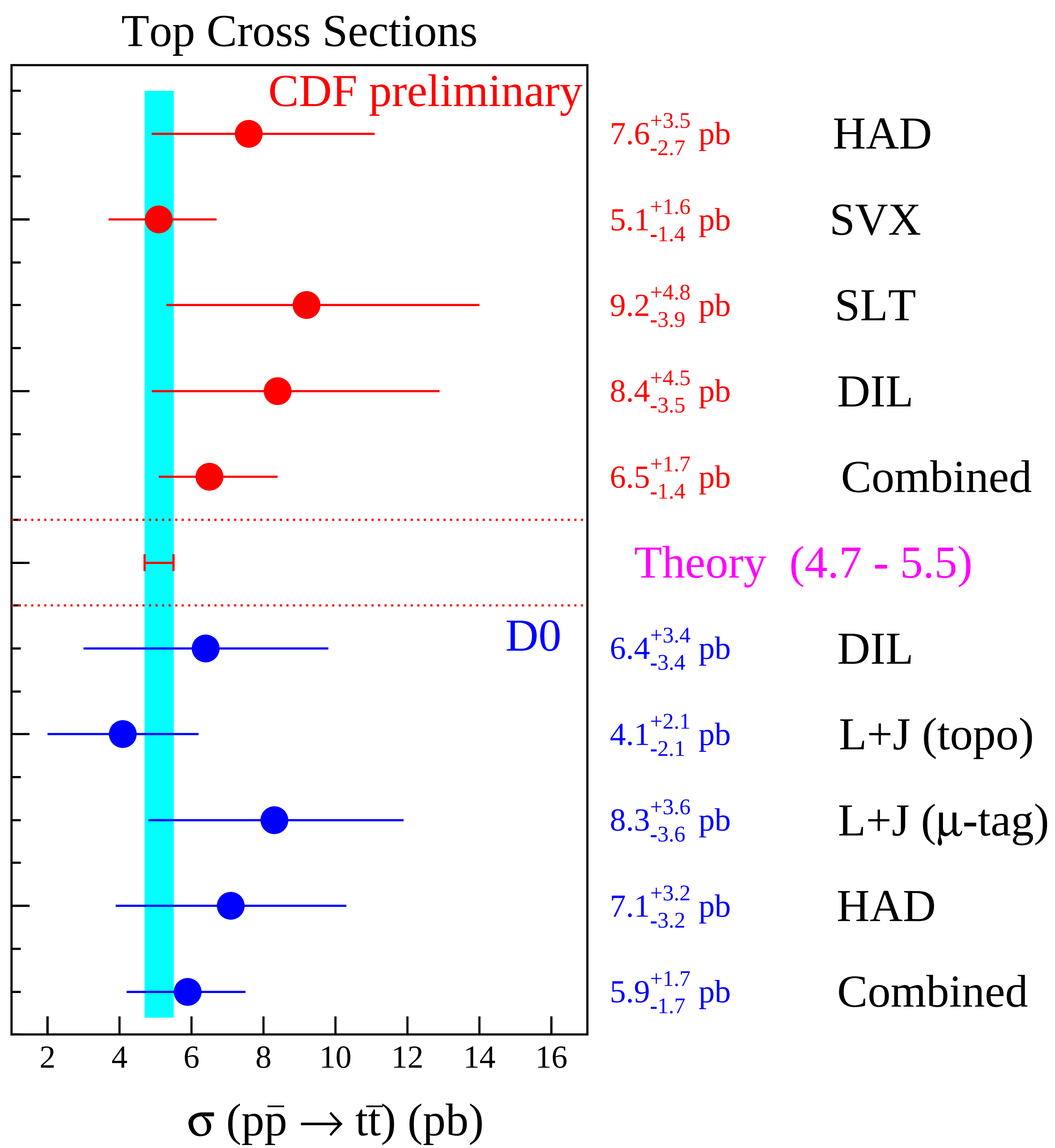
Heavy quark production

- Leading-order (LO) and next-to-leading-order (NLO) contributions:



