Plan of Talks

1. The Standard Model and (a Little) Beyond
2. Neutrinos (Mainly) from Heaven
3. The Numbers and What They Tell Us
4. The Flavor Puzzle(s)
5. Leptogenesis

Plan of Talk IV

The Flavor Puzzle(s)

1. The SM flavor puzzle
2. Approximate Horizontal Symmetries
3. Neutrino flavor puzzles
   - Large mixing and strong hierarchy
   - Anarchy

Quark Hierarchy

The SM Flavor Puzzle

The SM flavor parameters are small and hierarchical

U	$Y_t \sim 1$	$Y_c \sim 10^{-2}$	$Y_u \sim 10^{-5}$
D	$Y_b \sim 10^{-2}$	$Y_s \sim 10^{-3}$	$Y_d \sim 10^{-4}$
E	$Y_\tau \sim 10^{-2}$	$Y_\mu \sim 10^{-3}$	$Y_e \sim 10^{-6}$
CKM	$\lvert V_{us}\rvert \sim 0.2$	$\lvert V_{cb}\rvert \sim 0.04$	$\lvert V_{ub}\rvert \sim 0.004$
CPV	$\delta_{\rm KM} \sim 1$		

Approximate Horizontal Symmetries

- Horizontal symmetries = different generations carry different charges (unlike $G_{\rm SM}$)

- Approximate symmetry = broken explicitly by a small parameter of well defined charge (similar to the isospin symmetry of strong interactions) $\Longrightarrow$ 'Selection rules'

- Approximate horizontal symmetries can naturally explain the hierarchy in the quark and charged lepton Yukawa couplings

- The measured neutrino parameters test such flavor models and may shed new light on the flavor puzzles

Neutrino Flavor Parameters

- With three active neutrinos that have Majorana-type masses, there are nine new flavor parameters: three neutrino masses, three lepton mixing angles and three phases in the mixing matrix.
- Four have been measured.
- Are flavor models consistent with

$$
\begin{gathered}
|U_{\mu 3}U_{\tau 3}| \sim 0.47 - 0.50 \\
|U_{e1}U_{e2}| \sim 0.42 - 0.49 \\
|U_{e3}| \leq 0.23 \\
\frac{\Delta m^2_{21}}{|\Delta m^2_{32}|} \sim 0.02 - 0.04
\end{gathered}
$$

- (Inconsistencies might be in the eye of the beholder.)

Neutrino Hierarchy

Abelian or non-Abelian? (I)

- SU(3) with $Q(3), \bar{U}(\bar{3})$: $\Longrightarrow\ m_u = m_c = m_t$
- SU(3) otherwise: $\Longrightarrow\ m_t \ll \langle\phi\rangle$
- SU(2) with $Q(2+1), \bar{U}(2+1)$: $\Longrightarrow\ m_u = m_c$
- The data: $m_u \ll m_c \ll m_t \sim \langle\phi\rangle$

$\Longrightarrow$ The simplest non-Abelian models are excluded

Horizontal U(1) Symmetry

- $U(1)$ broken by $\lambda(-1)$ with $|\lambda| \sim 0.2$

Froggatt + Nielsen (1979)

- $Q(3,2,0),\ \bar{U}(5,2,0),\ \bar{D}(3,2,2)$

$$\Longrightarrow M_u \sim \langle \phi_u \rangle \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix}, \quad M_d \sim \langle \phi_d \rangle \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}$$

- Reproduces the data nicely
- The simplest models successfully predict for quarks:

$$\sin\theta_{13} \sim \sin\theta_{12}\sin\theta_{23}, \quad m_i/m_j \lesssim \sin\theta_{ij}, \quad V \sim \mathbf{1}$$

Leurer, Nir, Seiberg (1994)

- The simplest models predict for neutrinos:

$$\sin\theta_{13} \sim \sin\theta_{12}\sin\theta_{23}, \quad m_i/m_j \sim \sin^2\theta_{ij}$$

Abelian or non-Abelian? (II)

• The simplest models of Abelian horizontal symmetries predict:
$m_{\nu_i}/m_{\nu_j} \sim \sin^2\theta_{ij}, \quad \sin\theta_{13} \sim \sin\theta_{12}\sin\theta_{23}$

• The data:
$\sin\theta_{23} \sim 1, \quad \sin\theta_{12} \sim 1, \quad \sin\theta_{13} < 0.2, \quad m_2/m_3 \sim 0.2$

