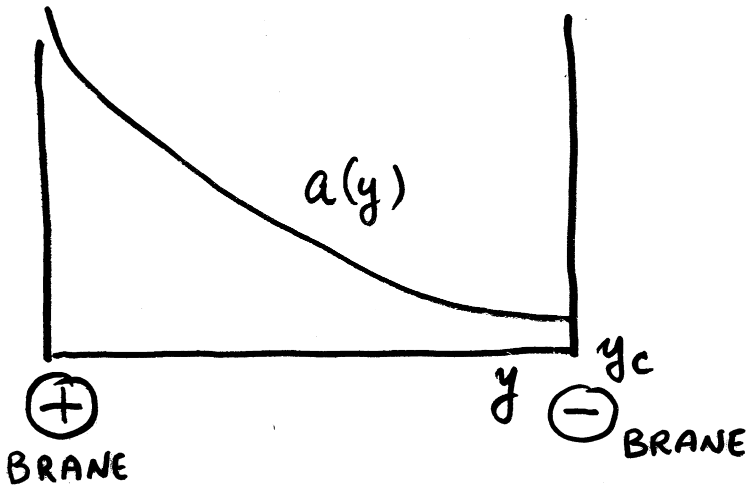
REMINDER:

RS1 SET UP



Gauge hierarchy solved due to

$$\frac{a(y_c)}{a(y=0)} \simeq e^{-ky_c}$$

if $y_c \simeq 35\,k^{-1}$.

This is not the only option.

E.G. PUT SM fields in BULK

$\Downarrow$

MASS SCALE $\sim M_{P\ell} \cdot e^{-ky_c}$ emerges,

even if $k \sim M_{P\ell}$, and $M_{P\ell}$ is THE ONLY MASS SCALE IN ACTION (i.e. $M = M_{P\ell}$).

This is BECAUSE K-K MODES BEHAVE AS $\cos\left(\frac{m}{k} e^{ky}\right)$

AWAY FROM $\oplus$ BRANE $\Rightarrow$ BOUNDARY CONDITIONS AT $\ominus$ BRANE GIVE ROUGHLY

$$\frac{m}{k} e^{ky_c} = n + \mathcal{O}(1)$$

The scale

$$M_{Pl} \cdot e^{-ky_c}$$

MAY WELL BE ASSOCIATED WITH WEAK SCALE.

- K-K EXCITATIONS OF SM FIELDS IN TeV ENERGY RANGE.

LIKEWISE, SUPERSYMMETRY BREAKING SCALE MAY ALSO BE

$$\sim M_{Pl}\, e^{-ky_c}$$

E.G., SUSY BROKEN AT $\ominus$ BRANE.

PHENOMENOLOGICALLY VIABLE.

BESIDES K-K GRAVITONS OF MASSES IN TeV RANGE, CONTAINS K-K partners of our particles (and superpartners) IN SAME RANGE.

TOO GOOD ...

—o—

EVEN IF GRAVITY IS STRONG AT TeV, ANY CONCRETE MODEL IS HARDLY THE TRUTH.

BUT THE MODELS INDICATE VARIETY OF PHENOMENA NOT BE MISSED.

EXTRA DIMENSIONS OF INFINITE SIZE : RS2

Consider RS set up again, but

- FORGET ABOUT HIERARCHY PROBLEM. LET IT BE SOLVED, E.G. IN A usual manner BY SUSY/GUT.
- FORGET ABOUT NEGATIVE TENSION BRANE. LET THE BULK EXTEND TO $|y| = \infty$.
- ASSUME OUR matter lives on remaining (+) Brane.

STILL SENSIBLE THEORY: GRAVITY LOCALIZED.

Recall

- GRAVITON ZERO mode

$$m^2 = 0 \qquad h_{\mu\nu}(x,y) = \sqrt{k}\, e^{-2k|y|}\, h^{(4)}_{\mu\nu}(x)$$

normalization factor — 4dim. graviton

- "K-K" modes WITH <u>CONTINUOUS</u> SPECTRUM OF m^2.

