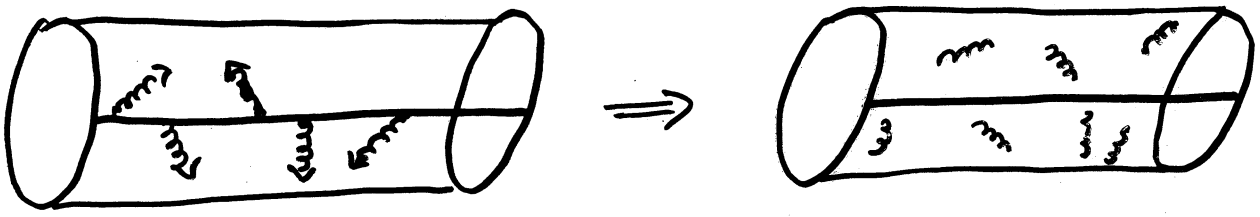
ADD: COSMOLOGY AND ASTROPHYSICS

OFTEN MAKES THINGS PROBLEMATIC (if not kills).

- COSMOLOGY: K-K gravitons (≡ BULK GRAVITONS) PRODUCED IN EARLY UNIVERSE. SHOULD NOT DOMINATE NUCLEOSYNTHESIS EPOCH (expansion AT)



PRODUCTION RATE/UNIT VOLUME AT TEMPERATURE T:

$$\frac{dn_{grav}}{dt} \sim \frac{1}{M^{N+2}} T^{N+6}$$

$$\Downarrow$$

PRODUCTION OF ENERGY DENSITY IN BULK GRAVITONS

$$\frac{d\rho_{grav}}{dt} \sim \frac{1}{M^{N+2}} T^{N+7}$$

HUBBLE EXPANSION RATE (RADIATION DOMINATED) $H(T) = \frac{T^2}{M_{Pl}^*}$ $M_{Pl} \sim 10^{18}$ GeV

ENERGY TRANSFERRED INTO BULK GRAVITONS IN HUBBLE TIME:

$$H^{-1}\frac{d\rho_{grav}}{dt} \sim \frac{M_{Pl}^{*}}{M^{N+2}} T^{N+5}$$

Let T_{max} = maximum temperature of Universe (e.g. reheat temperature after inflation)

Maximum production at $T \sim T_{max}$.

$$\rho_{grav}(T_{max}) \sim \frac{M_{Pl}^{*}}{M^{N+2}} T_{max}^{N+5}$$

M_{Pl} enhanced: slow expansion

$$H \sim \frac{1}{t_{universe}} \sim \frac{T^2}{M_{Pl}^{*}}$$

Then ρ_{grav} decreases due to expansion of Universe

At BEST AS T^4 (if all gravitons decay into something massless. If not, $\rho \propto T^3 \Rightarrow$ stronger limits)

$\Downarrow$

AT NUCLEOSYNTHESIS ($T_{NS} \sim 1$ MeV)

$$\rho_{GRAV}(T_{NS}) \sim \frac{M_{Pl}^{*}}{M} \cdot \left(\frac{T_{max}}{M}\right)^{N+1} \cdot T_{NS}^{4}$$

MUST BE SMALLER THAN T_{NS}^4 (ENERGY DENSITY OF STANDARD PARTICLE

$\Downarrow$

$$\frac{M_{Pl}^{*}}{M} \cdot \left(\frac{T_{max}}{M}\right)^{N+1} < 1$$

BOUND ON T_{max}:

$$T_{max} < M \left(\frac{M}{M_{Pl}^*} \right)^{\frac{1}{N+1}}$$

$M \sim 10^3$ GeV, $M_{Pl}^* \sim 10^{-15}$

$N=2 \Rightarrow T_{max} \lesssim 10$ MeV

$\vdots$

$N=6 \Rightarrow T_{max} \lesssim 10$ GeV

NO DIRECT CONTRADICTION: DATA ON EARLY UNIVERSE FOR $T \sim 1$ MeV ONLY (YET!)

