Beyond the Standard Model: Introduction

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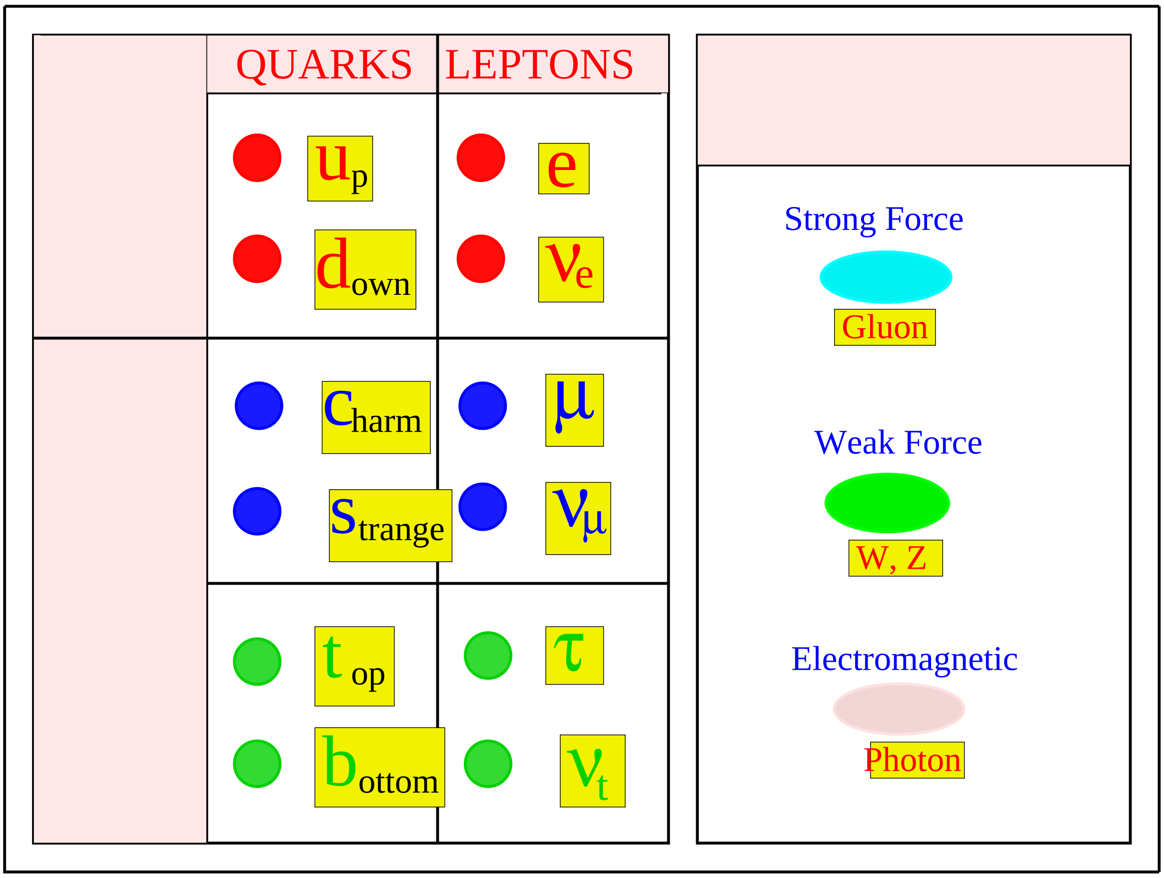
THE XXI CENTURY PERIODIC TABLE



