Quantum teleportation

Nicolas Gisin

H. Deriedmatten, I. Marcikic, R. Thew, W. Tittel, H. Zbinden

Groupe de Physique Appliquée-unité d'Optique

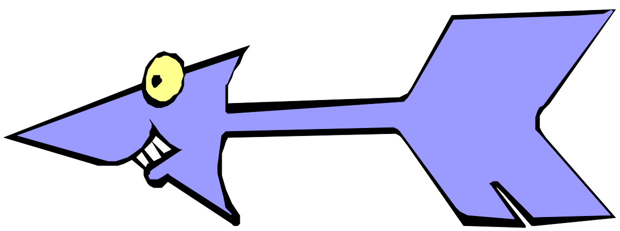
Université de Genène

- The science and the science-fiction of quantum teleportation
    - Intuitive and mathematical introduction
    - What is teleported ?
    - Q fax? The no-cloning theorem
    - The "teleportation channel": Entanglement
- The Geneva experiment
    - Telecom wavelengths
    - Time-bin qubits
    - partial Bell measurement and tests of Bell inequality
- Applications: Quantum Key Distribution
    - Simplifications, limitations
    - Q relays and Q repeaters

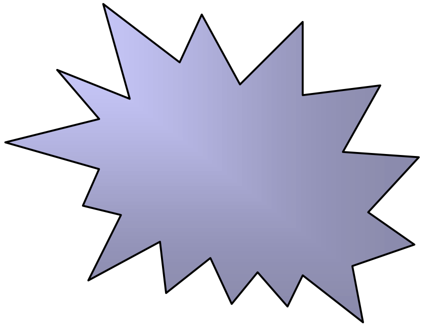
The Geneva Teleportation experiment over 3x2 km

Photon = particle (atom) of light

Polarized photon
(≈ structured photon)



Unpolarized photon
(≈ unstructured ≈ dust)



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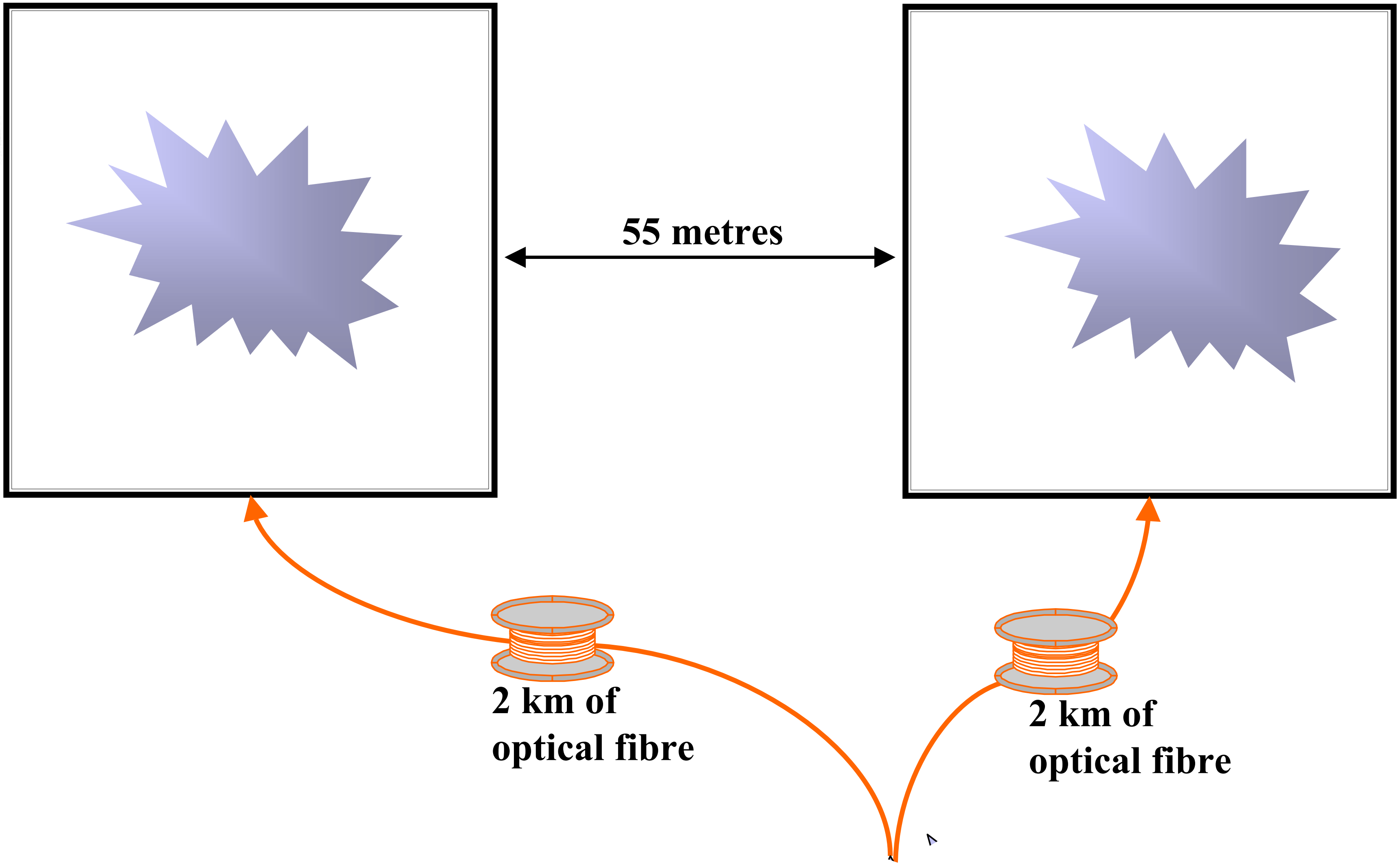

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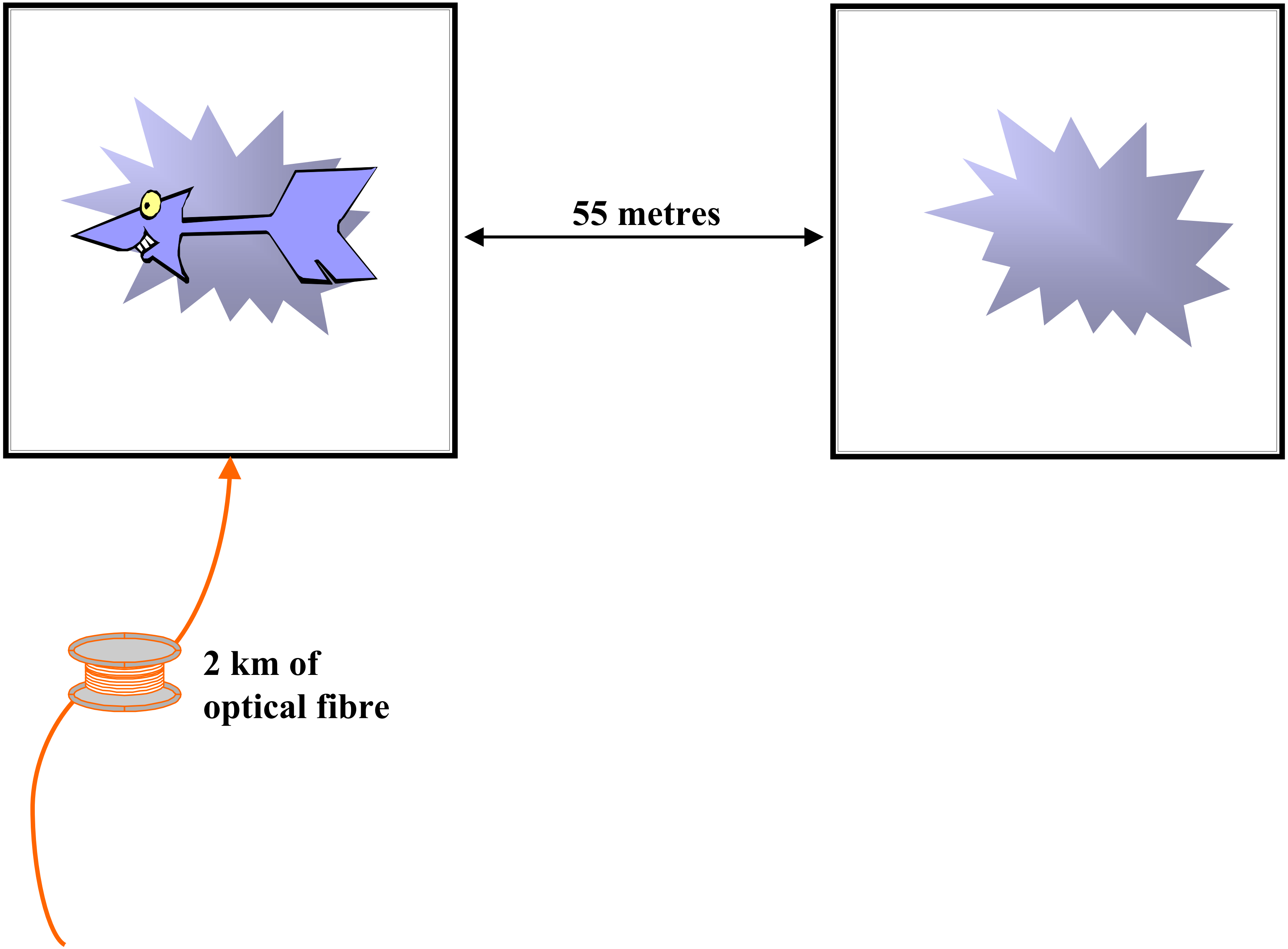

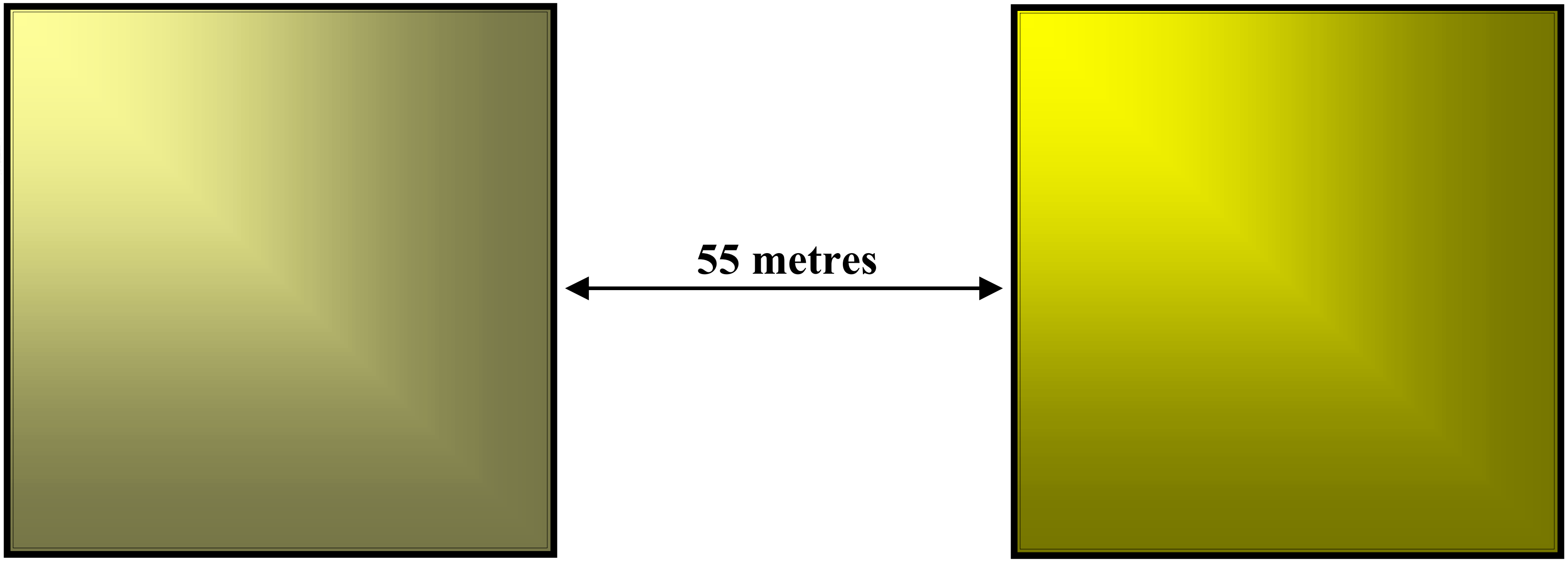

Bell measurement (partial)

the 2 photons interact

4 possible results: 0, 90, 180, 270 degrees

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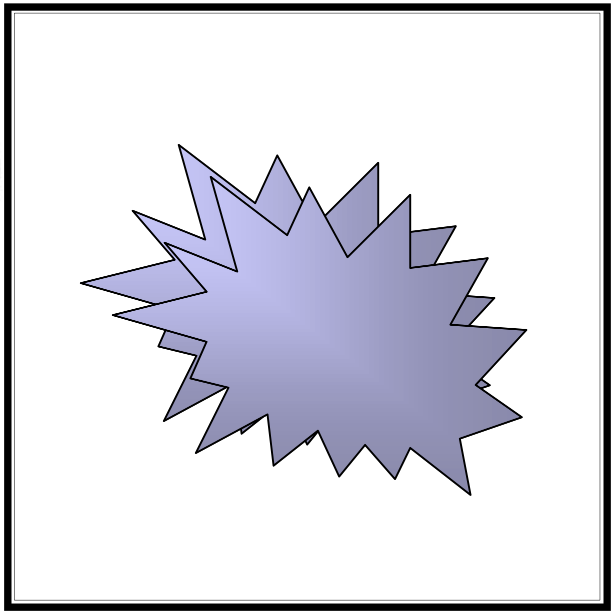
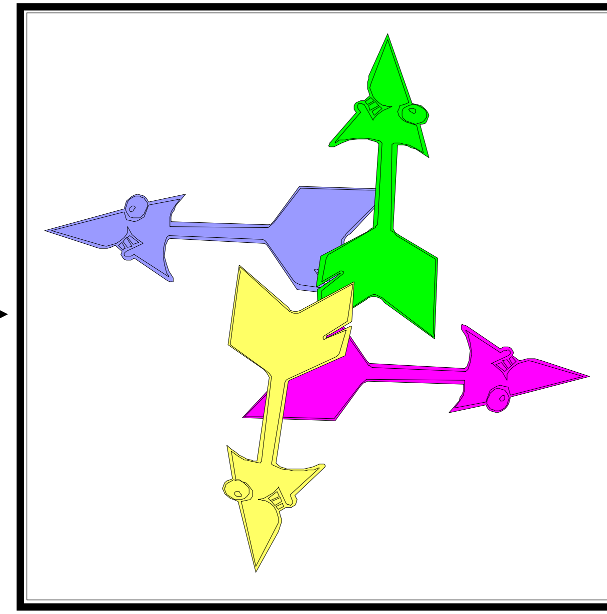


55 metres



Bell measurement
(partial)

the 2 photons interact

4 possible results:
0, 90, 180, 270 degrees

Perfect Correlation

The correlation is independent of the quantum state which may be unknown or even entangled with a fourth photon

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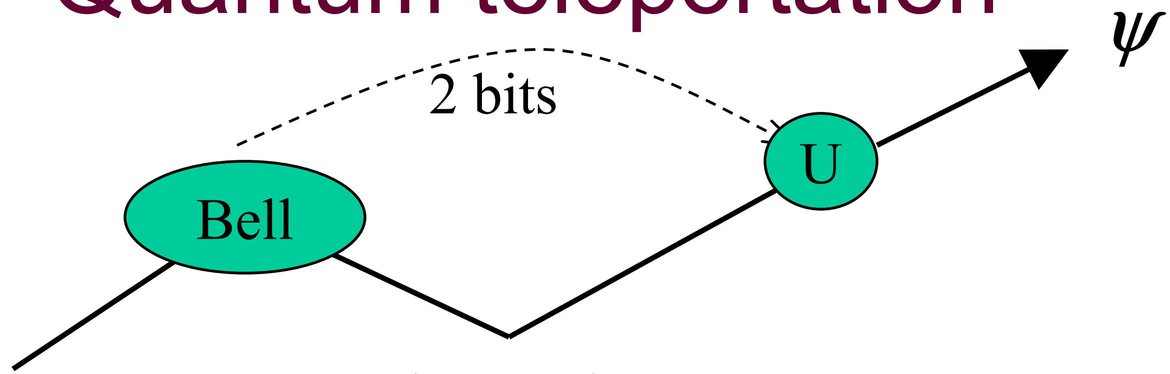
Quantum teleportation



$$\psi \quad \otimes \quad |EPR\rangle \quad =$$

$$(c_0|0\rangle + c_1|1\rangle) \otimes (|0,0\rangle + |1,1\rangle)/\sqrt{2}$$

$$= \frac{1}{2\sqrt{2}}(|0,0\rangle + |1,1\rangle) \otimes \underbrace{(c_0|0\rangle + c_1|1\rangle)} \rightarrow \Psi$$

$$+ \frac{1}{2\sqrt{2}}(|0,0\rangle - |1,1\rangle) \otimes \underbrace{(c_0|0\rangle - c_1|1\rangle)} \rightarrow \sigma_z\Psi$$

$$+ \frac{1}{2\sqrt{2}}(|0,1\rangle + |1,0\rangle) \otimes \underbrace{(c_1|0\rangle + c_0|1\rangle)} \rightarrow \sigma_x\Psi$$

$$+ \frac{1}{2\sqrt{2}}(|0,1\rangle - |1,0\rangle) \otimes \underbrace{(c_1|0\rangle - c_0|1\rangle)} \rightarrow \sigma_y\Psi$$

What is teleported ?

- **According to Aristotle, objects are constituted by *matter* and *form, ie* by *elementary particles* and *quantum states*.**

- **Matter and energy can not be teleported from one place to another: they can not be transferred from one place to another without passing through intermediate locations.**

- **However, quantum states, the ultimate structure of objects, can be teleported. Accordingly, objects can be transferred from one place to another without ever existing anywhere in between! But only the structure is teleported, the matter stays at the source and has to be already present at the final location.**

C.H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres and W. Wootters, *PRL* 70, 1895 (1993)

D. Boschi *et al.*, *Phys. Rev. Lett.* 80, 1121 (1998)

Y-K. Kim *et al.*, *Phys. Rev. Lett.* 86, 1370 (2001)

D. Bouwmeester *et al.*, *Nature* 390, 575 (1997)

I. Marcikic *et al.*, *Nature* 421, 509 (2003)

A Quantum Fax ?

- During a quantum teleportation process, the original system is destroyed.

- According to the basic law of quantum physics, this is a necessity since it is impossible to clone an unknown quantum state. If not:
  - one could violate Heisenberg's uncertainty relations (Quantum Physics would be deterministic !)
  - one could exploit entanglement and cloning to signal faster than light.
    (Relativity would have an absolute time).

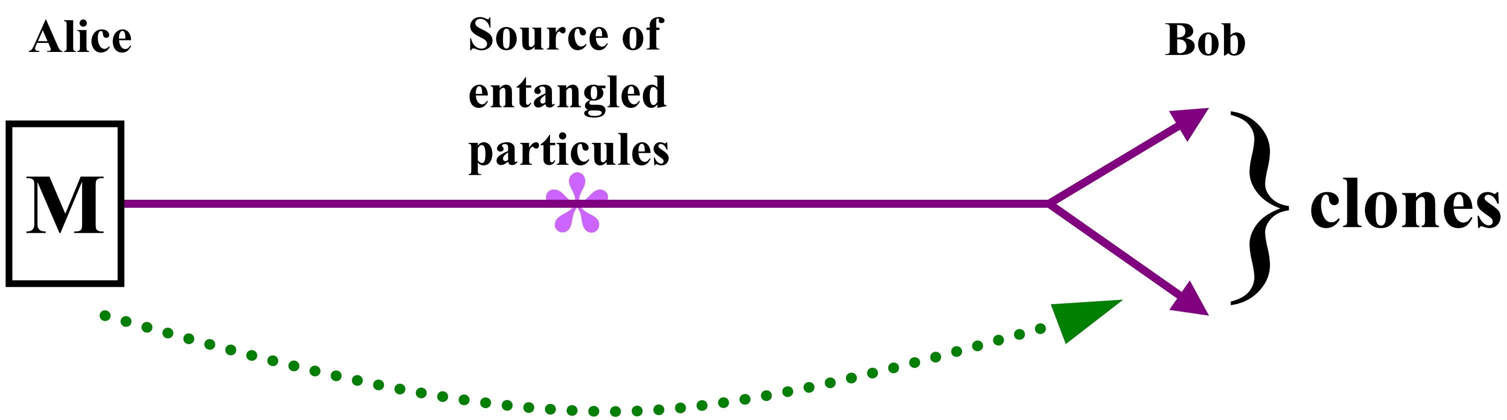
No cloning theorem and the compatibility with relativity

No cloning theorem: It is impossible to copy an unknown quantum state, $\psi \not\rightarrow \psi \cdot \psi$

Proof #1:

$$\begin{array}{l} |0\rangle \rightarrow |0,0\rangle \\ |1\rangle \rightarrow |1,1\rangle \end{array} \Rightarrow \begin{array}{l} |0\rangle + |1\rangle \rightarrow |0,0\rangle + |1,1\rangle \\ \neq (|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \end{array}$$

Proof #2: (by contradiction)



Arbitrary fast signaling !

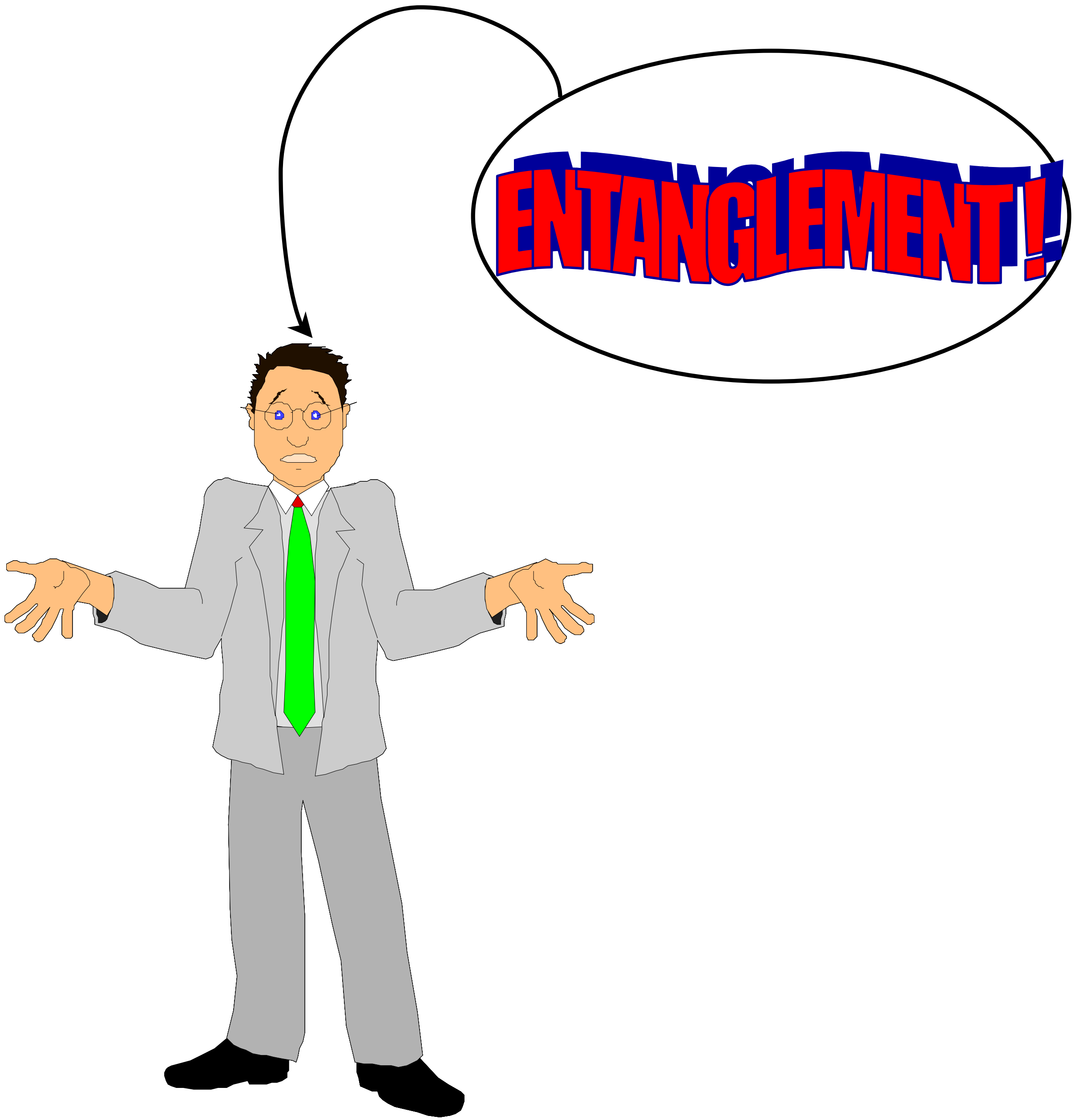
No cloning theorem and the compatibility with relativity

- The first account on quantum cloning was done by E.P. Wigner in his analysis of earlier work by W.M. Elsasser devoted to a discussion of the origin of life and the multiplication of organisms. Wigner has presented a quantum-mechanical argument according to which ``the probability is zero for existence of self-reproducing states''.