STILL NOT VERY PLAUSIBLE:

- BARYOGENESIS DIFFICULT
- DARK MATTER GENERATION DIFFICULT

NB: IF WE KNEW WHAT ARE DARK MATTER PARTICLES, WE WOULD RULE OUT LOW T_{max}
(STANDARD SCENARIO: DARK MATTER GENERATED AT $T \gtrsim 100$ GeV)

- ASTROPHYSICS

SN 1987a : NEUTRINOS EMITTED AT COLLAPSE

Central core heated to $T = (\text{a few}) \cdot 10\ \text{MeV}$ for time $\Delta t = (\text{a few}) \cdot 10\ \text{s}$. ENERGY IN ν's

BULK GRAVITONS SHOULD NOT TAKE AWAY ALL ENERGY (LEAVE GOOD PART TO ν's)

$$\frac{d\rho_{grav}}{dt} \sim \frac{1}{M^{N+2}} T^{N+7}$$

$$\rho_{grav} \sim \left(\frac{T}{M}\right)^{N+2} \cdot (T\Delta t) \cdot T^4 \quad < T^4$$

(T^4: energy density in ν's)

$$\Downarrow$$

$$M > T \cdot (T \cdot \Delta t)^{\frac{1}{N+2}}$$

($T \sim 10\ \text{MeV}$, $T \cdot \Delta t \sim 10^{23}$)

$N=2 \Rightarrow M > 10\ \text{TeV}$ (MORE ACCURATE $M > 30\ \text{TeV}$)

NB: $N=3 \Rightarrow M > 1\ \text{TeV}$, not RESTRICTIVE.

$N=2$: $M \gtrsim 30\ \text{TeV}$, quite a BIT HIGHER THAN M_{EW}

To summarize:

ADD:
- Interesting collider signatures
- Deviation from Newton's law at relatively large distances
- $N=2$: gravity scale $M \gtrsim 30$ TeV Difficult for all experiments
- Cosmology problematic

Does not need SUSY, but not inconsistent with SUSY.

EXAMPLE #2: WARPED EXTRA DIMENSION, RS

Until now: BRANE WITHOUT ENERGY DENSITY ASSOCIATED TO IT

NOT VERY REALISTIC.

WHAT HAPPENS WHEN BRANE CARRIES ENERGY DENSITY (TENSION)?

Introducing brane tension:

4 DIM'S: VACUUM IN FLAT SPACE-TIME IS LORENTZ-INVARIANT.

ENERGY-MOMENTUM OF VACUUM $\Leftrightarrow$ LORENTZ-INVARIANCE

$$T_{\mu\nu} = \sigma \cdot \eta_{\mu\nu}$$

σ: constant ≡ vacuum energy ≡ cosmological constant

$\eta_{\mu\nu}$: Minkowski tensor

NB: FLUID $\rightarrow T_{\mu\nu} = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} \Rightarrow$ VACUUM $p = -\rho$

ρ: energy density; p: pressure

In CURVED SPACE-TIME

$$T_{\mu\nu} = \sigma \cdot g_{\mu\nu}$$ FOR VACUUM.

BRANE IN EXTRA DIMENSIONS

If 4-dim. LORENTZ SYMMETRY IS NOT VIOLATED

$$T_{\mu\nu}(x,y) = \sigma \cdot g_{\mu\nu}(x) \cdot \delta^N(y)$$

- σ: constant: brane tension, energy per unit area
- $g_{\mu\nu}(x)$: METRIC ON BRANE, INDUCED FROM BULK
- $\delta^N(y)$: if brane THICKNESS is negligibly small

BRANE OF FINITE TENSION PRODUCES gravitational field in bulk $\Leftrightarrow$ self-gravitates.

IS THERE INDEED A SOLUTION WITH FLAT (LORENTZ-INVARIANT) BRANE?