AT $m < k$ BEHAVE like

$$h_m = \sqrt{\frac{m}{k}}\,, \qquad |y| \to 0$$

$$h_m = \dots \cos\left(\frac{m}{k}\, e^{k|y|}\right) \qquad |y| \to \infty$$

GRAVITATIONAL POTENTIAL BETWEEN UNIT MASSES ON ⊕ BRANE:

$$V(r) = -G_5 \, |\overset{k}{h_0(y=0)}|^2 \cdot \frac{1}{r}$$ ← zero mode contribution

$$- G_5 \int_0^\infty dm \, |\overset{m/k}{h_m(y=0)}|^2 \, \frac{e^{-mr}}{r}$$

Contribution of "K-K" continuum, decays faster than $\frac{1}{r}$

$$V(r) = -\frac{G_5 k}{r} - \frac{G_5}{k}\frac{1}{r^3}$$

$$= -\frac{G_5 k}{r}\left(1 + \frac{\text{const}}{k^2 r^2}\right)$$

4-dim. NEWTON'S LAW for $r \ll k^{-1}$

$$G_4 = G_5 \cdot k$$

$$\Updownarrow \qquad G_5 = \frac{1}{M^3}, \quad G_4 = \frac{1}{M_{Pl}^2}$$

$$M^3 = M_{Pl}^2 \cdot k$$

k may BE AS LARGE AS mm^{-1} (though no good reason for that)

EVEN FOR $k = mm^{-1}$ FUNDAMENTAL ENERGY SCALE LARGE, $M \sim 10^8$ GeV

FORGET ABOUT TeV QUANTUM GRAVITY

NOT SO INTERESTING FOR COLLIDER EXPERIMENTS

BUT MAY BE POTENTIALLY INTERESTING

- COSMOLOGY (ESPECIALLY IF k IS SMALL)
- BETTER UNDERSTANDING OF APPARENT ENERGY NON-CONSERVATION, ELECTRIC CHARGE NON-CONSERVATION
- RARE PHENOMENA AT LOW ENERGIES
- POSSIBILITY OF WEAK VIOLATION OF LORENTZ-INVARIANCE

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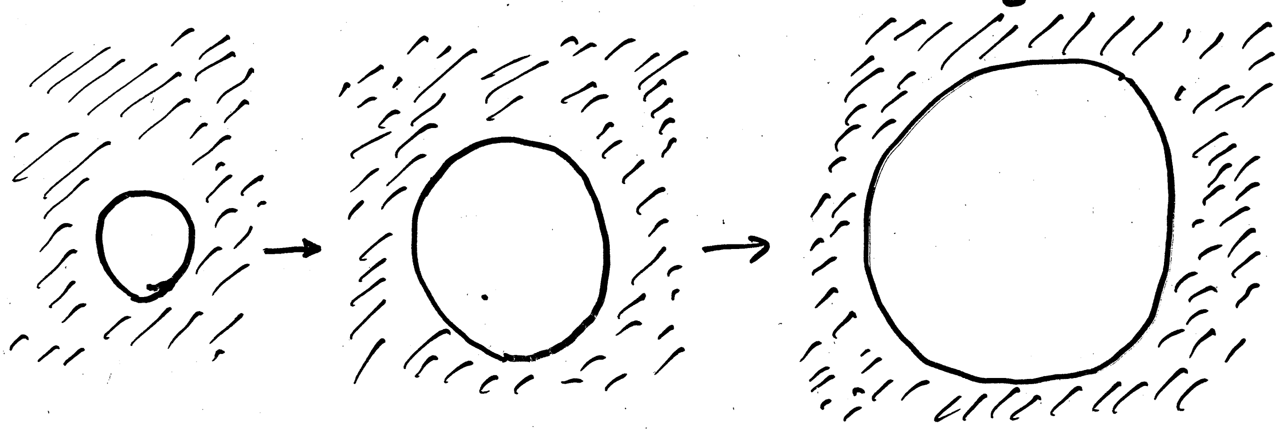
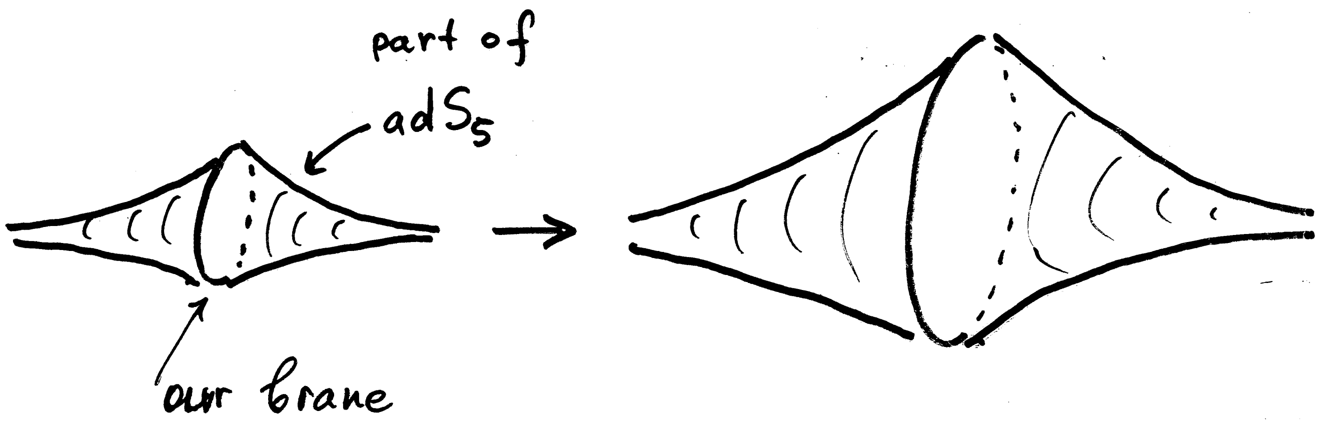
- COSMOLOGY:

OUR UNIVERSE LITERALLY EXPANDING BRANE IN "STATIC" BULK AdS_5.



BETTER PICTURE



Expansion rate:

Usual Friedmann equation
$$H^2 = \frac{8\pi}{3} G_4 \rho \quad , H \ll k$$
Neglecting spatial curvature

$$H^2 = \frac{16\pi^2}{9} G_5^2 \cdot \rho^2 \quad , \quad H \gg k$$
energy density of matter on our brane

i.e. (omitting 2's and π's)
$$H = \frac{\sqrt{\rho}}{M_{Pl}} \quad , \quad H \ll k$$

$$H = \frac{\rho}{M^3} \quad , \quad H \gg k$$

TRANSITION OCCURS (assuming Radiation domination, $\rho \sim T^4$)

AT $T_* \sim \sqrt{M_{Pl}^* \cdot k}$

$\sim$ (A few) $\cdot$ 100 GeV FOR $k^{-1} \sim$ mm

Higher temperatures at larger k.

AT EARLY TIMES, i.e., AT $T > T_*$

UNIVERSE COOLS DOWN FASTER THAN IN USUAL COSMOLOGY:

- FASTER EXPANSION RATE, $H \propto T^4$ INSTEAD OF $H \propto T^2$
- Emission OF K-K gravitons into BULK.

For $H > k$, temperature definitely larger than k.

$\Downarrow$

"K-K" gravitons from continuum have UNSUPPRESSED WAVE FUNCTIONS NEAR BRANE

$\Downarrow$

ENERGY LOSS

$$\left(\frac{d\rho}{dt}\right)_{LOSS} = -\frac{const}{M^3} \cdot T^8$$

ENERGY DILUTION DUE TO EXPANSION

$$\left(\frac{d\rho}{dt}\right)_{expansion} = -4H\rho = -\frac{Const}{M^3} \cdot T^8$$

$\uparrow$ $\frac{\rho}{M^3}$, $\rho \sim T^4$

ROUGHLY COMPARABLE

NB: AT CONVENTIONAL STAGE, $H \sim \frac{\sqrt{\rho}}{M_{Pl}} \sim \frac{T^2}{M_{Pl}}$

$$\frac{\text{DILUTION RATE}}{\text{LOSS RATE}} \sim \frac{T^6/M_{Pl}}{T^8/M^3} = \frac{M^3}{M_{Pl}T^2} = \frac{M_{Pl}k}{T^2} = \frac{T_*^2}{T^2}$$