THE STRUCTURE OF THE SM

Gauge bosons

$$SU(3) \times SU(2)_L \times U(1)_Y$$

Fermions

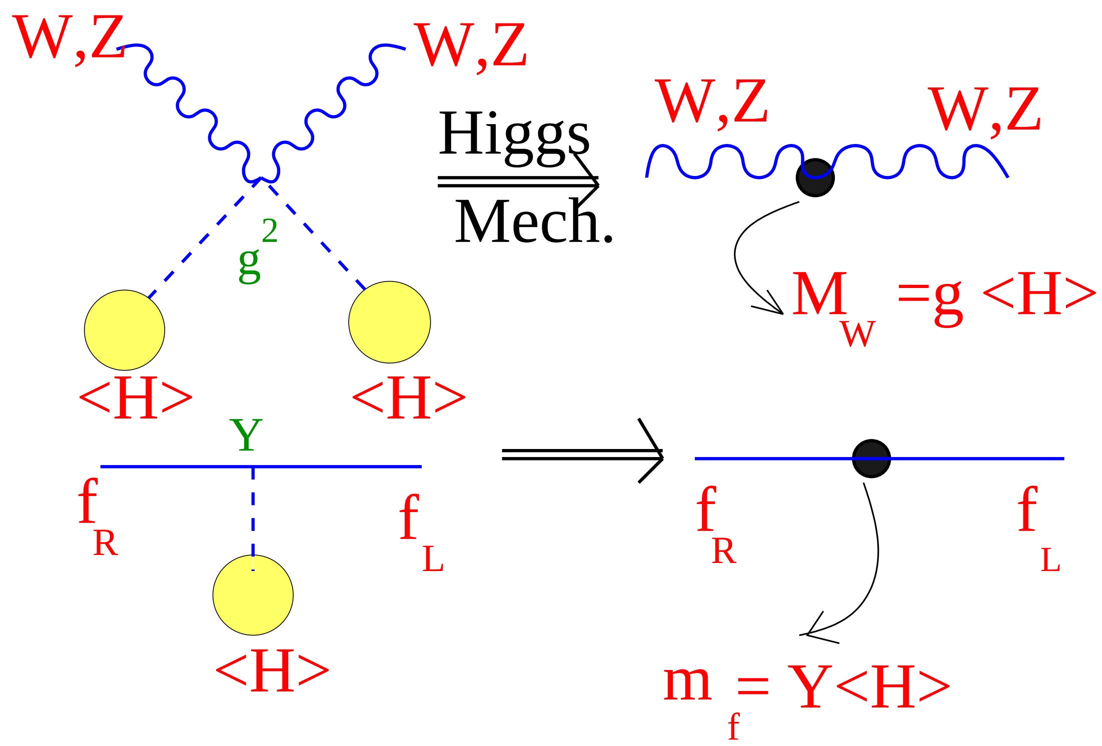
$$\begin{pmatrix} U_L^i \\ D_L^i \end{pmatrix} \; ; \; U_R^i, D_R^i \; ; \; \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix} \; ; \; e_R^i \; ; \; i = 1, 2, 3 \, generations$$

- They come in 3 generations of Weyl bi-spinors. Note there are NO right-handed neutrinos.

Scalars

$$\begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \; ; \; < H^0 > = \; 170 \, GeV$$

- The neutral component of the Higgs $SU(2)_L$ doublet H^0 gets a vev and breaks the symmetry down to $U(1)_{em}$.$W^{\pm}, Z^0$ become massive.
- Quarks and leptons get their masses from Yukawa couplings to the Higgs doublet :


W,Z
W,Z
g^2
<H>
<H>
Higgs
Mech.
W,Z
W,Z
M_W =g <H>
Y
f_R
f_L
<H>
f_R
f_L
m_f = Y<H>

CHIRALITY

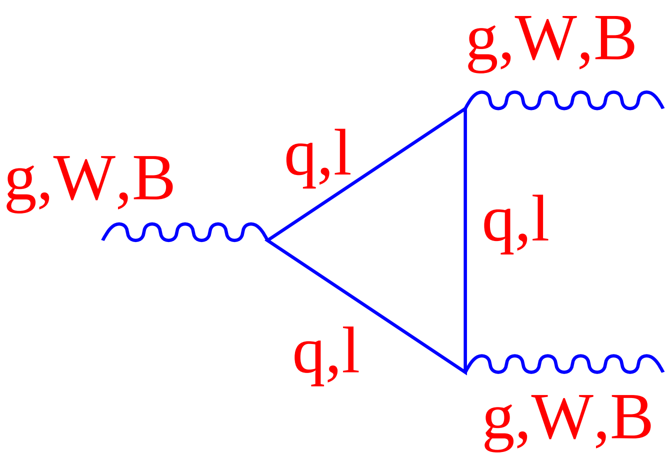
- This is perhaps the most remarkable property of the SM.
- Each generation of quark and leptons have the following quantum numbers under SM gauge group:

Fermion	$SU(3)$	$SU(2)_L$	$U(1)_Y$
$Q_L = (U, D)_L$	3	2	1/6
U_R=U_L^c	$\bar{3}$	1	-2/3
D_R= D_L^c	$\bar{3}$	1	+1/3
$L = (\nu, E)_L$	1	2	-1/2
E_R = E_L^c	1	1	+1

- The remarkable point is that left- and right-handed components of the same Dirac spinor have DIFFERENT quantum numbers under the SM group.
- That property is called CHIRALITY and corresponds to the V-A structure of weak interactions.
- (Mathematically this corresponds to the statement that the quarks and leptons live in a complex representation of the SM group).
- Note the peculiar values of hypercharges. They may be understood from *anomaly cancellation*.