5 DIM'S (ONE EXTRA)

- NEED COSMOLOGICAL CONSTANT Λ IN BULK
- Fine-tuning of Λ and σ

- EINSTEIN EQS. IN 5 DIM'S:

$$R^{(5)}_{AB} - \frac{1}{2} g^{(5)}_{AB} R^{(5)} = 8\pi G_5 \left(\Lambda \cdot g^{(5)}_{AB} + T^{\text{brane}}_{AB} \right)$$

NON-zero components

$$T_{\mu\nu} = \sigma \cdot g^{(4)}_{\mu\nu} \delta(y)$$

(in COORDINATES WITH brane SITTING AT $y=0$)

4-dim. LORENTZ-INV. SOLUTION EXISTS FOR

$$\Lambda = -\frac{4\pi}{3} G_5 \cdot \sigma^2$$

Fine-tuning, similar to fine-tuning of cosmological constant in 4-dim. THEORIES.

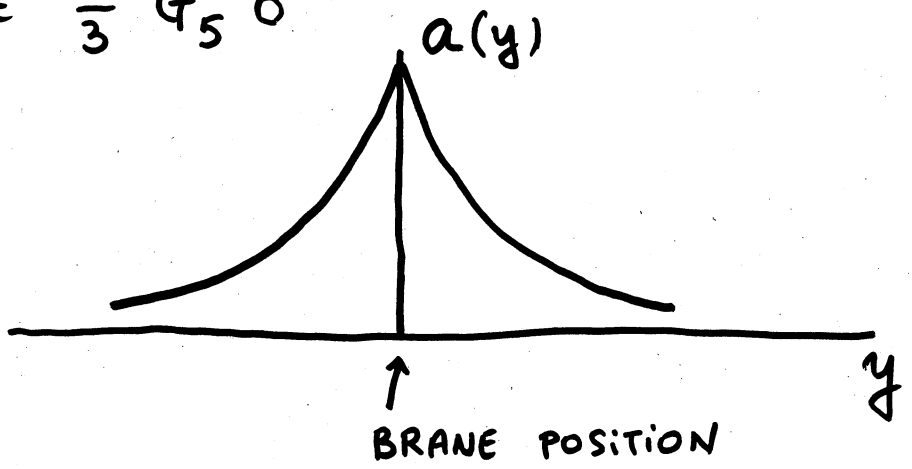
SOLUTION

$$ds^2 = a^2(y) \cdot \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$

↑ WARP FACTOR

$$a(y) = e^{-k|y|}$$

$$k = \frac{4\pi}{3} G_5 \sigma$$



NB: $\partial_y^2 a(y) \propto \delta(y) \Leftrightarrow T_{\mu\nu}^{\text{brane}} \propto \delta(y)$

NB: BULK METRIC IS PART OF adS_5

k = INVERSE "RADIUS" of adS_5

NB: Δx^μ = PHYSICAL DISTANCE, TIME INTERVAL AT THE BRANE ONLY

AWAY FROM BRANE PHYSICAL DISTANCE

$\Delta \ell^\mu = a(y) \cdot \Delta x^\mu$, smaller than Δx^μ

↑ $e^{-k|y|}$

SOLUTION TO FIELD EQS. IN RS BACKGROUND WILL CONTAIN FACTOR

$$e^{ip_\mu x^\mu}$$

p^μ = physical 4-momentum at brane only. AWAY FROM IT

$$p_\mu^{\text{phys}} = \frac{1}{a(y)} \cdot p_\mu$$ HARDER THAN p_μ.

GRAVITON PERTURBATIONS ABOUT RS BACKGROUND:

WRITE PERTURBED METRIC

Gauge choice ↓

$$ds^2 = \left[a^2(y)\cdot\eta_{\mu\nu} + h_{\mu\nu}(x,y) \right] dx^\mu dx^\nu - dy^2$$

$a^2(y) \to e^{-k|y|}$; $h_{\mu\nu}$ ↑ gravitational perturbations

PLUG INTO 5D EINSTEIN EQS., LINEARIZE

⇓ CHOOSING GAUGE, OMITTING INDICES IN $h_{\mu\nu}$

$$-a^2(y)\,\partial_y^2 h + \eta^{\mu\nu}\partial_\mu\partial_\nu h + 4k^2a^2 h - 2k\delta(y)\cdot h = 0$$

