PHYSICS OF EXTRA DIMENSIONS

V. Rubakov

INSTITUTE FOR NUCLEAR RESEARCH OF THE RUSSIAN ACADEMY OF SCIENCES, MOSCOW

- Motivation from string/M - THEORY:
  - SUPERSTRINGS MOST EASILY CONSTRUCTED IN (9+1) dimensions
  - M - THEORY : (10+1) Dimensions

EVEN WITHOUT STRINGS, INTERESTING TO UNDERSTAND PROPERTIES OF THEORIES WITH EXTRA DIMENSIONS.

- WHY EXTRA DIMENSIONS HAVE NOT BEEN OBSERVED?

A1: EXTRA DIMENSIONS COMPACT
KALUZA - KLEIN PICTURE

A2: We LIVE ON 3-BRANE

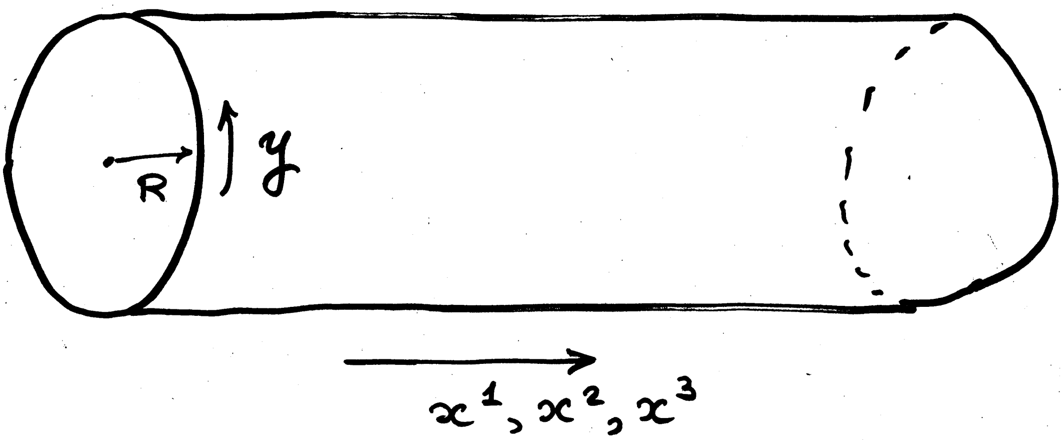
A3: BOTH.

<u>NB</u>: IN THESE LECTURES, EXTRA DIMENSIONS ALWAYS SPACE-LIKE.

ONE TIME.

KALUZA - KLEIN PICTURE: COMPACT EXTRA DIMENSIONS

$$\eta_{\mu\nu} = +1, -1, -1, \dots, -1\ ; \quad \Box = \partial_t^2 - \vec{\partial}_x^{\,2}$$



(4+1) - DIM. THEORY WITH COMPACT 5TH COORDINATE y

Massless SCALAR FIELD:

$$\Box^{(5)} \phi \equiv \left(\partial_\mu \partial^\mu - \partial_y^2 \right) \phi = 0 \qquad \mu = 0,1,2,3$$

ϕ periodic in y WITH PERIOD $2\pi R$

$$\Downarrow$$

SOLUTIONS $\quad \phi(x,y) = e^{i p_\mu x^\mu} e^{in\frac{y}{R}}$

$n = 0, \pm 1, \pm 2, \dots$ = angular momentum

$$p_\mu p^\mu = \frac{n^2}{R^2}$$

$n = 0$: Homogeneous MODE, PROPAGATING along x^1, x^2, x^3 WITH SPEED OF LIGHT $\Rightarrow$ (PROTOTYPE OF) KNOWN PARTICLES

$n \neq 0$: 4D MASSIVE PARTICLES, K-K MODES.

KK TOWER OF PARTNERS
(EXCITED ELECTRONS, PHOTONS, ETC.)

$$m_n = \frac{|n|}{R}$$

$$\Downarrow$$

ACCEPTABLE $R \lesssim (\mathrm{TeV})^{-1} \sim 10^{-17}$ cm.