CHIRALITY AND ANOMALIES

- Gauge theories with CHIRAL FERMIONS like the SM are typically sick due to one-loop inconsistencies,the ANOMALIES.
- They come from divergent triangle graphs which give rise to a breakdown of unitarity.....



- ....unless they cancel. This happens for certain choices of quantum numbers for the fermions.
- That is the case of the SM. If the hypercharge assignments are given PRECISELY as in the SM, anomalies cancel.

SOME QUESTIONS BEYOND THE SM

i) Values of couplings, masses and mixings

- Can they be computed in some new underlying theory?

ii) The origin of electroweak symmetry breaking

- Comes from an elementary Higgs? Composite? or?

iii) The fine-tuning problems:

- The cosmological constant puzzle
- The strong-CP problem
- The gauge hierarchy problem

iii) Unification with quantum gravity

- String theory?

i) ABOUT COUPLINGS, MASSES AND MIXINGS

- A first thing to understand would be the different intensity of $SU(3)$, $SU(2)$ and $U(1)$ interactions, i.e. the value of the three gauge coupling constants g_1, g_2, g_3.
- One should add the self-coupling λ of the Higgs field.
- On the other hand one of the most outstanding puzzles of the standard model is the structure of fermion masses and mixing angles.
- The masses of quark and leptons are clearly not random, showing a hierarchical structure.

U-quarks	u	c	t
	0.9-2.9 MeV	530-680 MeV	168-180 GeV
D-quarks	d	s	b
	1.8-5.3 MeV	35-100 MeV	2.8-3.0 GeV
Leptons	e	μ	τ
	0.51 MeV	105.6 MeV	1.777 GeV

Table 1: Masses of quarks and leptons at the M_Z scale (Fritzsch, Xing(2000)).

- Concerning mixing angles, the experimental measurements yield:

$$|V_{CKM}| = \begin{pmatrix} 0.9741 - 0.9756 & 0.219 - 0.226 & 0.0025 - 0.0048 \\ 0.219 - 0.226 & 0.9732 - 0.9748 & 0.038 - 0.044 \\ 0.004 - 0.014 & 0.037 - 0.044 & 0.9990 - 0.9993 \end{pmatrix} . \tag{1}$$

- This is close to a unit matrix with small off-diagonal mixing except for the Cabbibo (12) entry which is somewhat larger.
- Experiments also yield CP-violation which in terms of the Jarlskog invariant J is $J = (3.0 \pm 0.3) \times 10^{-5}$.
- The situation has become even more challenging after the experimental confirmation of neutrino oscillations.
- It has been found for neutrino mass^2 differences from solar ν's, KamLAND ant atmospheric:

$$\Delta m^2_{12} = 7.1 \times 10^{-5}\, eV^2 \,;\, \Delta m^2_{23} = 1.3 - 3.0 \times 10^{-3}\, eV^2$$

- The pattern for neutrino mixing seems to be quite different from the CKM case [a]

$$\begin{pmatrix} 0.79-0.86 & 0.50-0.61 & 0.0-0.16 \\ 0.24-0.52 & 0.44-0.69 & 0.63-0.79 \\ 0.26-0.52 & 0.47-0.71 & 0.60-0.77 \end{pmatrix} . \qquad (2)$$

- ν-mixing angles are typically large.
- All of these couplings, masses and mixings cannot be understood from just the SM. They are input parameters and their explanation should come from physics BSM.
- Vigorous attempts have been made in the last 25 years in order to understand these questions.
- Particularly attractive are Grand Unified Theories (GUT's). In GUT's one can:
  - Relate the three gauge coupling constants to each other
  - Understand the smallness of neutrino masses
  - Relate masses m_b,m_τ, m_t
  - Understand baryon number generation (e.g. Leptogenesis)

[a] e.g. Gonzalez-Garcia, Peña-Garay (2003)

- On the other hand not equally compelling theories exist to explain the full fermion spectra and mixings. Some attempts include:
  - Horizontal $U(1)$ symmetries (Frogatt-Nielsen)
  - GUT's supplemented with discrete symmetries
  - Specific string compactification models
  - Phenomenological models of extra dimensions *(Split Fermions)*
- I will not discuss much these possibilities but rather concentrate first on Grand Unified Theories.