FROM Einstein tensor

FROM $\Lambda(a^2\eta_{\mu\nu}+h_{\mu\nu})$

FROM $T^{brane}_{\mu\nu} = \sigma\cdot\delta(y)\left[a^2\eta_{\mu\nu}+h_{\mu\nu}\right]$

SOLUTIONS: $h = e^{ip_\mu x^\mu} h_m(y)$; $p_\mu p^\mu = m^2$

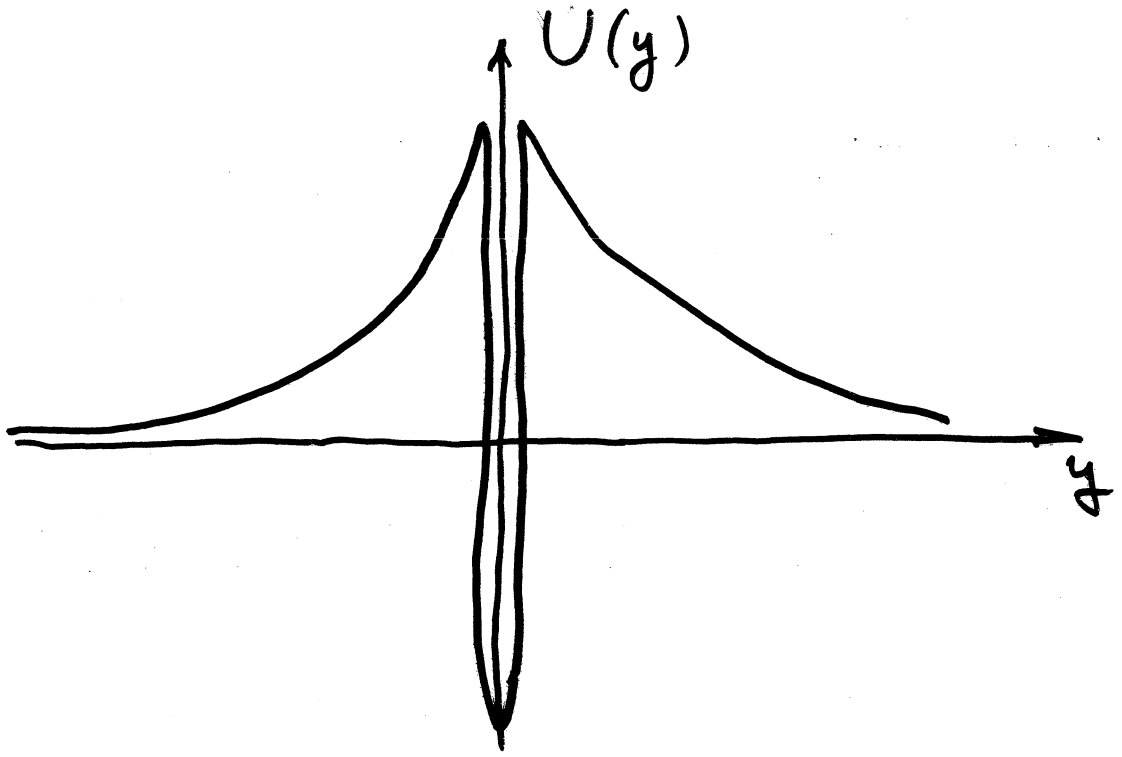
⇓ Eigenvalue equation

$$-a^2(y)\cdot\partial_y^2 h_m + \left[4k^2a^2(y) - 2k\delta(y)\right] h_m = m^2 h_m$$