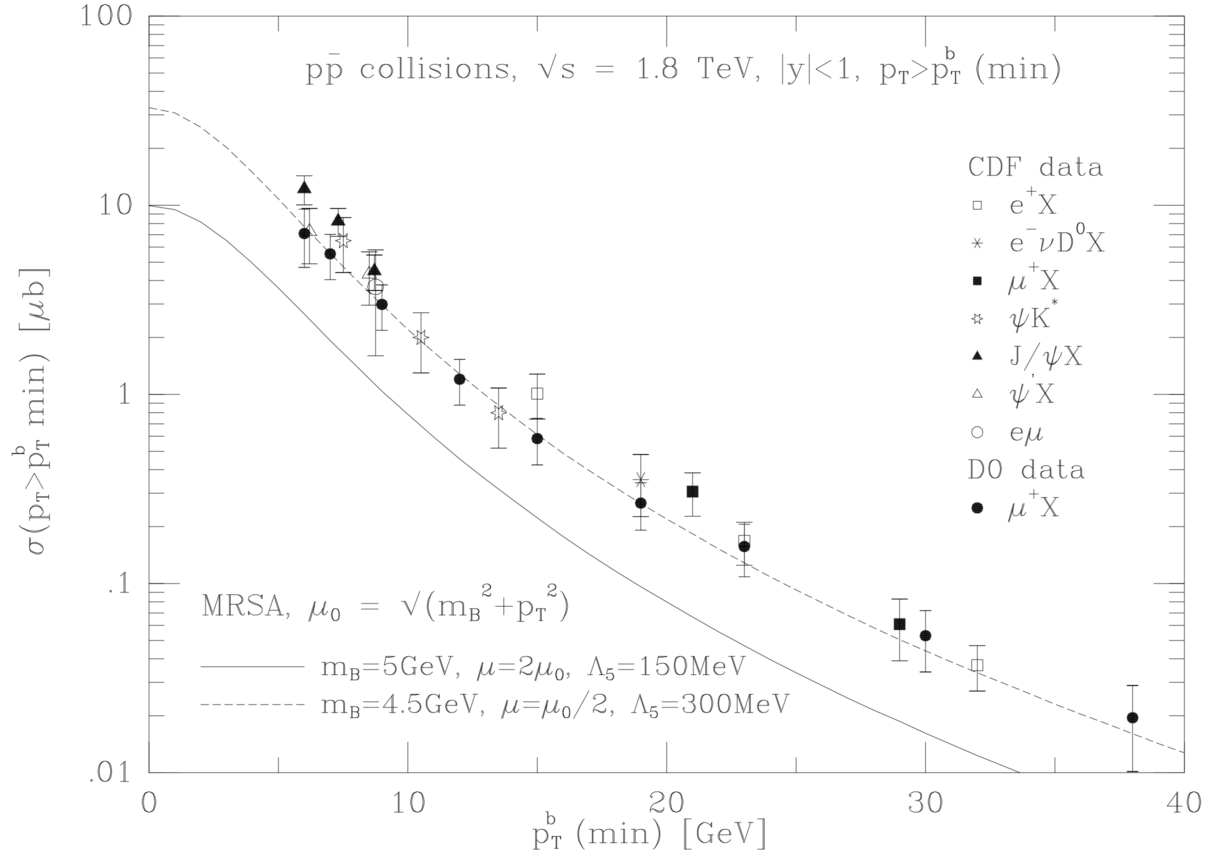


- In top quark production, NLO agrees with data.


Top Cross Sections
CDF preliminary
$7.6^{+3.5}_{-2.7}$ pb HAD
$5.1^{+1.6}_{-1.4}$ pb SVX
$9.2^{+4.8}_{-3.9}$ pb SLT
$8.4^{+4.5}_{-3.5}$ pb DIL
$6.5^{+1.7}_{-1.4}$ pb Combined
Theory (4.7 - 5.5)
D0
$6.4^{+3.4}_{-3.4}$ pb DIL
$4.1^{+2.1}_{-2.1}$ pb L+J (topo)
$8.3^{+3.6}_{-3.6}$ pb L+J (μ-tag)
$7.1^{+3.2}_{-3.2}$ pb HAD
$5.9^{+1.7}_{-1.7}$ pb Combined
2 4 6 8 10 12 14 16
$\sigma\ (p\bar{p} \rightarrow t\bar{t})$ (pb)