CHANCE TO DISCOVER EXTRA DIMENSIONS
IFF $R \sim (\mathrm{TeV})^{-1}$

NO HOPE FOR NATURAL (?) $R \sim M_{Pl}^{-1}$

NB: MORE THAN ONE EXTRA DIMENSIONS $\Rightarrow$
MORE K-K MODES.
E.G. $y_1, \dots, y_N$ ALL COMPACTIFIED
TO CIRCLE OF RADIUS $R_1, \dots, R_N$

$$\Downarrow$$

$$\phi = e^{ip_\mu x^\mu} e^{in_1 \frac{y_1}{R_1}} \dots e^{in_N \frac{y_N}{R_N}}$$

$$m^2_{\{n\}} = \sum \frac{n_i^2}{R_i^2}$$

One HOMOGENEOUS $\Leftrightarrow$ MASSLESS MODE.
MANY KK MODES. BELOW CERTAIN MASS.

EFFECTIVE 4D DESCRIPTION:

Quadratic (free) 5 dim ACTION

$$S^{(2)} = \int d^4x \int_0^{2\pi R} dy \, \frac{1}{2} \partial_A \phi \cdot \partial^A \phi$$

$$= \int d^4x \int_0^{2\pi R} dy \, \frac{1}{2} \left(\partial_\mu \phi \cdot \partial^\mu \phi - \partial_y \phi \, \partial_y \phi \right)$$

NB: DIMENSION $[\phi] = M^{3/2}$

PLUG IN KK DECOMPOSITION

$$\phi(x,y) = \sum_n \varphi_n(x) \, e^{in\frac{y}{R}}$$

$$\Downarrow$$

$$S^{(2)} = \int d^4x \sum_n \underbrace{(2\pi R)}_{\text{VOLUME OF EXTRA DIMENSIONS}} \left[\frac{1}{2} |\partial_\mu \varphi_n|^2 - \frac{1}{2} \frac{n^2}{R^2} |\varphi_n|^2 \right]$$

Free 4d action of infinitely many 4d fields.

Canonically normalized 4d fields:

$$\hat{\varphi}_n(x) = \frac{1}{\sqrt{2\pi R}} \varphi_n(x)$$

$[\hat{\varphi}_n] = M$ as usual!

$$\Downarrow$$

$$S^{(2)} = \sum_n \int d^4x \left[\frac{1}{2} |\partial_\mu \hat{\varphi}_n|^2 - \frac{1}{2} \frac{n^2}{R^2} |\hat{\varphi}_n|^2 \right]$$

Sum of standard 4d actions

INTERACTIONS:

5 Dim INTERACTION

$$S_{int} = \int d^4x \int_0^{2\pi R} dy \quad \lambda^{(5)} \phi^4$$

NB: $\left[\lambda^{(5)}\right] = \dfrac{1}{M}$

5 Dim THEORY STRONGLY COUPLED AT HIGH ENERGIES

$$E_{STRONG} \sim \frac{1}{\lambda^{(5)}} \equiv M_{(5)}$$

PLUG IN FOURIER DECOMPOSITION IN y, WITH CANONICALLY NORMALIZED FIELDS

$\Downarrow$

ZERO MODE ONLY, $n=0$ ("OUR" FIELD)

$$S_{int} = \int d^4x \underbrace{(2\pi R)}_{\int dy} \underbrace{\frac{\lambda^{(5)}}{(2\pi R)^2}}_{\text{normalization}} \hat{\varphi}_0^4(x)$$

$$\lambda^{(4)} = \frac{\lambda^{(5)}}{(2\pi R)} \longleftarrow \text{FUNDAMENTAL}$$

EFFECTIVE 4D COUPLING

Lesson #1:

4D couplings are effective, related to fundamental couplings of original, higher-dim. theory through volume of extra dimensions.

$$\lambda^{(4)} = \frac{\lambda^{(5)}}{(2\pi R)}$$

Weakly coupled 4D theory:

$$\lambda^{(4)} \ll 1$$

$$\Downarrow$$

$$R \gg \lambda^{(5)} \equiv \frac{1}{E_{STRONG}} \equiv \frac{1}{M_{(5)}}$$

Size of extra dimensions large compared to $(\text{energy scale})^{-1}$ of high-dim. theory

Lesson #2.

