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*Physics of shower simulation at LHC,  
at the example of GEANT4.*

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# *The Monte Carlo Roadmap*

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- Part 1: Introduction
  - LHC related use cases - LCG.
  - Analyzing showers and their development in matter.
  - Brief overview of hadronic models in geant4
- Part 2: Hadronic showers in bulk matter.
  - Selected topics on hadronic shower simulation:
    - Theory driven modeling of inelastic reactions.
- Part 3: ghad – how good is it really?
- Part 4: Modeling electromagnetic showers.
  - Examples of electromagnetic showers.
  - Selected topics on electromagnetic shower physics.

# *Selected topics of hadronic physics modeling*

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- Theory driven modeling of inelastic hadronic reactions at 'high' energies.
  - High here means above  $O(10 \text{ GeV})$  primary particle energy.

## *Initial notes*

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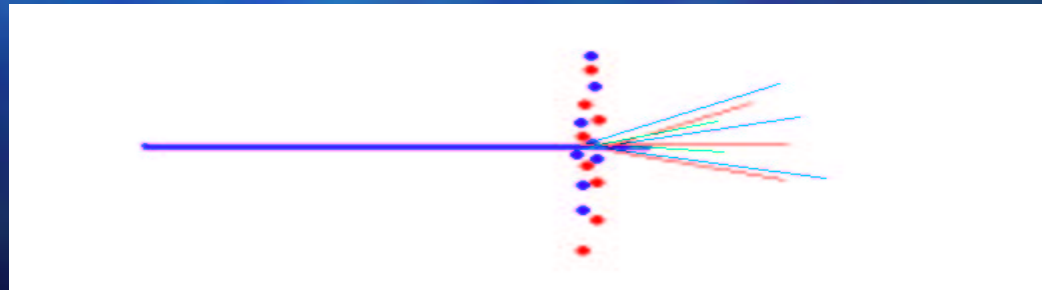
- For a particle energy above  $\sim 20$  GeV, the nucleus as 'seen' by the primary will be strongly Lorentz contracted.
- The nuclear thickness seen by a projectile is a canonical  $O(1)$  fermi.
- This thickness is small in comparison with the typical hadronization time for products of the high energy, soft reaction.

# *The basic process is soft*

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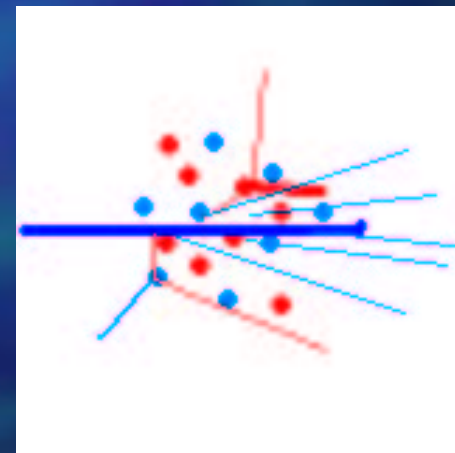
- Where  $X$  can be (literally) anything.
- The reaction begins with the primary undergoing a (series of) soft reactions with nucleons from the nucleus.



# *The intra-nuclear cascade part*

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- It continues with particles passing through the nucleus, with the possibility to undergo secondary interactions
- Many particles are formed outside the nuclear volume, though.



## *The pre-equilibrium part*

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- After some time, of all scattered particles, only slow or still bound nucleons and holes are left in the nuclear volume.
- The nuclear medium has not equilibrated their energy, but these particles (and holes) carry a substantial part of the momentum of the residual system.
- We can view them as excitons of the nuclear matter. We call the nucleus a pre-compound system.

## *The evaporation part*

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- At times of  $ct \sim 100\text{fm}$ , the nucleus has equilibrated the residual excitation energy.
- The system is now a 'compound system'.



# *Evaporation*

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- This compound system can carry significant excitation energy, while the nucleons contained are in statistical equilibrium.
- Most of the neutrons in a hadronic shower stem from evaporation from such compound systems.
- The first statistical model for compound evaporation was proposed by Weisskopf and Ewing, published in *Phys.Rev.*57,472 (1940).

# *The phases of a reaction*

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- Soft interactions
- Intra-nuclear cascade
- Pre-equilibrium phase
- Evaporation

– Note: In recent years, several authors concluded that the pre-equilibrium phase is not necessary. These conclusions seem not fully justified in all cases.

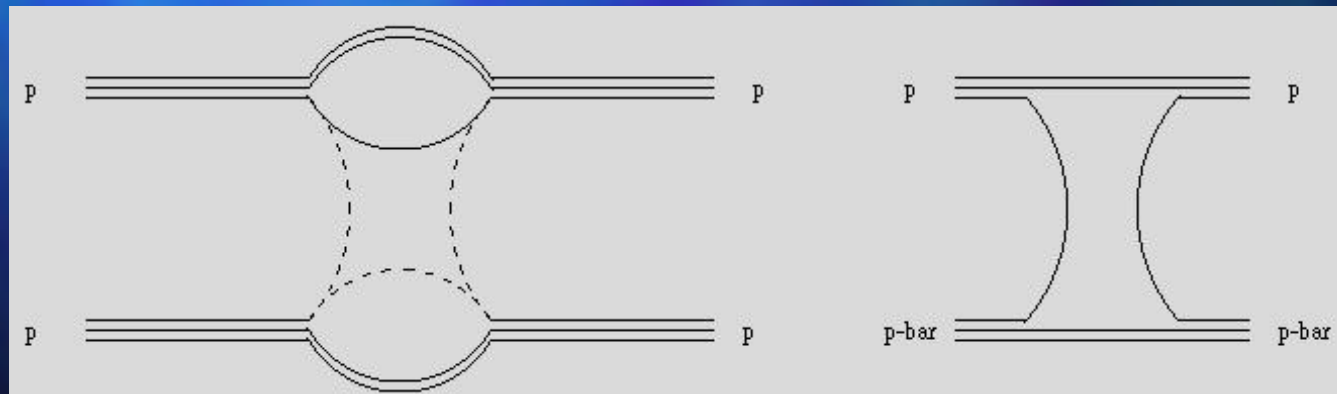
# *Theoretical approaches towards modeling the various phases: examples*

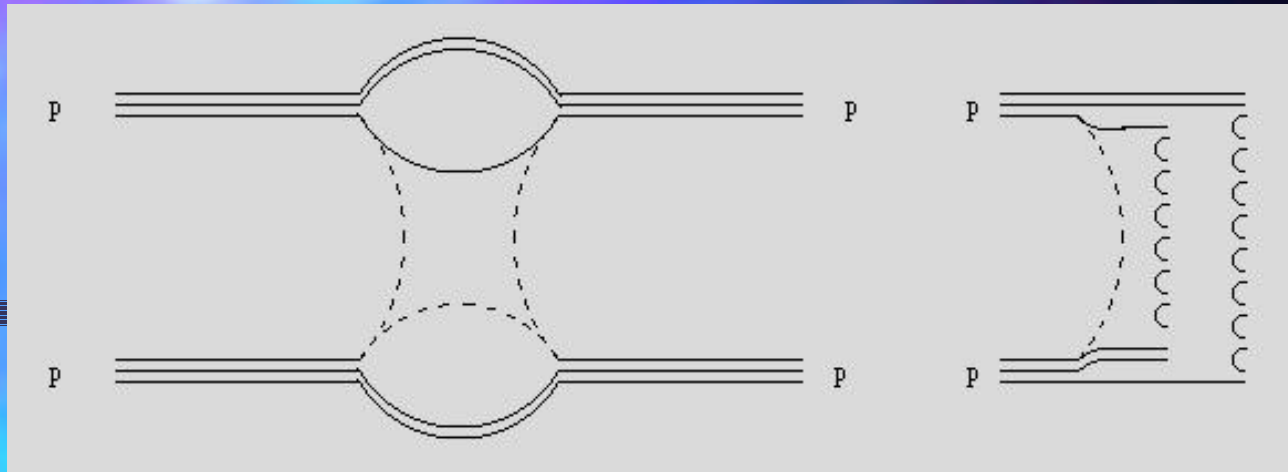
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- Initial soft interactions :
  - Dual parton, or quark gluon string model
- Secondary scattering :
  - Classical or binary cascade, QMD.
- Slow nucleons and fragments :
  - Exciton, or hybrid pre-equilibrium decay models
- compound nucleus
  - Statistical evaporation models

# Dual parton or quark gluon string model – hadron hadron scattering

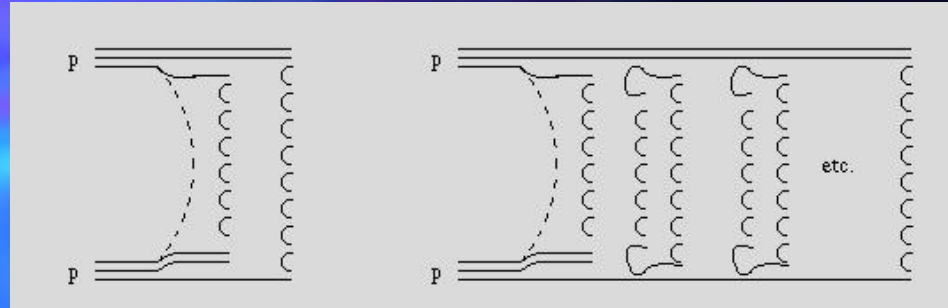
- In the approach based on the topological expansion, the Pomeron pole is described by graphs of the cylindrical type, while the secondary Reggeons are described by planar graphs
- The planar case involves annihilation of valence quarks of the colliding hadrons, and a qq-bar string.