$\Downarrow$

DILUTION $\gg$ LOSS

HOT BIG BANG GROSSLY MODIFIED AT $T > T_*$, POSSIBLY AT $T \sim \text{TeV}$

WILL BE INTERESTING IF WE EVER HAVE DATA ON UNIVERSE AT $T \gtrsim \text{TeV}$

(HOMEWORK: CALCULATE DARK MATTER DENSITY IN YOUR FAVORABLE EXTENSION OF SM)

— o —

INFLATION

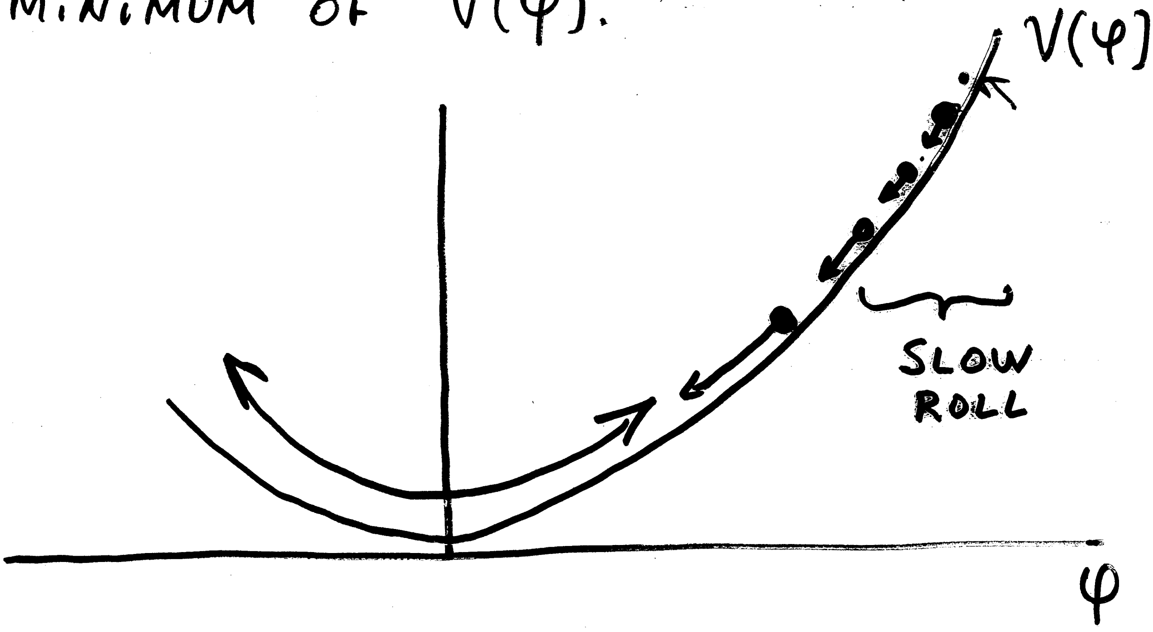
- IS IT AT ALL POSSIBLE, WITH $H > k$ AT INFLATION?

If yes:
- WHAT DENSITY PERTURBATIONS AND GRAVITATIONAL WAVES ARE GENERATED? OFF HAND: NO REASON TO BE SIMILAR TO 4-dim. THEORY.

INFLATION IN NUTSHELL

- INFLATION IS DRIVEN BY A SCALAR FIELD, INFLATON, φ WITH SUITABLE SCALAR POTENTIAL $V(\varphi)$ ALTHOUGH OTHER MECHANISMS EXIST
- DURING INFLATION, φ IS (CLASSICALLY) HOMOGENEOUS, $\varphi = \varphi(t)$

- φ SLOWLY ROLLS TOWARDS MINIMUM OF $V(\varphi)$.



- WHEN φ BECOMES SUFFICIENTLY SMALL, IT STARTS TO ROLL FAST, INFLATION ENDS

- DURING INFLATION, ENERGY DENSITY COMES FROM $V(\varphi)$, CHANGES VERY SLOWLY IN TIME.