GRAND UNIFIED THEORIES

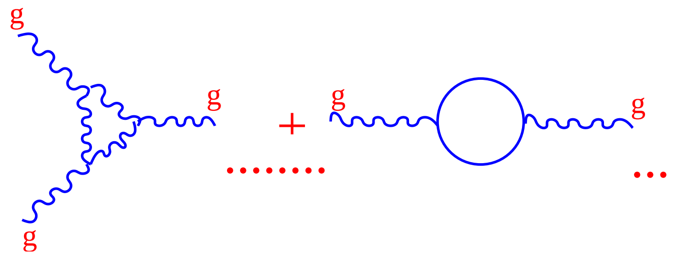
GRAND UNIFIED THEORIES

- In the SM there are FIVE multiplets per generation: $Q_L, U^c_L, D^c_L, L, E^c_L$.
- Gran Unified Theories [a] assume there is an underlying gauge symmetry G larger than $SU(3) \times SU(2) \times U(1) \in$ G.
- Quarks and leptons then unify into a smaller number of multiplets (e.g. two in $SU(5)$, one in $SO(10)$).
- Instead of the three independent gauge couplings $g_{1,2,3}$ of the SM, there is a single one g_{GUT}. This leads to predictions for gauge couplings.
- The unification takes place at a very large scale $\propto 10^{15}$ GeV. Its effects can only be checked indirectly.
- A particularly compelling motivation is the running of the SM gauge couplings

[a] Georgi,Glashow;Pati,Salam (1974)

THE SM GAUGE COUPLINGS RUN

- Due to renormalization effects:



- Renormalization group equations :

$$\mu \frac{\partial}{\partial \mu} \alpha_i(\mu) \;=\; \frac{1}{2\pi} b_i \alpha_i^2(\mu)$$

μ = energy scale ; b_i= β-function coefficients.

- For $SU(N)$ gauge theory one has:

$$b_i \;=\; -\frac{11}{3}N + \frac{1}{3}n_f + \frac{1}{6}n_s$$

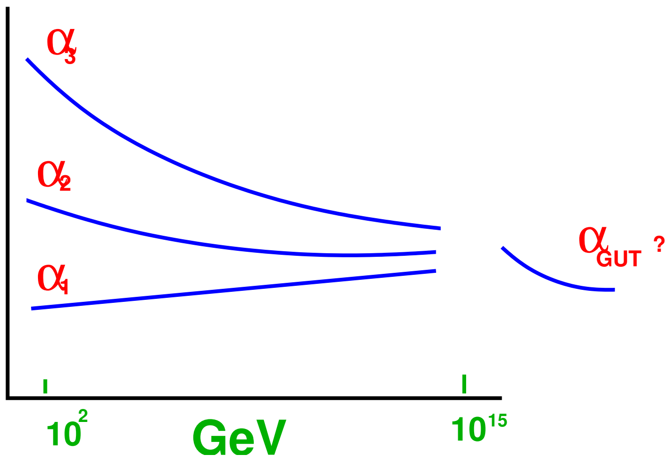
with $n_{f,s}$ = number of fermion(scalar) N-plets.

- For the Standard Model one has:

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + N_{gen} \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{higgs} \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}$$

- Integrating the renormalization group equations [a](one loop):

$$\frac{1}{\alpha_i(q^2)} = \frac{1}{\alpha_i(\mu^2)} + \frac{b_i}{2\pi} log(\frac{\mu^2}{q^2}) \; ; \; i = 1,2,3$$



- Suggests the existence of a unified theory at scales of order 10^{15} GeV.
- Simplest Lie groups containing $SU(3) \times SU(2) \times U(1)$ and having chiral fermions: $SU(5), SO(10)$.