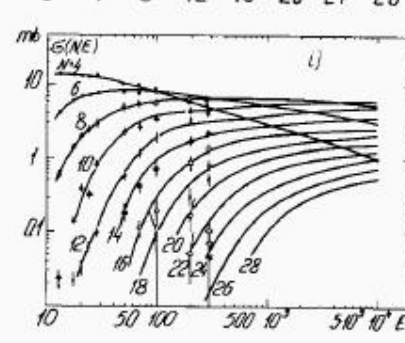
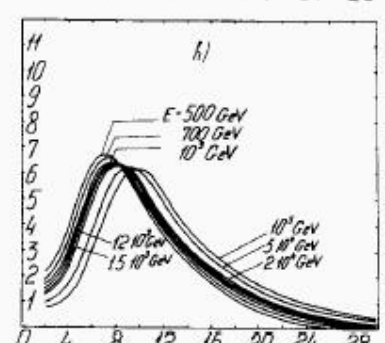
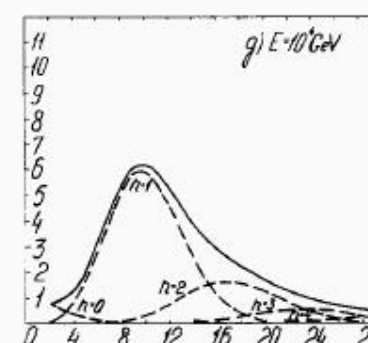
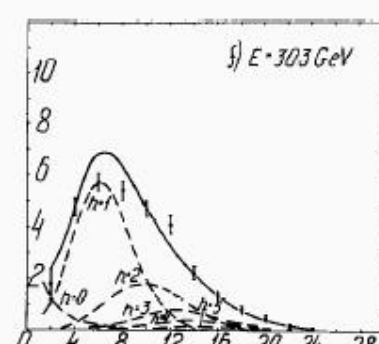
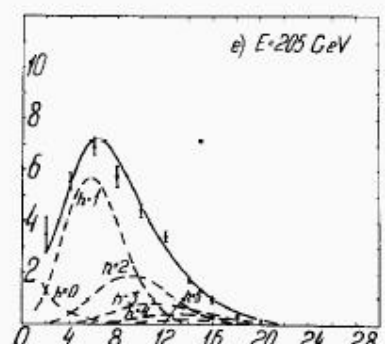
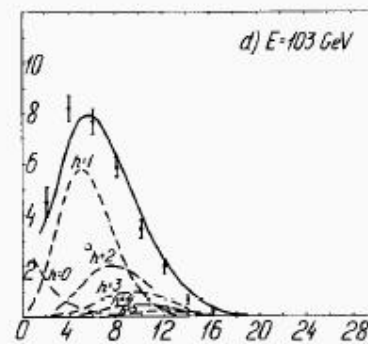
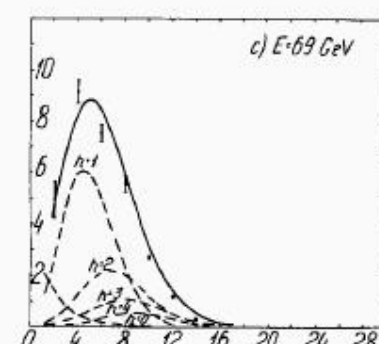
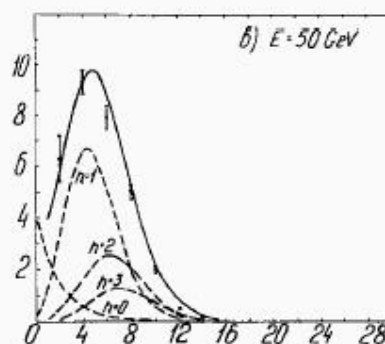
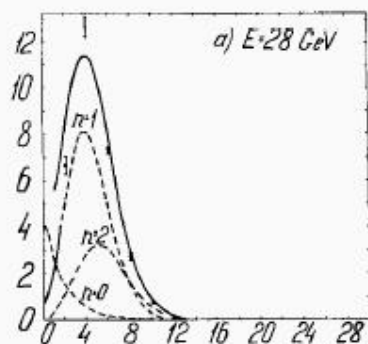
- In the cylindrical (Pomeron) case, the colliding hadrons simply exchange one or several gluons, resulting in color coupling between the valence quarks of the hadrons. They are connected by quark gluon strings.
- Breaking the strings leads to the appearance of white hadrons.

# Multiple Pomeron exchange



- The parameters of the Pomeron trajectory cannot at present be calculated, but are taken from fits to experimental data.
- Kaydalov  $\Delta \equiv 1 - \alpha_p = 0.07$ , and  $\alpha'_p \approx 0.25(\text{GeV} / C)^{-2}$
- For the supercritical Pomeron,  $\Delta > 0$ , multi-Pomeron exchange becomes important, since the contribution to the total cross-section grows approximately like  $(s / s_0)^{n\Delta}$   
(Ter-Martyrosian, Phys.Lett.44B,377ff)

# $n$ -pomeron exchange contributions to charged multiplicities (Ter-Martirosyan, PLB44, 377, 1973).



# *N-Pomeron exchange probabilities*

- contributions from enhanced graphs are small, the cross-section for the production of n Pomeron showers can be written as

$$\sigma_n(\xi) = \frac{\sigma_P}{nz} \left( 1 - e^{-z} \sum_{k=0}^{n-1} \frac{z^k}{k!} \right)$$

- Where z describes the size of the successive re-scatterings

$$z = \frac{2C\gamma_P}{R^2 + \alpha_P} \xi e^{\xi\Delta}$$



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- Here we have

$$\sigma_P = 8\pi\gamma_P e^{\xi\Delta},$$

$$\xi = \ln(s/s_0),$$

$$s_0 = 1\text{GeV}^2$$

- The parameters  $R$  (radius), and  $\gamma$  (product of couplings) describe the Pomeron hadron vertices.

# *From n-Pomeron exchange probabilities to hadrons*

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- Given the knowledge of the n-Pomeron exchange probabilities, and the factorization of these in the cross-sections, we can define the cross-section for n-Pomeron exchange, and hence the number of strings in a hadron-hadron collision.

$$\frac{d\sigma^h}{dy} = \sum_{n=0}^{\infty} \sigma_n \varphi_n^h(s, y).$$

- What we need in addition is the momenta carried by the individual partons, and the hadronization mechanism for the strings.

# Parton densities and fragmentation functions

- The functions  $\varphi$  can be expressed in terms of functions that describe the string ends, which themselves can be written in terms of parton density functions, and hadronisation functions. For the pp case this gives for example:

$$\varphi_n^h = a^h \left\{ F_v^{h,n}(x_+) F_{qq}^{h,n}(x_-) + F_{qq}^{h,n}(x_+) F_v^{h,n}(x_-) + 2(n-1) F_s^{h,n}(x_+) F_s^{h,n}(x_-) \right\}$$

# Parton densities and hadronization functions

$$F_v^{h,n} = \frac{2}{3} \int_x^1 f_p^{u,n}(\xi) G_u^h(x/\xi) d\xi + \frac{1}{3} \int_x^1 f_p^{d,n}(\xi) G_d^h(x/\xi) d\xi$$

$$F_{qq}^{h,n} = \frac{2}{3} \int_x^1 f_p^{ud,n}(\xi) G_{ud}^h(x/\xi) d\xi + \frac{1}{3} \int_x^1 f_p^{uu,n}(\xi) G_{uu}^h(x/\xi) d\xi$$

$$F_s^{h,n} = \frac{1}{4 + 2\delta_S} \int_x^1 f_p^{u_s,n}(\xi) G_u^h(x/\xi) + f_p^{\bar{u}_s,n}(\xi) G_{\bar{u}}^h(x/\xi) + f_p^{d_s,n}(\xi) G_d^h(x/\xi) + f_p^{\bar{d}_s,n}(\xi) G_{\bar{d}}^h(x/\xi) + \delta_S [f_p^{s_s,n}(\xi) G_s^h(x/\xi) + f_p^{\bar{s}_s,n}(\xi) G_{\bar{s}}^h(x/\xi)] d\xi$$

- Here the functions G can be related to the fragmentation functions:

$$G_{parton}^h(z) = z D_{parton}^h(z) / a^h$$

# Parton densities are motivated from Regge theory

- We use

$$f_p^{u,n} = C_u x^{-\alpha_R} (1-x)^{\alpha_R - 2\alpha_B + n - 1}$$

$$f_p^{d,n} = C_d x^{-\alpha_R} (1-x)^{\alpha_R - 2\alpha_B + n}$$

$$f_p^{uu,n} = C_{uu} x^{\alpha_R - 2\alpha_B + 1} (1-x)^{-\alpha_R + n - 1}$$

$$f_p^{ud,n} = C_{ud} x^{\alpha_R - 2\alpha_B} (1-x)^{-\alpha_R + n - 1}$$

$$f_p^{s,n} = f_p^{\bar{s},n} = C_s (1-x)^{-\alpha_B + n - 1}$$

- Where  $\alpha_R = \alpha_R(0) \approx 0.5$ ,  $\alpha_B = \alpha_B(0) \approx -0.5$
- The coefficients C are determined from sum rules.

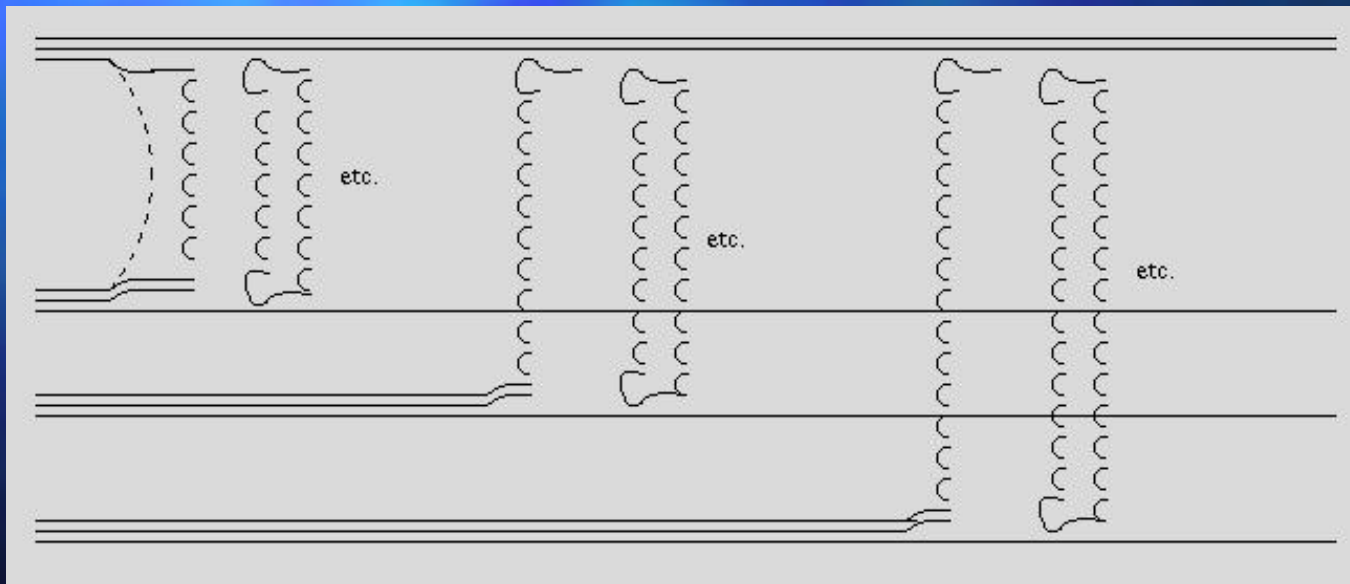
# *Hadronization functions*

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- The hadronization functions are based on measurements of particle spectra from hadron-hadron collisions.
- These are phenomenological functions. (not to be confused with the Lund hadronization function)

# *Hadron nucleus collisions*

- With respect to hadron hadron collisions, hadron nuclear collisions offer the additional twist of multiple participating target nucleons.



# *Quark gluon string model in geant4*

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- The algorithm
  - A 3-dimensional nuclear model is built up
  - It is collapsed into 2 dimensions
  - The impact parameter is calculated
  - Hadron-nucleon collision probabilities calculated based on the quasi-eikonal model, using Gaussian density distributions for hadrons and nucleons.
  - Sampling of the number of pomerons exchanged in each collision.
  - String formation from pairs of partons and hadronization.