Bottom quark production

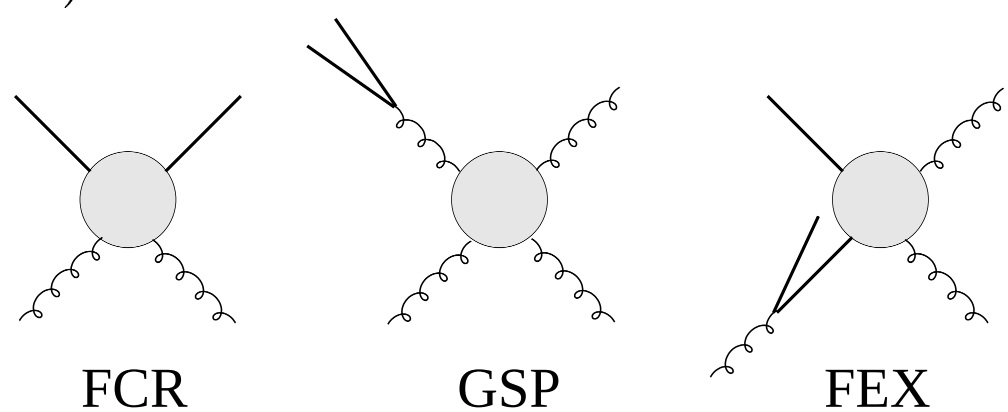
- For bottom quark production, NLO prediction is too low:



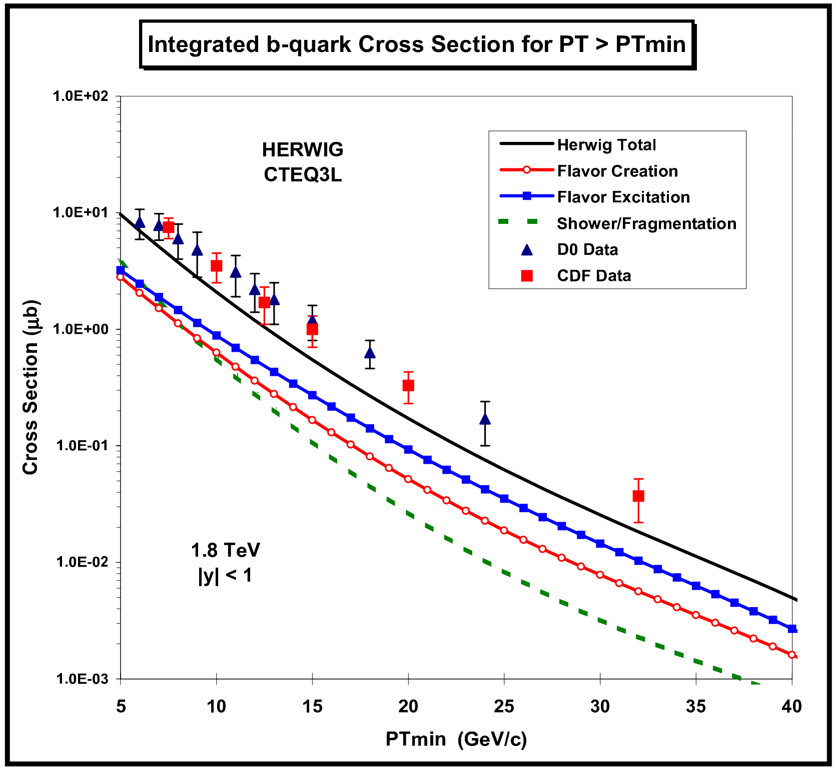
- Fit requires low b mass, low scale, high α_S
- Higher orders, e.g. $\alpha_S^2[\alpha_S \log(p_T/m)]^k$?
- Bad b fragmentation model? (M Cacciari & P Nason)