[a]Georgi,Quinn,Weinberg (1974)

GRAND UNIFICATION: SU(5)

- 24 GAUGE BOSONS (Adjoint of $SU(5)$)

$$A^{\mu}_{SU(5)} = \begin{pmatrix} & & & X_1^- & Y_1^- \\ & 8\ gluons & & X_2^- & Y_2^- \\ & & & X_3^- & Y_3^- \\ X_1^+ & X_2^+ & X_3^+ & Z^0 & W^- \\ Y_1^+ & Y_2^+ & Y_3^+ & W^+ & \gamma \end{pmatrix}$$

$X_i^{\pm}$, $Y_i^{\pm}$ very massive, $M_{X,Y} \propto 10^{15}$ GeV.

- FERMIONS: each generation: 10+$\overline{5}$

$$\overline{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ \nu_e \end{pmatrix} \ ; \ 10 = \begin{pmatrix} 0 & u_3^c & u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & d_2 \\ & & 0 & u_3 & d_3 \\ & & & 0 & e^+ \\ & & & & 0 \end{pmatrix}$$

- One generation of quarks and leptons JUST FIT!
- The 10+$\overline{5}$ combination is ANOMALY FREE

$SU(5)$ Symmetry Breaking

- $SU(5)$ symmetry is broken spontaneously at a large scale by the vacuum expectation value of scalars Φ_a in the adjoint of $SU(5)$:

$$<\Phi_a> \; = \begin{pmatrix} 2V & 0 & 0 & 0 & \\ 0 & 2V & 0 & 0 & 0 \\ 0 & 0 & 2V & 0 & 0 \\ 0 & 0 & 0 & -3V & 0 \\ 0 & 0 & 0 & 0 & -3V \end{pmatrix}$$

V $\propto 10^{15}$ GeV $\rightarrow M_{X,Y} \;\propto 10^{15}$

$SU(5) \longrightarrow SU(3) \times SU(2) \times U(1)$

- Further symmetry breaking $SU(2) \times U(1) \rightarrow U(1)_{em}$ by scalars H_5 in a 5-plet:

$$H_5 \; = \begin{pmatrix} H_1^c \\ H_2^c \\ H_3^c \\ H_{WS}^{\pm} \\ H_{WS}^{0} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix}$$

SOME IMPLICATIONS OF GUT's

1) Charge quantization

- Electromagnetic charge Q_{em} is an $SU(5)$ generator: Trace (Q_{em}) = 0 :

$$\begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ \nu_e \end{pmatrix} \rightarrow Q_{em} = \begin{pmatrix} Q(d^c) & 0 & 0 & 0 & \\ 0 & Q(d^c) & 0 & 0 & 0 \\ 0 & 0 & Q(d^c) & 0 & 0 \\ 0 & 0 & 0 & Q(e^-) & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\longrightarrow Q(d) = \frac{1}{3} Q(e^-)$$

- Explains an important fact of the observed world: charges of the proton and electron should be equal and opposite.

2) Prediction of Weak Mixing Angle

- There is only one gauge coupling $g_5 \longrightarrow \alpha_{em}, \alpha_s$, $sin^2\theta_W$ are RELATED

- $$sin^2\theta_W = \frac{Tr(T_3^2)}{Tr(Q_{em}^2)}$$

$$Q_{em}^2 = \begin{pmatrix} 1/9 & 0 & 0 & 0 & \\ 0 & 1/9 & 0 & 0 & 0 \\ 0 & 0 & 1/9 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad T_3^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 1/4 \end{pmatrix}$$

$\longrightarrow$

$$sin^2\theta_W = \frac{1/2}{12/9} = \frac{3}{8}$$

- That is the value at the GUT scale!
- Below the GUT scale the couplings run differently in a way which is computable using the renormalization group equations.

$$\frac{1}{\alpha_i(M_W)} = \frac{1}{\alpha_{GUT}} + \frac{b_i}{2\pi} log(\frac{M_{GUT}}{M_W}) \ ; \ \ i = 1, 2, 3$$

- Combining the three equations one gets:

$$\frac{1}{\alpha_3(M_W)} = \frac{3}{8}(\frac{1}{3\alpha_{em}(M_W)} - \frac{1}{2\pi}(b_1 + b_2 - \frac{8}{3}b_3)\, log(\frac{M_{GUT}}{M_W}))$$

$$sin^2\theta_W(M_W) = \frac{3}{8} + \frac{5\alpha_{em}(M_W)}{16\pi}(b_2 - \frac{3}{5}b_1)\, log(\frac{M_{GUT}}{M_W})$$