# The nuclear model

- The nuclear density distributions used are of the Saxon-Woods form for high A (Grypeos, J.Phys.G17,1093,1991)

$$\rho(r_i) = \frac{\rho_0}{1 + \exp[(r_i - R) / a]}$$

- And the harmonic oscillator form for light nuclei (A<17, Elton, Nuclear Sizes, Ox.Un.Press 1961)

$$\rho(r_i) = (\pi R^2)^{-3/2} \exp(-r_i^2 / R^2)$$

- The nucleon momenta are randomly chosen between zero and the Fermi momentum

## *The nuclear model, cont.*

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- The sampling is done in a correlated manner
  - such the local phase-space densities stay within what is allowed by Pauli's principle, and
  - such that the sum of all nucleon momenta equals zero.

# Collision criterion

- In the Regge Gribov approach, the collision probability can be written as

$$p_{ij}(\delta b_{ij}, s) = 1/c(1 - \exp[-2u(\delta b_{ij}, s)]) = \sum_{n=0}^{\infty} p_{ij}^{(n)}(\delta b_{ij}, s)$$

- where

$$p_{ij}^{(n)}(\delta b_{ij}, s) = 1/c \exp[-2u(\delta b_{ij}^2, s)] \frac{[2u(\delta b_{ij}^2, s)]^n}{n!}$$

- And

$$u(\delta b_{ij}^2, s) = \frac{z(s)}{2} \exp(\delta b_{ij}^2 / 4\lambda(s)), \quad \lambda = R_p^2 + \alpha' \ln(s/s_0)$$

(Capella, et al, Phys,Rev.D18,4120,1978)

# Diffraction

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- Diffraction is split off using the shower enhancement factor  $c$  (Baker '76 , Ter-Martirosyan '72).

$$p_{ij}^{diff}(\delta b_{ij}, s) = \frac{1-c}{c} (p_{ij}^{tot}(\delta b_{ij}, s) - p_{ij}(\delta b_{ij}, s))$$

# String formation

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- String formation is done via the parton exchange mechanism, sampling the parton densities, and ordering pairs of partons into color coupled entities.

$$f^h(x_1, x_2, \dots, x_{2n-1}, x_{2n}) = f_0 \prod_{i=1}^{2n} u_{p_i}^h(x_i) \delta(1 - \sum_{i=1}^{2n} x_i)$$

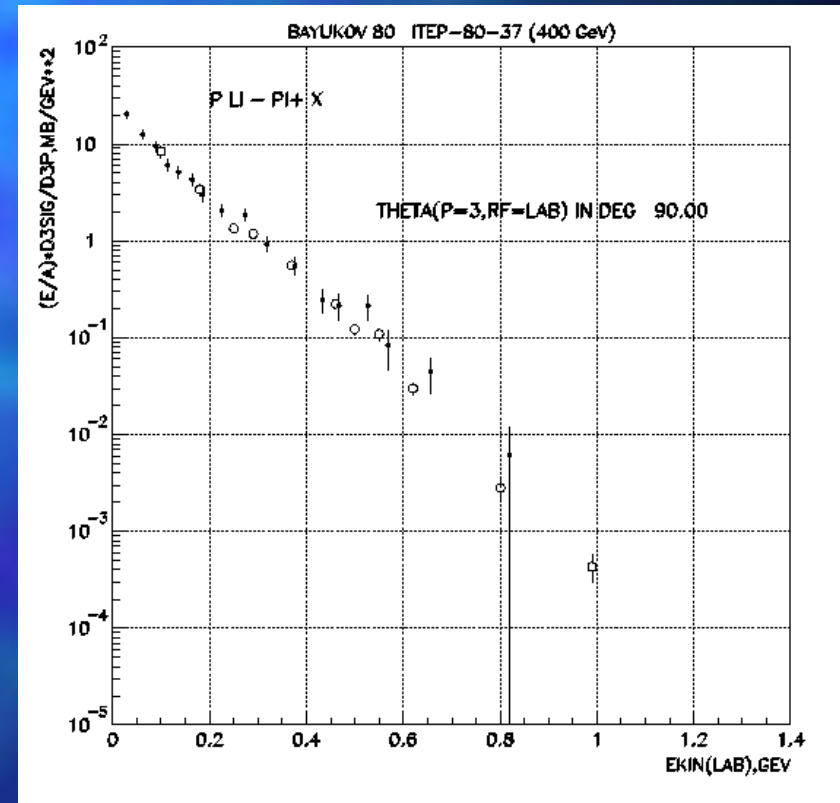
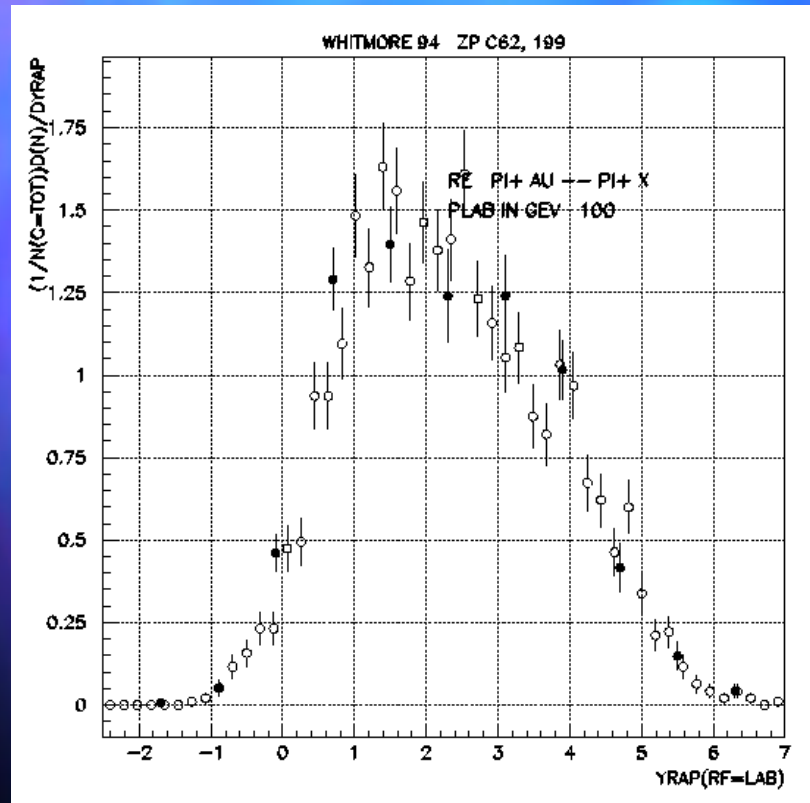
# *QGS model for $\pi$ , N, and K induced reactions*

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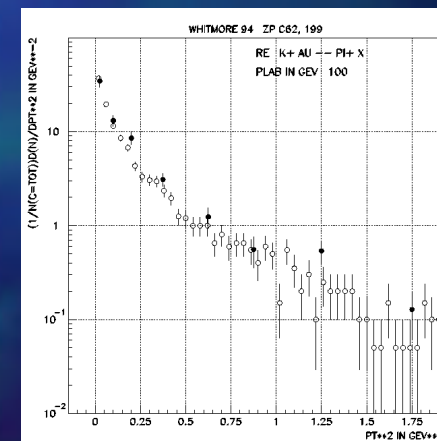
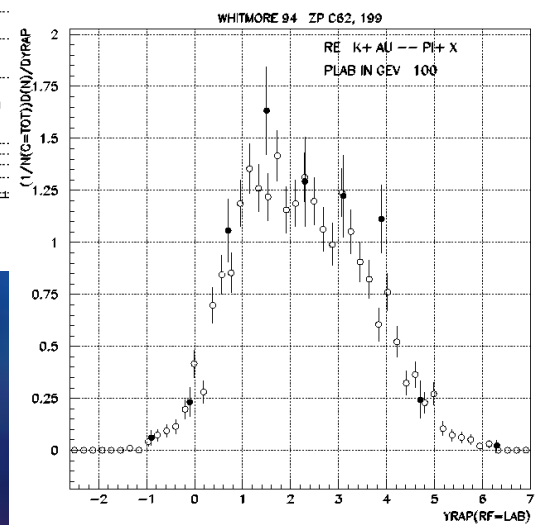
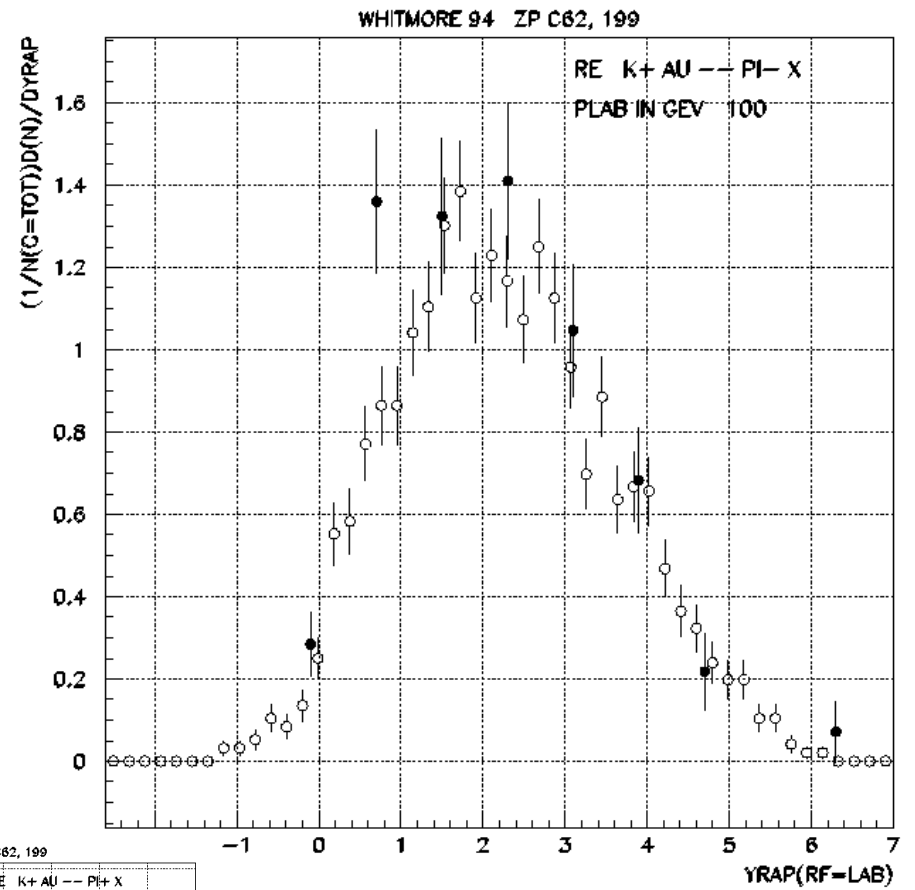
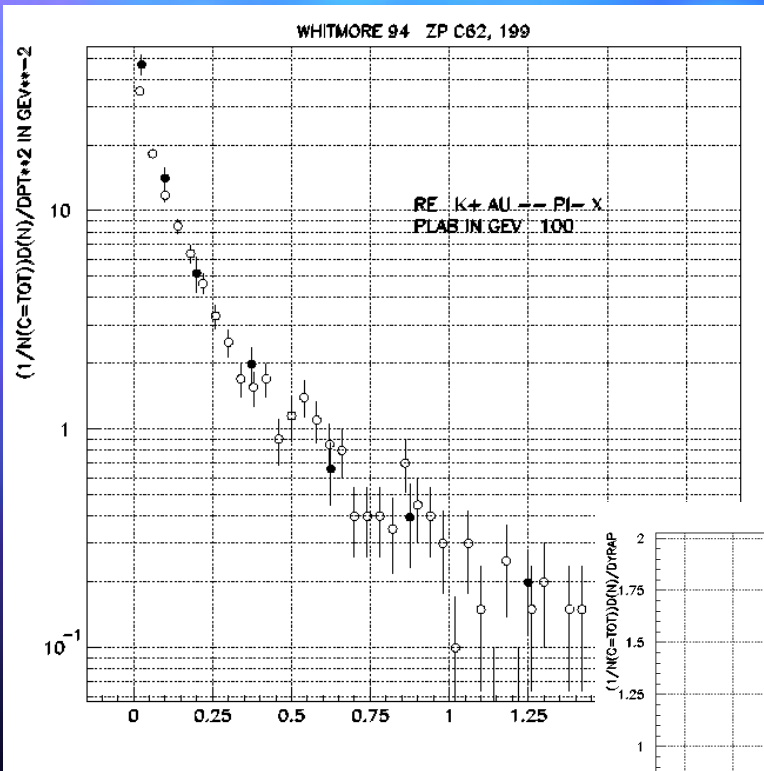
- Pomeron trajectory and vertex parameters found in a global fit to elastic, total and diffractive (6% assumed) cross-sections for nucleon, kaon and pion scattering off nucleons.