$\Longrightarrow$ The simplest Abelian models are excluded

• It is particularly difficult to explain $\sin\theta_{23} \sim 1$ with $m_2/m_3 \ll 1$
(large mixing $\leftrightarrow$ strong hierarchy)

Abelian or non-Abelian? (III)

The data: $\sin\theta_{23} \sim 1, \quad \sin\theta_{12} \sim 1, \quad \sin\theta_{13} < 0.2, \quad m_2/m_3 \sim 0.2$

Some options:

- $L(1,0,0)$ with an $\mathcal{O}(\lambda)$ accidental cancellation:
  $m_2/m_3 \sim 1(\to \lambda)$, $\sin\theta_{12} \sim \lambda(\to 1)$
- $L(0,0,0)$ with an $\mathcal{O}(\lambda)$ accidental cancellation:
  $m_2/m_3 \sim 1(\to \lambda)$, $\sin\theta_{13} \sim 1(\to \lambda)$
- Very specific Abelian models
- A combination of Abelian (charged fermion) and non-Abelian [neutrino (and sfermion)] symmetries

The Data

In the 2-3 generation sector, there is large mixing, but the corresponding masses are hierarchical

- $|U_{\mu 3} U_{\tau 3}| = \mathcal{O}(1)$.
- $\Delta m^2_{21}/\Delta m^2_{32} \ll 1$ suggests $m_2/m_3 \ll 1$.

The Puzzle

$$M_\nu^{2-3} = \frac{\langle\phi\rangle^2}{\Lambda_{\rm NP}}\begin{pmatrix} A & B \\ B & C \end{pmatrix}$$

- $\tan 2\theta_{23} = \frac{2B}{C-A}$
  Large mixing $\Longrightarrow$ $|B| \sim |C-A|$
- $\frac{m_2 m_3}{(m_2+m_3)^2} = \frac{AC-B^2}{(A+C)^2}$
  Strong hierarchy $\Longrightarrow$ $|AC-B^2| \ll |A+C|^2$
- Large mixing + strong hierarchy: $\Longrightarrow$ $AC-B^2 \ll AC, B^2$

Fine Tuning?