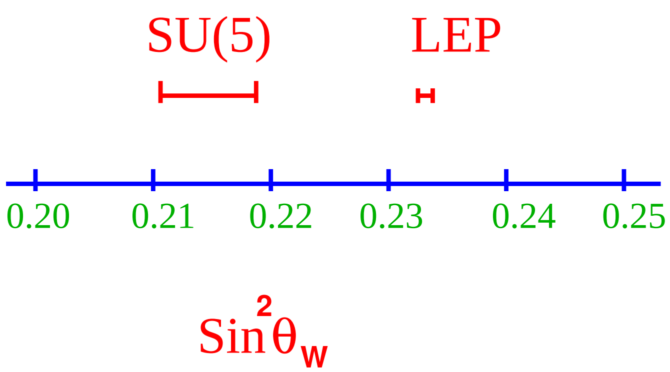
- With e.g. the first of those equations one can compute:

$$M_{GUT} = 10^{14} - 10^{15} GeV$$

- Then one gets a prediction for the weak angle :

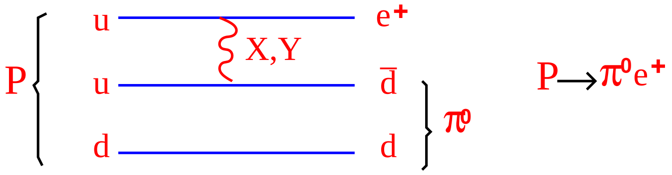
$$sin^2\theta_W(M_W) = 0.214 \pm 0.004(3)$$

- This is ruled out by LEP results:



3) Baryon Number Violation

- Since quarks and leptons live inside same multiplets, X,Y massive gauge bosons can mediate baryon number violating transitions. E.g. $SU(5)$:



- On dimensional grounds, since amplitude goes like $1/M_X^2$, proton time-life goes like:

$$\tau_P \propto \frac{M_X^4}{m_P^5}$$

A more sophisticated theoretical computation gives

$$\tau(P \to e^+\pi^0) = 4 \times 10^{29\pm.7} years$$

Compared to experimental (Super Kamiokande) bounds:

$$\tau(P \to e^+\pi^0) > 5.4 \times 10^{33} years$$

- Thus (minimal) $SU(5)$ is excluded

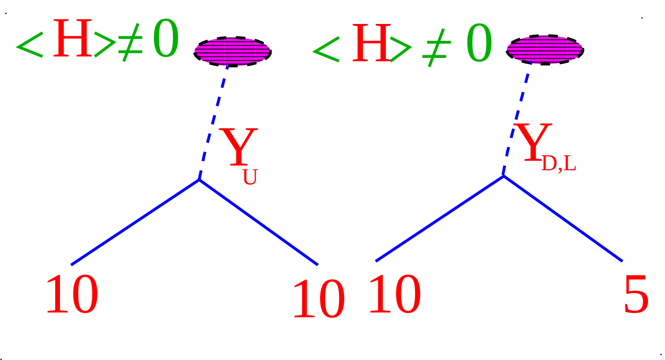
4) Relationships among fermion masses

- Recall that in the SM the fermion masses arise from three independent sets of Yukawa couplings of the fermions to the Higgs doublet H_{WS}:

$$L_{Yuk} = Y_U^{ij}\overline{Q}_L^i U_R^j H^* + Y_D^{ij}\overline{Q}_L^i D_R^j H + Y_L^{ij}\overline{L}^i E_R^j H + h.c.$$

- In e.g. $SU(5)$ the number of independent Yukawa couplings is reduced:

$$L_{Yuk}^{SU(5)} = Y_U^{ij}\overline{\psi}_{10}^i \psi_{10}^j H_{\bar{5}}^* + Y_{D,L}^{ij}\overline{\psi}_{\bar{5}}^i \psi_{10}^j H_{\bar{5}} + h.c.$$

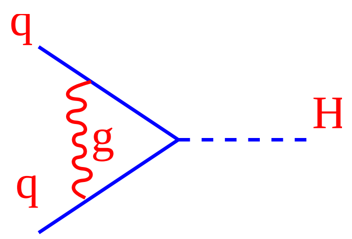


- Thus at the GUT scale one has a relationship between the Yukawas of charged leptons and D-quarks: $Y_D^{ij} = Y_L^{ij}$

- In particular this means [a]

$$Y_b(M_X) = Y_\tau(M_X)\,;\, Y_s(M_X) = Y_\mu(M_X)\,;\, Y_d(M_X) = Y_e(M_X)$$

- However the Yukawa couplings run with the energy. In particular $SU(3)$ loop corrections enhance the quark Yukawas compared to the charged leptons as one goes down in energies.