# QGS Model

## Pion and proton scattering



# $K^+$ , scattering off Au





# *Intra-nuclear transport*

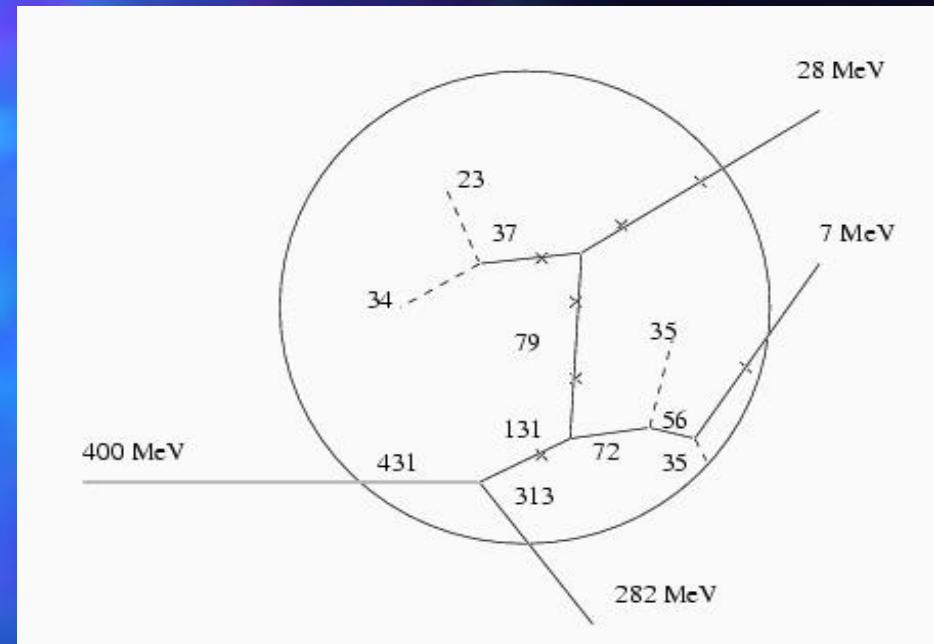
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## ■ Notes:

- In inelastic particle nucleus collisions, there is a phase of fast particle emission (  $10^{-23} - 10^{-21} s$  ).
- A Boltzmann equation needs to be solved to treat the physical process of the collision in detail.
- The original intra-nuclear cascade model developed by Bertini solves the Boltzmann equation on average.
- Much later other cascade-type models have been proposed to the same purpose.

# *Cascade type models*

- Bertini cascade
- Quantum molecular dynamics (QMD)
- Binary cascade



# *Some history of the intra-nuclear cascade*

- First proposed by R. Serber:
  - Nuclear reactions at high energies, Phys.Rev.72,v11,p1114f(1947)
- Statistical calculations by M. Goldberger
  - The interaction of high energy neutrons and heavy nuclei, Phys.Rev.74,v10, p1269ff(1948).
- Monte Carlo computer code by Metropolis et al
  - Monte Carlo simulations of inter-nuclear cascades, I, II, Phys.Rev.110,v1, p185ff, p204ff, 1958.
- 1963 H.W. Bertini published standard methods to be used in many cascade implementations
  - Phys.Rev.131,p1801, 1963 (ORNL-3383)

# *Canonical components of an intra-nuclear cascade*

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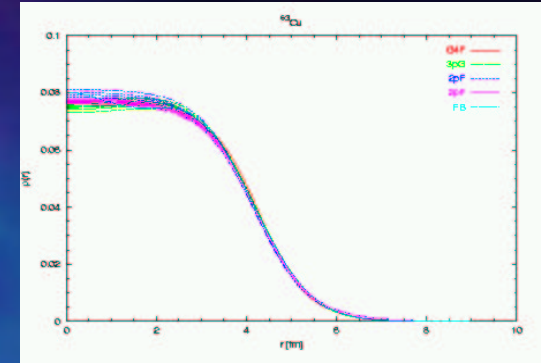
- A description of the nucleus
  - $A$ ,  $Z$ , densities, etc..
- A description of the scattering of moving particles off nucleons in the nucleus
  - Cross-sections, angular distributions
- A description of the coherent interaction of moving particles with the nuclear field.
  - Hamiltonian, equations of motion, potential
- A description of the effects of the nuclear medium on scattering cross-sections.
  - Pauli's principle, effective masses and width, etc..

# *Bertini cascade in geant4*

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- The basic steps
  1. The impact parameter of the projectile is selected uniformly over the projected area of the nucleus.
  2. Total, free cross-sections are used to determine the reactions locations of the cascade particles in a nuclear medium with density  $\rho$ .
    - A set of concentric spherical shells with constant density are used to model the nuclear medium.
    - At shell boundaries, cascade particles are refracted or reflected.
  3. The momenta of collision partners are calculated.
  4. If their states are allowed by Pauli's exclusion principle, these become cascade particles, and we continue with step 2.

# The nuclear model



- Nucleons are assumed to have Fermi gas momentum distributions, with Fermi momentum calculated in local density approximation.

$$p_F = \left( \frac{3\pi^2 \rho(r)}{2} \right)^{1/3}$$

- If  $A < 5$ , a nucleus consisting of one shell with 8fm radius is used. For  $4 < A < 11$ , three concentric spheres with radii

$$r_i(\alpha_i) = \sqrt{C_1(1 - 1/A) + 6.4\sqrt{-\log(\alpha_i)}}$$

- Where  $C_1 = 3.3836A^{1/3}$  and  $\alpha_i = \{0.01, 0.3, 0.7\}$
- For larger A we use

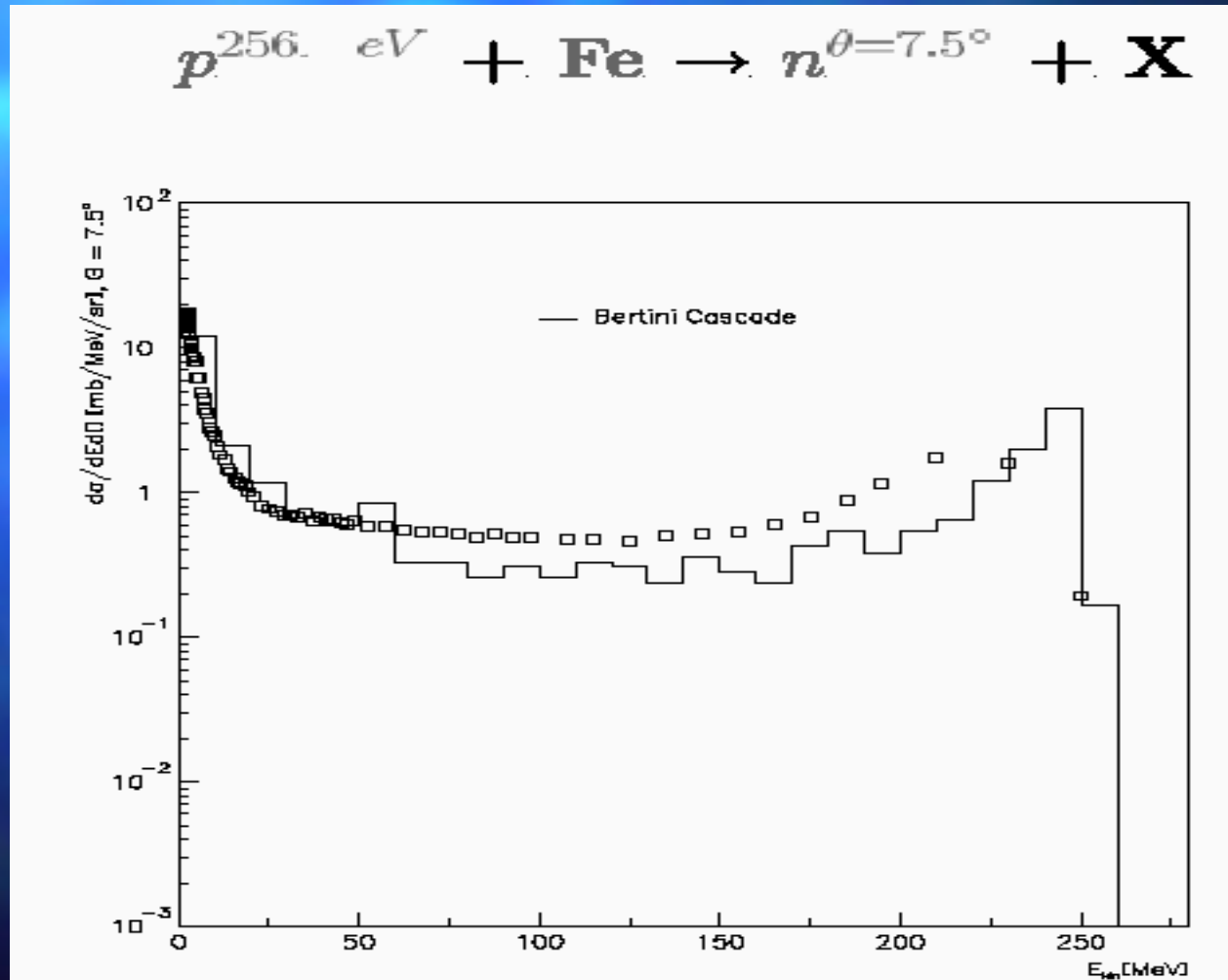
$$r_i(\alpha_i) = C_2 \log \left( \frac{1 - e^{C_1/C_2}}{\alpha_i} - 2 \right) + C_1, \quad C_2 = 1.7234$$