- NORMALIZATION $\int \frac{1}{a^2}|h_m|^2\,dy$
- UPTO SOMEWHAT UNUSUAL DERIVATIVE TERM SCHRÖDINGER EQUATION IN POTENTIAL

$$U(y) = 4k^2a^2(y) - 2k\delta(y)$$

$$= 4k^2 e^{-k|y|} - 2k\delta(y)$$



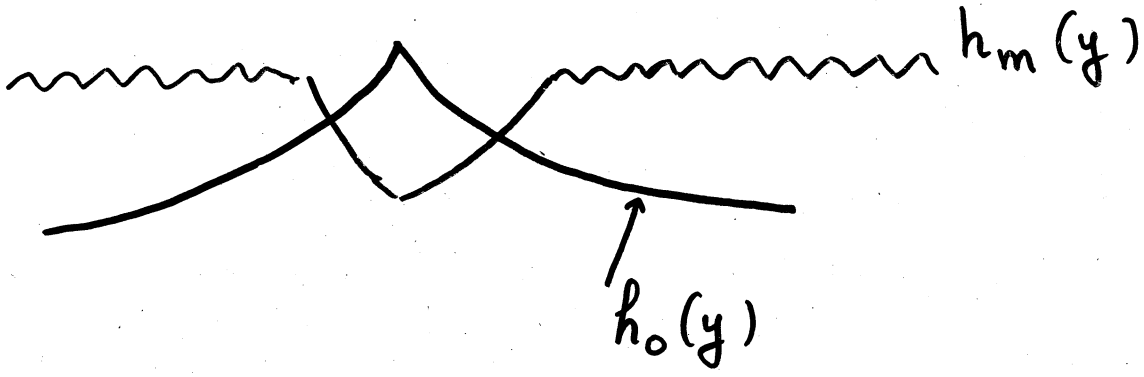
- ZERO MODE: $m = 0$
  MASSLESS 4-DIM. GRAVITON, DESCRIBES OUR GRAVITY.

$$h_0 = a^2(y) = e^{-2k|y|}$$

- OTHER MODES: DAMPED NEAR BRANE AT $m \ll k$
  OSCILLATE AWAY FROM BRANE,

$$h_m \propto \cos\left(\frac{m}{k} e^{ky}\right) \quad , \; y \gg \frac{1}{k} \ln \frac{m}{k}$$

HOW CAN ONE USE THIS SET UP TO CONSTRUCT INTERESTING BRANE-WORLD MODELS?


$h_m(y)$
$h_o(y)$

RS1: COMPACT EXTRA DIMENSIONS, EXPONENTIALLY EASIER TWISTED HIERARCHY PROBLEM.

Put another, NEGATIVE TENSION BRANE AT $y = y_c$.

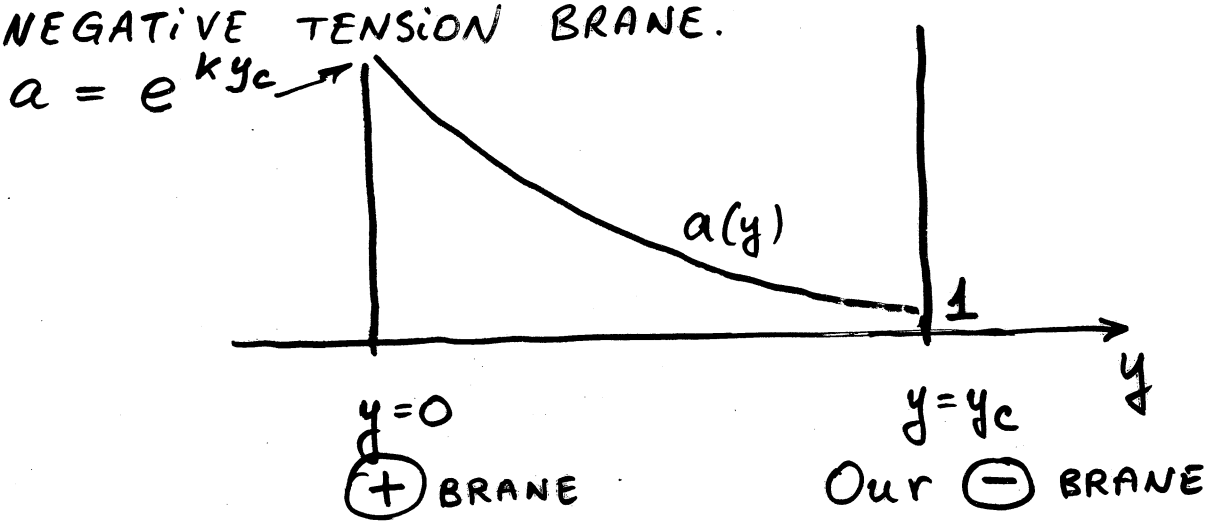
↑ weird, BUT HAS STRING THEORY interpretation

$$ds^2 = a^2(y)\,\eta_{\mu\nu}\,dx^\mu dx^\nu - dy^2$$

$$a(y) = e^{-k(|y| - y_c)}$$

COORDinates x^μ rescaled for convenience, measure physical distances on negative tension BRANE

ASSUME OUR MATTER LIVES ON NEGATIVE TENSION BRANE.