- The leading correction yields e.g. for the 3-d generation:

$$\frac{m_b(M_W)}{m_\tau(M_W)} = \left(\frac{\alpha_3(M_W)}{\alpha_3(M_X)}\right)^{-\gamma/2b_3} \simeq 3$$

with γ the *anomalous dimension* coefficient.

- This result is quite good for the 3-d generation. However, the identity fails for the first and second generations

- More complicated GUT models involving several other Higgs multiplets beyond $H_{\bar{5}}$ are able to fit the data.

[a] Buras et al. (1978)

GRAND UNIFICATION: SO(10)

- $SO(10) \supset SU(5) \times U(1)$
- GAUGE BOSONS: In adjoint representation, which has dimension 45 and transforms under $SU(5)$ like

$$45 \ = \ 24 + 10 + \overline{10} + 1$$

- FERMIONS: all quarks and leptons fit into a single spinorial multiplet of dimension 16

$$16 \ = \ \left(\nu_L \ u_L^1 \ u_L^2 \ u_L^3 \ ; \ e_L \ d_L^1 \ d_L^2 \ d_L^3 \ ; \ d_R^3 \ d_R^2 \ d_R^1 \ e_R \ ; \ u_R^3 \ u_R^2 \ u_R^1 \nu_R\right)$$

$$\Longleftrightarrow$$

LEFT-RIGHT SYMMETRY

- The theory predicts the existence of a right-handed neutrino ν_R.

- GAUGE SYMMETRY BREAKING DOWN TO SM is more complicated and requires not only adjoint scalars Φ_{45} but also χ_{16} or Σ_{126}. Breaking may occur in steps

$$SO(10) \longrightarrow SU(3) \times SU(2) \times U(1)$$
$$SO(10) \longrightarrow SU(5) \times U(1) \rightarrow SU(3) \times SU(2) \times U(1)$$
$$SO(10) \rightarrow SU(4) \times SU(2)_L \times SU(2)_R \rightarrow SU(3) \times SU(2) \times U(1)$$

- Electroweak symmetry breaking proceeds through scalars in the $SO(10)$ fundamental , H_{10}.
- If $SO(10)$ is broken directly to the SM the predictions are similar to those of $SU(5)$.
- In the simplest $SO(10)$ model Yukawa couplings are more constrained since there is a single Yukawa coupling

$$L^{SO(10)}_{Yuk} = Y^{ij} \overline{\psi}^i_{16} \psi^j_{16} H_{10} + h.c.$$

- This leads to a unification (at the GUT scale):

$$Y^{ij}_U = Y^{ij}_D = Y^{ij}_L = Y^{ij}_N$$

(However a single Higgs field H_{10} is not enough to get realistic spectrum since there would be no mixing).

NEUTRINO MASSES AND GUTS

- Neutral fermions like neutrinos can have three types of mass terms:

Type	Term	ΔL	ΔY
LH-Majorana	$M_L \nu_L \nu_L$	2	1
Dirac	$M_D \bar{\nu}_L \nu_R$	0	1/2
RH-Majorana	$M_R \nu_R \nu_R$	2	0