# *Cross-sections and Pauli principle*

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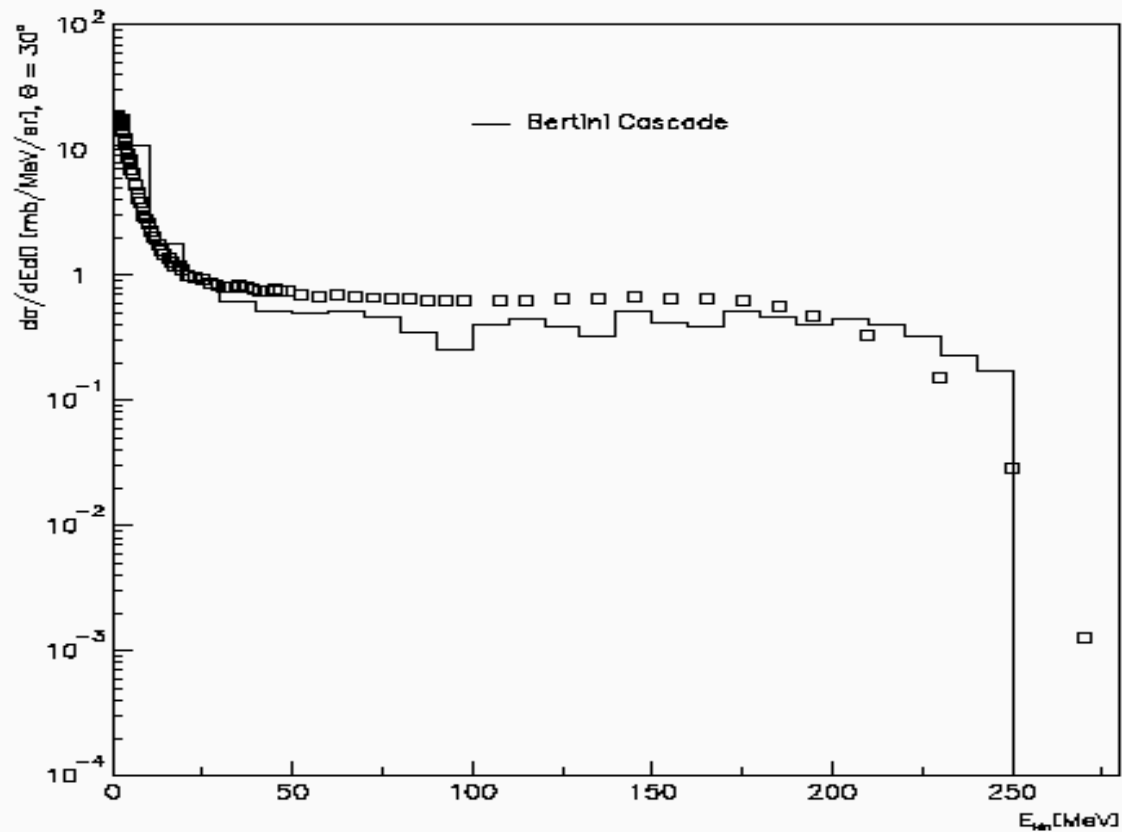
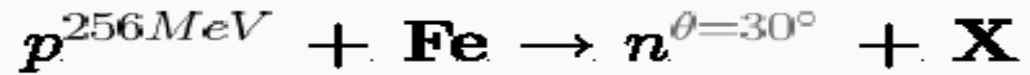
- Pauli's exclusion principle is modeled by accepting only scattered nucleons with energy above Fermi energy  $E_N > E_{Fermi}$
- Path length are sampled according to the free nucleon-nucleon or pion-nucleon cross-sections.
- Secondary kinematics is also determined from experimental differential cross-sections.
- S-wave pion absorption is included for all pions, using experimental data on the channel cross-sections for producing final states.

# Example plots – Iron 7.5 degrees

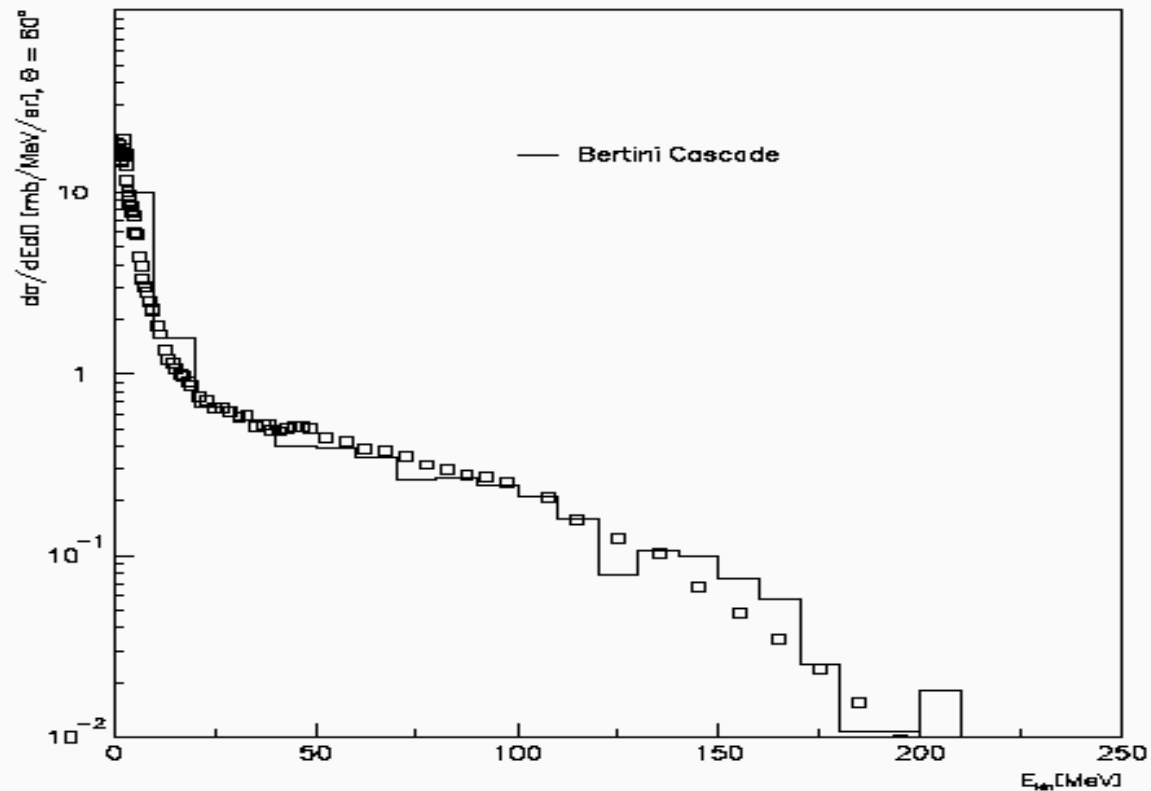
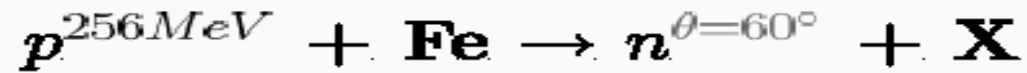




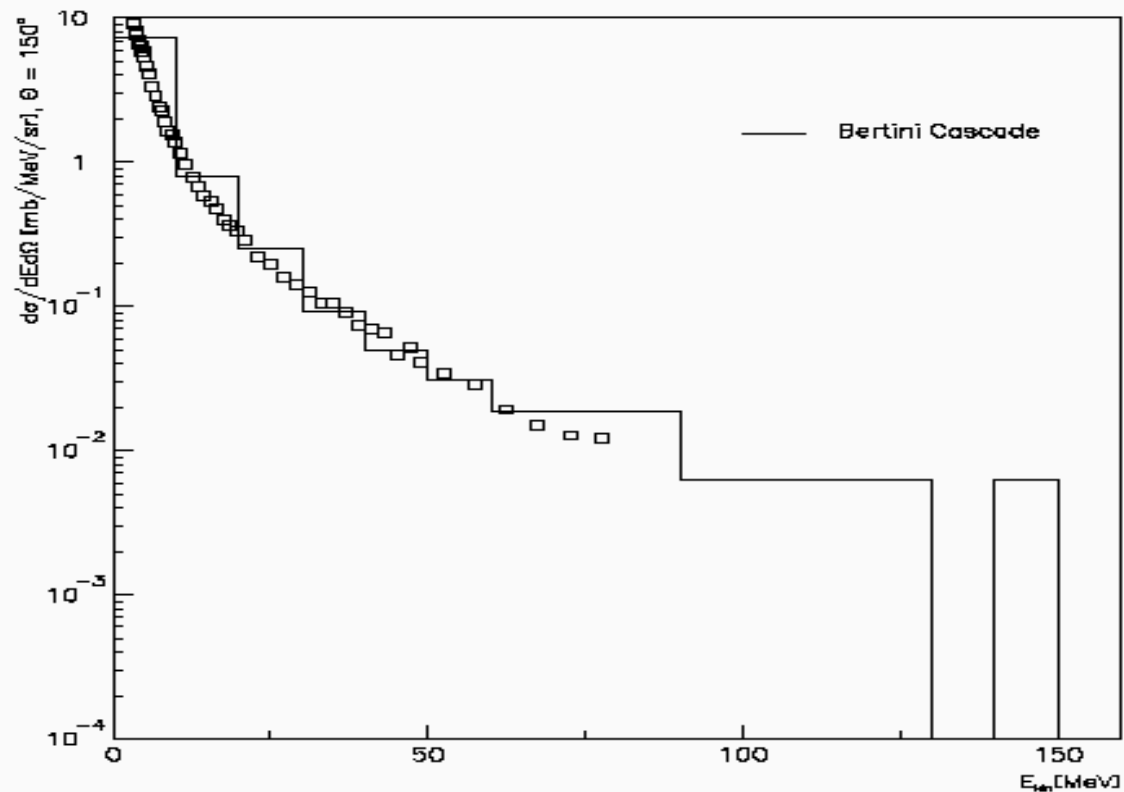
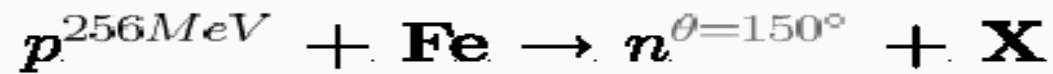
# Example plots – Iron 30 degrees



# Example plots – Iron 60 degrees



# Example plots – Iron 150 degrees



# Binary cascading

- Some characteristics of binary cascading:
  - In binary cascading, like in QMD, each nucleon participant is described by

$$\phi(x, q_i, p_i, t) = (2/(L\pi))^{3/4} \exp(-2/L(x - q_i(t))^2 + ip_i(t)x)$$

- And the total wave function is assumed to be the direct product of these (no anti-symmetrization).
- The equations of motion for this wave-form are identical in structure to the classical Hamilton equations, and can be solved by numerical integration.
- In QMD, the Hamiltonian is calculated from 2- and 3-body interactions of the particles in the system. In Binary cascade, the Nuclear Hamiltonian is calculated from optical potentials based on the target nucleus' property.