- Today's standard references are:
  W. K. Wootters and W. H. Zurek, Nature 299, 802, 1982.
  P.W. Milonni and M.L.Hardies, Phys. Lett. A 92, 32, 1982.

- The connection to "no signaling" appeared in:
  N. Gisin, Phys. Lett. A 242, 1, 1998.

The Quantum Teleportation channel ?



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Entanglement

A density matrix ρ is <u>separable</u> iff it decomposes in product states with probabilité coefficients p_j>0 :

$$\rho = \sum_j p_j \, \rho_j^A \otimes \rho_j^B$$

ρ is <u>entangled</u> iff it is not separable.

Given a ρ, one knows of no constructive method to determine wheher ρ is separable or entanglement !

The <u>partial transpose</u> test :
If ρ is seprable, then its partial transpose is $\rho^{pt} \geq 0$

whre $\rho^{pt}_{ab,\alpha\beta} \equiv \rho_{a\beta,\alpha b}$ **(this test is exhaustive en dim 2x2)**

Non-locality: definition

$$P(a,b \mid A,B)$$

probability

Events. Not under human control

Experimental settings. Under human control

The correlations $P(a,b|A_n,B_m)$ are local iff there is a random variable λ such that:

$$P(a,b|A_n,B_m,\lambda) = P(a|A_n,\lambda) \cdot P(b|B_m,\lambda)$$

Historically λ was called a "local hidden variable". Today, one measures the amount of nonlocality by the minimum communication required to reproduce $P(a,b|A_n,B_m)$.

Implications of entanglement

- The world can't be understood in terms of "little billiard balls".
- The world is nonlocal (but the nonlocality can't be used to signal faster than light).
- Quantum physics offers new ways of processing information.

What can carry the Q info to be teleported ?

What can be teleported ?

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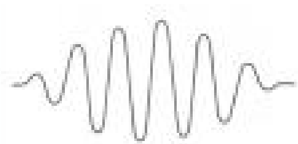
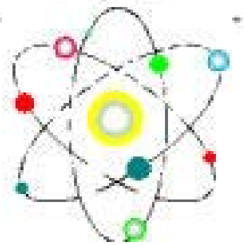
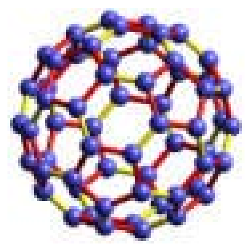


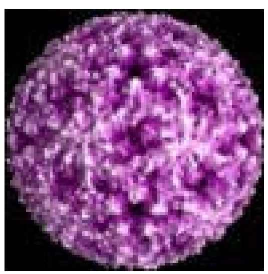
Photon — **Done**



Atom — **Probably soon !**



Molecule — **Likely some day**



Virus — **?? Possibly ??**

Large object — **Still science-fiction**

Q communication in optical fibres

Two problems : Losses and decoherence.
How to minimize them ?

- The transmission depends on the wavelength
  - Lower attenuation : 1310 nm (0.3 dB/km) and 1550 nm (0.2dB/km) (telecom wavelengths)

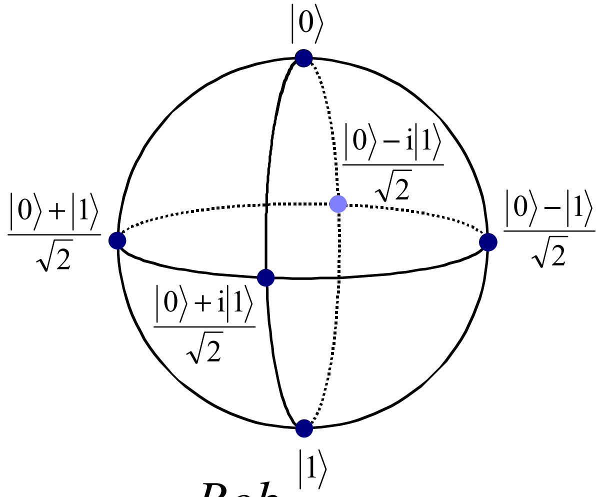
- Decoherence due to birefringence :
Polarization Mode Dispersion

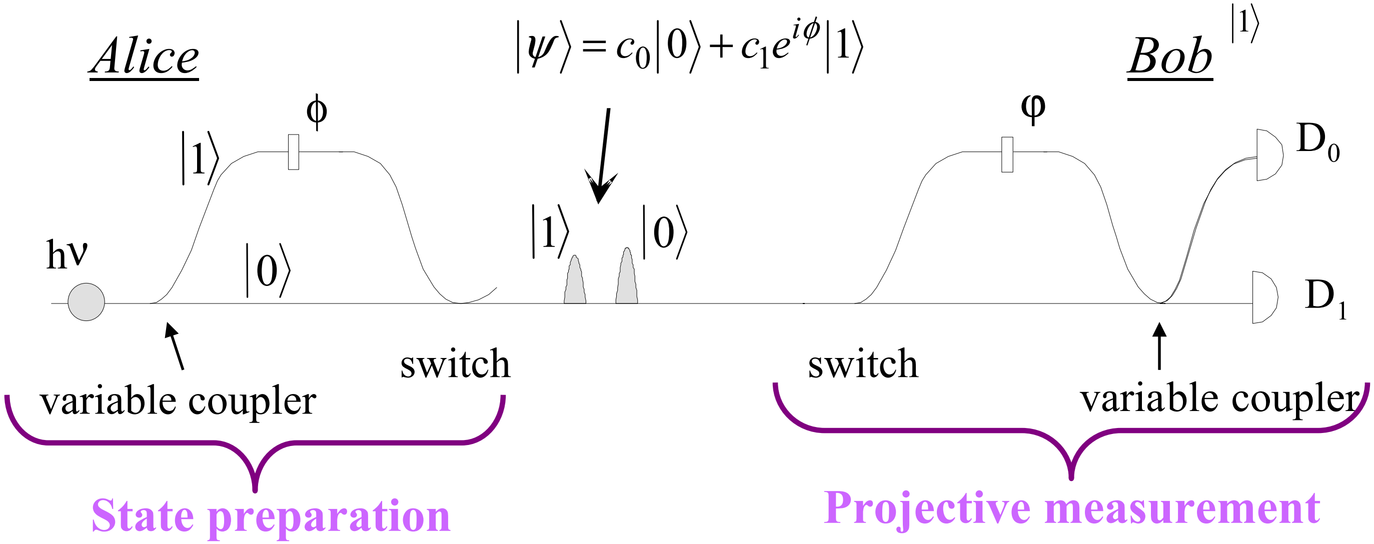
Time-bin coding with photons at telecom wavelength

Time-bin qubits

- **qubit :** $|\psi\rangle = c_0|0\rangle + c_1 e^{i\phi}|1\rangle$

- **any qubit state can be created and measured in any basis**





W. Tittel & G. Weihs, *Quant. Inf. Comput.* 1, Number 2, 3 (2001)

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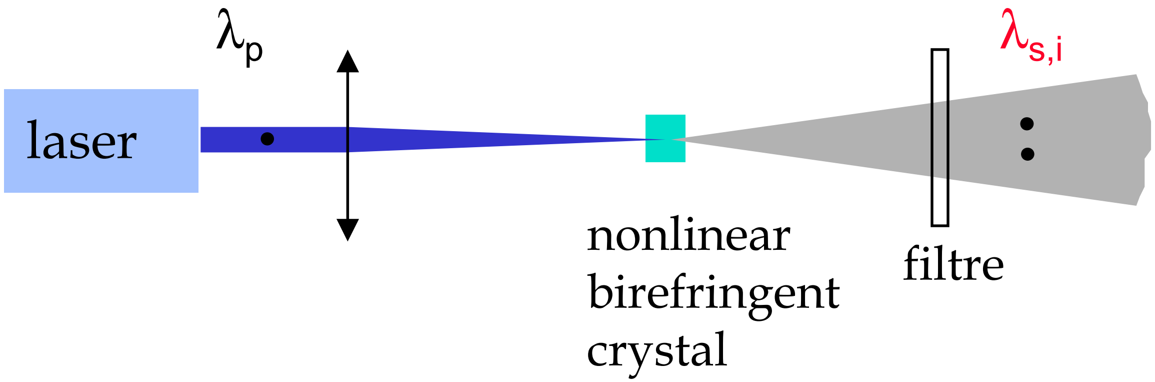
Photon pairs source



- Parametric fluorescence
- Energy and momentum conservation

$$\omega_p = \omega_s + \omega_i \qquad \vec{k}_p = \vec{k}_s + \vec{k}_i$$

- Phase matching determines the wavelengths and propagation directions of the down-converted photons

Energy conservation:

⇒ **each photon from the pair has an uncertain frequency, but the sum of the two frequencies is precisely that of the pump laser**

⇒ **each photon from the pair has an uncertain age, but the age's difference is precisely zero**

⇒ **similar to the original EPR state**

Time-bin entanglement

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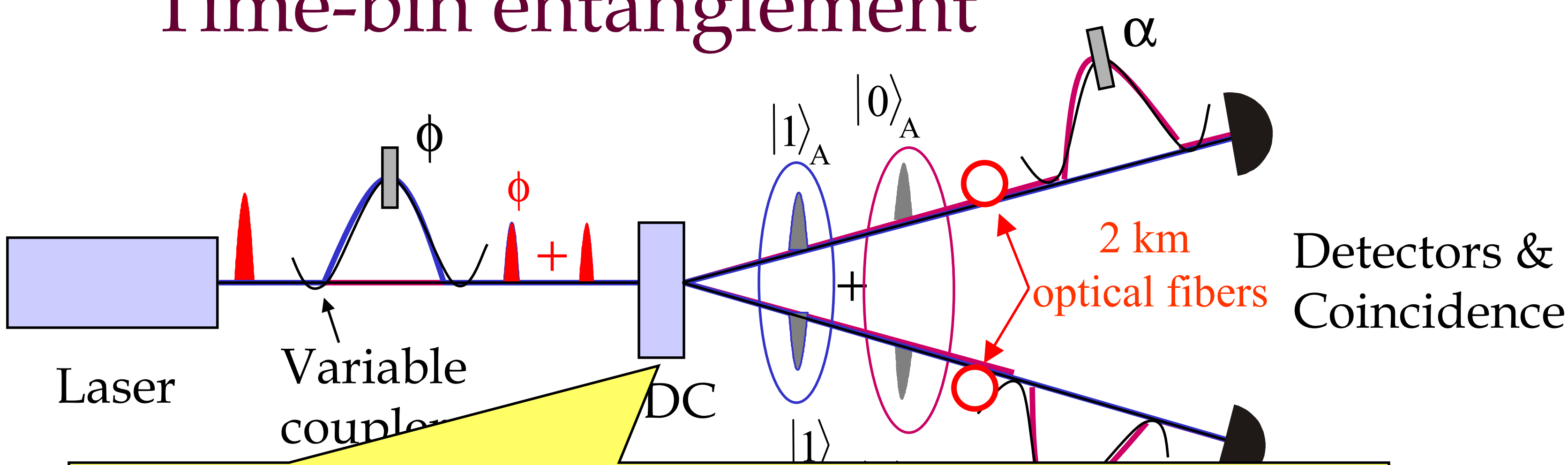


- Photon pair creation in a non-linear crystal
- Parametric down-conversion (PDC)
- Energy and momentum conservation

$$\omega_p = \omega_s + \omega_i \qquad \vec{k}_p = \vec{k}_s + \vec{k}_i$$

$$\lambda_p = 710\text{nm} \rightarrow \begin{cases} \lambda_s = 1310\text{nm} \\ \lambda_i = 1550\text{nm} \end{cases}$$

$|\psi\rangle =$

Ro
d
in

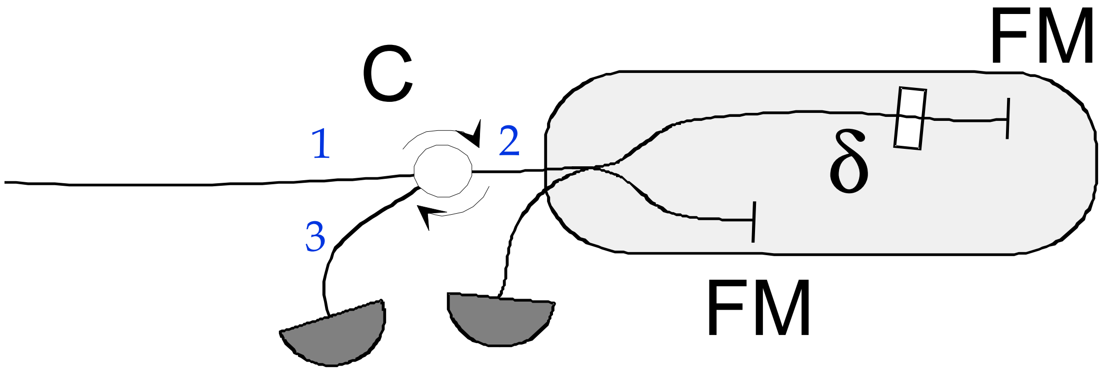
2km
l fibers

el

0 20 40 60 80 100
phase[arb unit]

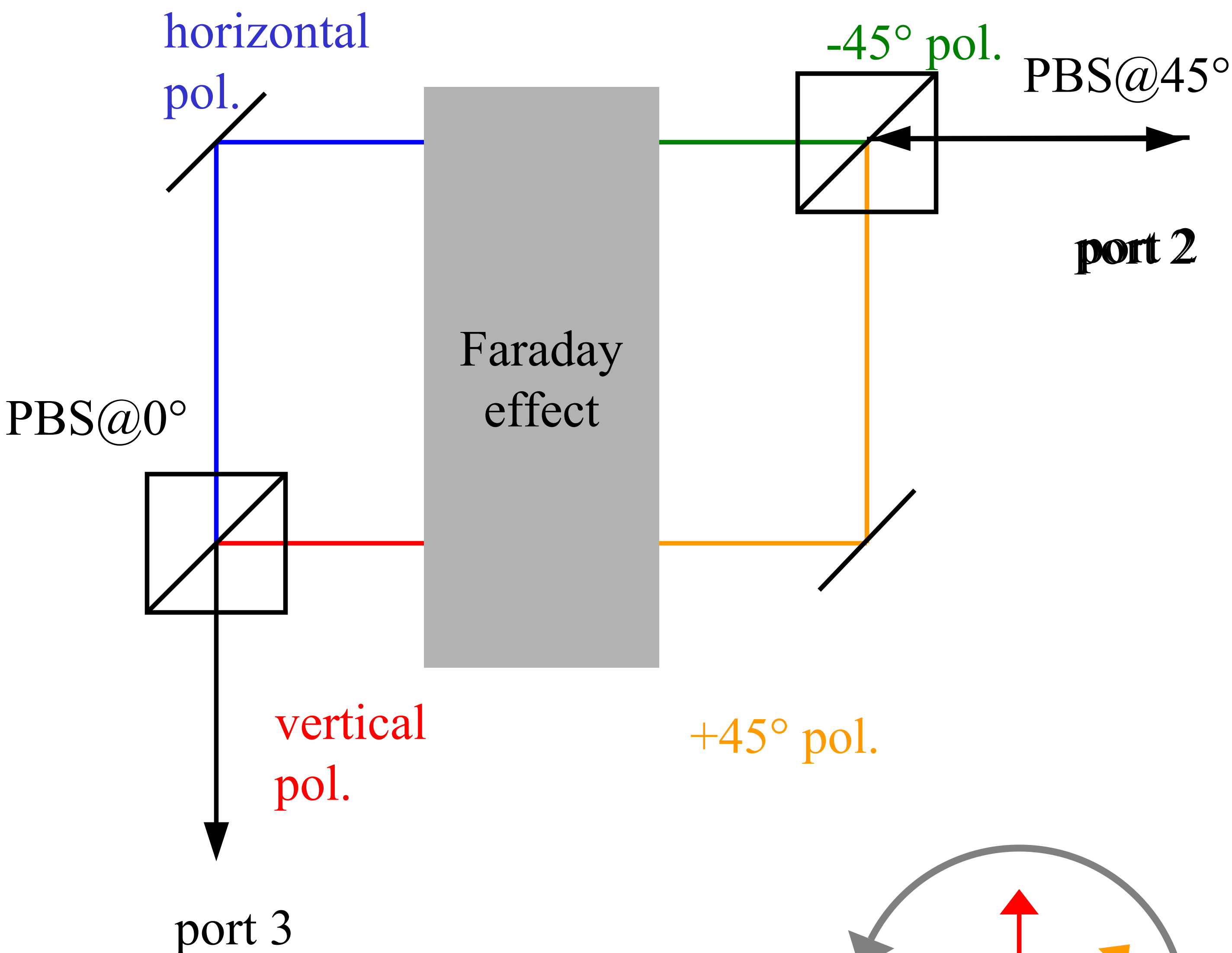
R. Thew *et al.*, *Phys. Rev. A* 66, 062304 (2002)

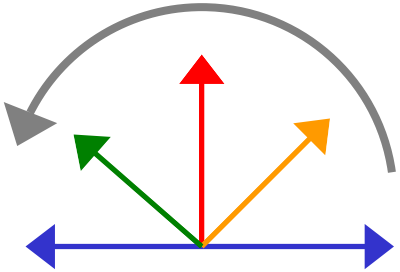
The interferometers



- single mode fibers
- Michelson configuration
- circulator C : second output port
- Faraday mirrors FM: compensation of birefringence
- temperature tuning enables phase change

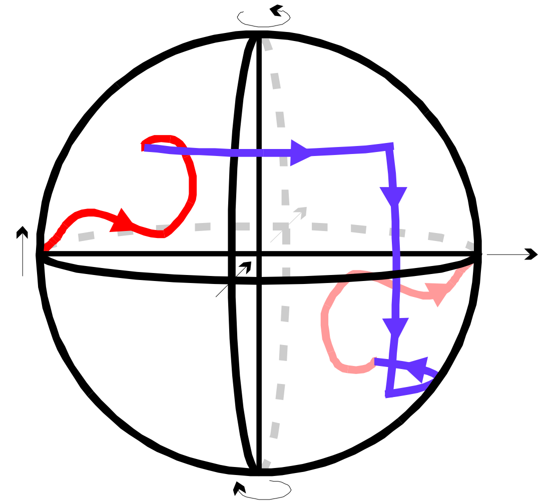
Optical circulator





Faraday mirrors

- $\lambda/4$ **Faraday rotator**
- **standard mirror (⊥ incidence)**
- $\lambda/4$ **Faraday rotator**