- HUBBLE PARAMETER $H = \frac{\dot{a}}{a}$

STAYS ALMOST CONSTANT (SLOWLY DECREASES), UNIVERSE EXPANDS EXPONENTIALLY

EQUATION OF MOTION FOR HOMOGENEOUS INFLATON FIELD

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi} \equiv -V'$$

FRIEDMANN EQN.

$$H^2 = \frac{8\pi}{3} G_4 \rho \overset{2's,\,\pi's}{=} \frac{\rho}{M_{pl}^2}$$

$$\rho = V(\varphi) + \frac{1}{2}\dot{\varphi}^2$$

SLOW ROLL CONDITIONS

$$\ddot{\varphi} \ll H\dot{\varphi} \; ; \qquad \dot{\varphi}^2 \ll V(\varphi)$$

GIVE ALMOST THE SAME FOR POWER-LAW $V(\varphi)$

$$\Downarrow$$

EQ'S AT INFLATIONARY STAGE

$$\begin{cases} 3H\dot{\varphi} = -V' \\ H^2 = \dfrac{V(\varphi)}{M_{pl}^2} \end{cases}$$

INFLATION ENDS WHEN

$$\dot{\varphi}^2 \sim V(\varphi)$$

$$\dot{\varphi}^2 = \frac{V'^2}{H^2} = \frac{V'^2 M_{pl}^2}{V} \sim V$$

$$\Downarrow$$

$$\frac{V'}{V} \sim \frac{1}{M_{pl}}$$

FOR $V = \lambda\varphi^4$

$\varphi \sim M_{pl}$, STILL $V \sim \lambda M_{pl}^4 \ll M_{pl}^4$

✓ SAME

$$H = \frac{4\pi}{3} G_5 \rho = \frac{\rho}{M^3}$$

✓

✓

✓ SCALAR FIELD EQ

$$H = \frac{V}{M^3}$$

✓

$$\dot{\varphi}^2 = \frac{V'^2}{H^2} = \frac{V'^2}{V^2} \cdot M^6 \sim V$$

$$\Downarrow$$

$$\frac{V'^2}{V^3} \sim \frac{1}{M^6}$$

$$\varphi \sim \lambda^{-1/6} M$$

OK : $V \sim \lambda^{1/3} M^4 \ll M^4$

DURING INFLATION, THE INFLATON FIELD DEVELOPS QUANTUM FLUCTUATIONS, ALMOST SCALE INVARIANT, OF AMPLITUDE

$$\delta\varphi \sim H$$

(at interesting length scales, i.e., beyond Hubble horizon of inflationary epoch).

$\Downarrow$

AT DIFFERENT PLACES OF UNIVERSE, THE INFLATON ROLLS DOWN EFFECTIVELY FROM DIFFERENT VALUES OF $\varphi = \varphi(t) \pm \delta\varphi$

$\Downarrow$

INFLATION ENDS IN DIFFERENT PLACES AT DIFFERENT TIMES, WITH TIME DISPERSION

$$\delta t = \frac{\delta\varphi}{\dot{\varphi}}$$

$\Downarrow$

Matter density different after inflation (if inflation ends earlier, energy density gets diluted more than on average, due to expansion)

$$\delta\rho \sim \dot{\rho}\cdot\delta t \approx -H\rho\cdot\delta t$$

$\Downarrow$

$$\frac{\delta\rho}{\rho} \sim H\delta t \sim \frac{H\delta\varphi}{\dot{\varphi}}$$

$$\frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot{\varphi}}$$

$$\frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot\varphi}$$ ← THESE SHOULD BE TAKEN TOWARS END OF INFLATION, some 60 e-folds before end

$$H\dot\varphi = -V'$$
$$H^2 = \frac{V}{M_{Pl}^2}$$
$$\Downarrow$$
$$\left(\frac{\delta\rho}{\rho}\right)^2 \sim \frac{V^3}{M_{Pl}^6 V'^2}$$
$$\frac{V'}{V} = \epsilon \frac{1}{M_{Pl}}$$

ϵ SMALL FROM SLOW ROLL CONDITION

$$\Downarrow$$
$$\boxed{\left(\frac{\delta\rho}{\rho}\right)^2 \sim \frac{V}{\epsilon^2 M_{Pl}^4}}$$

TOWARDS END OF INFLATION

Likewise, GRAVITON FIELD DEVELOPS QUANTUM FLUCTUATIONS

$\hat h \sim H$ WHERE $\hat h = M_{Pl} \cdot h$

$$\Downarrow$$

GRAVITY WAVES

$$h^2 = \frac{H^2}{M_{Pl}^2} \qquad H^2 = \frac{V}{M_{Pl}^2}$$

$$h^2 \sim \frac{V}{M_{Pl}^4}$$ SOMEWHAT SMALLER THAN $\left(\frac{\delta\rho}{\rho}\right)^2$.