Recall $m_n = \frac{|n|}{R} \Rightarrow m_n \ll M_{(5)}$ for $n \sim 1$

$$\Downarrow$$

Trust KK modes with relatively low n.

Lesson #3

Interaction of "our" field $\hat{\varphi}_0$ WITH KK modes $\hat{\varphi}_n$

Also comes from

$$S_{int} = \int d^4x \int_0^{2\pi R} dy \, \lambda^{(5)} \Phi^4$$

Decompose:

$$\Phi^4 = \left(\hat{\varphi}_0 + \sum_{n\neq 0} \hat{\varphi}_n e^{in\frac{y}{R}} \right)^4 \frac{1}{(2\pi R)^2}$$

zero in $\int dy$

$$= \frac{1}{(2\pi R)^2} \Big[\hat{\varphi}_0^4 + 4 \hat{\varphi}_0^3 \sum_{n\neq 0} \hat{\varphi}_n e^{in\frac{y}{R}} + 6 \hat{\varphi}_0^2 \sum_{n\neq 0} \sum_{n'\neq 0} \hat{\varphi}_n \hat{\varphi}_{n'} e^{i(n+n')\frac{y}{R}} \Big]$$

$$\int dy \to \delta_{n+n'}$$

$$S_{int} \Rightarrow \int d^4x \cdot 12 \lambda^{(4)} \sum_{n\neq 0} \hat{\varphi}_0^2 \hat{\varphi}_n \hat{\varphi}_{-n}$$

- PAIR PRODUCTION OF KK MODES

NOT TRUE IN BRANE WORLD MODELS

$\Updownarrow$

CONSERVATION OF ANGULAR MOMENTUM ALONG y

- UNIQUE EFFECTIVE 4d coupling $\lambda^{(4)}$

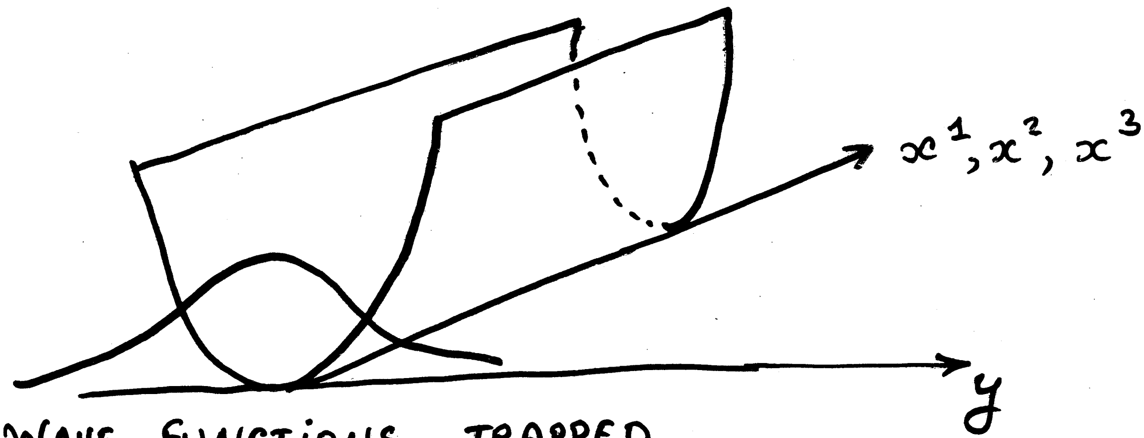
Strong coupling $\equiv$ LARGE CROSS SECTION AT $E \sim E_{strong}$: MANY CHANNELS OPEN (RATHER THAN ENHANCEMENT OF INDIVIDUAL CHANNEL)

BRANE WORLD

OUR MATTER LOCALIZED ON 3 dim. HYPERSURFACE (BRANE) EMBEDDED IN HIGHER DIMENSIONAL SPACE.