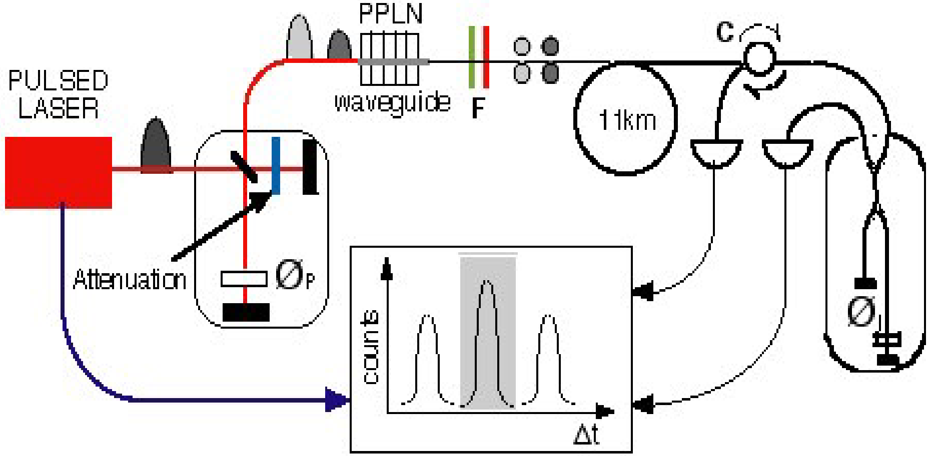


$$\vec{m} \rightarrow R(\omega)\vec{m} \rightarrow (-R(\omega)\vec{m})$$

$$-\vec{m} \leftarrow R^{-1}(\omega)(-R(\omega)\vec{m}) \leftarrow$$

Independent of ω

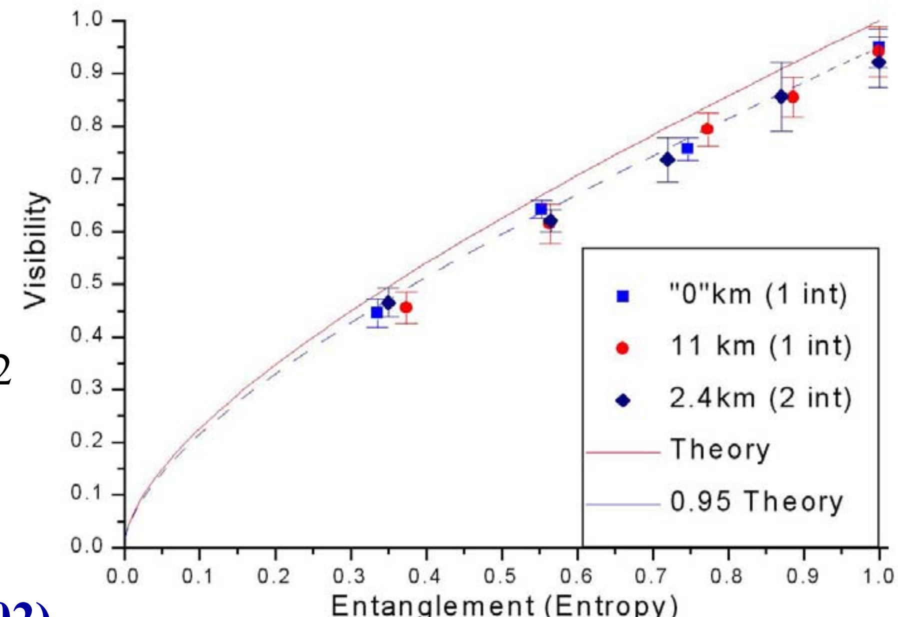
Partially Entangled Time-Bin Qubits



$$|\psi\rangle = c_0|00\rangle + c_1 e^{i\phi_p}|11\rangle$$

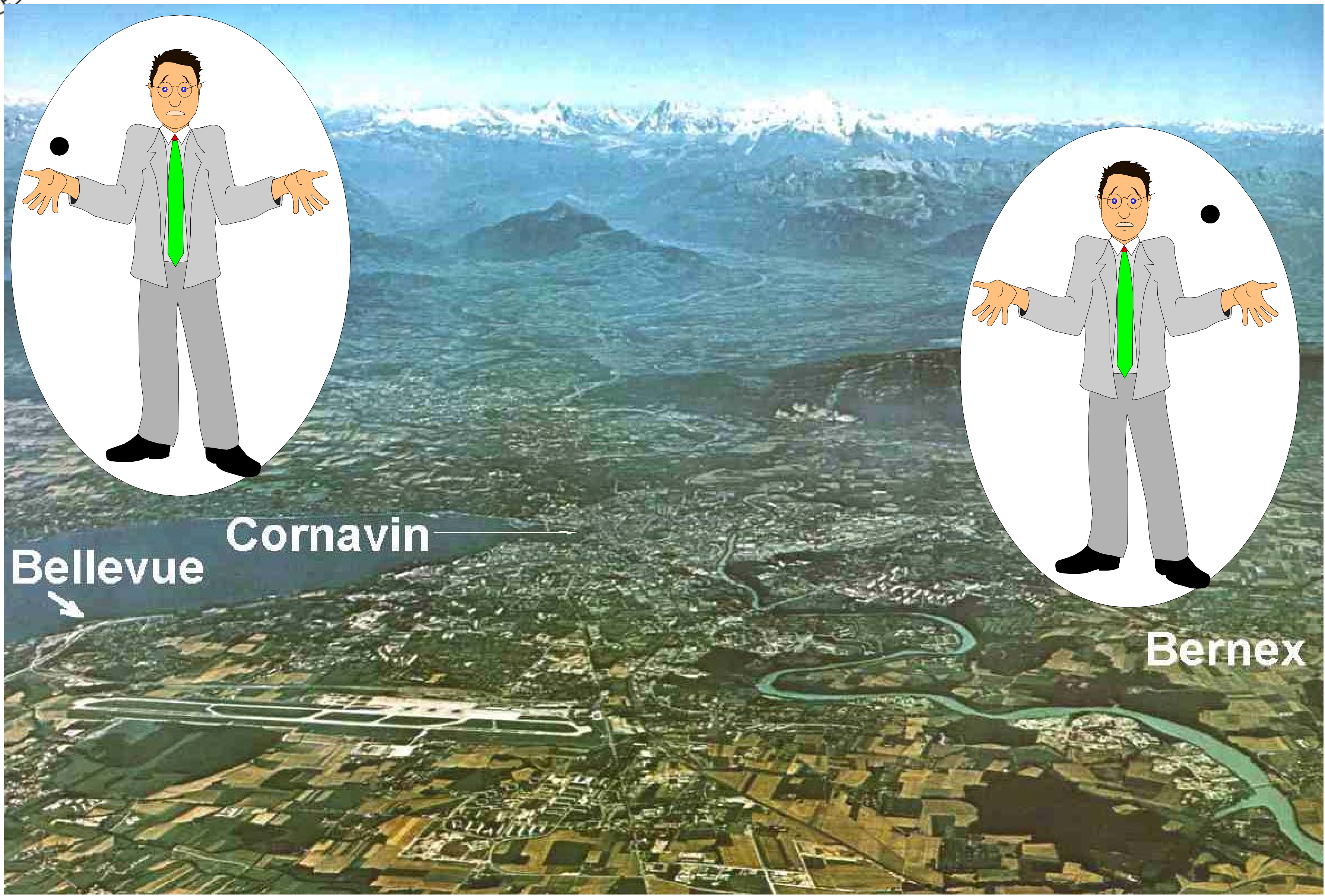
$$V = 2c_0c_1$$

$$E = -c_0^2\log_2 c_0^2 - c_1^2\log_2 c_1^2$$

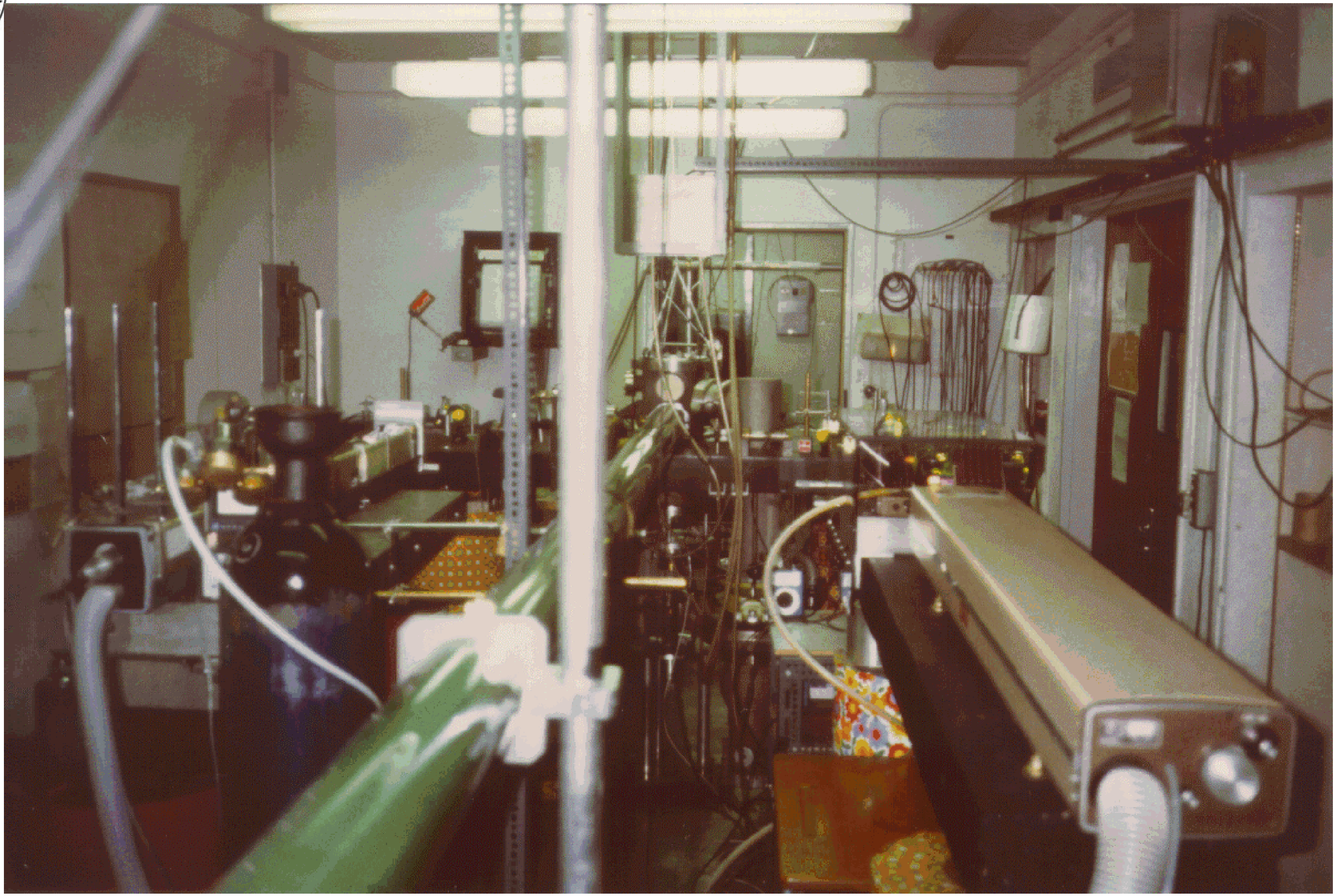


R. Thew *et al.*, *Phys. Rev. A* 66, 062304 (2002)

Geneva 1997



2-υ source of Aspect's 1982 experiment



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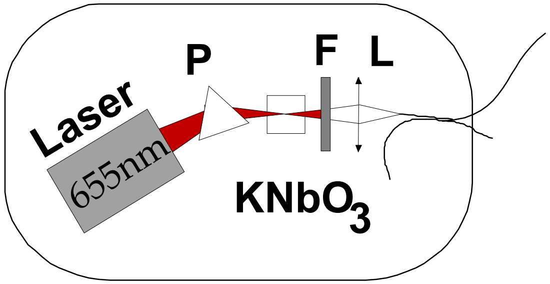
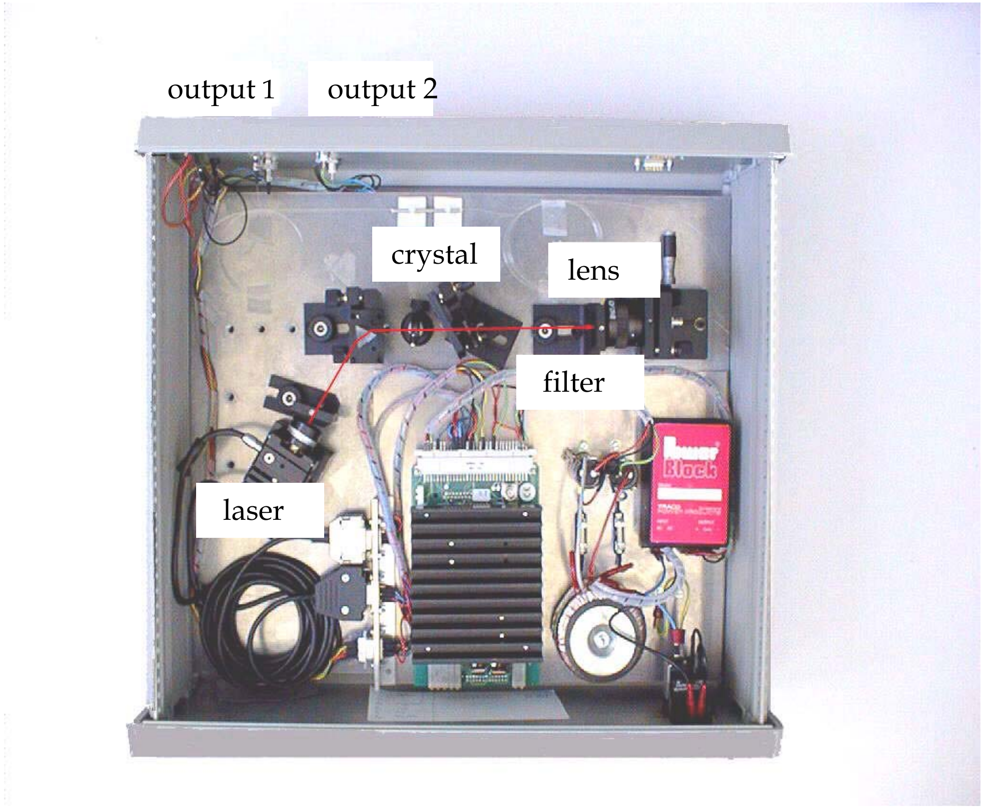
Photon pairs source (Geneva 1997)





- Energy-time entanglement
  - $\lambda_p = 655\,\text{nm};\ \overline{\lambda}_{s,i} = 1310\,\text{nm}$
- diode laser
- simple, compact, handy

  40 x 45 x 15 cm^3
- I_{pump} = 8 mW
- with waveguide in $LiNbO_3$ with quasi phase matching, $I_{pump} \approx 8\ \mu W$

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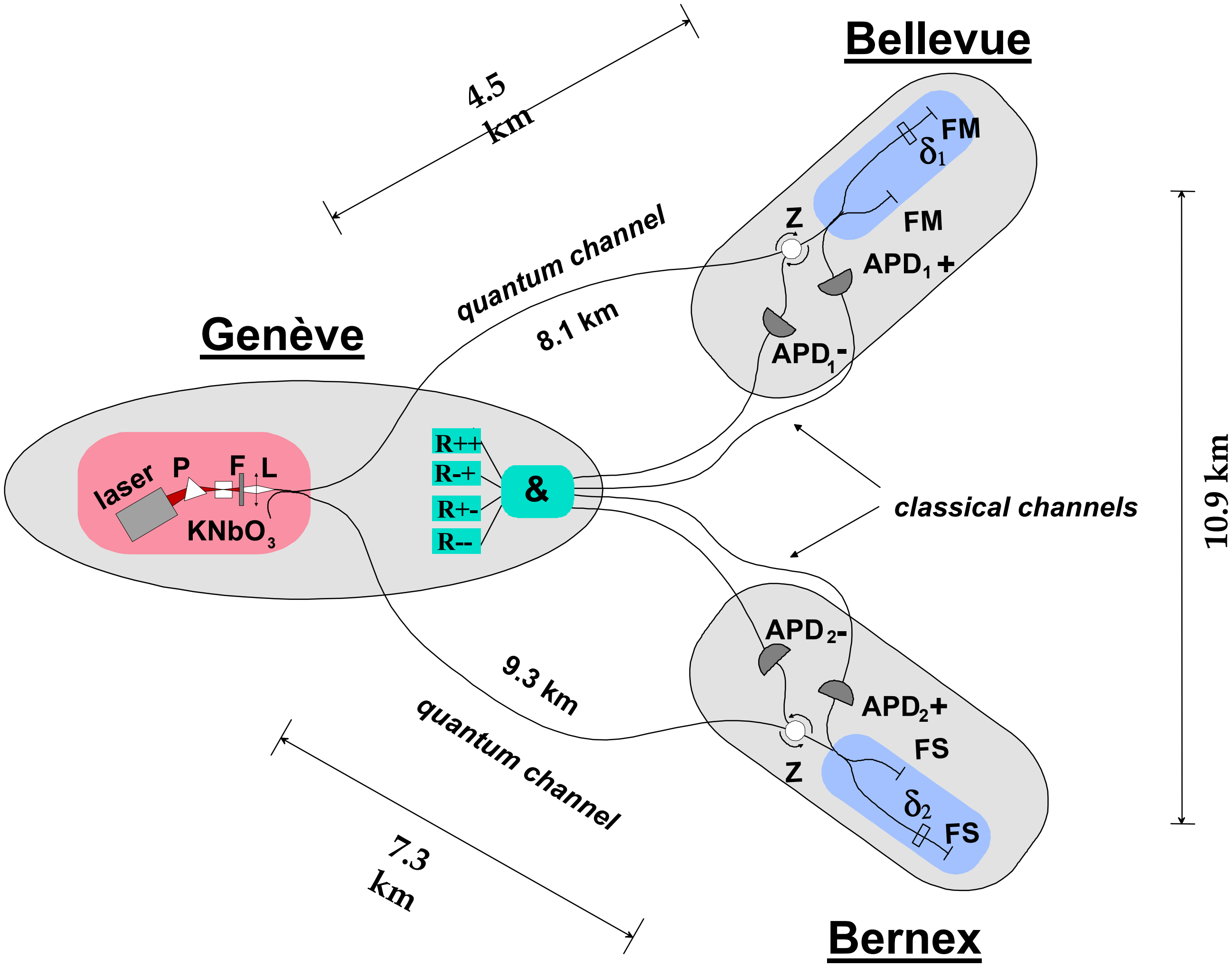
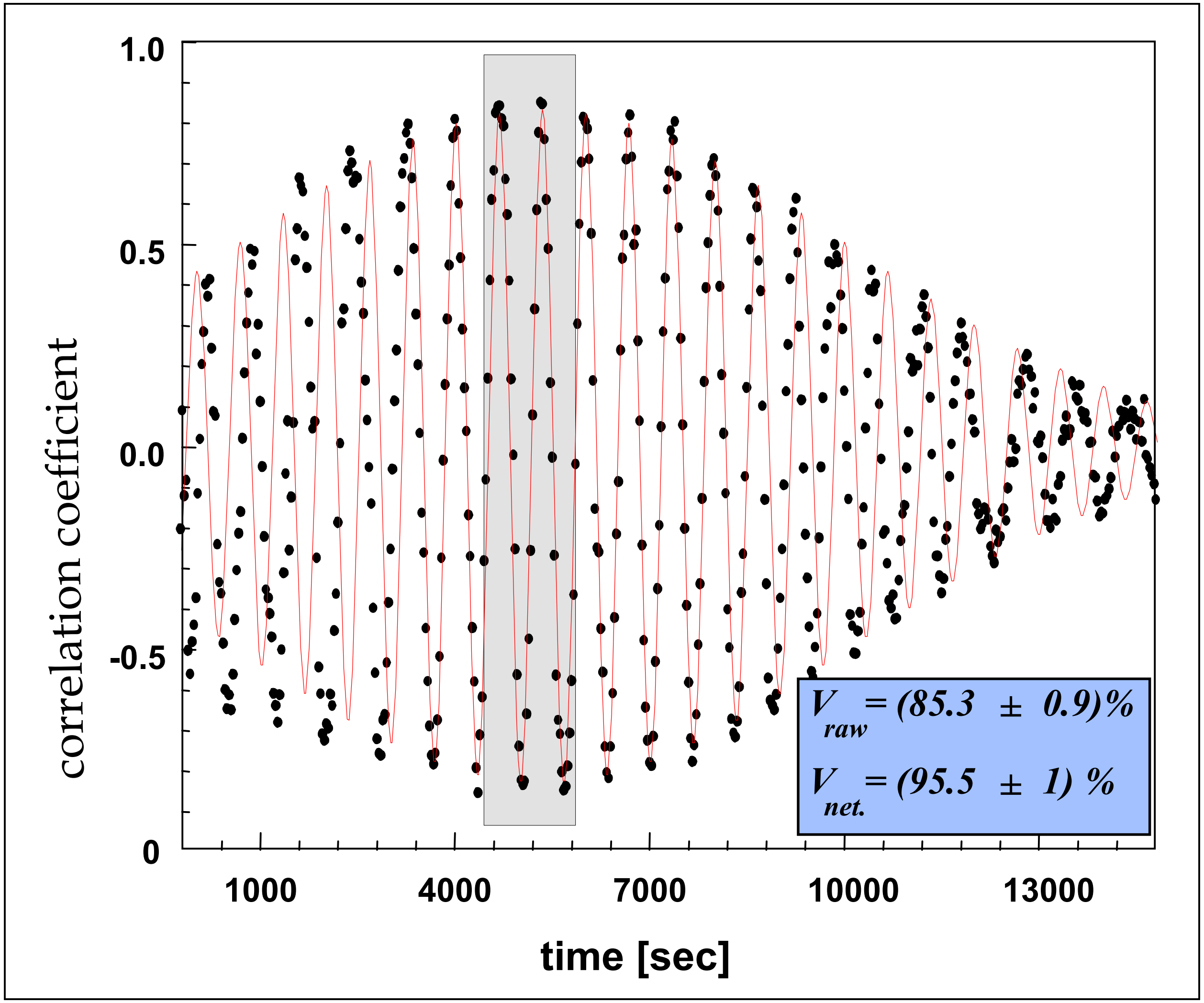
test of Bell inequalities over 10 km



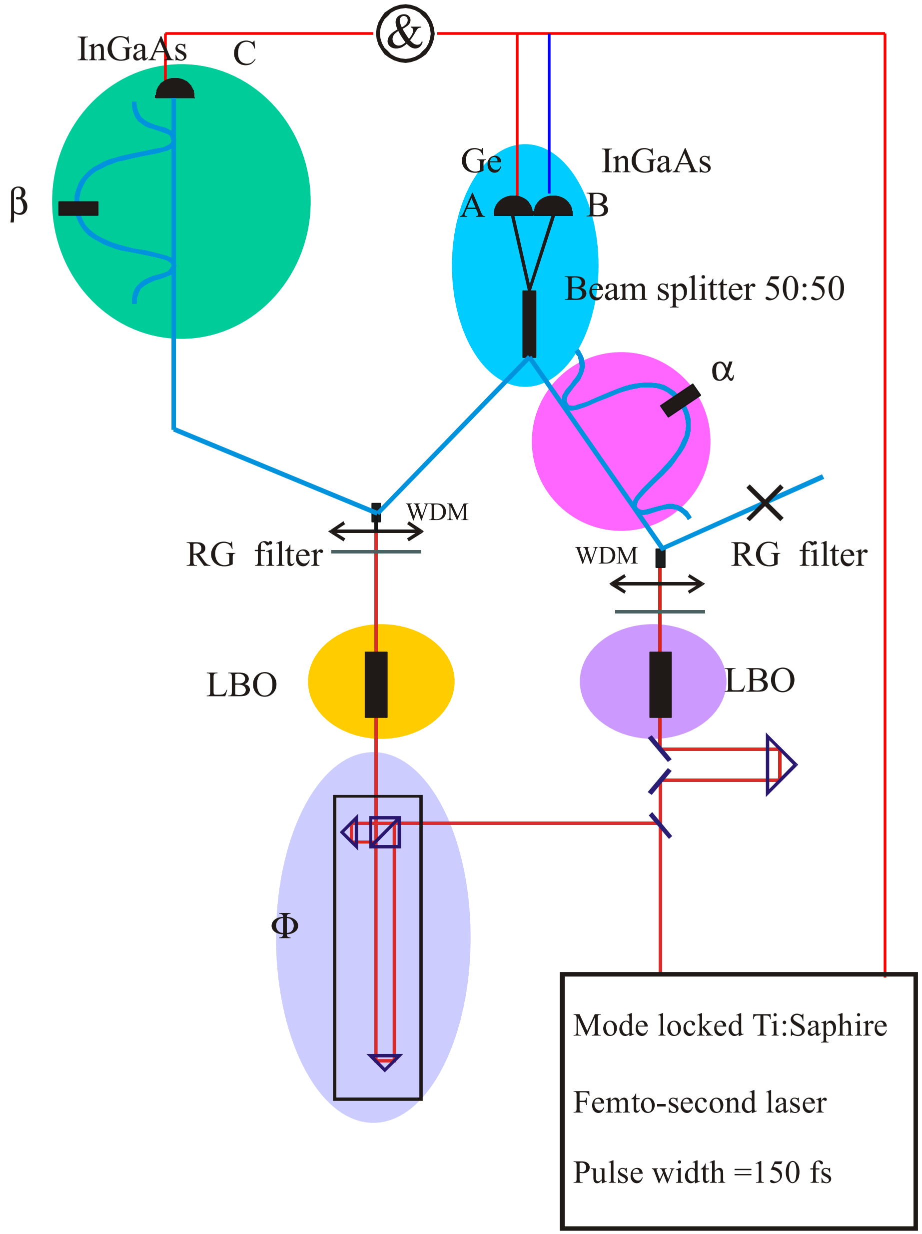
results



- 15 Hz coincidences
- $S_{raw} = 2.41$
  $S_{net} = 2.7$
- violation of Bell inequalities by 16 (25) standard-deviations
- close to quantum-mechanical predictions
- same result in the lab

teleportation setup

I. Marcikic et al., Nature, 421, 509-513, 2003



Creation of a qubit

Creation of an entangled pair

Creation of a photon

Creation of any qubit to be teleported

The Bell measurement

Analysis of the teleported qubit

Coincidence electronics

4-fold

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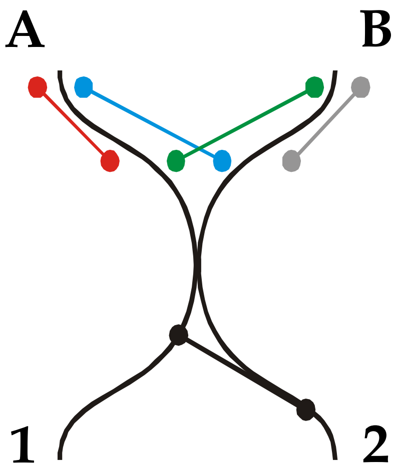
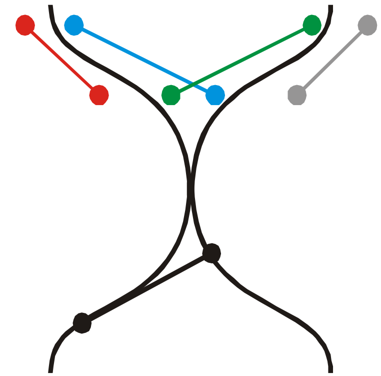
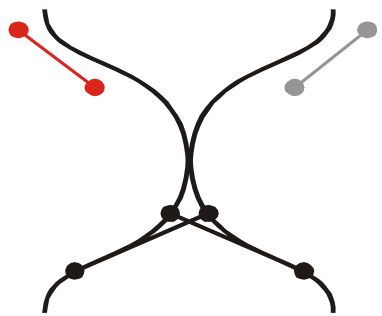
Bell state measurement