# *The nuclear model*

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- A 3-dimensional model of the nucleus is constructed
  - positioning individual nucleons
  - in local density approximation
  - Using the Fermi gas model.
- The sampling is done in a correlated manner
  - such the local phase-space densities stay within what is allowed by Pauli's principle, and
  - such that the sum of all nucleon momenta equals zero.

# The nuclear density

- The nuclear density distributions used are the Saxon-Woods form for high A

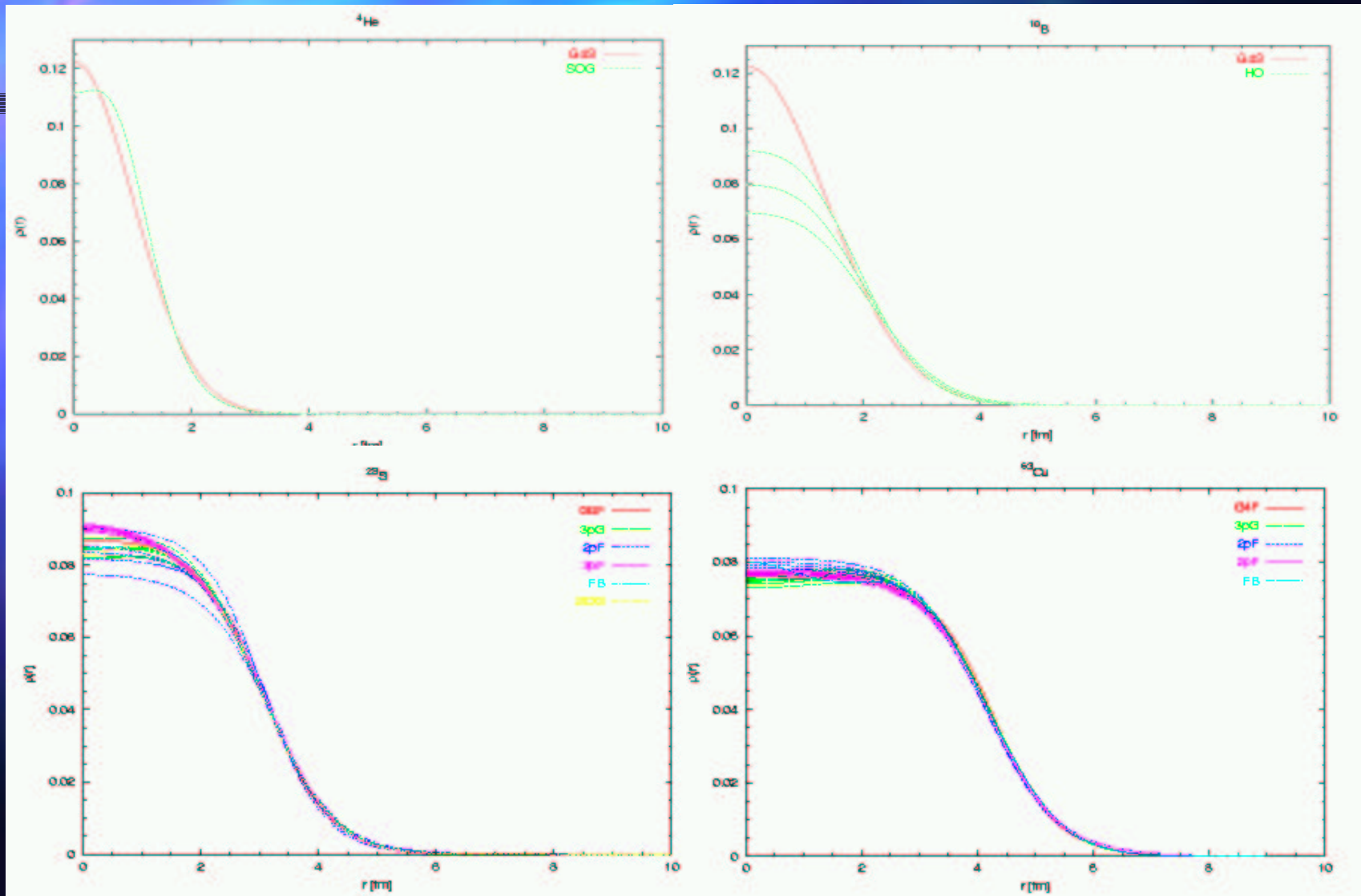
$$\rho(r_i) = \frac{\rho_0}{1 + \exp[(r_i - R)/a]}, \quad \rho_0 = \frac{3}{4\pi R^3} \left( 1 + \frac{a^2 \pi^2}{R^2} \right)$$

- Here  $a = 0.545 \text{ fm}$ ,  $R = r_0 A^{1/3}$ ,  $r_0 = 1.16(1 - 1.16A^{-2/3}) \text{ fm}$
- And the harmonic oscillator form for light nuclei ( $A < 17$ )

$$\rho(r_i) = (\pi R'^2)^{-3/2} \exp(-r_i^2 / R'^2)$$

- with  $R' = \frac{2}{3} \langle r^2 \rangle = 0.8133 A^{2/3} \text{ fm}^2$

# Nuclear densities: Ex. ${}^4\text{He}$ , ${}^{10}\text{B}$ , ${}^{28}\text{Si}$ , and ${}^{63}\text{Cu}$



# *The transport algorithm*

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1. The impact parameter is chosen randomly across the projected area of the nucleus.
2. The path of all cascade particles is projected as straight lines, to determine the time to the earliest collision
  - Based on geometrical cross-section interpretation, and free cross-sections, as described later
3. The collision is simulated, and the final state is subject to Pauli's principle. If rejected, the procedure is repeated with the next collision.
4. If accepted, the system is transported in the nuclear potential, using a 4'th order Runge-Kutta integration algorithm, and we go to step 2.



# *The imaginary part of the G-matrix*

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- Acts like a scattering term
- Described as 2-body, point-like collisions, and resonance decay
  - Collision assumption of black disk cross-section

# Why binary cascade?

- The name binary cascade comes from the fact that only binary collisions (and decay) are considered, like



- No further details on the mathematics, but the nucleon and delta resonances taken into consideration are these

- $\Delta_{1232}, \Delta_{1600}, \Delta_{1620}, \Delta_{1700}, \Delta_{1900}, \Delta_{1905}, \Delta_{1910}, \Delta_{1920}, \Delta_{1930}, \Delta_{1950}$
- $N_{1400}, N_{1520}, N_{1535}, N_{1650}, N_{1675}, N_{1680}, N_{1700}, N_{1710}, N_{1720}, N_{1900}, N_{1990}, N_{2090}, N_{2190}, N_{2220}, N_{2250}$

# Channel cross-sections

- Many cross-sections in meson-nucleon scattering can be modeled as resonance formation in the S-channel.

$$\sigma_{res}(\sqrt{s}) = \sum_{FS} \frac{2J+1}{(2S_1+1)(2S_2+1)} \frac{\pi}{k^2} \frac{\Gamma_{IS}\Gamma_{FS}}{(\sqrt{s} - M_R)^2 + \Gamma^2/4}$$

- Where the partial width for decay then will depend on the stochastic mass of the resonance

$$\Gamma_{R \rightarrow 12}(M) = (1+r) \frac{\Gamma_{R \rightarrow 12}(M_R) M_R}{p(M_R)^{2l+1}} \frac{p(M)^{2l+1}}{M [1+r[p(M)/p(M_R)]]^{2l}}, \quad r = 0.2$$

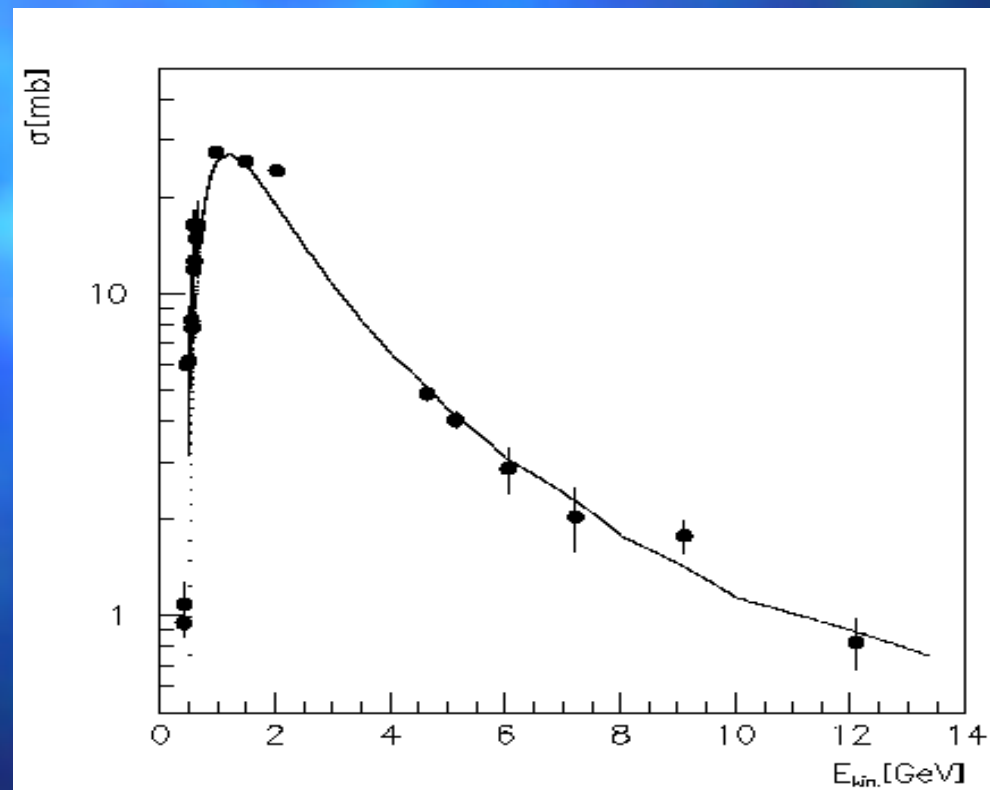
# *Resonance production in the t-channel*

- The cross-section for resonance formation in the t-channel was parameterized based on experimental data. The parametrization is motivated from the form of the  $\Delta 1232$  cross-section.

$$\sigma_{pp \rightarrow AB} = 2\alpha_{AB}\beta_{AB} \frac{\sqrt{s} - \sqrt{s_0}}{(\sqrt{s} - \sqrt{s_0})^2 + \beta_{ab}^2} \left( \frac{\sqrt{s_0} + \beta_{AB}}{\sqrt{s}} \right)^{\gamma_{AB}}$$