V: φ and ρ are BRANE fields

$$H\dot{\varphi} = -V' \; ; \; H = \frac{V}{M^3}$$

$$\Downarrow$$

$$\left(\frac{\delta\rho}{\rho}\right)^2 \approx \frac{V^6}{M^{18} V'^2}$$

$$\frac{V'^2}{V^3} = \epsilon^2 \cdot \frac{1}{M^6}$$

$$\Downarrow$$

$$\left(\frac{\delta\rho}{\rho}\right)^2 = \frac{V^3}{\epsilon^2 M^{12}}$$

GRAVITON IS BULK FIELD. DOES NOT CARE ABOUT M_{Pl}.

$$h^2 = \frac{H^3}{M^3} \sim \frac{V^3}{M^{12}}$$

$$\swarrow$$

INFLATON LIVING ON THE BRANE.

HOW DOES ALL THIS GETS MODIFIED FOR $H > k$ AT INFLATION?

Q1: DOES INFLATION OCCUR AT LOW ENERGY DENSITY,

$$V(\varphi) \ll M^4$$

(NB: M may be much smaller than M_{Pl}).

A1: YES, FOR SMALL $\lambda^{1/3}$

Q2: WHAT IS THE RELATION BETWEEN AMPLITUDES OF DENSITY PERTURBATIONS AND GRAVITY WAVES?

A2: CONSPIRACY !

GROSS FEATURES SIMILAR.

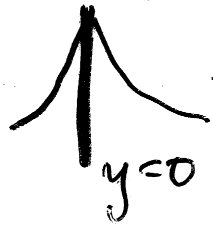
Details (tilt - tensor RELATION, ETC) MAY BE SLIGHTLY DIFFERENT

BUT NO SMOKING GUN.

FOR NO OBVIOUS GOOD REASON...

NATURE (THEORY, RATHER) IS NOT ALWAYS KIND.

QUASI-LOCALIZATION IN RS-2.



PROTOTYPE EXAMPLE:

SCALAR FIELD BOUND TO BRANE DUE TO A POTENTIAL

$$S_\phi = \int d^4x\, dy \left[\frac{1}{2} g^{AB} \partial_A \phi \partial_B \phi - \frac{1}{2} V(y) \phi^2 \right]$$