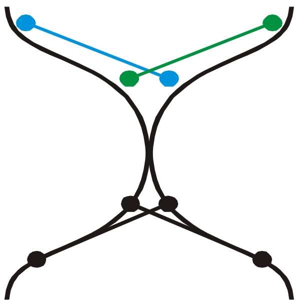




$$|0\rangle_1|1\rangle_2 \mapsto i|0\rangle_A|1\rangle_A + i|0\rangle_B|1\rangle_B + |0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B$$

$$|1\rangle_1|0\rangle_2 \mapsto i|0\rangle_A|1\rangle_A + i|0\rangle_B|1\rangle_B - |0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B$$

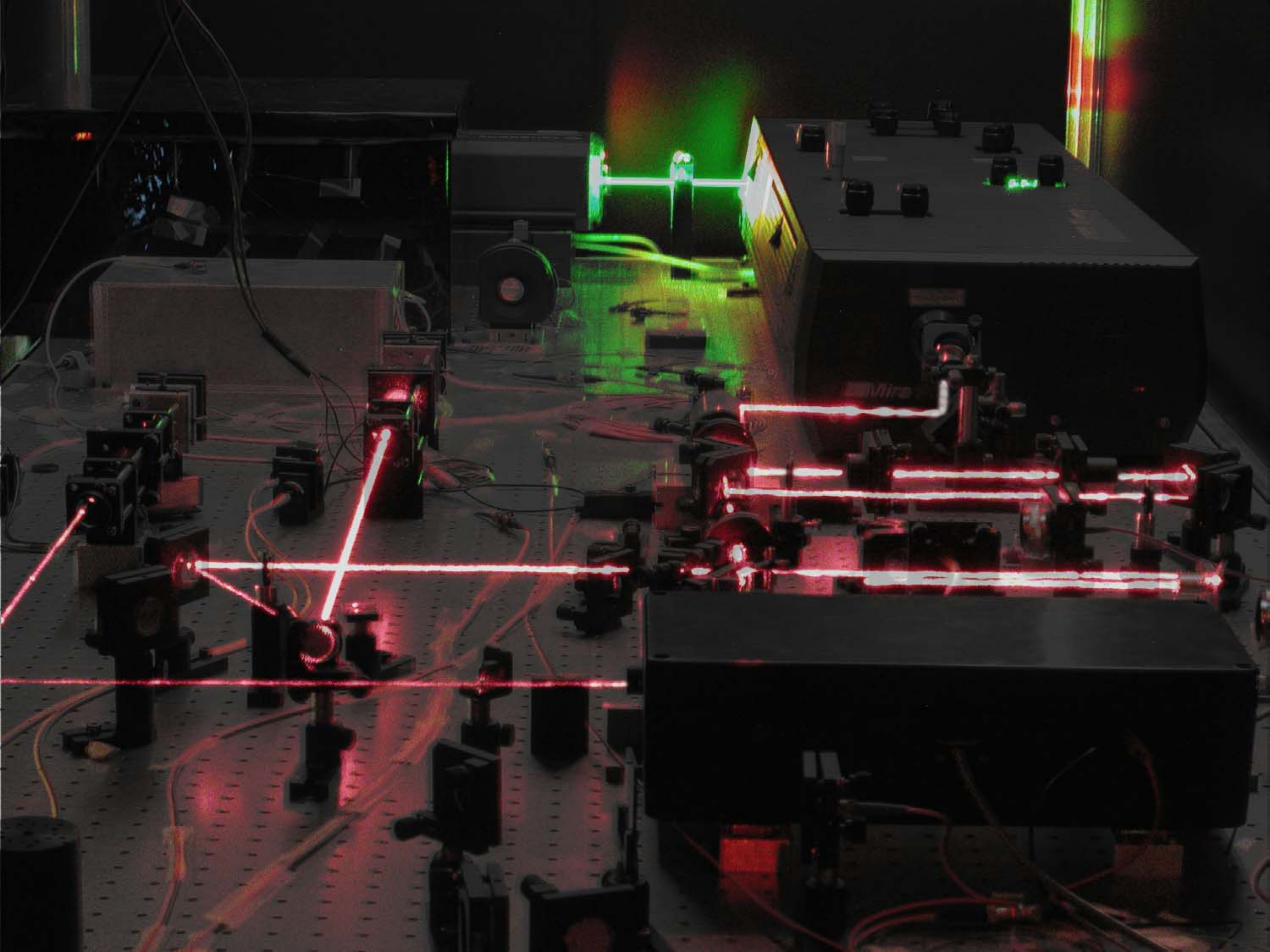




$$|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2 \mapsto i(|0\rangle_A|1\rangle_A + |0\rangle_B|1\rangle_B)$$

$$|0\rangle_1|1\rangle_2 - |1\rangle_1|0\rangle_2 \mapsto |0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B$$

H. Weinfurter, Europhysics Letters <u>25</u>, 559-564 (1994)
H. de Riedmatten *et al.*, *Phys. Rev. A* <u>67</u>, 022301 (2003)



Teleportation of a time-bin qubit
equatorial states

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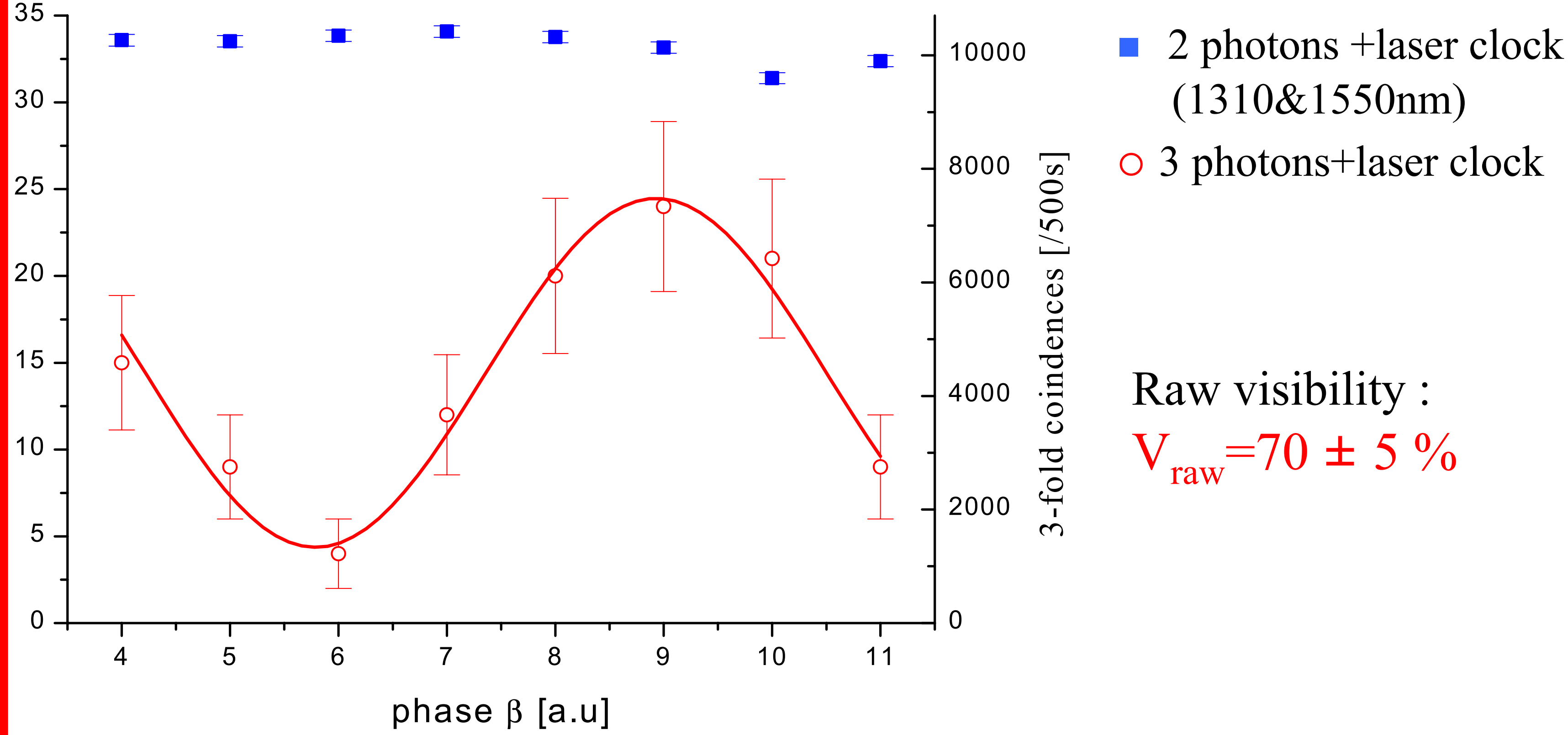


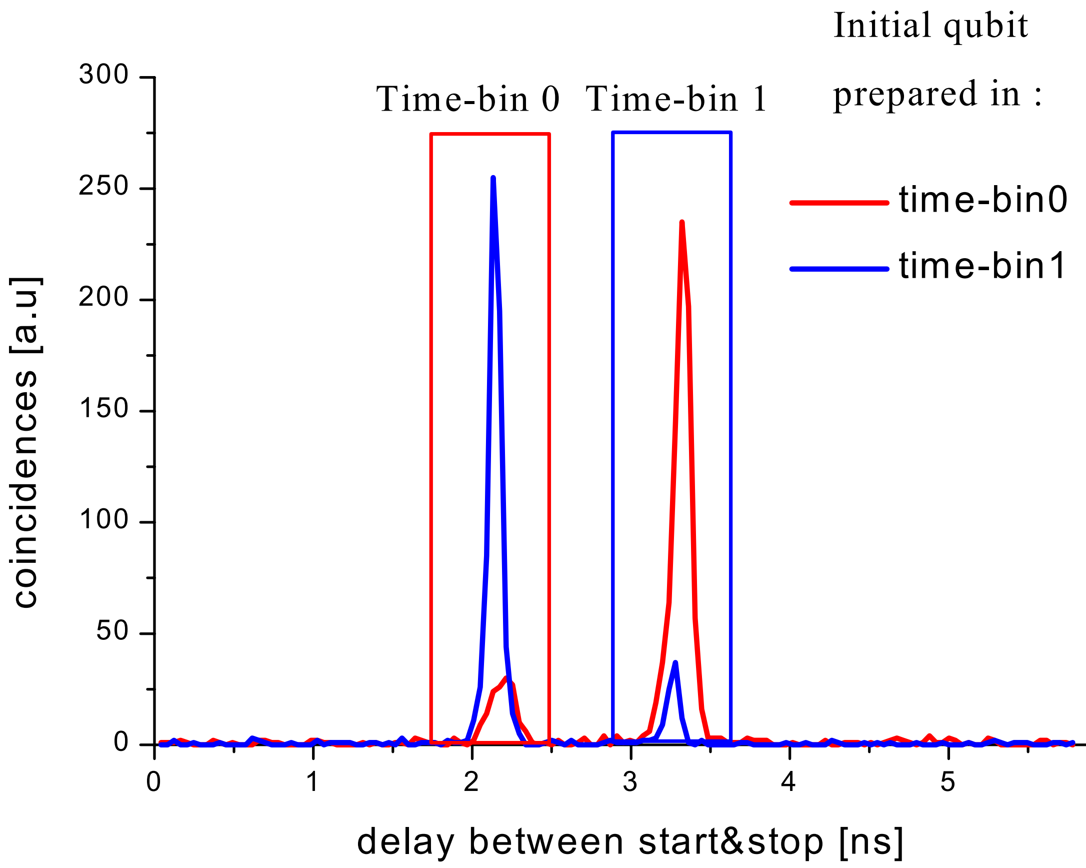
Raw visibility :

V_{raw}=70 ± 5 %

Fidelity for equatorial states : $F_{eq} = \frac{1+V_{raw}}{2}$ =85 ± 2.5 %

Teleportation of a time-bin qubit

North&South poles



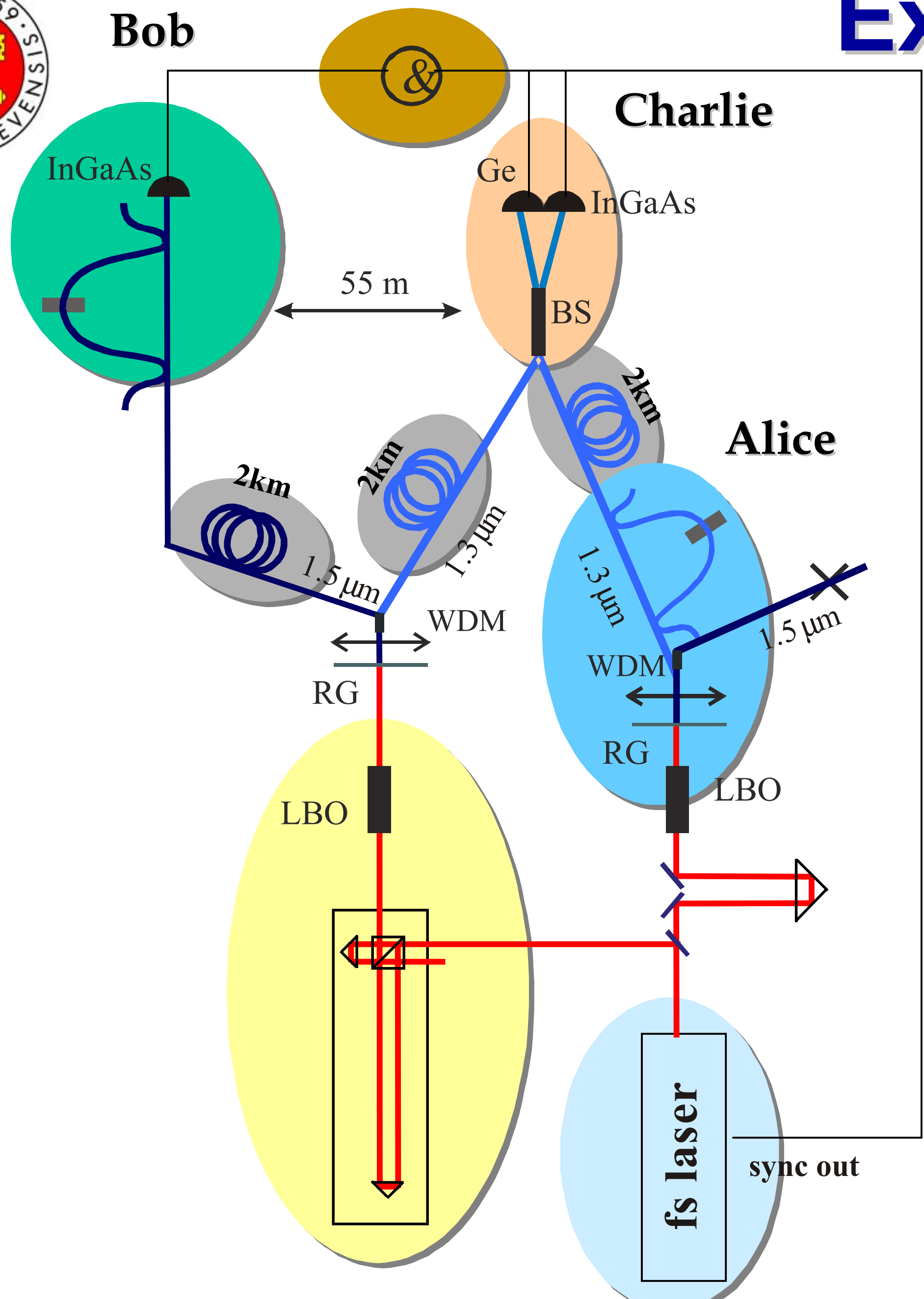
$$F(|0.1\rangle) = \frac{P(|1,0\rangle)}{P(|1,0\rangle) + P(|0,1\rangle)}$$

$$F(|0,1\rangle) = 88\% \pm 4\%$$

$$F(|1,0\rangle) = 84\% \pm 4\%$$

$$F_{tot} = \frac{2}{3}F_{eq} + \frac{1}{3}F_p = 85\ \% \pm 3\ \%$$

Experimental setup



fs laser @ 710 nm

Alice:creation of qubits to be teleported

creation of entangled qubits

Charlie:the Bell measurement

Bob:analysis of the teleported qubit, 55 m from Charlie

2 km of optical fiber

coincidence electronics

I. Marcikic et al., Nature, 421, 509-513, 2003

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results

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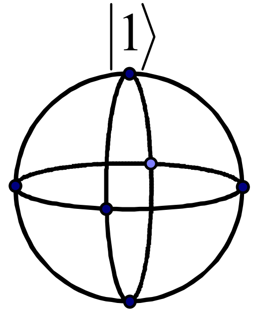


Equatorial states

$F = \frac{C_{correct}}{\ldots}$

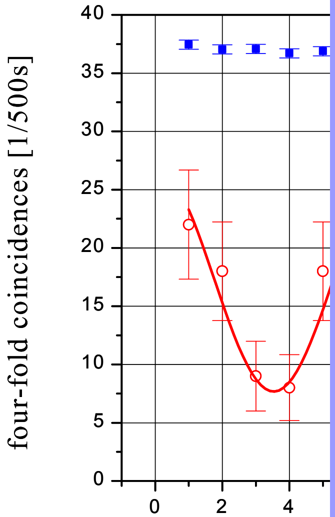
wrong

± 3%

± 3%

77.5 ± 3 %

th poles

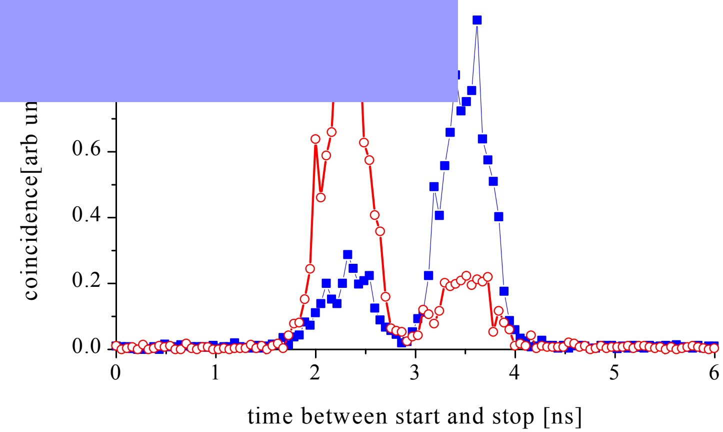


Mean Fidelity

$$F_{mean} = \frac{2}{3} F_{eq} + \frac{1}{3} F_p$$

$$= 77.5 \pm 2.5\ \%$$

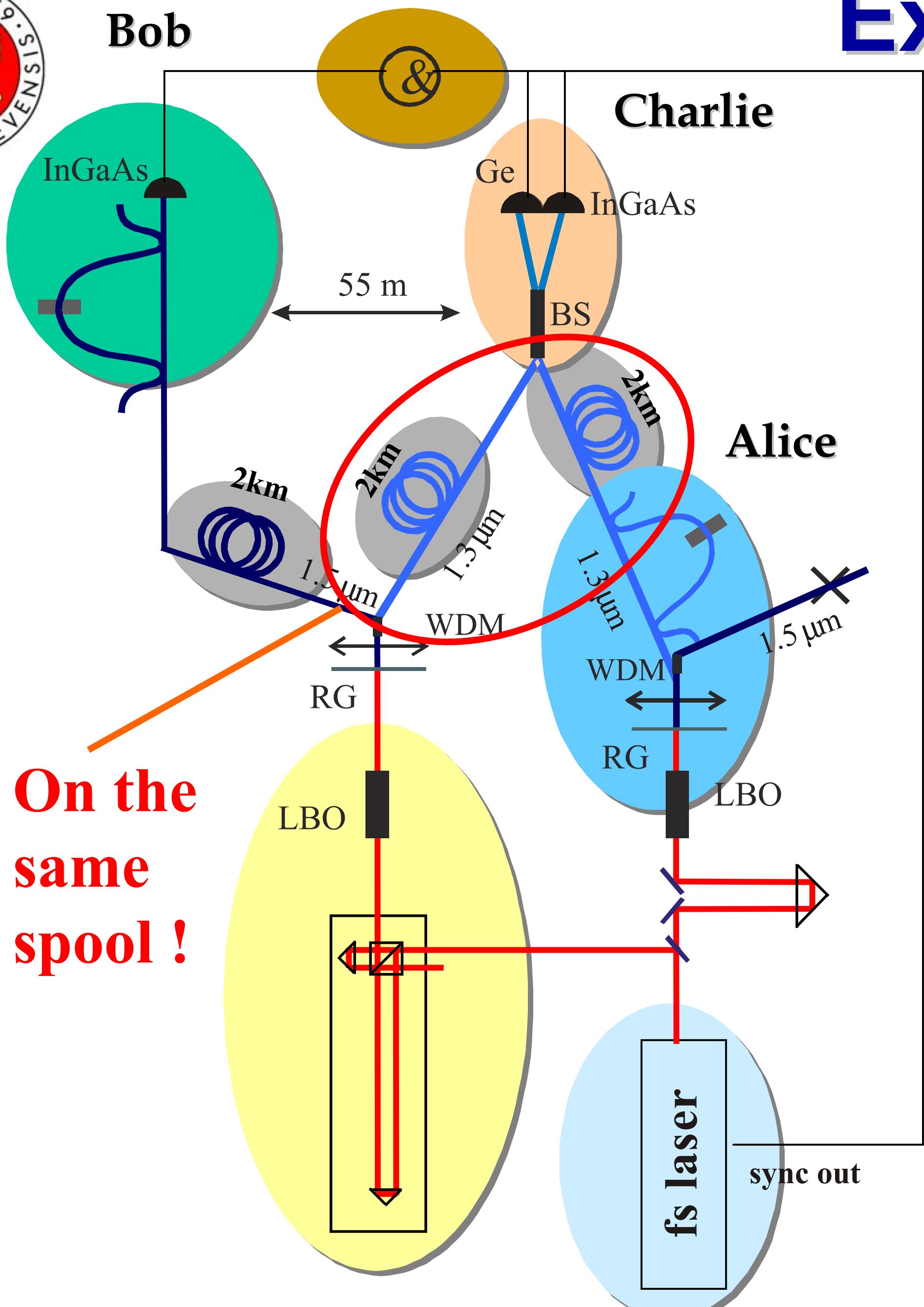
» 67 % (no entanglement)