- Single and double resonance excitation in nucleon nucleon scattering are taken into the model

# *Predicting the $\Delta^{++}$ production cross-section in $pp$ scattering by binary cascade*



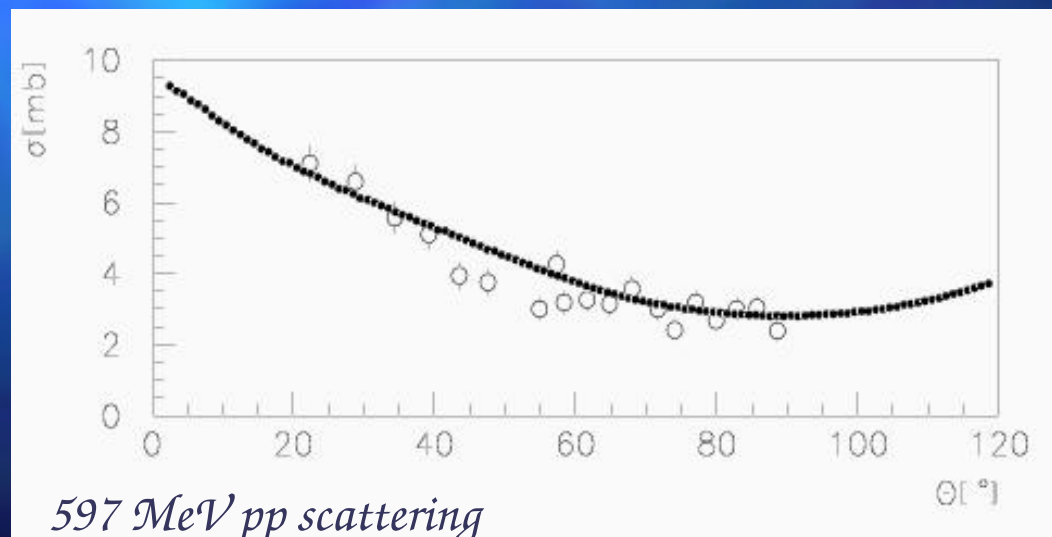
# *Non pp induced t-channel cross-sections*

- Other cross-sections are derived from this using detailed balance, while isospin invariance is assumed

$$\sigma_{cd \rightarrow ab} = \sum_{J,M} \frac{\langle j_c m_c j_d m_d \parallel JM \rangle^2}{\langle j_a m_a j_b m_b \parallel JM \rangle^2} \frac{(2S_a + 1)(2S_b + 1) \langle p_{ab}^2 \rangle}{(2S_c + 1)(2S_d + 1) \langle p_{cd}^2 \rangle} \sigma_{ab \rightarrow cd}$$

# *Nucleon nucleon elastic scattering final states*

- These are taken from the phase-shift analysis of R. Arndt,
  - Mod.Phys.A 18,449(2003)



# Angular distributions for resonance re-scattering in the $t$ -channel

- These are calculated analytically from the collision term of the in-medium, relativistic Boltzmann-Uehling-Uhlenbeck equation, via scaling of the center of mass energy:

$$\sigma_{NN \rightarrow NN}(s, t) = \frac{1}{(2\pi)^2 s} \left( D(s, t) + E(s, t) + (\text{inverted } t, u) \right)$$

- with

$$D(s, t) = \frac{(g_{NN}^\sigma)^4 (t - 4m^2)^2}{2(t - m_\sigma^2)^2} + \frac{(g_{NN}^\omega)^4 (2s^2 + 2st + t^2 - 8m^2 s + 8m^4)}{2(t - m_\omega^2)^2} \\ + \frac{24(g_{NN}^\pi)^4 m^4 t^2}{2(t - m_\pi^2)^2} + \frac{(g_{NN}^\sigma g_{NN}^\omega)^2 (2s + t - 4m^2) m^2}{(t - m_\sigma^2)(t - m_\omega^2)}$$

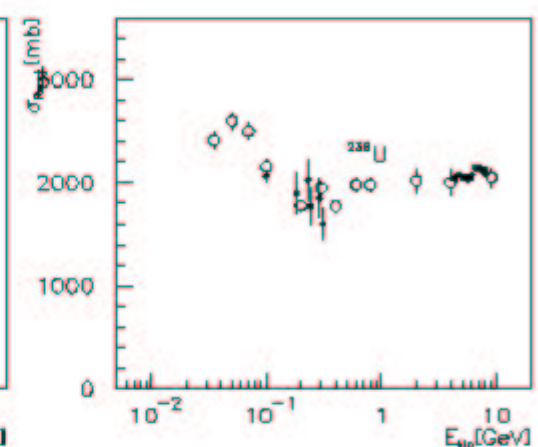
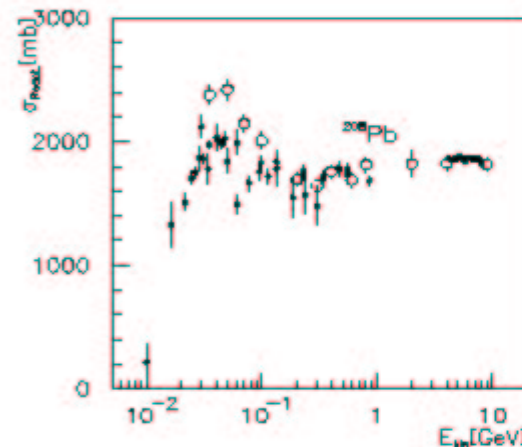
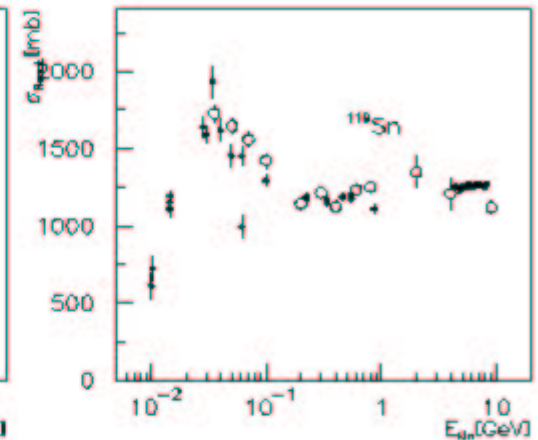
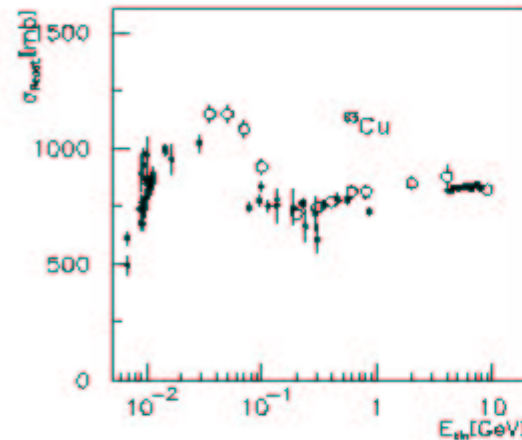
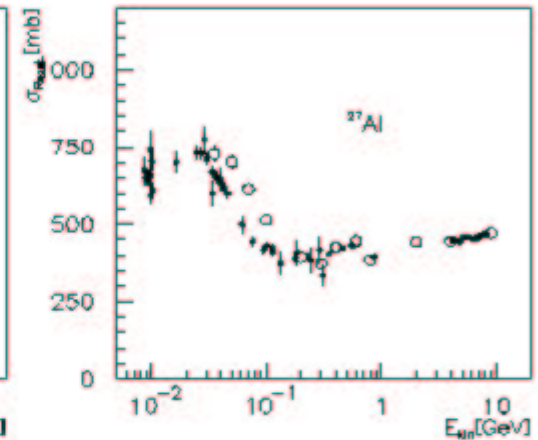
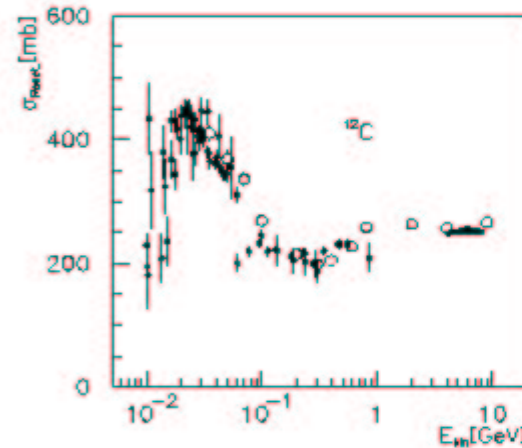