NB: WAVE FUNCTION OF ZERO MODE GRAVITON (CARRYING 4-dim. gravity)

$h_0 \propto a^2(y)$ DECAYS TOWARDS OUR BRANE.

4-dim. Newton constant :

$$S_{eff} = \frac{1}{16\pi G_5} \int_{-y_c}^{y_c} \frac{dy}{a^2(y)} \, d^4x \, (\partial_\mu h)^2$$

$h = a^2(y) \cdot h^{(4)}(x)$

recall measure in normalization of graviton modes

$$= \underbrace{\frac{1}{16\pi G_5} \left[\int_{-y_c}^{y_c} a^2(y) dy \right]}_{\frac{1}{16\pi^2 G_4}} \cdot \int d^4x \left[\partial_\mu h^{(4)}(x) \right]^2$$

$\frac{1}{k}(e^{2ky_c} - 1)$

$$G_4 = \frac{G_5 \cdot k}{e^{2ky_c} - 1}$$

($G_5 \sim 1/M^3$, $G_4 \sim \frac{1}{M_{Pl}^2}$)

FOR $y_c > \frac{1}{k}$, G_4 and $G_5 k$ exponentially different,

$$\boxed{M_{Pl}^2 = \frac{M^3}{k} e^{2ky_c}}$$

$ky_c > 1$

Exponential twist OF HIERARCHY PROBLEM:
FOR M, k IN TeV RANGE
CORRECT $M_{Pl} = 10^{16}$ TeV IS OBTAINED FOR
$y_c \sim 35 k^{-1}$
Still: WHY y_c is quite a bit larger than (TeV)$^{-1}$?

KEY PROPERTY : WAVE FUNCTION OF GRAVITON (ZERO MODE) IS exponentially smaller on our brane than on (+) BRANE.

—o—

K-K GRAVITON MODES

$$h_m \propto \ldots \cos\left(\frac{k}{m} e^{k(|y|-y_c)}\right)$$

at large enough $|y|$.

- BOUNDARY CONDITIONS AT OUR BRANE, $y=y_c$ RELATE $\partial_y h_m$ and $k h_m$

  ⇒ DISCRETE SPECTRUM

  $m_n \approx n \cdot k$

  n: integer

TRUST FIRST K-K MODES FOR $k < M$

FOR $k \sim$ TeV, MASSES IN TeV RANGE.

- WAVE FUNCTIONS OF K-K GRAVITONS DO NOT DECREASE TOWARDS OUR BRANE

  ⇓

  K-K gravitons interact with SM particles at strength $\frac{1}{M}$

  (TeV scace interactions)

- K-K GRAVITONS = RESONANCES, e.g. in

$pp \to \text{GRAVITON} \to \mu^+\mu^-, e^+e^-$

$pp \to \text{GRAVITON} \to \text{jet} + \text{jet}$

Fig.

DIFFERENT FROM OTHER NEW PARTICLES, since

- K-K gravitons = TENSOR PARTICLES
  ANGULAR DISTRIBUTION of LEPTON PAIRS, JETS WRT BEAM direction

$gg \to G \to \ell^+\ell^- : 1 - \cos^4\theta$

$q\bar{q} \to G \to \ell^+\ell^- : 1 - 3\cos^2\theta + 4\cos^4\theta$

cf. $1 + \cos^2\theta$ for vector resonance
const for scalar.