Raw visibility : $V_{raw} = 55 \pm 5\ \%$

$$F_{eq} = \frac{1 + V_{raw}}{2} = 77.5 \pm 2.5\ \%$$

Experimental setup



fs laser @ 710 nm

Alice:creation of qubits to be teleported

creation of entangled qubits

Charlie:the Bell measurement

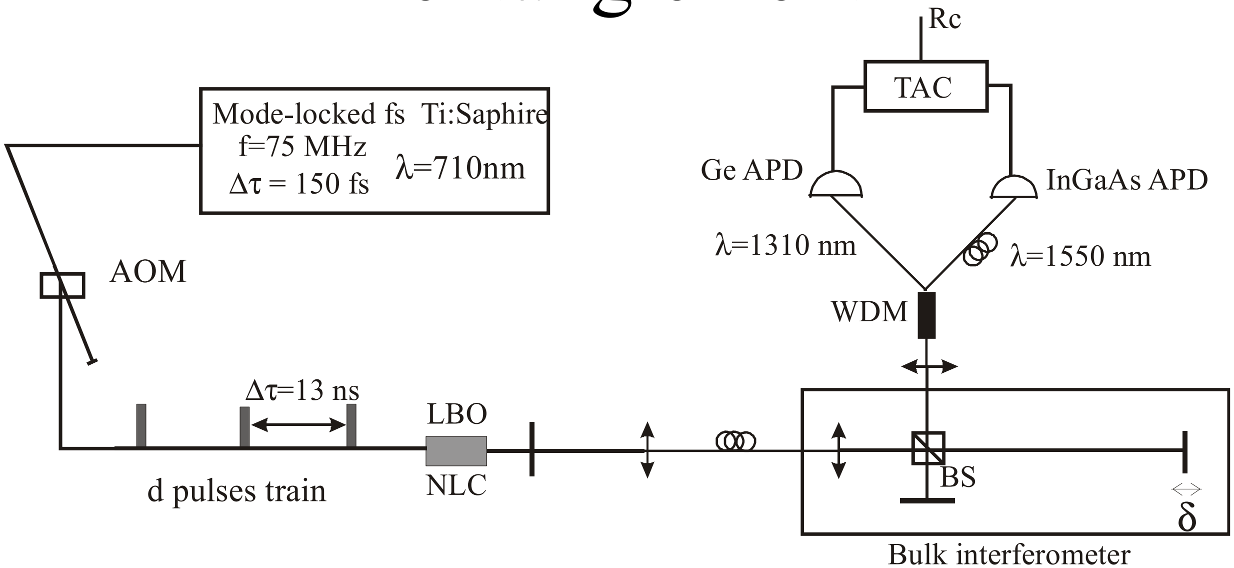
Bob:analysis of the teleported qubit, 55 m from Charlie

2 km of optical fiber

coincidence electronics

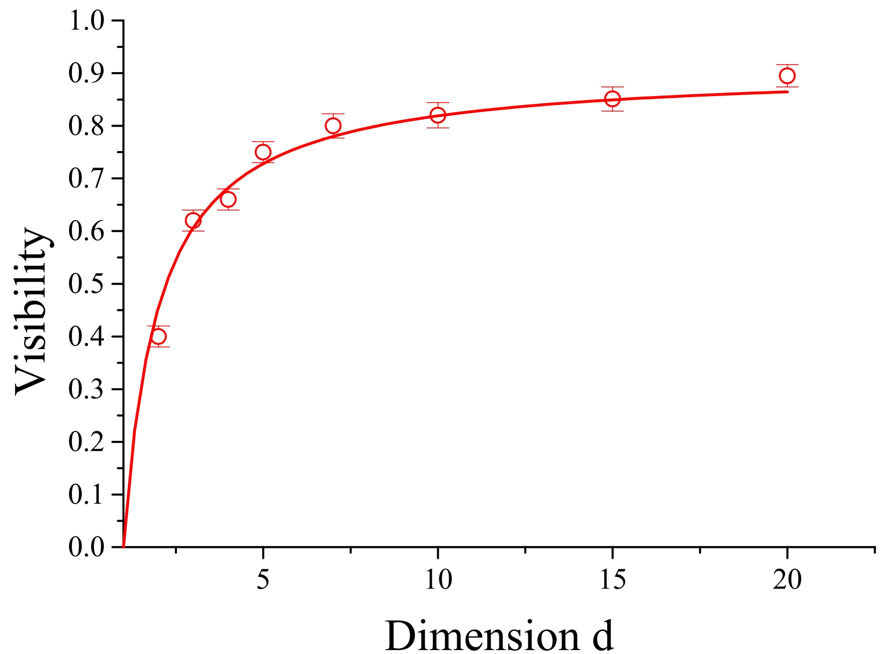
I. Marcikic et al., Nature, 421, 509-513, 2003

Tailoring high dimensional time-bin entanglement



Analysis with 2-arms interferometer

$$V = V_{max}\frac{d-1}{d}$$



de Riedmatten et al,
Quant. Inf. Comput, 2, 425 (2002)

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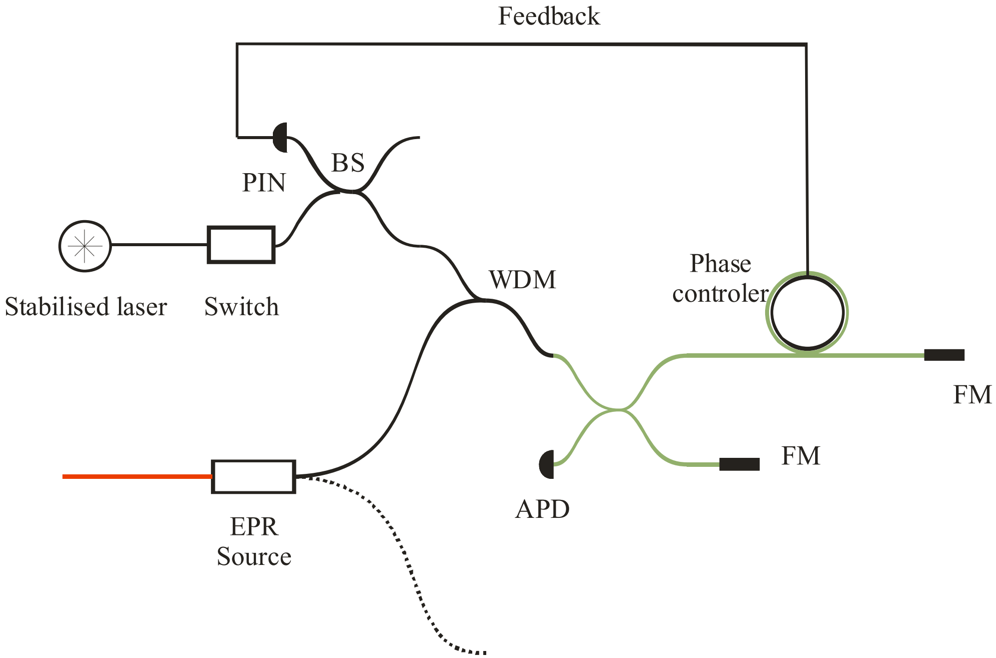
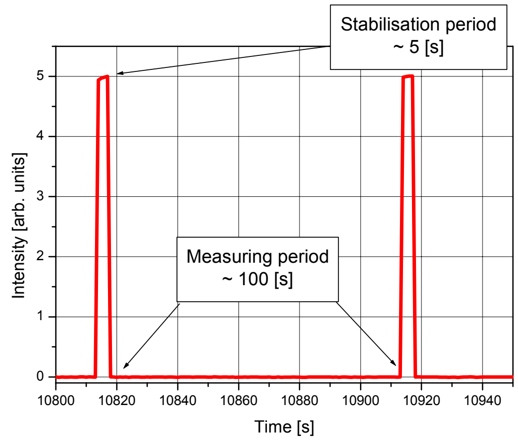
Stabilisation of the interferometers

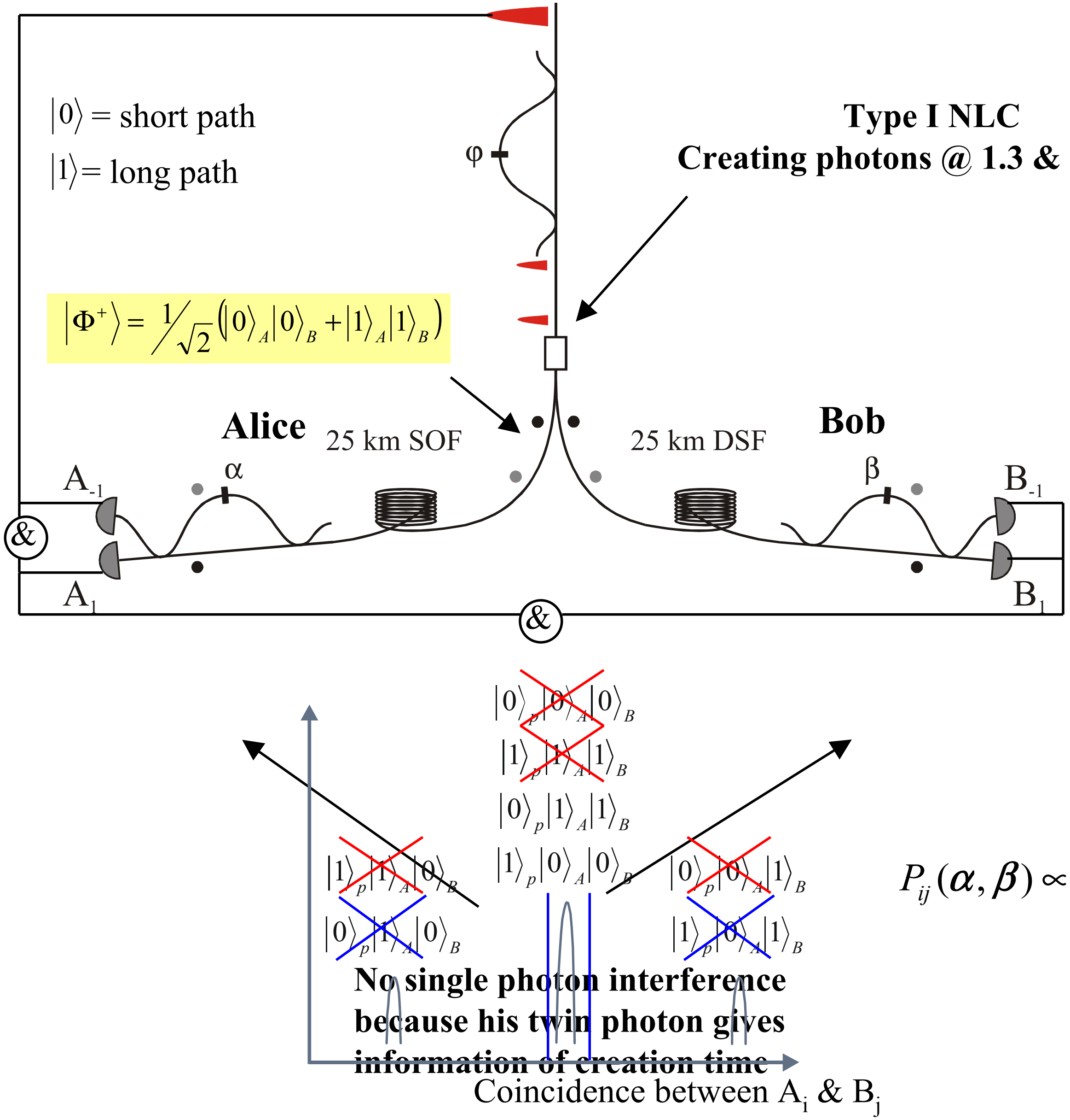
- Idea: verify from time to time the phase





Every 100 s the phase is brought back to a given value

Bell test over 50 km



$$P_{ij}(\alpha,\beta) \propto 1 + ijV\cos(\alpha+\beta)$$

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Bell test over 50 km

- Until now no phase control
- Correlation function: $E(\alpha,\beta) = V\cos(\alpha+\beta)$
- Violation of Bell inequalities depends on visibility:

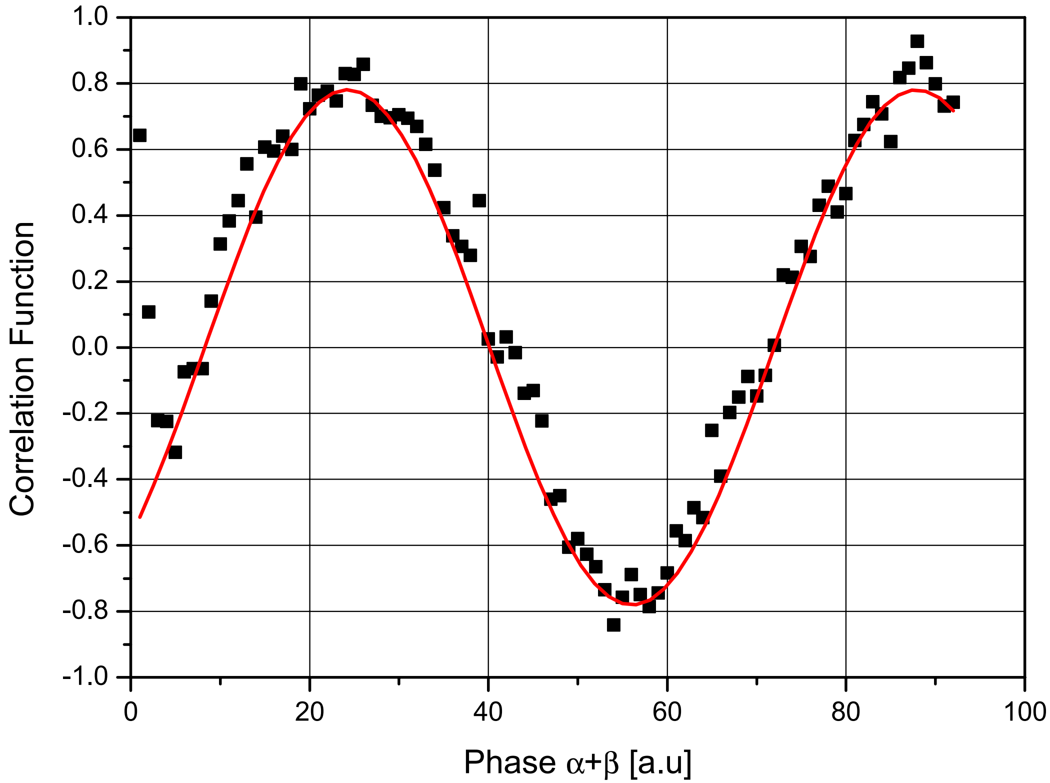
$$S = \frac{4}{\sqrt{2}} V$$

Row visibility after 50 km

$$V_{raw} = 78 \pm 1.6\%$$

$$S = 2.21 \pm 0.04$$

Violation of Bell inequalities by more than 4.5 σ



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Bell test over 50 km

- With phase control we can choose four different settings $\alpha = 0^\circ$ or 90° and $\beta = -45^\circ$ or 45°
- Violation of Bell inequalities:

$$S = E(\alpha = 0^\circ, \beta = -45^\circ) + E(\alpha = 90^\circ, \beta = -45^\circ) + E(\alpha = 0^\circ, \beta = 45^\circ) - E(\alpha = 90^\circ, \beta = 45^\circ)$$

$$E(\alpha = 0^\circ, \beta = -45^\circ) = 0.518 \pm 0.006$$

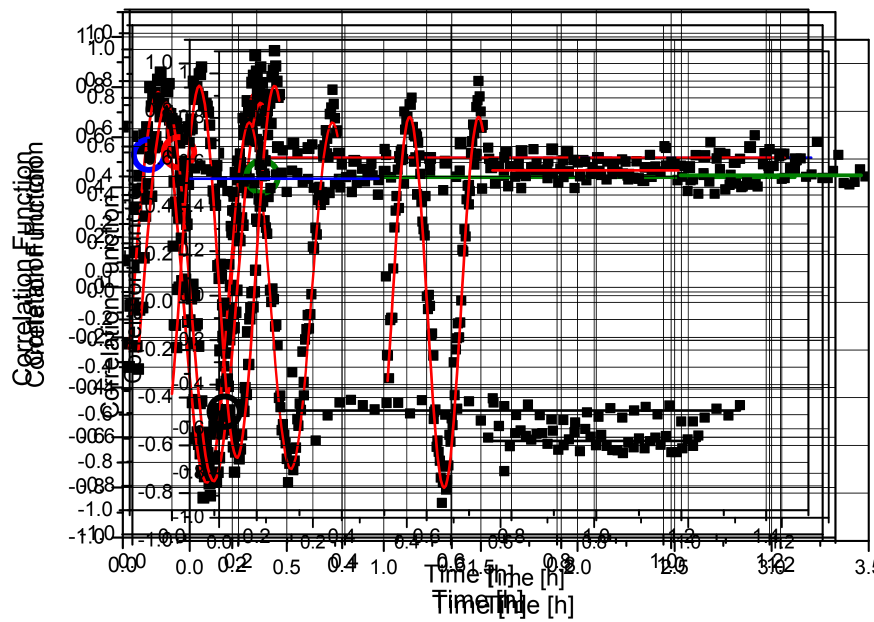
$$E(\alpha = 90^\circ, \beta = -45^\circ) = 0.554 \pm 0.005$$

$$E(\alpha = 0^\circ, \beta = 45^\circ) = 0.533 \pm 0.006$$

$$E(\alpha = 90^\circ, \beta = 45^\circ) = 0.581 \pm 0.007$$

$$S = 2.185 \pm 0.012$$

Violation of Bell inequalities by more than 15 σ



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For what could Q teleportation be useful ?

- For the physicist's fascination !
- For teaching Q physics !
- For the secure communications of tomorrow !

If the structure to be teleported contains a message, then no adversary can intercept it, since it doesn't exist anywhere inbetween the emitter and the receiver !

Let's exploit this idea

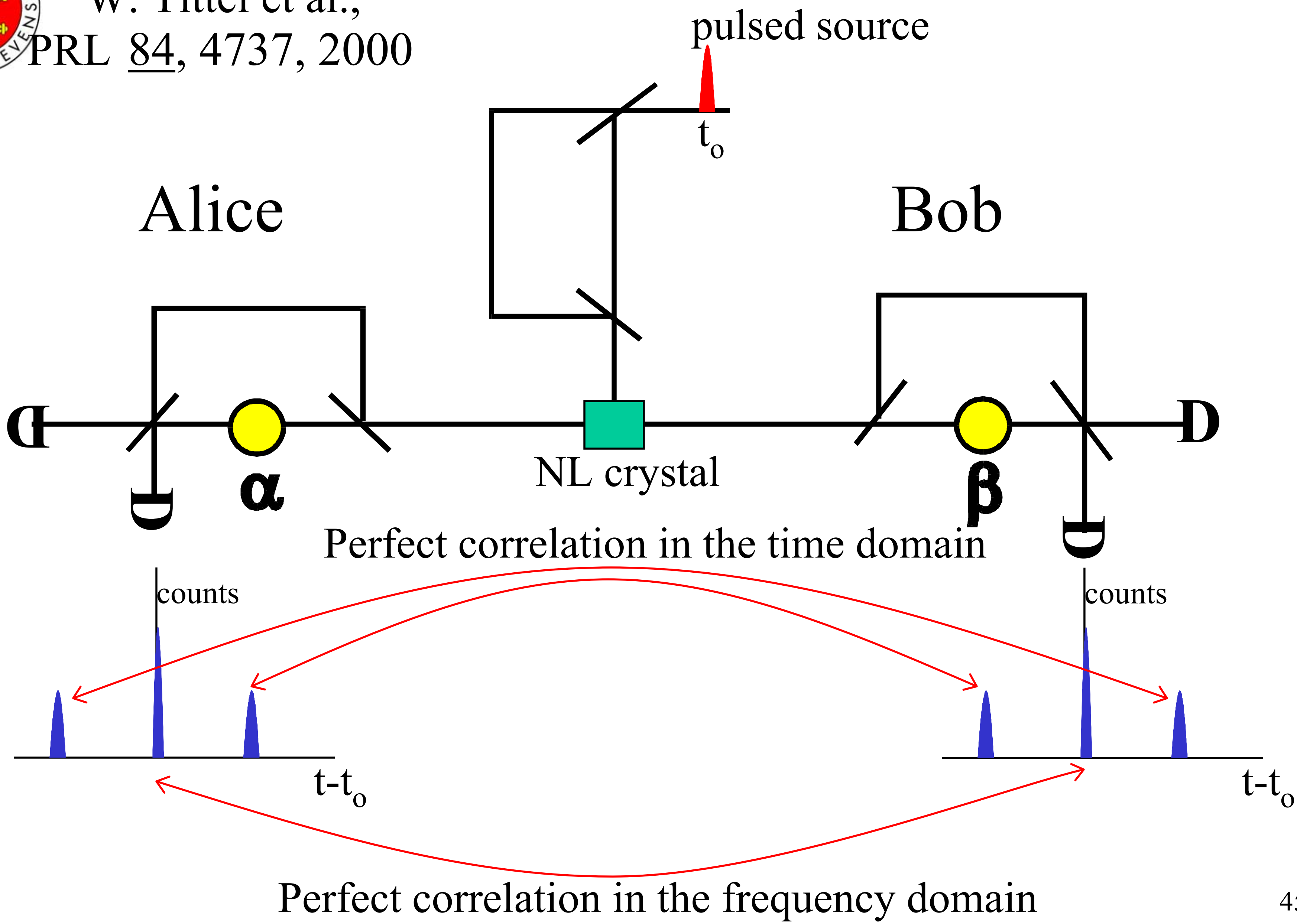
→ **to make the idea practical with today's technology**

→ **to improve tomorrow's quantum cryptography**

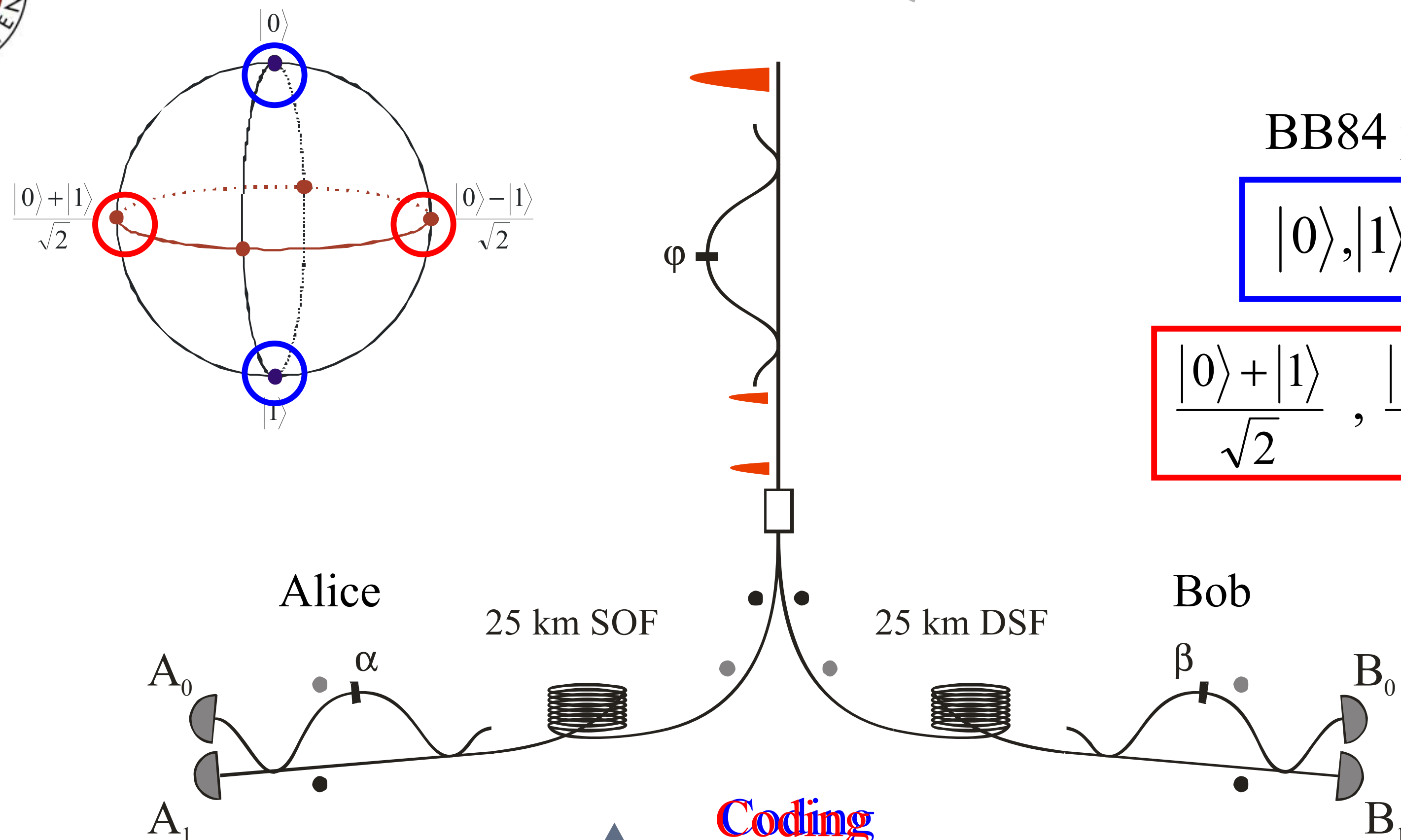

St-Gervais

Experimental QKD with entanglement

W. Tittel et al.,
PRL 84, 4737, 2000



Simulation of QKD over 50 km



BB84 protocol

$|0\rangle, |1\rangle$ basis

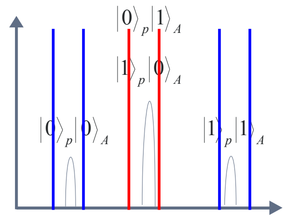
$\frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}}$ basis