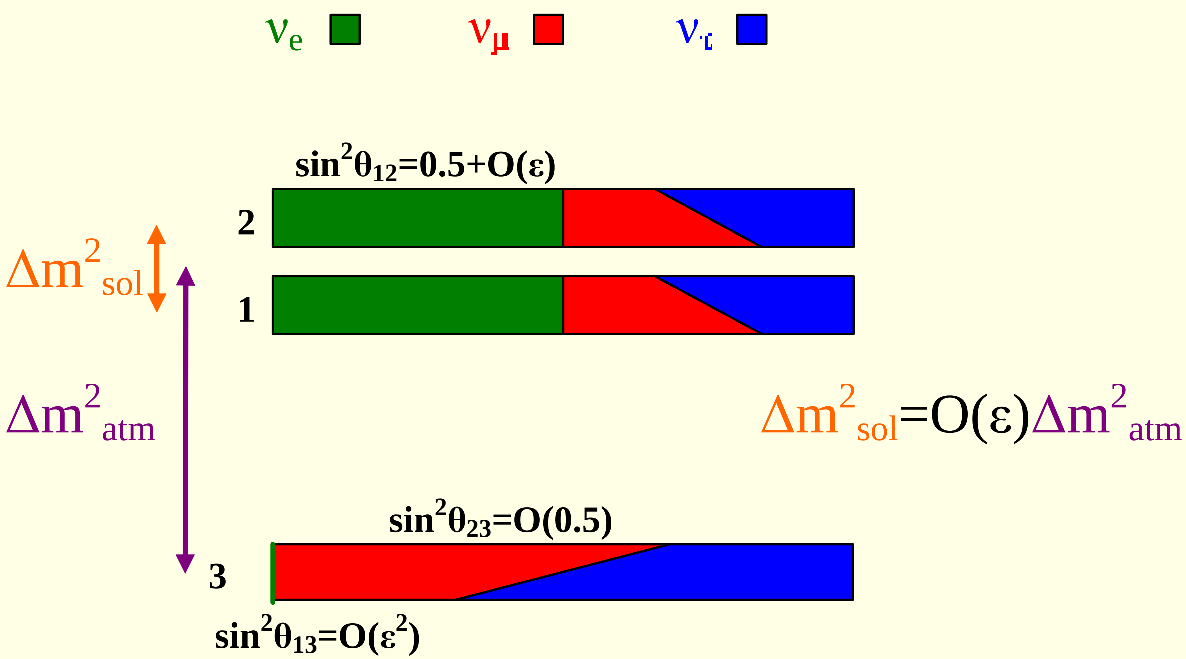
Solutions

1. Accidental hierarchy
2. Several sources for neutrino masses
   (a) A single right-handed neutrino dominance
   (b) Supersymmetric models without R-parity
3. Large mixing from the charged lepton sector
4. Large mixing from the see-saw mechanism
5. (A three generation mechanism ($L_e - L_\mu - L_\tau$ symmetry))

$L_e - L_\mu - L_\tau$ (I)

- An example of an approximate horizontal Abelian symmetry:
- $U(1)_{e-\mu-\tau}$ broken by $\epsilon_+(+2)$ and $\epsilon_-(-2)$, with $|\epsilon_\pm| \ll 1$.
- $M_\nu \sim \frac{\langle\phi\rangle^2}{\Lambda_{\rm NP}} \begin{pmatrix} \epsilon_- & 1 & 1 \\ 1 & \epsilon_+ & \epsilon_+ \\ 1 & \epsilon_+ & \epsilon_+ \end{pmatrix}, \quad M_\ell \sim \langle\phi\rangle \begin{pmatrix} \lambda_e & \lambda_\mu\epsilon_- & \lambda_\tau\epsilon_- \\ \lambda_e\epsilon_+ & \lambda_\mu & \lambda_\tau \\ \lambda_e\epsilon_+ & \lambda_\mu & \lambda_\tau \end{pmatrix}$
- $\lambda_{e,\mu,\tau}$ affect neither mixing angles nor neutrino masses
- Can estimate all ν-parameters in terms of $m \sim \frac{\langle\phi\rangle^2}{\Lambda_{\rm NP}}$, ϵ_+, ϵ_-.

$L_e - L_\mu - L_\tau$ (II)



$$m_{1,2} = m[1 \pm \mathcal{O}(\epsilon_\pm)], \quad m_3 = m\mathcal{O}(\epsilon_+)$$

$$|U_{\mu 3} U_{\tau 3}| = \mathcal{O}(1), \quad |U_{e1} U_{e2}| = 1/2 - \mathcal{O}(\epsilon_\pm^2), \quad |U_{e3}| = \mathcal{O}(\epsilon_\pm)$$

$L_e - L_\mu - L_\tau$ (III)

$$m_{1,2} = m[1 \pm \mathcal{O}(\epsilon_\pm)], \quad m_3 = m\mathcal{O}(\epsilon_+)$$
$$|U_{\mu 3}| = \mathcal{O}(1), \quad |U_{e3}| = \mathcal{O}(\epsilon_\pm), \quad |U_{e1}U_{e2}| = 1/2 - \mathcal{O}(\epsilon_\pm^2)$$

- Large mixing ($|U_{\mu 3}| \sim 1$) with strong hierarchy ($\frac{\Delta m^2_{21}}{\Delta m^2_{23}} \sim \epsilon_\pm$)
- One small + two large angles: $\sin\theta_{13} \ll \sin\theta_{12}\sin\theta_{23}$
- Pseudo-Dirac SN ($\theta_{12} \approx \pi/4, \;\Delta m^2_{21} \ll m^2_{2,1}$)
- Inverted hierarchy ($m_{1,2} \gg m_3$)
- Relations between $\Delta m^2_{\rm SN}/\Delta m^2_{\rm AN}, \;\; |U_{e1}U_{e2}| - \frac{1}{2}, \;\; |U_{e3}|$.
- Solar mixing near-maximal:
  $|U_{e1}U_{e2}| = \frac{1}{2} - \mathcal{O}\left[\left(\frac{\Delta m^2_{\rm SN}}{\Delta m^2_{\rm AN}}\right)^2\right] = 0.500(2)$
  $\Longrightarrow$ EXCLUDED

Neutrino Anarchy

The Data

None of the measured neutrino flavor parameters is $\ll 1$

- $|U_{\mu 3} U_{\tau 3}| = \mathcal{O}(1)$
- $\frac{m_2}{m_3} \geq \sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{32}}} = \mathcal{O}(0.15)$
- $|U_{e1} U_{\mu 1}| = \mathcal{O}(1)$
- $|U_{e3}| \leq 0.23$

Future tests:

- $|U_{e3}| \ll 1$?
- $m_3 \gg \sqrt{\Delta m^2_{\rm AN}}$?
- $|U_{\mu 3} U_{\tau 3}| - 1/2 \ll 1$?