MOTIVATION FROM STRING THEORY : D-BRANES, ...

Overall picture: POTENTIAL WELL IN EXTRA DIMENSIONS



WAVE FUNCTIONS TRAPPED

EXTRA DIMENSIONS MAY BE OF INFINITE SIZE.

NEED PROPERTIES OF SPECTRUM:

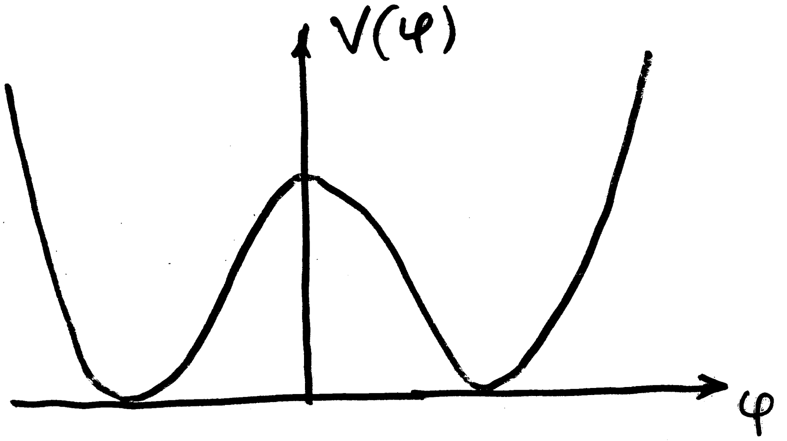
- LOWEST 4 dim. mass $\simeq 0$
- EXCITED STATES HAVE LARGE 4 dim. masses. $\Leftrightarrow$ NARROW WELL

ARE THESE POSSIBLE/NATURAL ?

FIELD THEORY ($\equiv$ TOY MODEL) EXAMPLES: DEFECTS.

E.G. 5 DIM THEORY OF REAL SCALAR FIELD

$$S = \int d^4x \, dy \left[\frac{1}{2} \partial_A \varphi \cdot \partial^A \varphi - V(\varphi) \right]$$