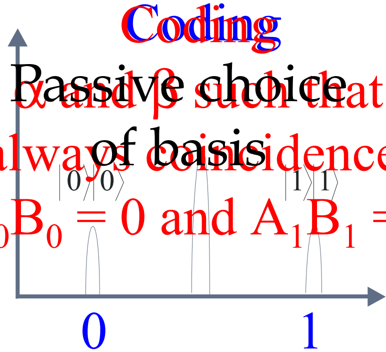
Coding

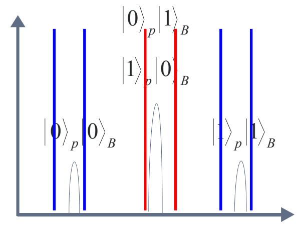
Passive choice of basis

α and β such that always coincidence

$A_0B_0 = 0$ and $A_1B_1 = 1$







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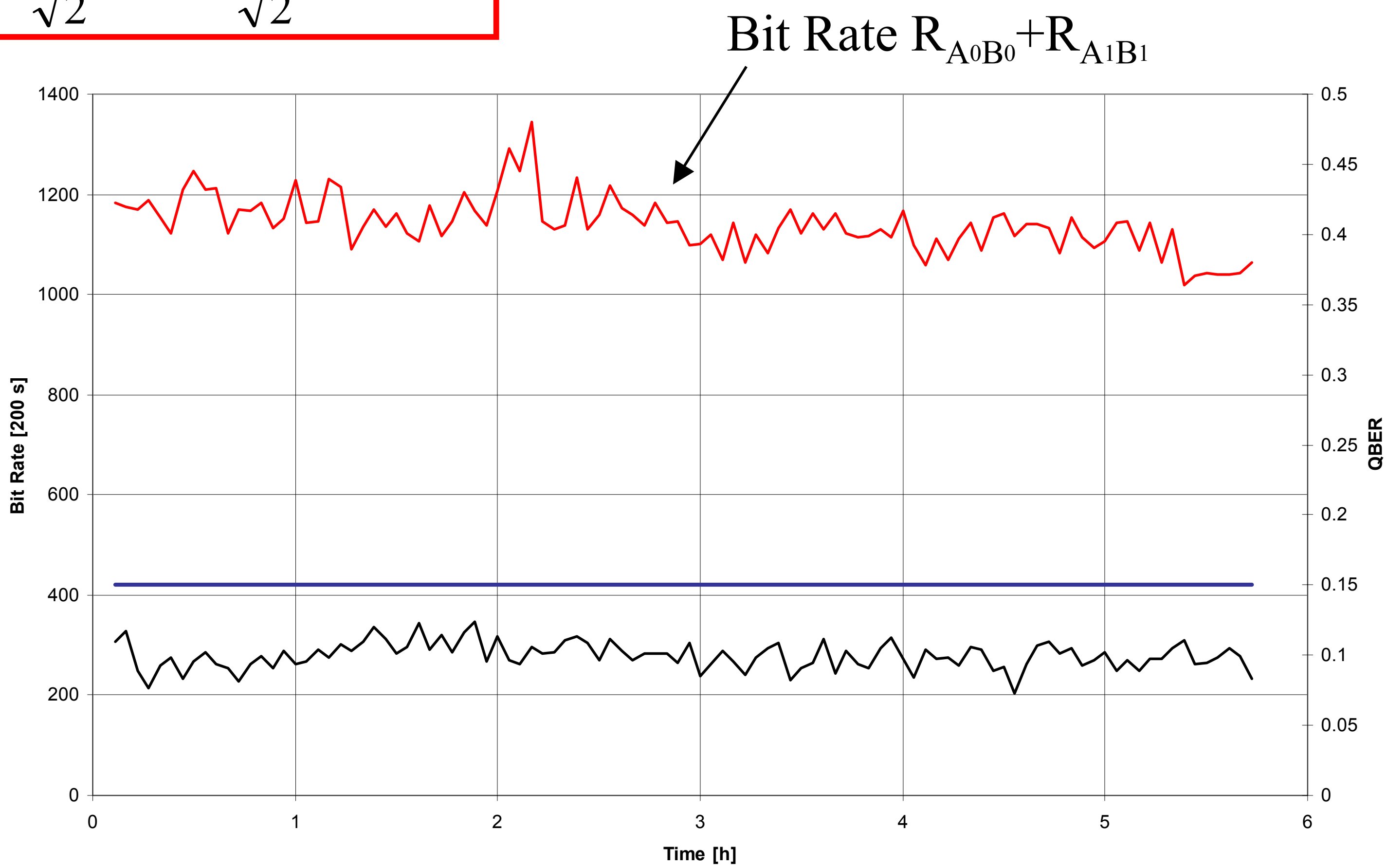
Simulation of QKD over 50 km

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}}, \frac{|0\rangle - |1\rangle}{\sqrt{2}} \text{ basis}$$



Mean value of QBER = 10 %

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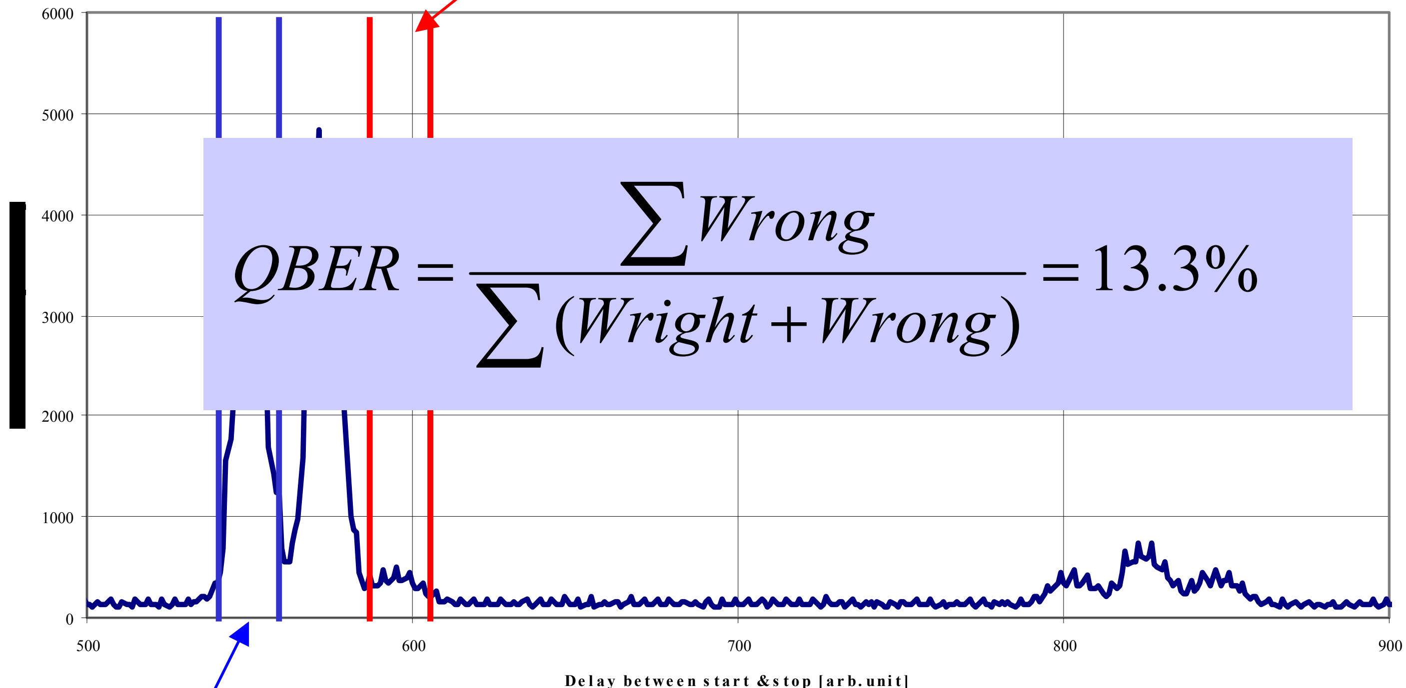
Simulation of QKD over 50 km

$|0\rangle, |1\rangle$ basis

Wrong counts



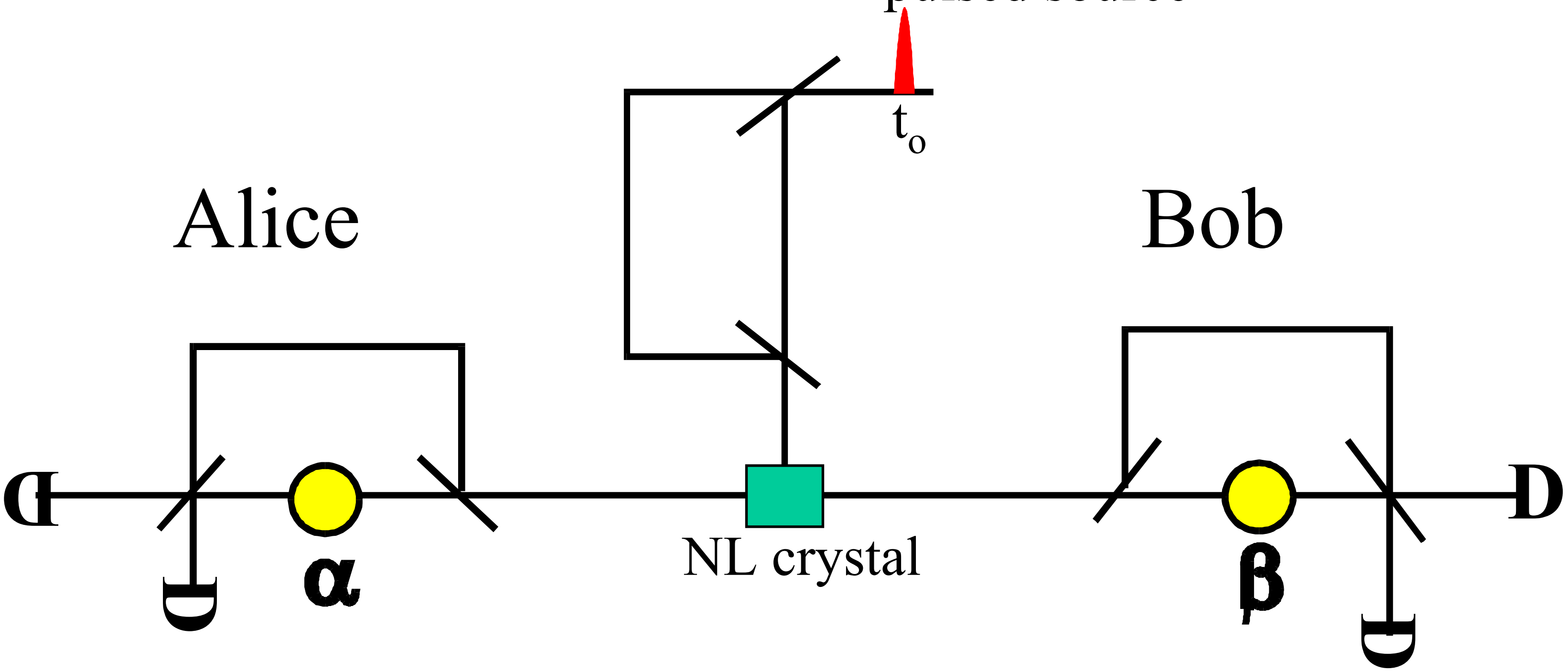
$$QBER = \frac{\sum Wrong}{\sum (Wright + Wrong)} = 13.3\%$$

Wright counts

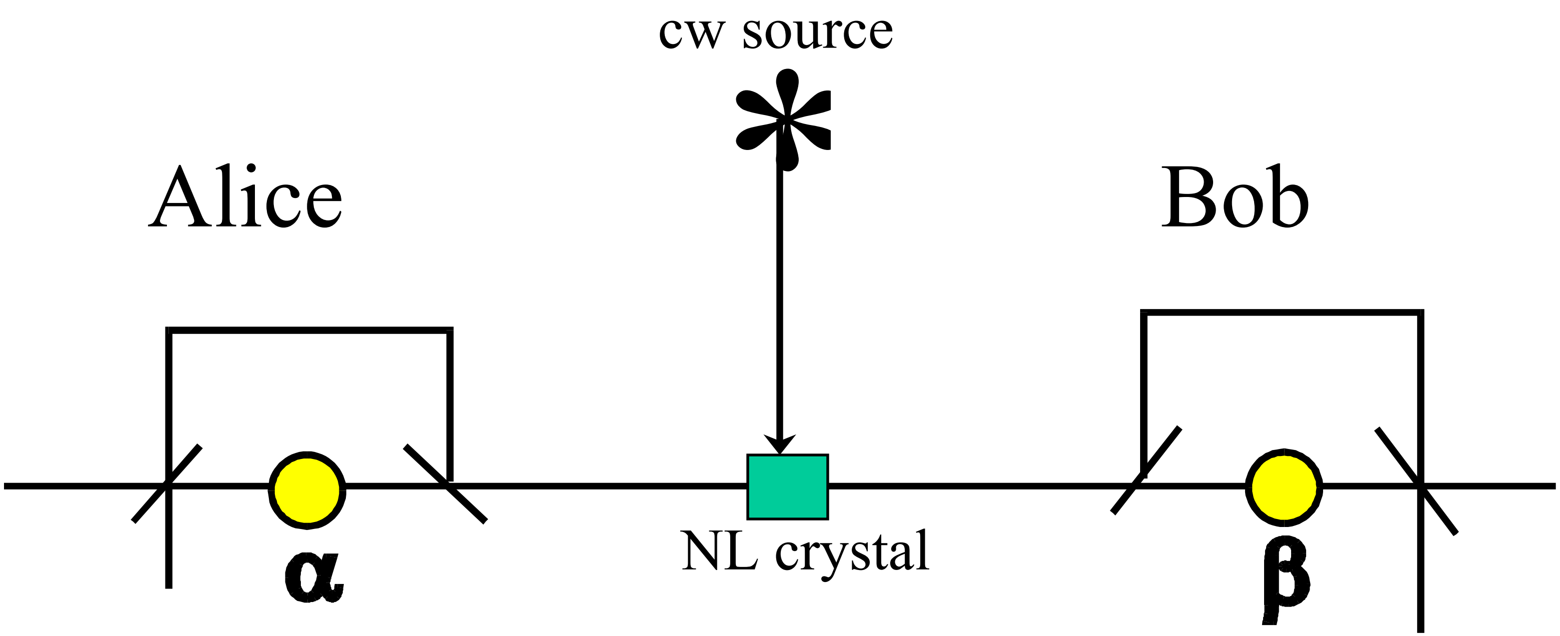
Coincidences between A_0 and B_0

Experimental QKD with entanglement

W. Tittel et al.,
PRL 84, 4737, 2000



Experimental QKD with entanglement



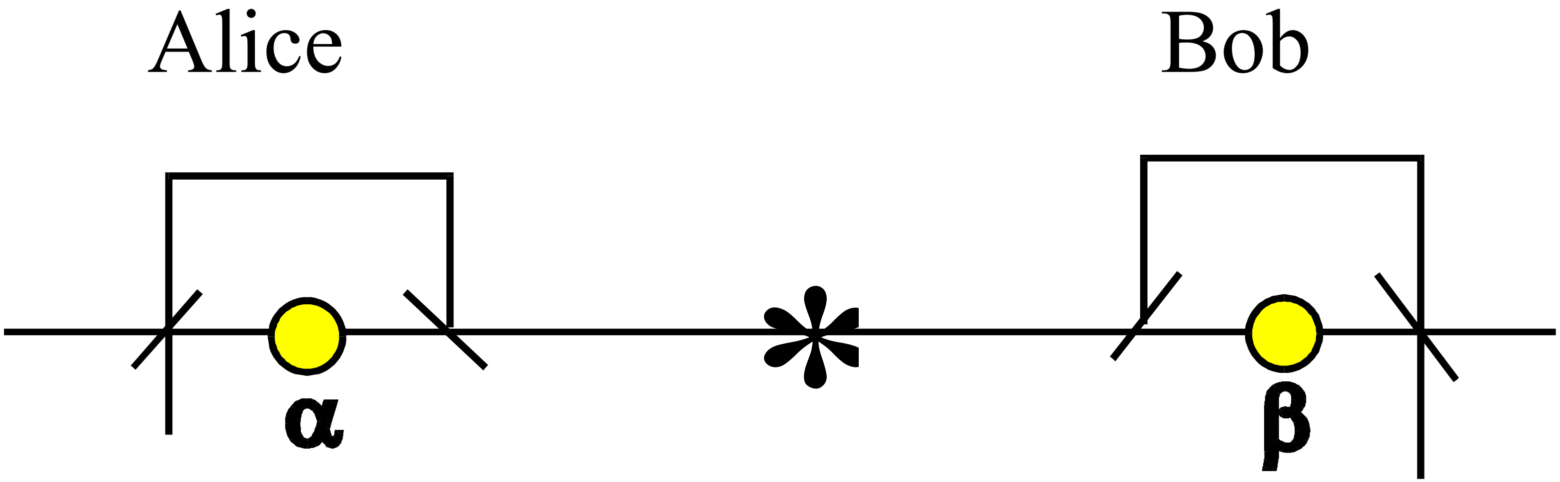
J. Franson, PRL 62, 2205, 1989

W. Tittel et al., PRL 81, 3563-3566, 1998

From Bell tests to Quantum cryptography

100% correlation $\Rightarrow$ perfect key

Fragile correlation $\Rightarrow$ secret key



W. Tittel et al., PRL 81, 3563-3566, 1998

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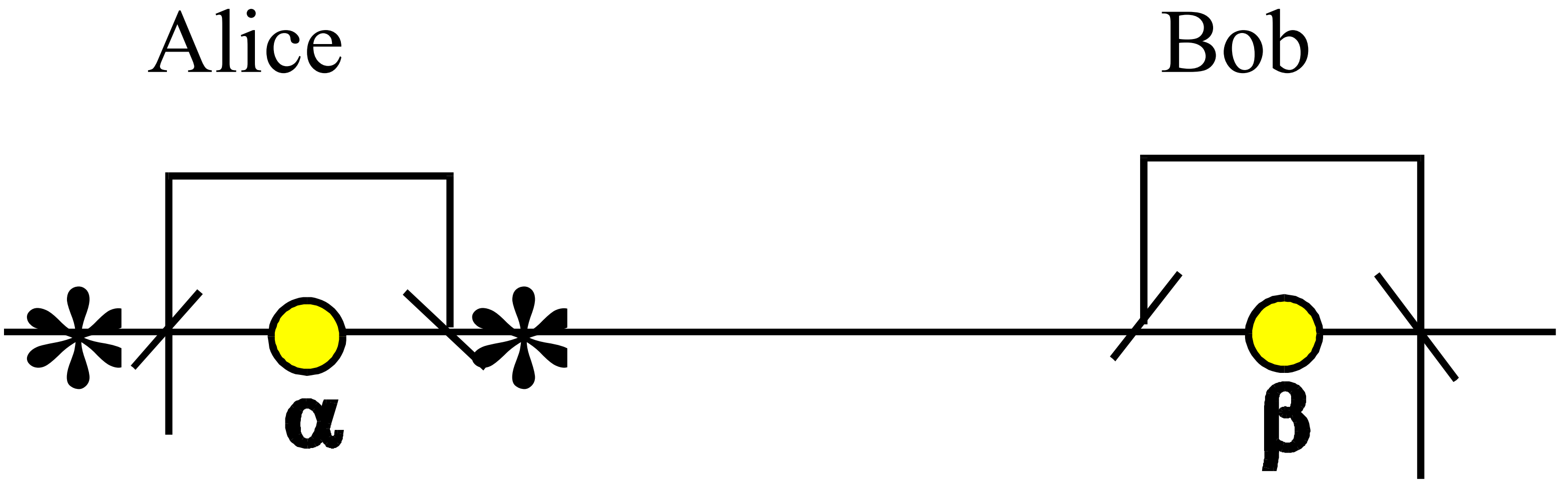
From Bell tests to Quantum cryptography

100% correlation ⇒ perfect key

Fragile correlation ⇒ secret key

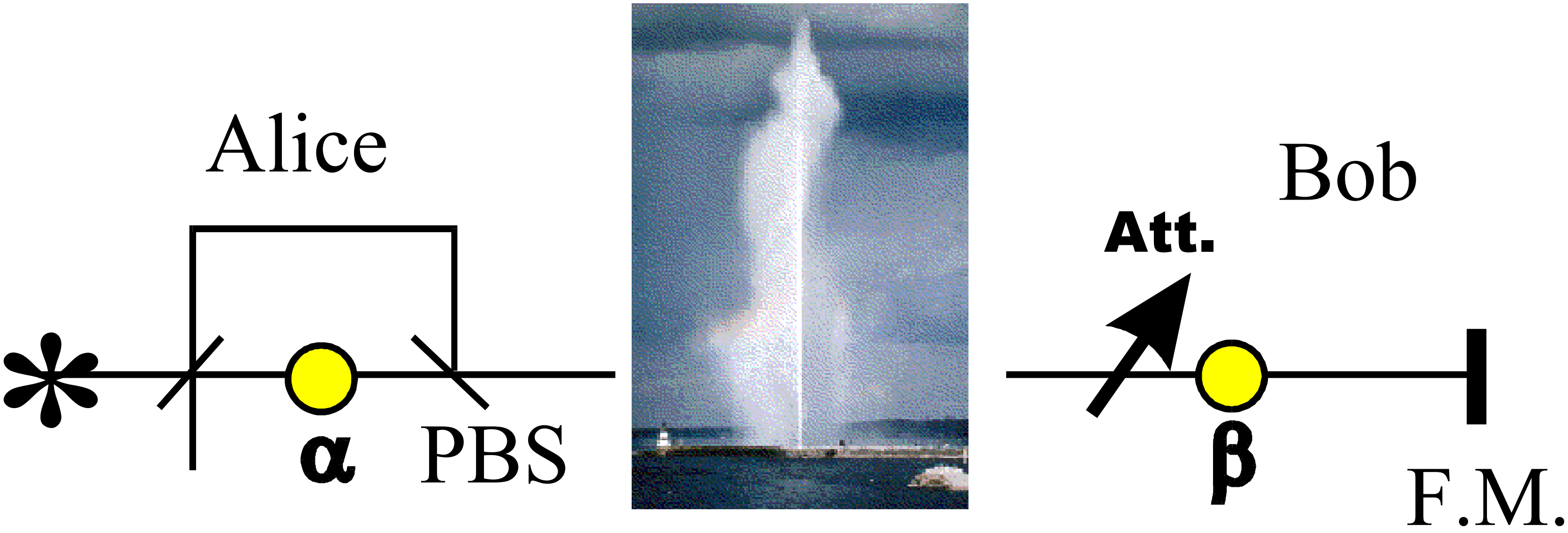


G. Ribordy et al., Phys. Rev. A 63, 012309, 2001

P.D. Townsend et al., Electr. Lett. 30, 809, 1994

R. Hughes et al., J. Modern Opt. 47, 533-547 , 2000

Quantum cryptography below lake Geneva



Applied Phys. Lett. 70, 793-795, 1997.
Electron. Letters 33, 586-588, 1997; 34, 2116-2117, 1998.
J. Modern optics 48, 2009-2021, 2001.