BINDING POTENTIAL DUE TO DOMAIN WALL

- FLAT EXTRA DIMENSION, $g_{AB} = \eta_{AB}$

$$\partial_A \partial^A \phi + V(y) \phi = 0$$

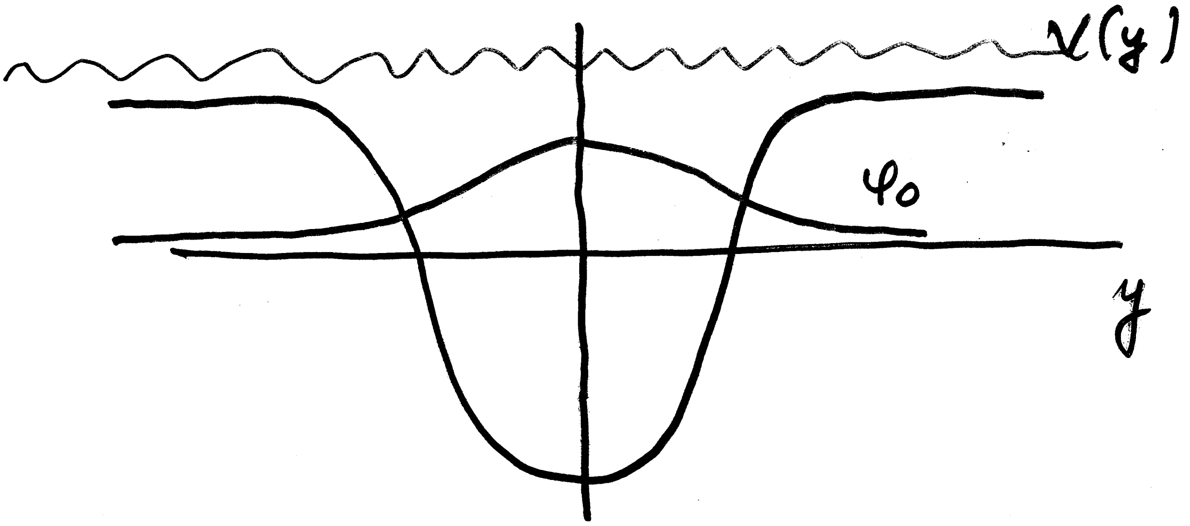
$$\Downarrow$$

$$m^2 \phi = \partial_\mu \partial^\mu \phi = \left[-\partial_y^2 + V(y) \right] \phi$$

$\phi = e^{i p_\mu x^\mu}$; $p_\mu p^\mu = m^2$

BRANE-WORLD SCENARIO WORKS, provided THERE IS one small EIGENVALUE $m^2 = m_0^2$ WHILE OTHERS ARE LARGE ($\gtrsim (\mathrm{TeV})^2$).

TAKE $m_0^2 \neq 0$: AFTER ALL, MOST SM PARTICLES ARE MASSIVE



- Continuum STARTS AT $m^2 = V(\infty)$.
  ESCAPE INTO EXTRA DIMENSIONS IN HIGH ENERGY COLLISIONS.

- WARPED EXTRA DIMENSION:

$$g_{\mu\nu} = a^2(y)\,\eta_{\mu\nu} \qquad g_{yy} = -1$$

$$g^{\mu\nu} = \frac{1}{a^2(y)}\,\eta^{\mu\nu} = e^{2k|y|}\,\eta^{\mu\nu}$$

$$\Downarrow$$

$$S_\phi = \int d^4x\, dy \left[\frac{1}{2} g^{\mu\nu} \partial_\mu\phi\, \partial_\nu\phi + \frac{1}{2} g^{yy} (\partial_y \phi)^2 - \frac{1}{2} V(y)\phi^2 \right] \cdot \sqrt{g^{(5)}}$$

$$= \int d^4x\, dy \left[\frac{1}{2} e^{2k|y|}\, \partial_\mu\phi\, \partial_\nu\phi \cdot \eta^{\mu\nu} - \frac{1}{2} V(y)\phi^2 + \dots \right] \sqrt{g^{5}}$$

SMALL COMPARED TO KINETIC TERM.