VACUA: $\varphi = v$

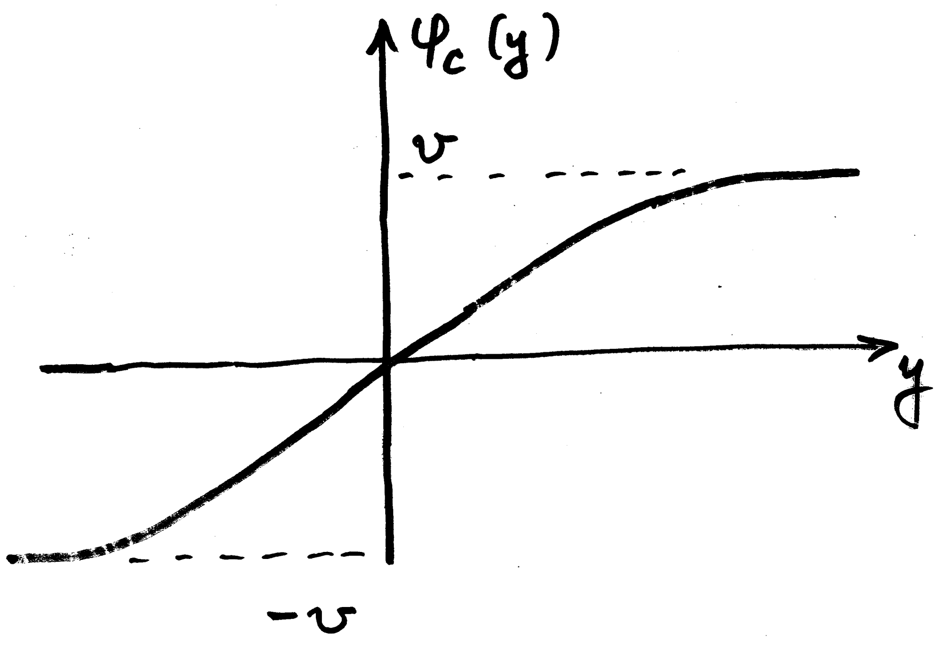
$\varphi = -v$

$$V(\varphi) = \lambda^{(5)} \left(\varphi^2 - v^2 \right)^2$$

FIELD EQUATION: $$\partial_A \partial^A \varphi + \frac{\partial V}{\partial \varphi} = 0$$

Domain WALL SOLUTION: depends only on y

$$\varphi_c = \varphi_c(y) = v \tanh\left(\sqrt{2\lambda^{(5)}} \, v y \right)$$ kink



BREAKS TRANSLATION ALONG y;
DOES NOT BREAK 4D LORENTZ-INV.

SCALAR MODES:

$$\varphi(x,y) = \varphi_c(y) + \delta\varphi(x,y)$$

LINEARIZED FIELD EQUATION

$$\partial_A \partial^A (\delta\varphi) + \frac{\partial^2 V}{\partial\varphi^2}[\varphi = \varphi_c] \cdot \delta\varphi = 0$$

$$\partial_\mu \partial^\mu - \partial_y^2 \qquad\qquad U(y) \text{, POTENTIAL WELL}$$

$$\delta\varphi = e^{ip_\mu x^\mu} \delta\varphi_m(y)$$

$p_\mu p^\mu \equiv m^2$: 4 dim. mass

$$m^2 \delta\varphi_m = \left[-\partial_y^2 + U(y)\right] \delta\varphi_m$$

- Zero mode: $m^2 = 0$

$$\delta\varphi_0(y) = \varphi_c'(y) \propto \frac{1}{\cosh\left(\sqrt{2\lambda^{(5)}}\, v y\right)}$$

Our particles localized on a brane, traveling along the brane with speed of light.

NB: translational zero mode (modulus)

$\Updownarrow$

VIBRATIONS OF THE BRANE.

Higher modes

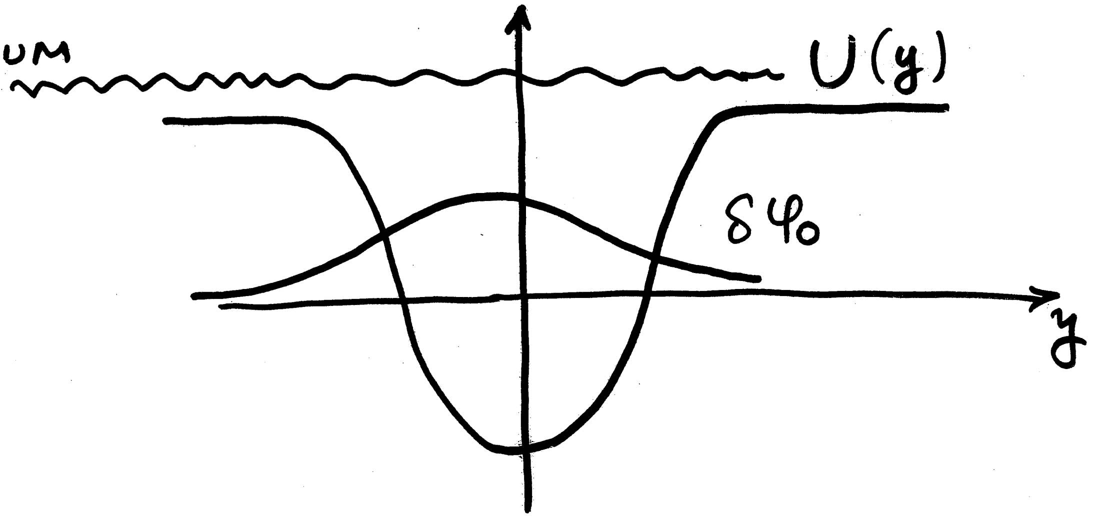
- DISCRETE, $m \sim \sqrt{\lambda^{(5)}} \cdot v$