Pseudo single-photon Q cryptography: the Plug-&-Play configuration

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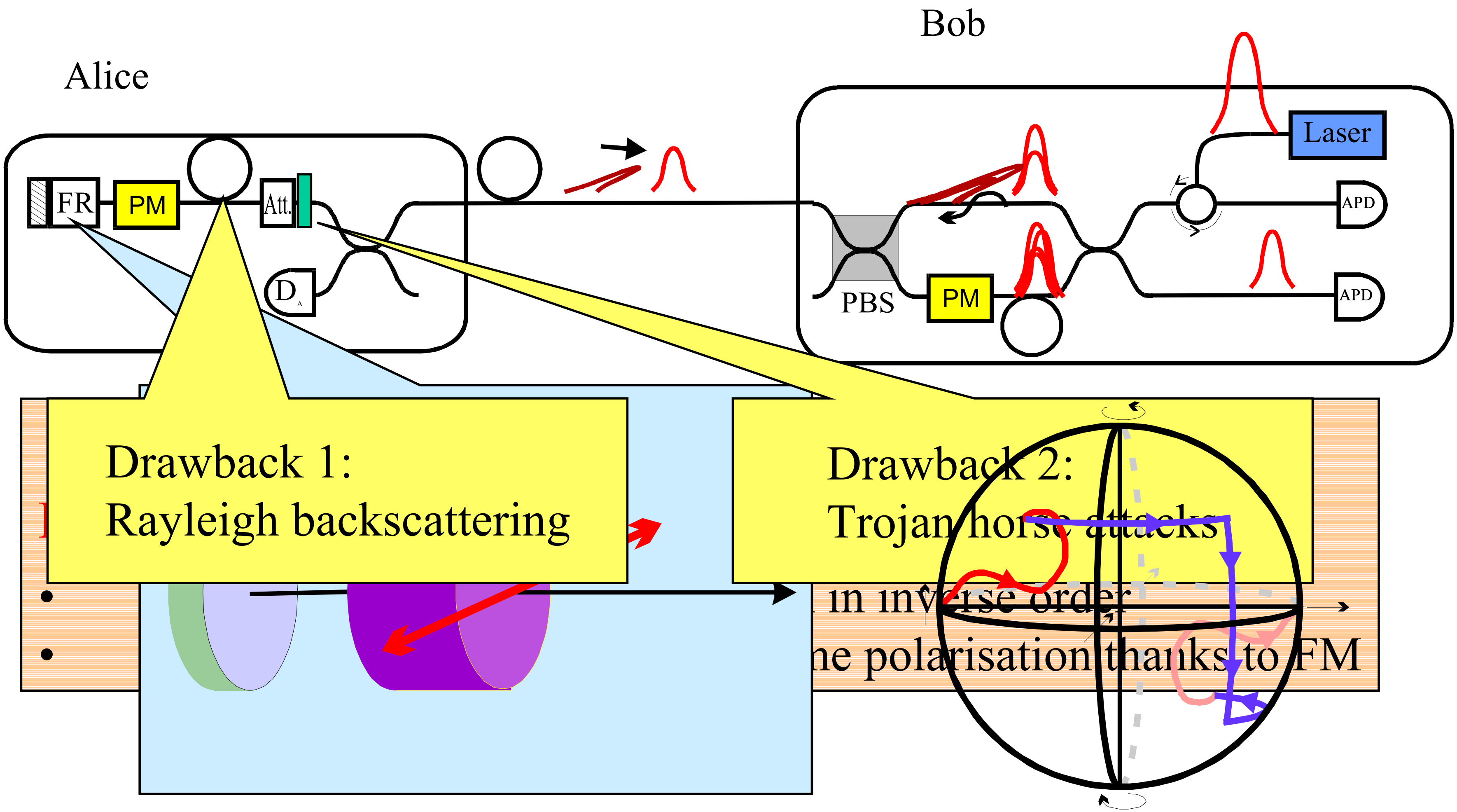

QC over 67 km, QBER ≈ 5%

RMP 74, 145-195, 2002,
Quant-ph/0101098

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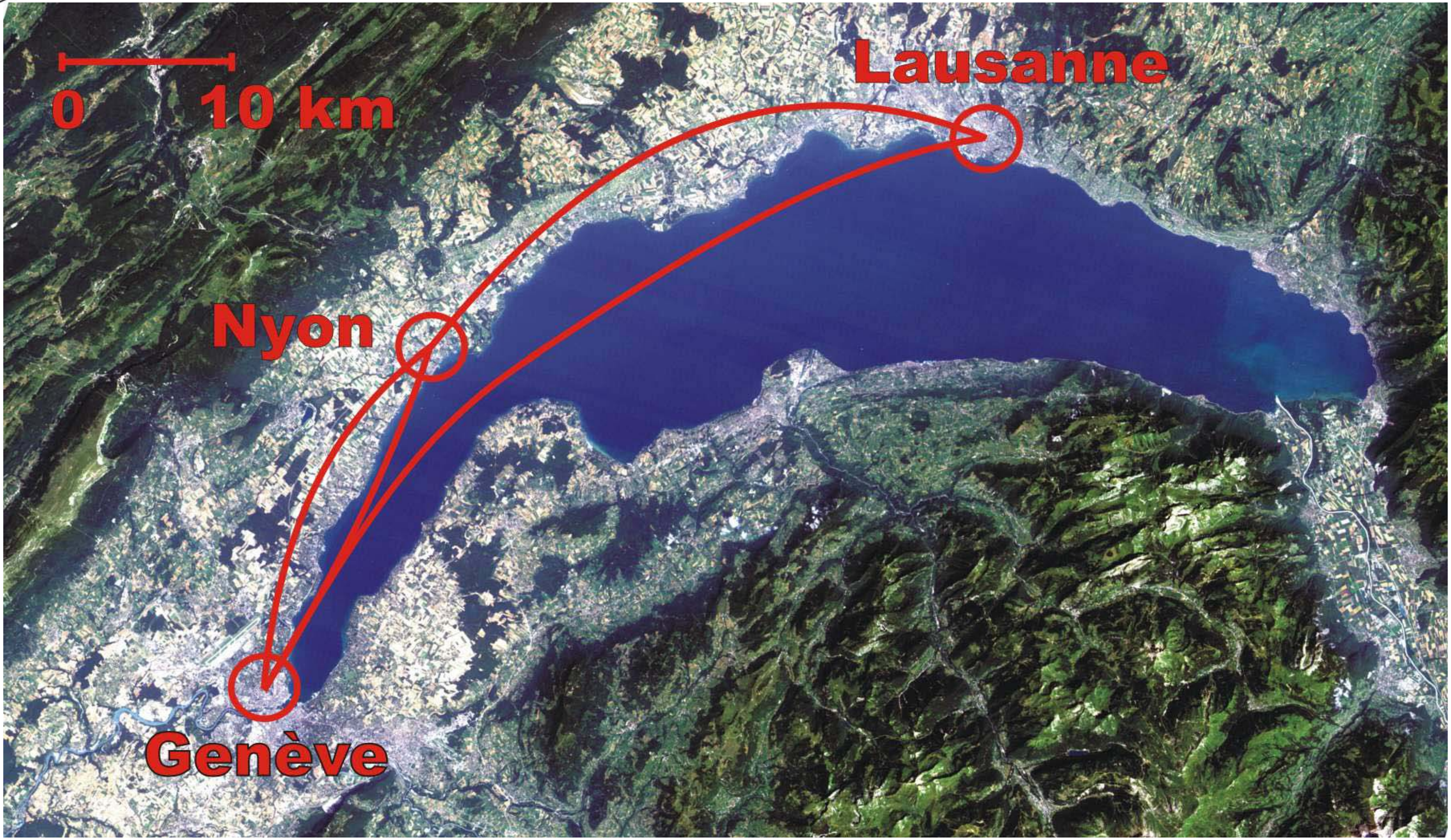
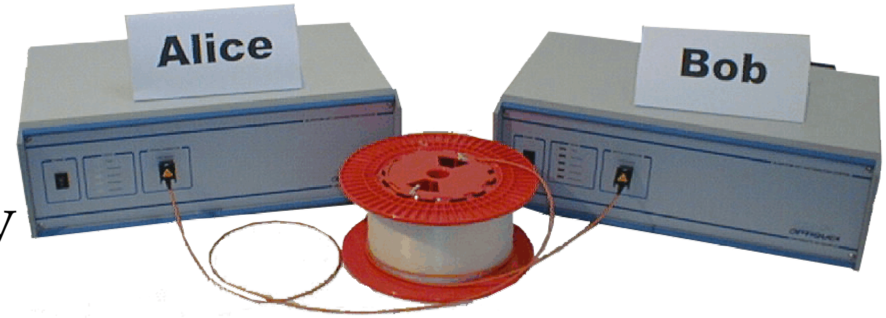


+ aerial cable (in Ste Croix, Jura) !

D. Stucki et al., New Journal of Physics 4, 41.1-41.8, 2002. Quant-ph/0203118

- Company established in 2001
  - Spin-off from the University of Geneva
- Products
  - Quantum Cryptography (optical fiber system)

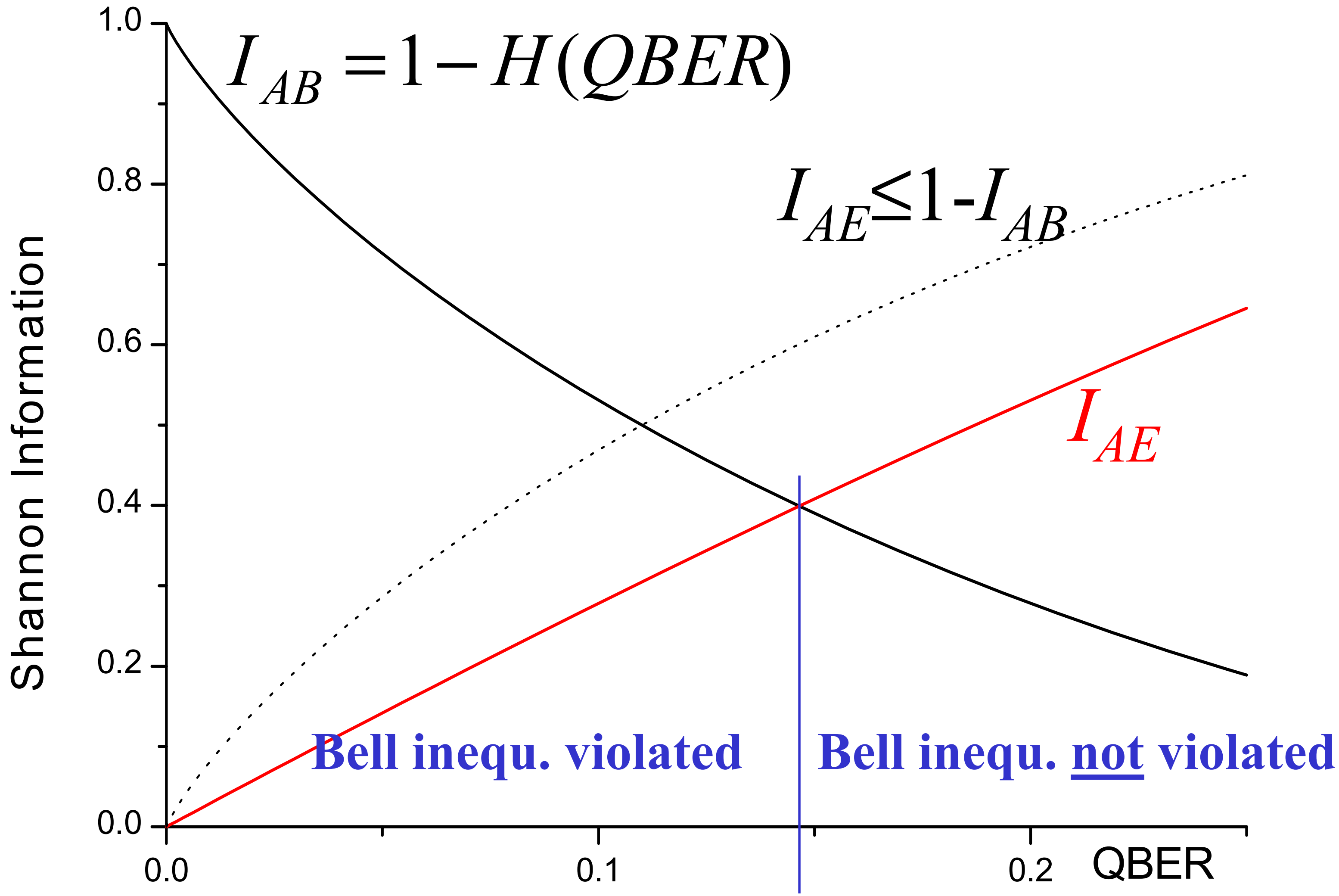


  - Quantum Random Number Generator
  - Single-photon detector module (1.3 µm and 1.55 µm)

- Contact information

  email: info@idquantique.com

  web: http://www.idquantique.com


1.0
0.8
0.6
0.4
0.2
0.0
Shannon Information
0.0
0.1
0.2
QBER
$I_{AB} = 1 - H(QBER)$
$I_{AE} \leq 1 - I_{AB}$
I_{AE}
Bell inequ. violated
Bell inequ. not violated

Limits of Q crypto

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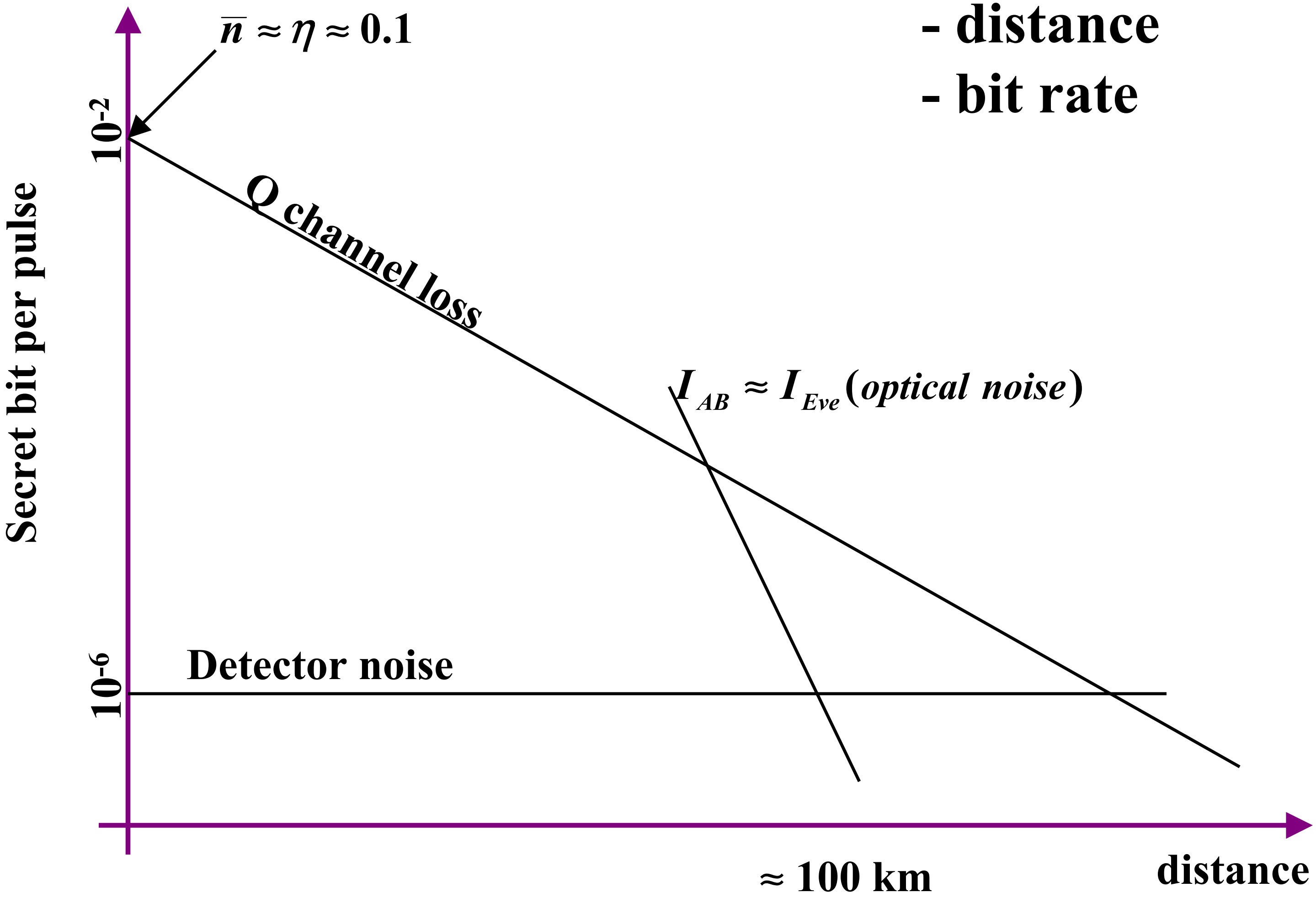
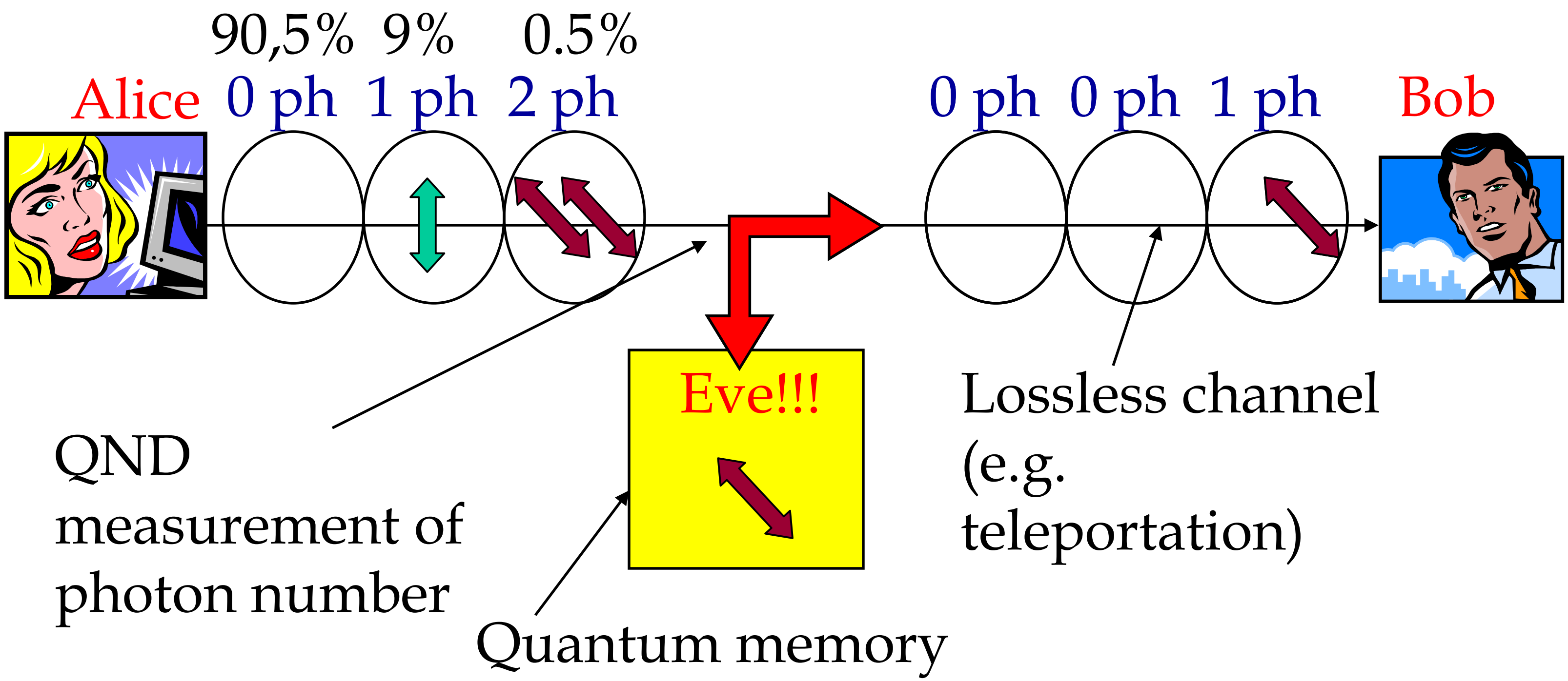
- distance
- bit rate



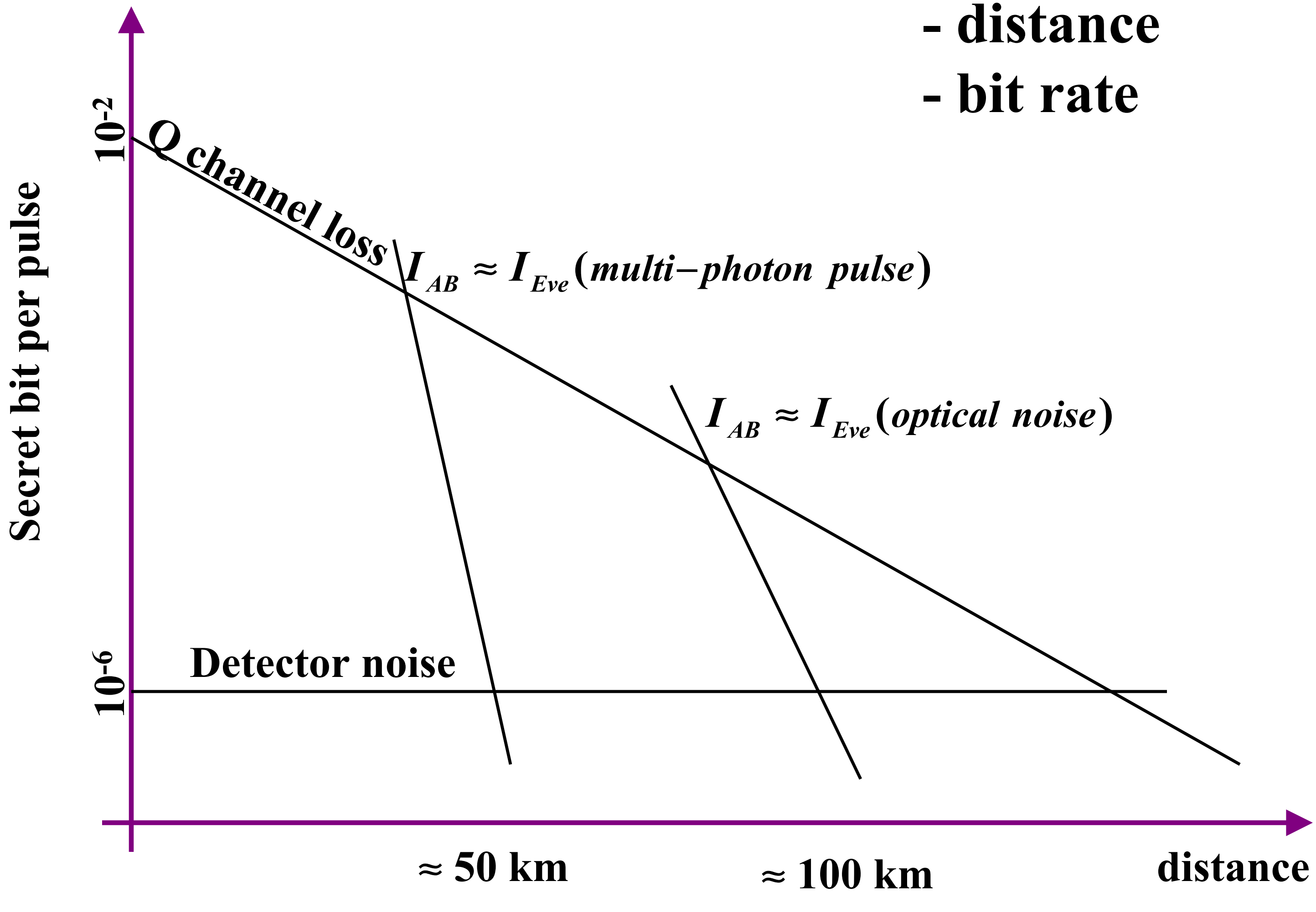
PNS Attack: the idea



- PNS (photon-number splitting):
  - The photons that reach Bob are unperturbed
  - Constraint for Eve: do not introduce more losses than expected (PNS important for long-distance QKD).