Bottom quark production in Monte Carlos

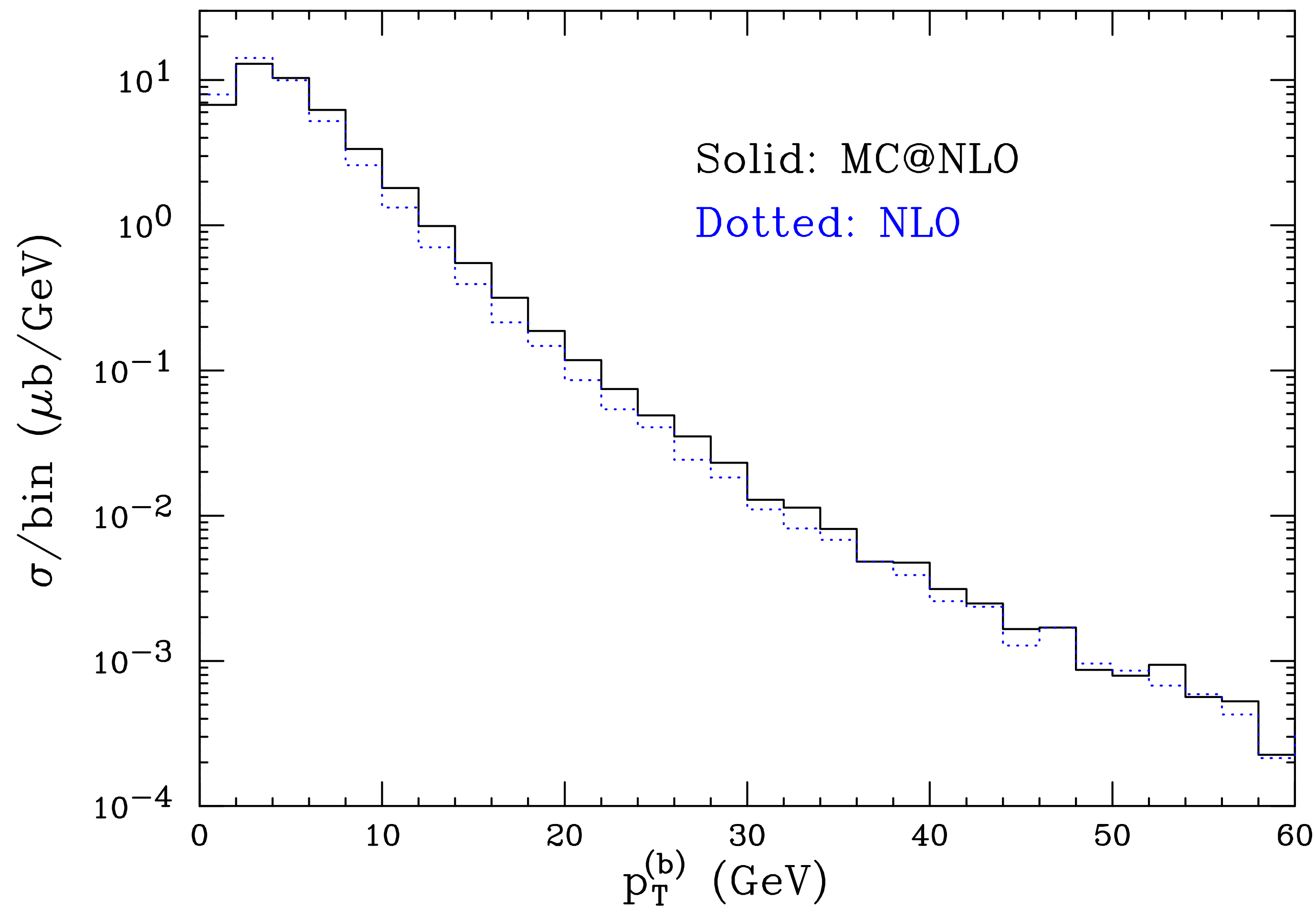
- 3 types of processes contribute: flavour creation (FCR), gluon splitting (GSP), flavour excitation (FEX)



- GSP and FEX are higher order, cutoff dependent, effectively enhancing NLO [R Field, PRD65, 094006]



- Avoid these problems by using MC@NLO, which matches Monte Carlo to NLO without double-counting [S Frixione, P Nason & BW, JHEP0308, 007]



Summary of Lecture 5

- Deep inelastic scattering (DIS) measures parton distribution functions, which show expected scaling violation.
- Small-x PDFs rise rapidly with increasing Q^2 or decreasing x, in contrast to small-x fragmentation functions, due to different higher-order corrections to splitting functions.
- Hadron-hadron processes can be predicted from PDFs measured in DIS using factorization:
  - Lepton pair production (Drell-Yan process)
  - Jet production (QCD $2 \to 2$ scattering)
  - Heavy quark production ($t\bar{t}$ & $b\bar{b}$)