■ and

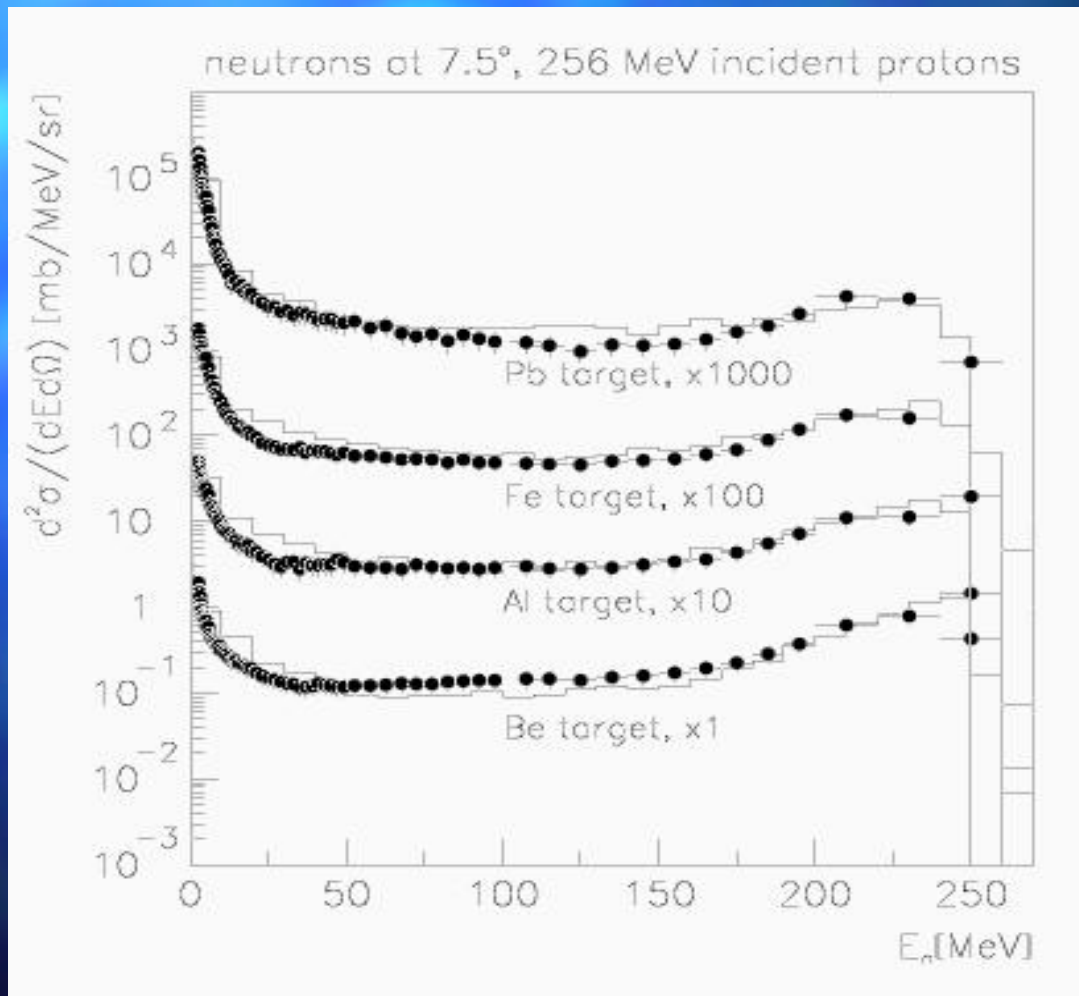
$$\begin{aligned}
 E(s,t) = & \frac{(g_{NN}^\sigma)^4 (t(t-s) + 4m^2(s-t))}{8(t-m_\sigma^2)(u-m_\sigma^2)} + \frac{(g_{NN}^\omega)^4 (s-2m^2)(s-6m^2)}{2(t-m_\omega^2)(u-m_\omega^2)} + \\
 & \frac{6(g_{NN}^\pi)^4 (4m^2 - s - t)m^4 t}{(t-m_\pi^2)(u-m_\pi^2)} + \\
 & \frac{3(g_{NN}^\pi g_{NN}^\sigma)^2 m^2 (4m^2 - s - t)(4m^2 - t)}{(t-m_\sigma^2)(u-m_\pi^2)} + \frac{3(g_{NN}^\pi g_{NN}^\sigma)^2 m^2 t(t+s)}{2(u-m_\sigma^2)(t-m_\pi^2)} + \\
 & \frac{(g_{NN}^\omega g_{NN}^\sigma)^2 (t^2 - 4m^2 s - 10m^2 t + 24m^2)}{4(t-m_\sigma^2)(u-m_\omega^2)} + \frac{(g_{NN}^\omega g_{NN}^\sigma)^2 ((t+s)^2 - 2m^2 s + 2m^2 t)}{4(u-m_\sigma^2)(t-m_\omega^2)} + \\
 & \frac{(g_{NN}^\omega g_{NN}^\pi)^2 m^2 (t+s-4m^2)(t+s-2m^2)}{4(t-m_\omega^2)(u-m_\pi^2)} + \frac{(g_{NN}^\omega g_{NN}^\pi)^2 m^2 (t^2 - 2tm^2)}{4(t-m_\pi^2)(u-m_\omega^2)}
 \end{aligned}$$

# Binary cascade prediction

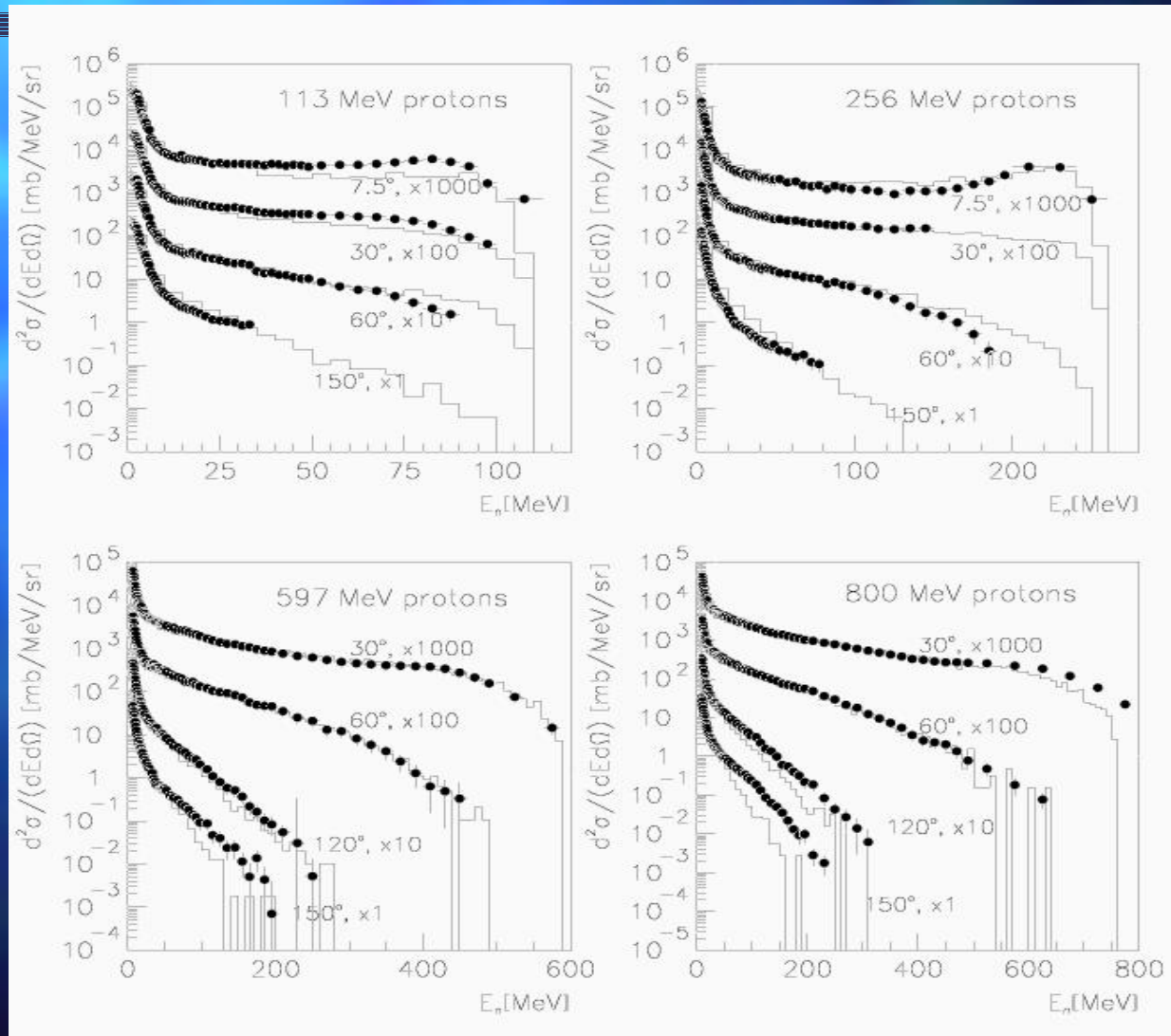
Sample the  
Impact parameter over  
A large area.  
Make the ratio of 'hits'  
To trials, times the area  
sampled



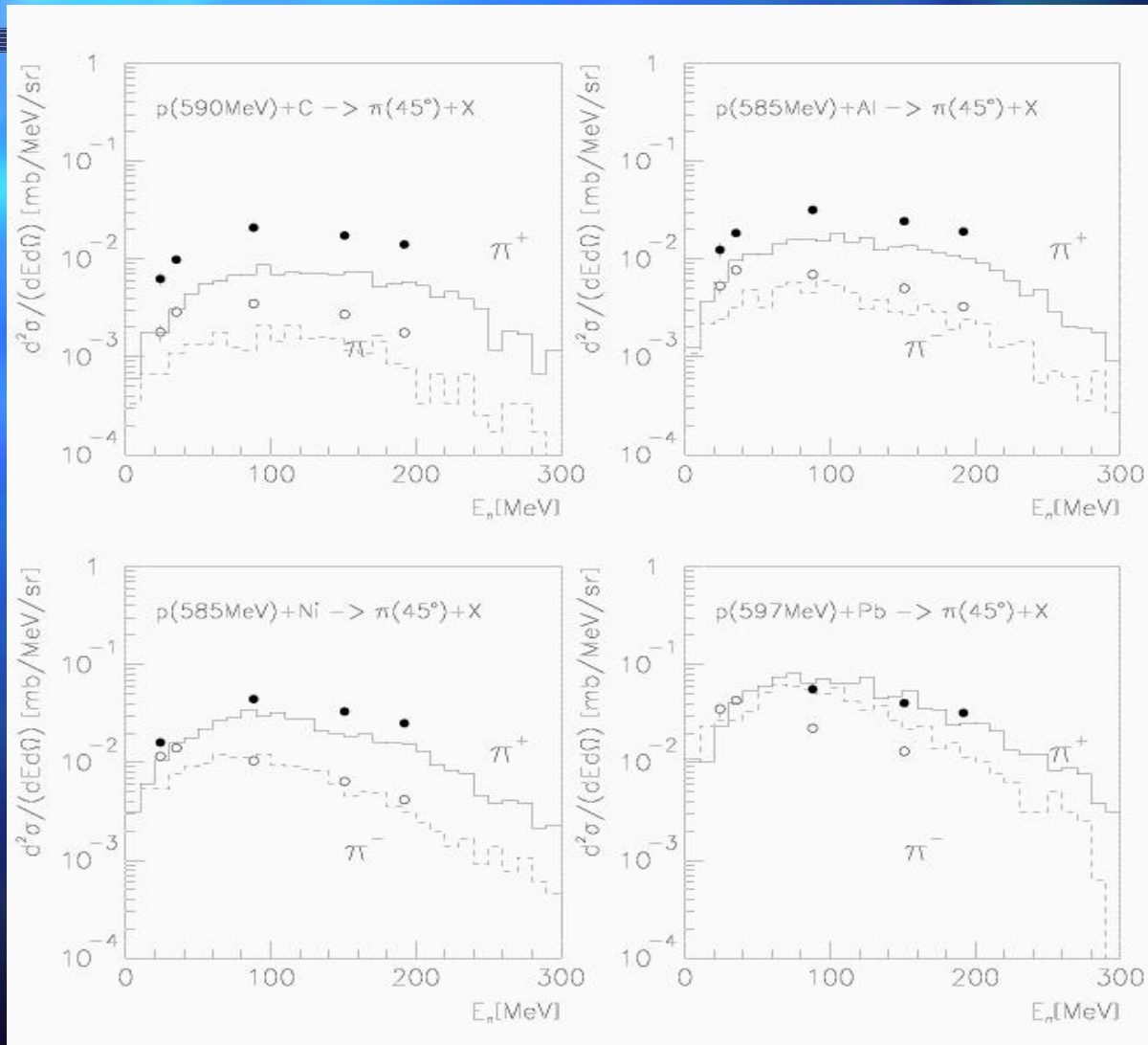
# Forward scattering in proton scattering (256 MeV)



# Scattering off lead at various angles and energies



# Pion production at $\sim 600\text{MeV}$ on various targets





The End?

# *Tomorrow*

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- Pre-equilibrium decay and evaporation models
- Verification of the hadronic models of geant4



The END