- UNIVERSAL COUPLING TO ALL SM PARTICLES
- SERIES OF RESONANCES

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$m_{1\text{-st KK graviton}} = 1.5$ TeV

$\ell^+\ell^-$-pairs at LHC

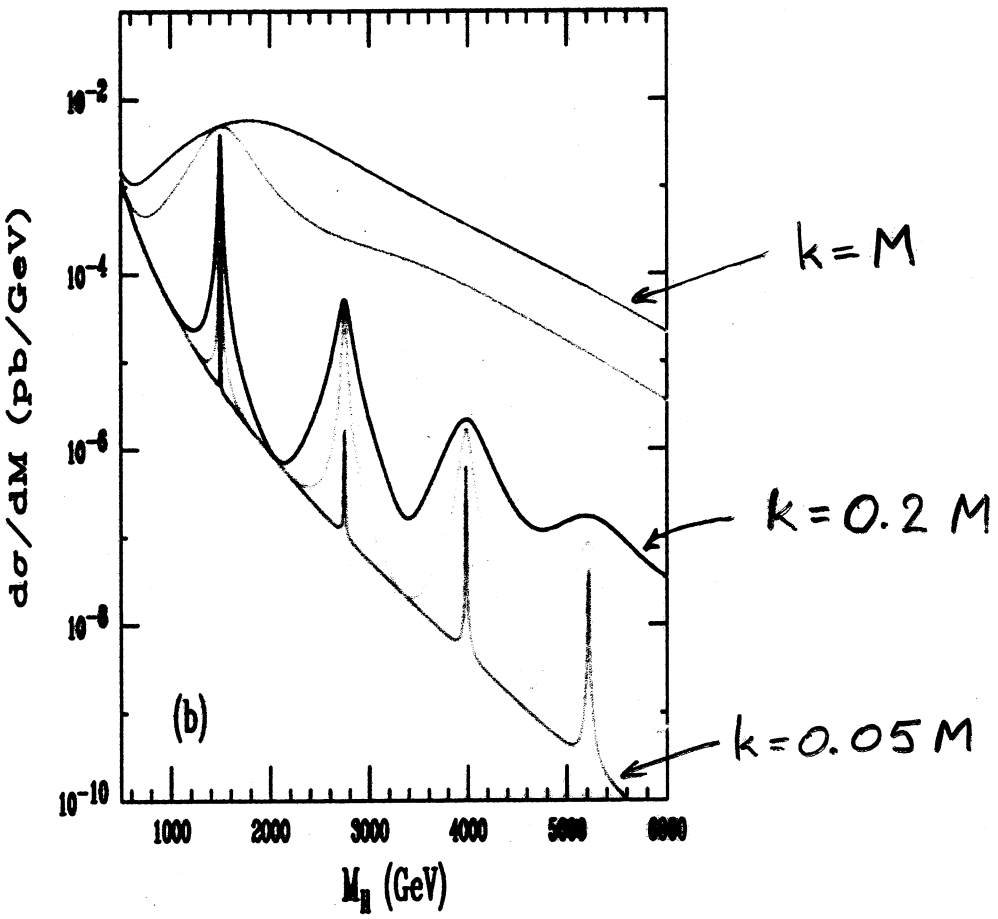

Figure 13: Drell-Yang production of the KK graviton with $M_1 = 1500$ GeV and its subsequent tower states at the LHC for $\eta = 1, 0.5, 0.1, 0.05, 0.2$, and 0.01, respectively, from top to bottom [72].

Davoudiasl, Hewett, Rizzo

- Cosmology much easier than in ADD: maximum temperature of Universe just below k is allowed.
- No astrophysical constraints for $k \sim$ TeV.

RS1 in some sense more similar to Kaluza-Klein with $(\text{TeV})^{-1}$ size of extra dimension(s). Except for hierarchy relation

$$G_4 = G_5 \cdot k \cdot e^{-2kyc}$$

To summarize:

- with compact extra dimensions, fundamental gravity scale may be in TeV range
  (also models with non-compact extra dim's)
  Quantum gravity may be within reach

- UNIFICATION OF COUPLINGS IN 4-DIM. SUSY AT GUT SCALE IS AN ACCIDENT