  "excited electrons", still traveling along the brane

- CONTINUUM: STARTS AT

$$m_{cont} = \sqrt{8 \lambda^{(5)}}\, v$$

Particles moving out to $y \to \pm\infty$



Particles escaping THE BRANE $\Leftrightarrow$ missing energy,

$e^+e^- \to$ NOTHING THRESHOLD EFFECT

$e^+e^- \to \gamma$ + NOTHING APPARENT ENERGY NONCONSERVATION

SINGLE PARTICLE POSSIBLE TO EMIT INTO BULK

ARRANGE PARAMETERS: $\sqrt{\lambda^{(5)}}\, v > \text{TeV}$

NO FIRM PREDICTION FOR THRESHOLD ENERGY...

BUT KEEP AN EYE

LOCALIZED FERMIONS

SAME KINK BACKGROUND, BUT INCLUDE 5-DIM. FERMIONS

5-DIM. gamma MATRICES Γ^A,

$$\{\Gamma^A, \Gamma^B\} = \eta^{AB}$$

Minimal choice: 4×4 matrices,

$$\Gamma^\mu = \gamma^\mu \quad \text{4-dim.}$$
$$\Gamma^5 = -i\gamma^5$$

Fermion = 4-column, like in 4-dim.

Choose fermion action

$$S_\Psi = \int d^4x\, dy\, \left(i\bar{\Psi}\Gamma^A\partial_A\Psi - h\varphi\bar{\Psi}\Psi\right)$$

DIRAC EQUATION IN KINK BACKGROUND

$$i\Gamma^A\partial_A\Psi - h\varphi_c(y)\Psi = 0$$

AGAIN, SPECTRUM 4DIM. LORENTZ-INVARIANT

4-dim. MASSLESS MODE:

$$i\gamma^\mu\partial_\mu\Psi_0 = 0$$
$$\Downarrow$$
$$i\Gamma^5\partial_5\Psi_0 \equiv \gamma^5\partial_y\Psi_0 = h\varphi_c(y)\Psi_0$$

UNIQUE SOLUTION

$$\gamma^5 \Psi_0 = \pm \Psi_0$$

$$\Downarrow$$

$$\Psi_0 \propto e^{\pm \int_0^y h \varphi_c(y') dy'}$$

$$\hookrightarrow e^{\pm h v |y|} \quad \text{as } |y| \to \infty$$

NORMALIZABLE ZERO MODE: LEFT-HANDED

$$\gamma^5 \Psi_0 = - \Psi_0$$

$$\Psi_0(x,y) = e^{-\int_0^y h \varphi_c(y') dy'} \psi_L(x)$$

MASSLESS 4-DIM. FERMION
CHIRAL (BONUS)
TRAVELS ALONG BRANE.

NO ACCIDENT:
INDEX THEOREMS $\Longrightarrow$ CHIRAL FERMION ZERO MODES IN BACKGROUNDS OF TOPOLOGICAL DEFECTS.

SCALARS AND FERMIONS ARE EASY TO LOCALIZE

MORE THAN ONE EXTRA DIMENSIONS

- ABRIKOSOV-NIELSEN-OLESEN STRING (COSMIC STRING) IN (5+1) dim.
- 't Hooft-Polyakov monopole in (6+1)-dim
- NONCOMMUTATIVE SOLITONS, D-BRANES

LOCALIZING GAUGE FIELDS: NOT AS STRAIGHTFORWARD.