Limits of Q crypto

- distance
- bit rate



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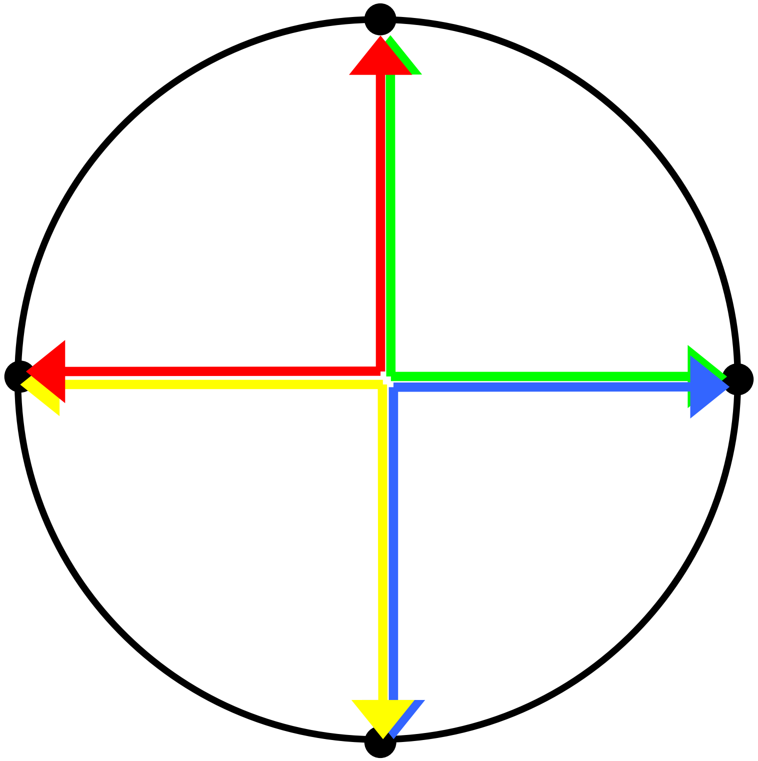
How to improve Q crypto ?

	Effect on distance	Effect on bit rate	Feasibility
Detectors	↑	↗	↑
1-υ source	↗	→	↗
Q channel	↑	↑	→
Protocols	→	↗	↑
Q relays	↗	→	↗
Q repeater	↑	↑	→

A new protocol: SARG



The quantum protol is identical to the BB84

During the public discussion phase of the new protocol Alice doesn't announce bases but sets of non-orthogonal states

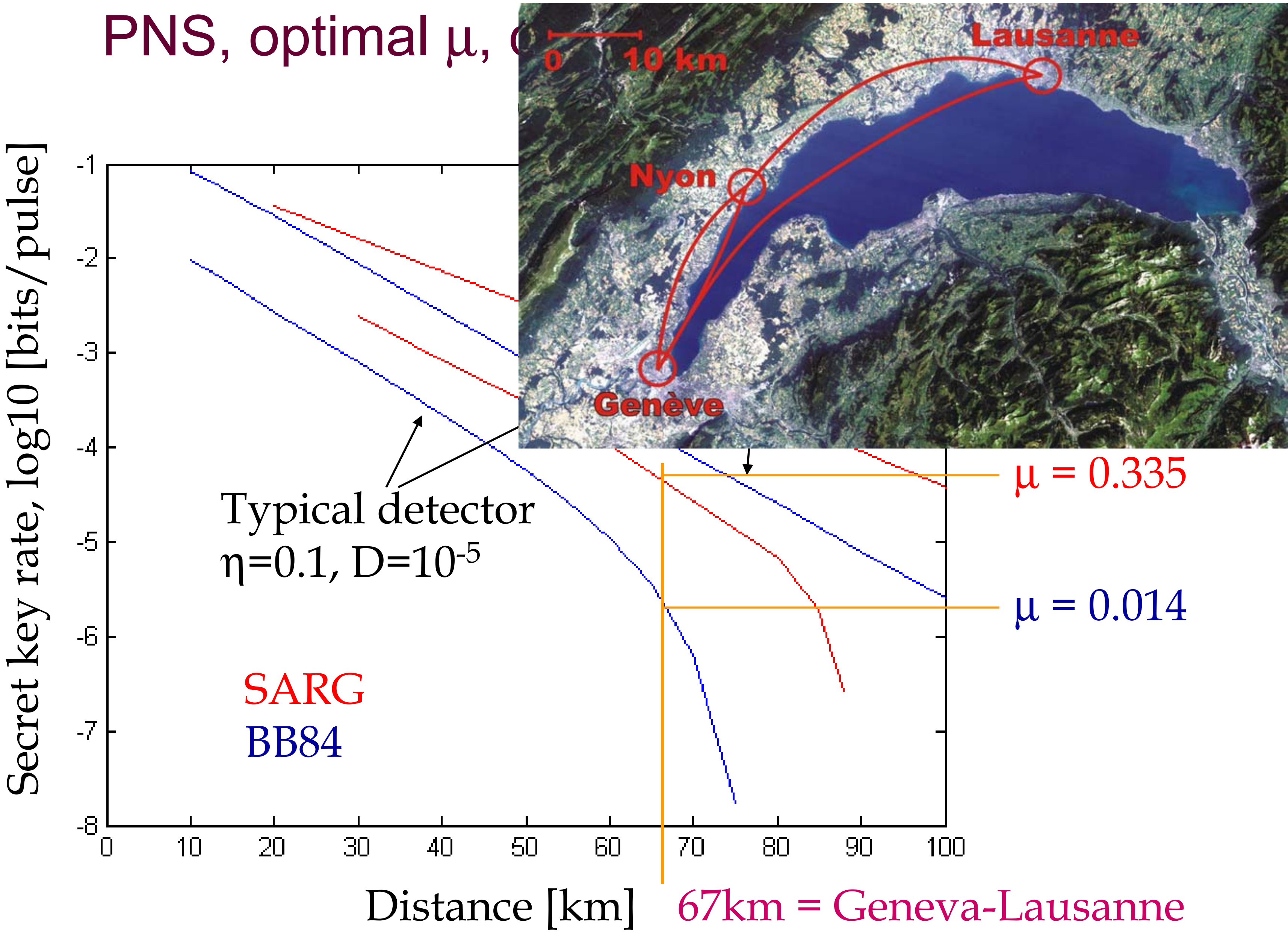
⇒ even if Eve hold a copy, she can't find out the bit with certainty

⇒ **More robust against PNS attacks !**

Joint patent UniGE + id Quantique pending
Phys. Rev. A, 2003; quant-ph/0211131& 0302037

SARG vs BB84

PNS, optimal μ, d



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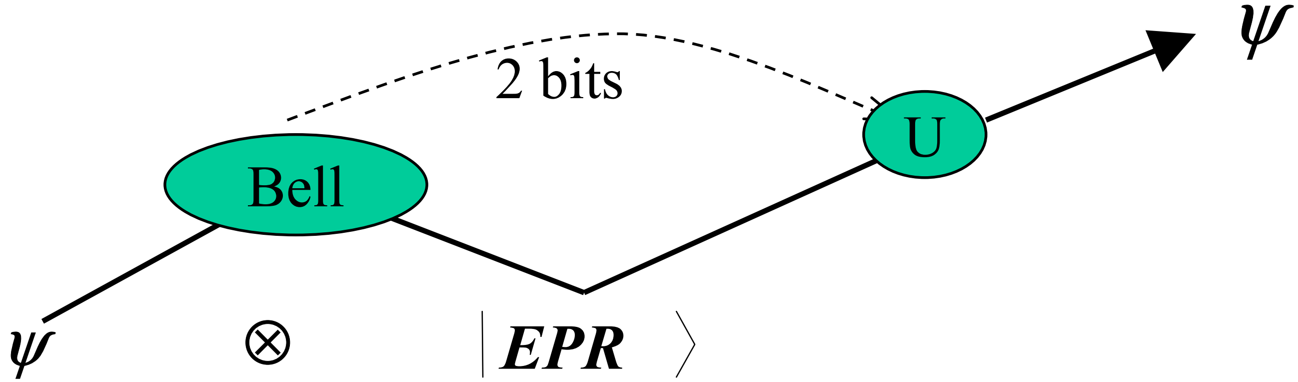
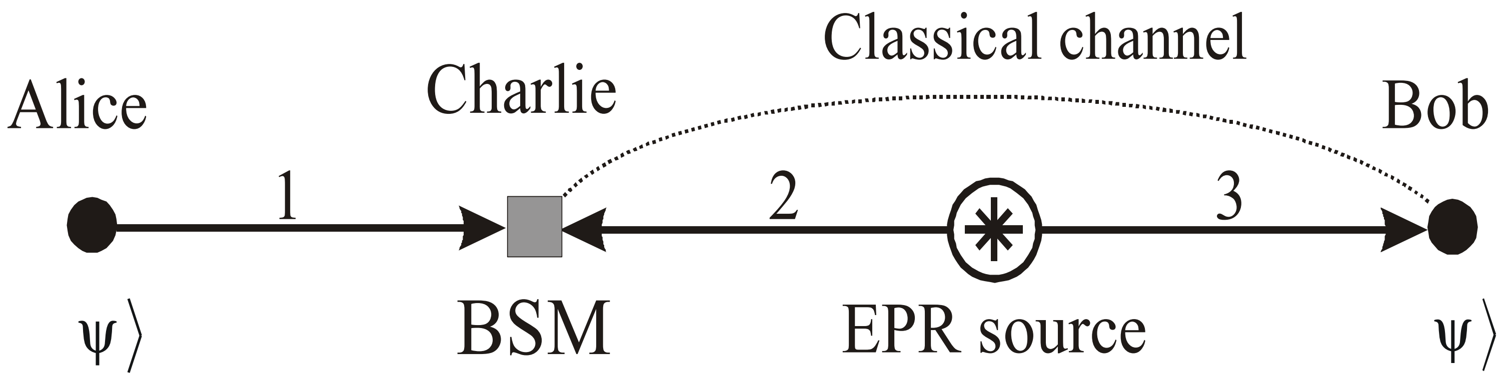
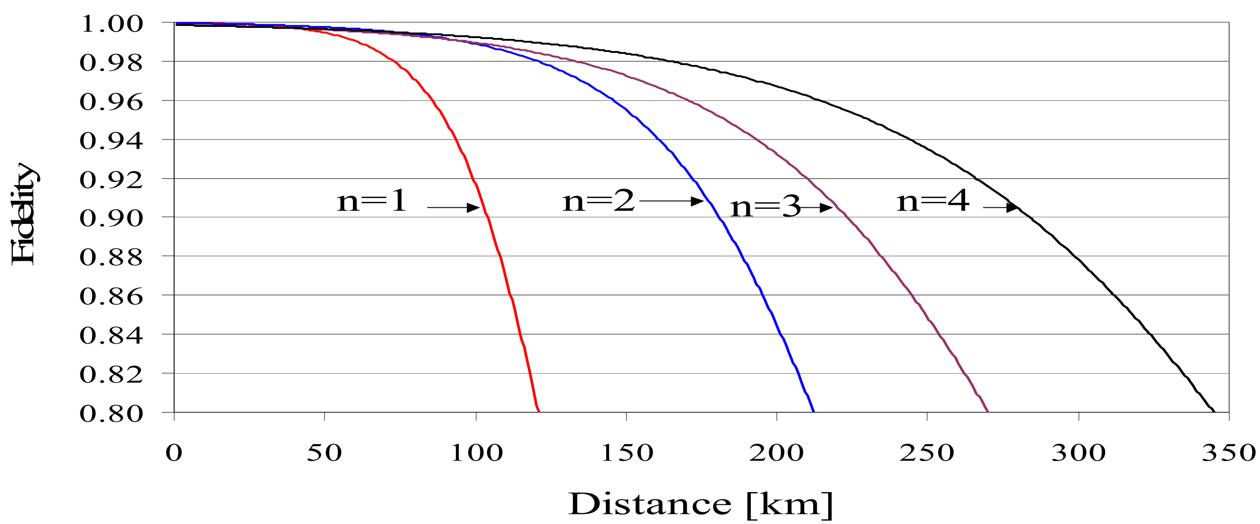
3-photon: Q teleportation & Q relays







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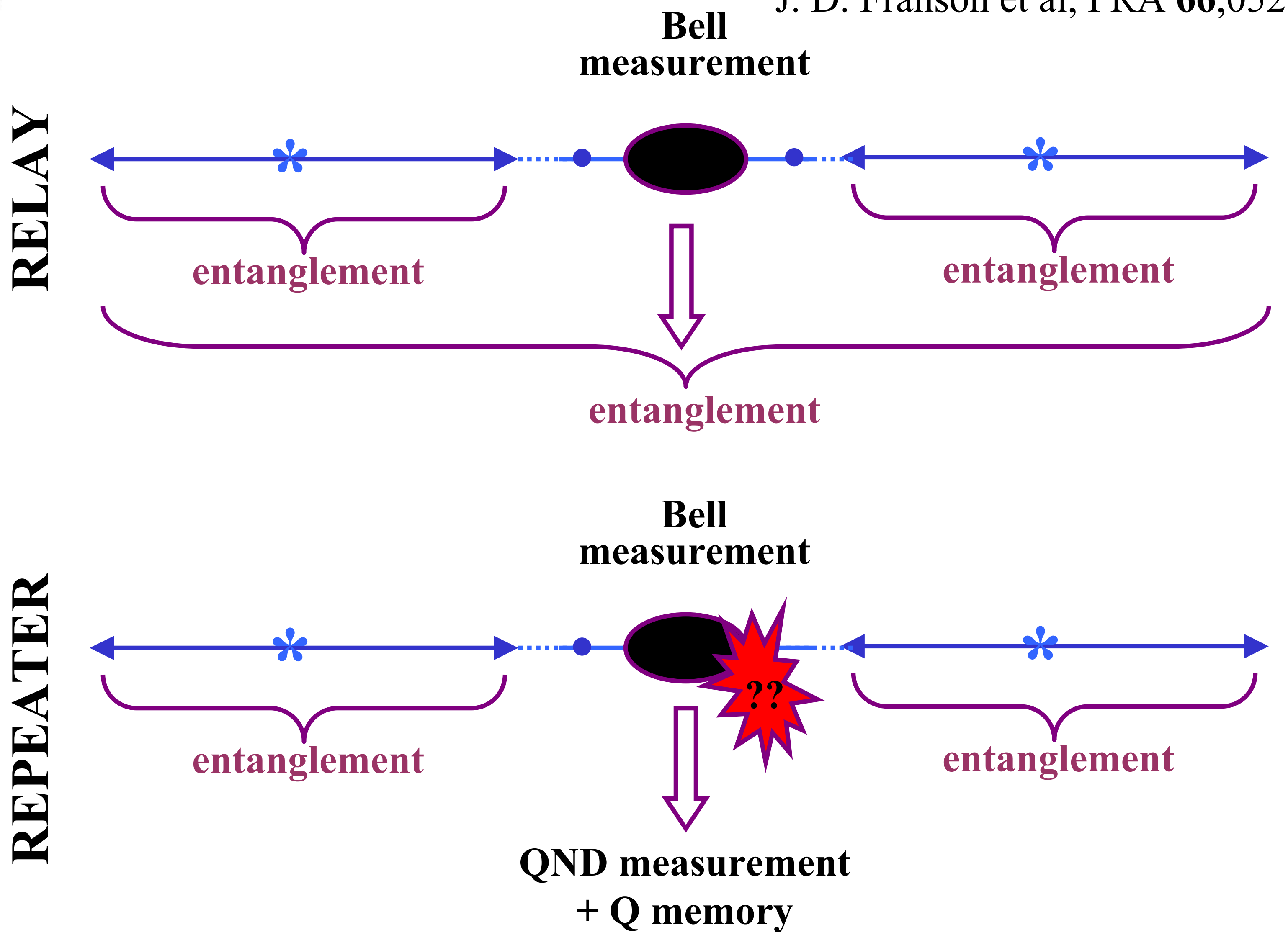
Q repeaters & relays

J. D. Franson et al, PRA **66**,052307,2002



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Conclusions

- **Q teleportation with:**
  - *telecom wavelength*
  - *two different crystals (spatially separated sources)*
  - *from one wavelength (1300 nm) to another (1550 nm)*
  - *first time with time-bins (ie insensitive to polarization fluctuations)*
  - *over 3x2 km of fiber and 55 meters of physical distance*
  - *mean fidelity : ≈ 85% both in the lab and at a distance of 2 km*
  - *mean fideity 77.5% in a 3x2km quantum relay configuration*

= the possibility to teleport the "ultimate structure" of an object from one place to another, without the object ever being anywhere in between

- **Q teleportation raises questions about the meaning of basic concepts like: object, information, space & time.**
- **Elementary Q processor can extend today's Q crypto systems**

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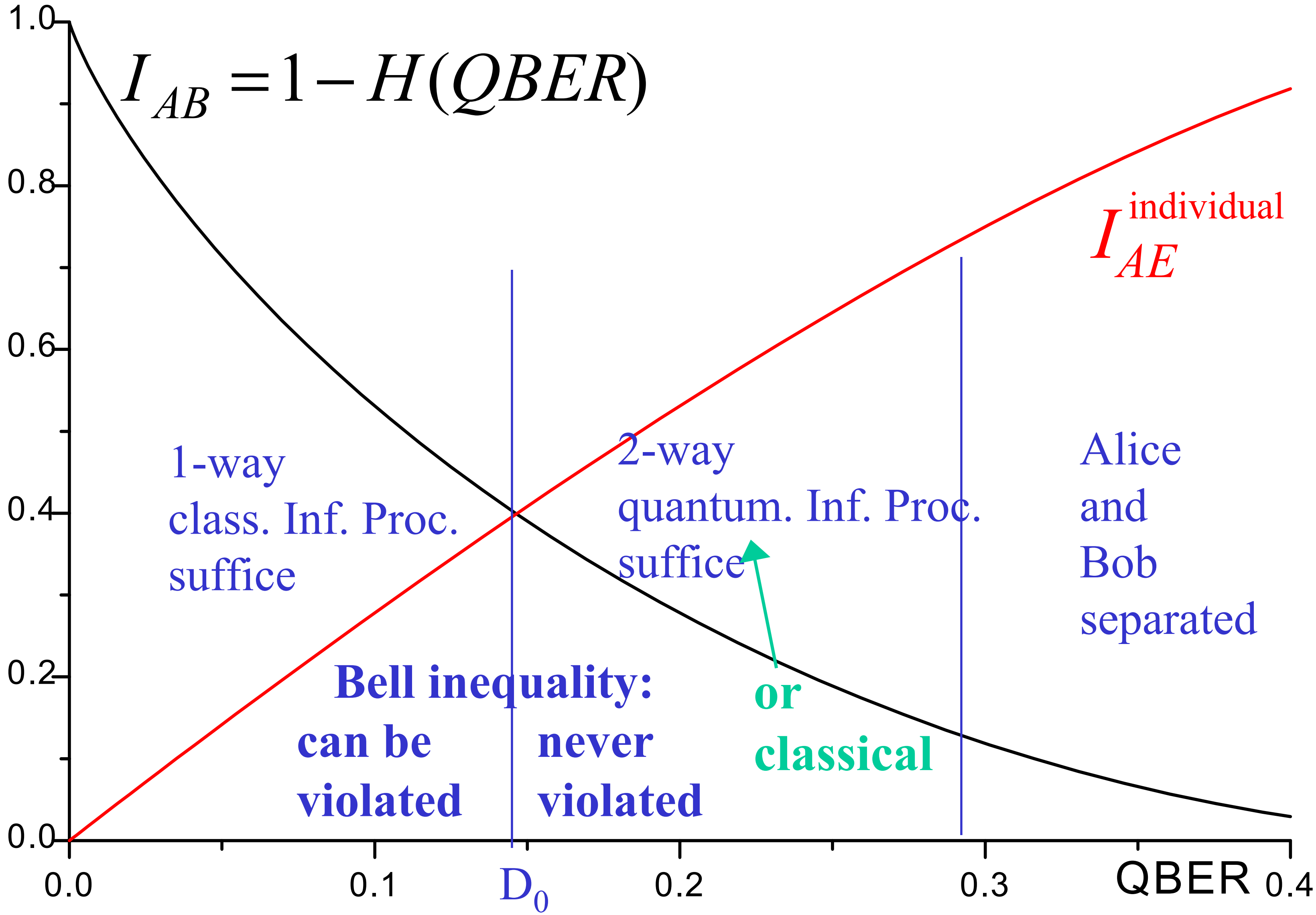
Shannon Information

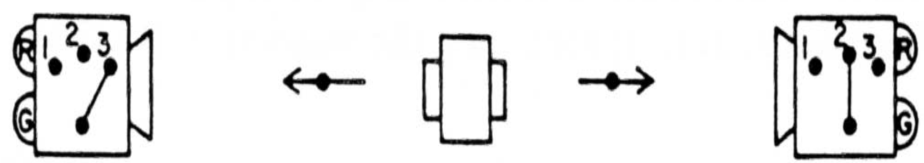


Bell's inequality: (D. Mermin, Am. J. Phys. 49, 940-943, 1981)

Bob / Alice	Left same	Left different	Middle same	Middle different	Right same	Right different
Left	100 %	0 %	1/4	3/4	1/4	3/4
Middle	1/4	3/4	100 %	0 %	1/4	3/4
Right	1/4	3/4	1/4	3/4	100 %	0 %

LMR	if ≠ settings Prob(results =)
GGG	100 %
GGR	1/3
GRG	1/3
RGG	1/3
GRR	1/3
RGR	1/3
RRG	1/3
RRR	100 %
Arbitr. mixture	≥ 1/3

Quantum Mechanics	= 1 / 4



Alice **Bob**

Bell Inequality

Quantum non-locality

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