The Idea

The charged fermion flavor parameters have a special structure, that is, they are small and hierarchical

$$\Updownarrow$$

Could it be that the neutrino flavor parameters have no special structure, that is, they are anarchical?

Hall, Murayama, Weiner (2000)

Lessons for Theory

A simple (but, perhaps, disappointing) explanation:

- All doublet-lepton fields L_i carry the same horizontal charge

$\Longrightarrow$ The Yukawa couplings and the singlet-neutrino masses are hierarchical, but $m_\nu^{\rm light} \sim \langle\phi\rangle^2 Y M^{-1} Y^T$ is not.

An intriguing possibility:

- The special, Majorana nature of ν's makes them flavor blind

Goswami, Indumathi, Shadmi, Nir (in progress)

An explicit example:

- Horizontal symmetries broken at the same scale as L

$\Longrightarrow$ Y, M (and, consequently, $m_\nu^{\rm light}$) are all anarchical.

Summary

- Neutrino flavor parameters have features that are very different from the charged fermion parameters.
- Some of the features are surprising:
  - Large mixing $\leftrightarrow$ strong hierarchy in the $2-3$ sector;
  - Near-maximal $2-3$ mixing;
  - Two large (s_{12}, s_{23}) and one small (s_{13}) mixing angles.
- Most of the simplest and most predictive flavor models - excluded.
- Quite likely, the neutrino flavor structure involves the heavy, singlet neutrinos in a significant way (the LEET is not enough)
- Neutrino mass anarchy?
  Interplay between flavor and Majorana/Dirac?

Prospects

There is still a lot to be learnt:

1. $|U_{e3}| \ll 1$?
   Small or tiny?

2. $|U_{\mu 3} U_{\tau 3}| \neq \frac{1}{2}$?
   Large or maximal?

3. $m_i =$?
   Hierarchical or degenerate?

4. $\Delta m^2_{32} > 0$?
   Normal or inverted?

5. $m_{ee} =$?
   Majorana or Dirac?

The NP Flavor Puzzle

- Generically, no GIM mechanism to explain the smallness of FCNC
- In SUSY, many new sources of flavor violation and of CPV

$$K-\bar{K}\ \text{mixing}: \Longrightarrow\ \frac{1\ TeV}{\tilde{m}}\frac{\tilde{m}^2_{d_2}-\tilde{m}^2_{d_1}}{\tilde{m}^2}K^d_{12}\leq 10^{-3}.$$

$\Longrightarrow$ The flavor (and phase) structure of SUSY is highly non-generic

- Universality: degeneracy between sfermion generations
- Alignment: smallness of supersymmetric mixing angles
- Heaviness: first two squark generations heavier than TeV
- (Approximate CP: CPV phases are small)

- Universality $\leftrightarrow$ non-Abelian
- Alignment $\leftrightarrow$ Abelian

L_2, L_3, single N

$$M_\nu = \begin{pmatrix} 0 & 0 & Y_2\langle\phi\rangle \\ 0 & 0 & Y_3\langle\phi\rangle \\ Y_2\langle\phi\rangle & Y_3\langle\phi\rangle & M \end{pmatrix}$$

$\Longrightarrow$ One heavy $[\mathcal{O}(M)]$ and two light $[\mathcal{O}(\langle\phi\rangle^2/M)]$ neutrinos:

$$m_N = M, \quad m_\nu^{\rm light} = \frac{\langle\phi\rangle^2}{M}\begin{pmatrix} Y_2^2 & Y_2Y_3 \\ Y_2Y_3 & Y_3^2 \end{pmatrix}$$

$\Longrightarrow$ Strong hierarchy and (for $Y_2/Y_3 = \mathcal{O}(1)$) large angle:

$$\tan 2\theta_{23} = \frac{2Y_2Y_3}{Y_3^2-Y_2^2} = \mathcal{O}(1), \quad m_2 = 0, \quad m_3 = \frac{(Y_2^2+Y_3^2)\langle\phi\rangle^2}{M}$$

- $\Delta m^2_{32} \neq 0$ and $\Delta m^2_{21} \neq 0 \Longrightarrow$ at least two N's required

$\Longrightarrow$ 'Single right-handed neutrino dominance'