Suppose THERE WERE ZERO MODE of gauge field, $A^{(0)}(y)$

4-dim. EFFECTIVE INTERACTION WOULD BE PROPORTIONAL TO OVERLAP

$$\int dy \; \Psi_0^+(y)\, A_0(y)\, \Psi_0(y)$$

Generally, depends on shape of $\Psi_0(y)$

$\Downarrow$

CHARGE UNIVERSALITY NOT GUARANTEED

<u>But</u> NON-ABELIAN GAUGE THEORIES MUST OBEY CHARGE UNIVERSALITY!

WAYS OUT:

A. ZERO MODES ALL HAVE SAME SHAPES

- NONCOMMUTATIVE SOLITONS

$\Updownarrow$

D-BRANES WITHIN D-BRANES

B. $A^{(0)}$ INDEPENDENT OF y

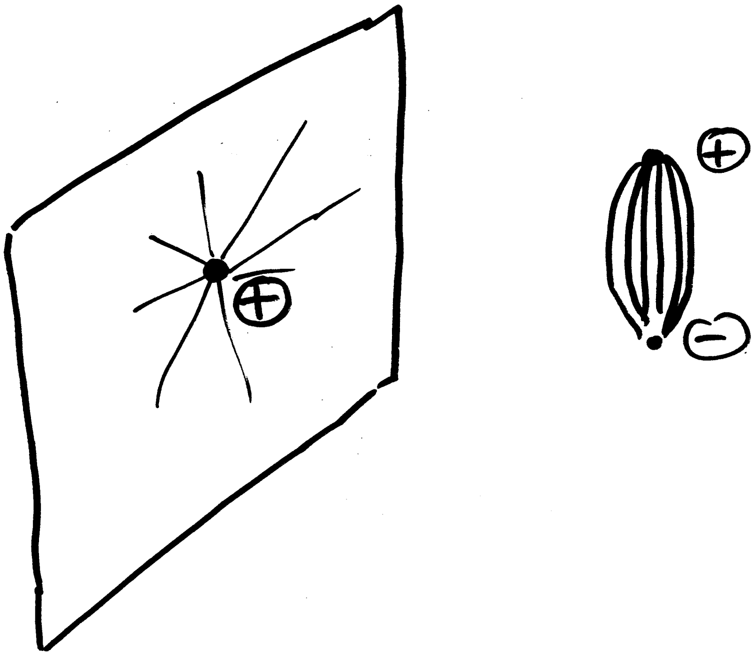
$\Downarrow$

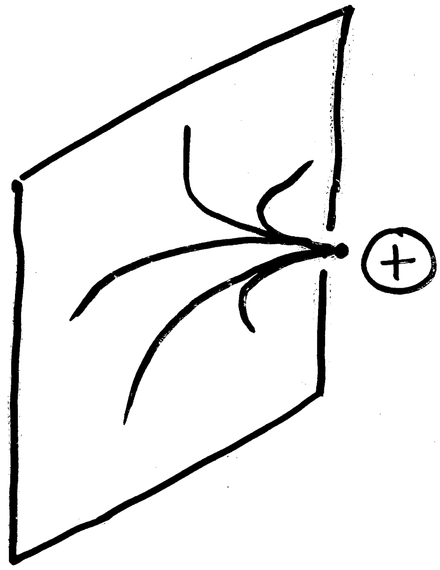
$$\text{OVERLAP} = \int dy \; \Psi_0^+ \Psi_0 = 1$$

- WARPED BULK, $A^{(0)} = \text{const}$ NORMALIZABLE

C NO GAUGE FIELDS IN BULK AT ALL

• Confinement in bulk, no confinement on brane





Same Coulomb field on brane, irrespectively of position of charge in bulk.

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CHARGE UNIVERSALITY.

PHENOMENOLOGY DIFFERENT:

A, C : CHARGE CONSERVATION ON BRANE

B: (APPARENT) CHARGE NON-CONSERVATION FOR BRANE-BASED OBSERVER.

$e^- \longrightarrow$ NOTHING.

D GAUGE FIELDS LIVE IN BULK. BULK IS COMPACT. BACK TO KALUZA-KLEIN.