BINDING POTENTIAL NEGLIGIBLE AT LARGE $|y|$.
EXPECT CONTINUUM STARTING FROM $m^2 = 0$

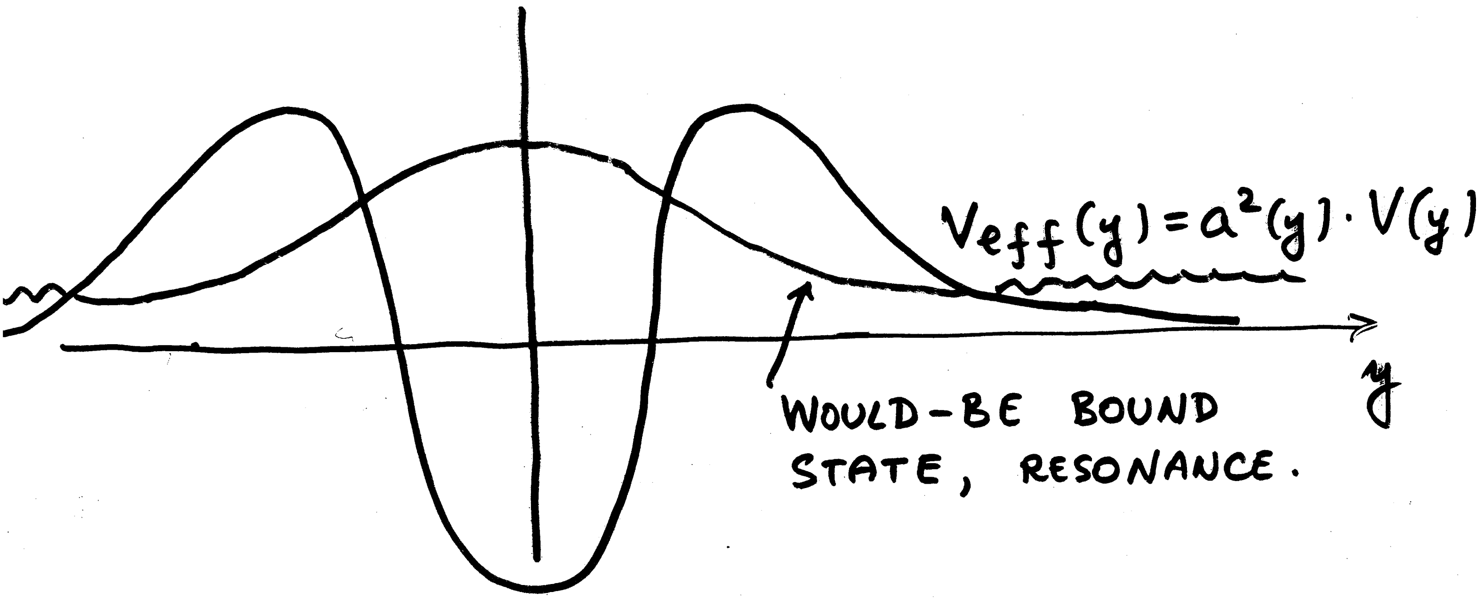
INDEED, EIGENVALUE EQUATION IS NOW

$$m^2\phi = -\frac{1}{a^2}\partial_y\left(a^4\partial_y\phi\right) + \underbrace{\underset{\uparrow\ e^{-2k|y|}}{a^2(y)}V(y)\phi}_{\text{NEGLIGIBLE AT LARGE } y}$$

Continuum indeed starts from $m^2=0$.
WOULD-BE BOUND STATE WITH $m^2=m_0^2$
EMBEDDED IN CONTINUUM

$\Downarrow$

IT BECOMES QUASI-LOCALIZED STATE,
RESONANCE.



FINITE WIDTH OF A MASSIVE BRANE PARTICLE AGAINST ESCAPE INTO BULK.

MAY BE LONG: TUNNELING.

EXAMPLE:

SCALAR FIELD BOUND TO BRANE BY GEOMETRY ITSELF

$$V(y) = 0$$

EIGENVALUE EQN:

$$m^2 \phi = - \frac{1}{a^2} \partial_y \left(a^4 \partial_y \phi \right)$$

NB: Normalization $\int a^2 dy \, |\phi|^2$

CONSTANT MODE, $\phi_0 = \text{const}$, NORMALIZABLE. MASSLESS BOUND STATE.

- ADD MASS IN BULK.

$$V(y) = \mu^2$$

$$\Downarrow$$

ϕ_0 BECOMES MASSIVE BRANE MODE (in 4d sense)

$$m_0^2 = \frac{1}{2} \mu^2$$

WITH FINITE WIDTH AGAINST ESCAPE FROM BRANE

$$\Gamma = \frac{\pi}{16} m_0 \left(\frac{m_0}{k} \right)^2$$

NB: FINITE AT small m_0. NOT A THRESHOLD EFFECT.

LESSON:

ESCAPE INTO EXTRA DIMENSION(S) POSSIBLE EVEN AT <u>LOW</u> ENERGIES,

AT SMALL PROBABITITY. THE HEAVIER THE PARTICLE, THE LARGER THE WIDTH.

$$(\text{positronium } e^+e^-) \longrightarrow \text{nothing}$$
$$\text{nucleon} \longrightarrow \text{nothing}$$
$$\text{Higgs}, Z_0 \longrightarrow \text{nothing}$$

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ESTIMATES VERY MODEL-DEPENDENT

NEW FEATURE:

LOW ENERGY / LARGE TIME INTERVAL, LARGE DISTANCE

EFFECT OF EXTRA DIMENSIONS.