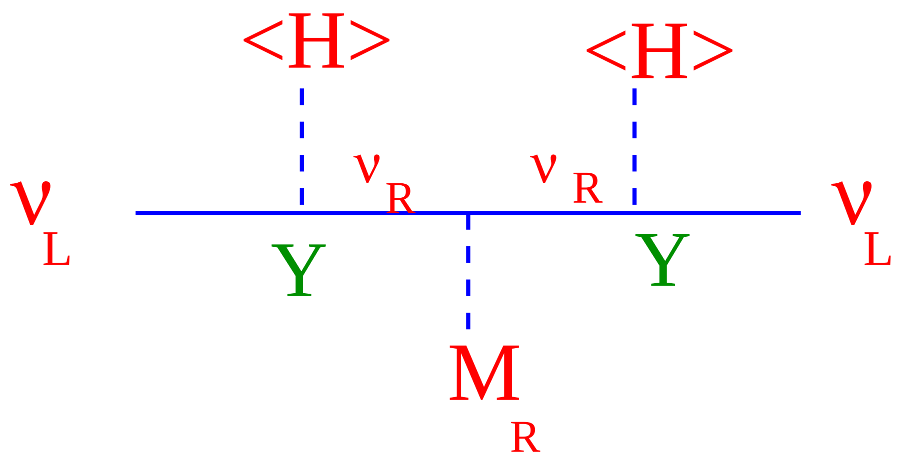
- We saw that in the SM there are only left-handed neutrinos ν_L. This means, that in the SM there are no possible Dirac masses.
- On the other hand if there is a new mass scale M_X where lepton number is broken the effective dim=5 operator may appear in the theory:

$$\frac{h_{ij}}{M_X} \nu_L^i \nu_L^j H H \ + \ h.c.$$

- Once the Higgs field gets a vev $< H >= M_W/g$, LH-Majorana neutrino masses M_L appear of order:

$$M_L^{ij} = \frac{h_{ij} < H >^2}{M_X}$$

- If $h_{ij} \simeq 1$ and $M_X \simeq 10^{14} - 10^{15}$ GeV, neutrino masses $\simeq 0.1 - 0.1$ eV's are obtained.
- GUT's provide for a natural explanation for such a large lepton number violating scale M_X [a].
- Indeed, in GUT's like $SO(10)$:
  - Lepton number symmetry is broken
  - There are right-handed neutrinos ν_R with masses of order M_X
  - The dim=5 operator mentioned above naturally appears:



[a] Yanagida;Gelman,Ramond,Slansky (1979)

- Neutrinos may have normal Dirac mass terms M_D as any other fermion. However, since the right-handed neutrino has very large mass of order M_X, the induced Majorana mass for the left-handed ν is of order $M_L \simeq M_D^2/M_X$. This is the SEE-SAW mechanism.
- Thus one can claim that GUT's naturally predict the smallness of neutrino masses
- Many $SO(10)$ models supplemented by some extra symmetries have been proposed in order to describe the observed neutrino oscillation patterns.
- In my opinion, none of them are extremely compelling. They often include Higgs representations with 126 components and or poorly motivated non-renormalizable couplings.
- One interesting point of the see-saw models of neutrino masses is that they can give rise to the observed baryon asymmetry of the universe.
- Indeed, out-of-equilibrium decay of right-handed neutrinos can give rise to a lepton asymmetry in the universe. The latter will then be transformed by non-perturbative $SU(2)_L$ effects into a baryon asymmetry [b].
- This mechanism for generating the observed baryon asymmetry in the universe is called LEPTOGENESIS.

[b] Fukugita, Yanagida (1986)

SUMMARY OF (non-SUSY) GUT's

- Charge quantization and the unification of quarks and leptons in simpler multiplets are very attractive.
- Also attractive is the understanding of the smallness of neutrino masses.
- However simpler non-SUSY GUT's are ruled out both by proton stability and coupling unification.
- We will see that going to SUSY-GUT's will avoid those two problems. In addition it will help in solving the notorious hierarchy problem that we will discuss soon.
- Nevertheless we will eventually see that some problems of GUT's will still remain, even in the SUSY case.
- Note also that GUT's give no clue about questions like existence of three generations or the full structure of fermion masses and mixings.
- A lot of work has been done in trying to write down models of fermion masses and mixings:

THE PUZZLE OF FERMION MASSES AND MIXINGS

- We described before the peculiar hierarchical structure of fermion masses and mixings in the SM
- We would like to have a theory of flavor explaining:
  - Why there are 3 generations
  - Why the hierarchical structure of fermion masses
  - Why CKM matrix is close to unity
  - Understand origin and size of CP violation
- At the moment we do not have a totally compelling theory of fermion masses
- There are plenty of models which combine a GUT symmetry with some flavor symmetry in generation space.
- It is impossible to review all approaches attempted.
- Let me just qualitatively describe a general approach based on flavor $U(1)$ symmetries : the Froggatt-Nielsen scenario [a].

[a] Froggatt,Nielsen (1979);Leurer,Nir, Seiberg (1993)

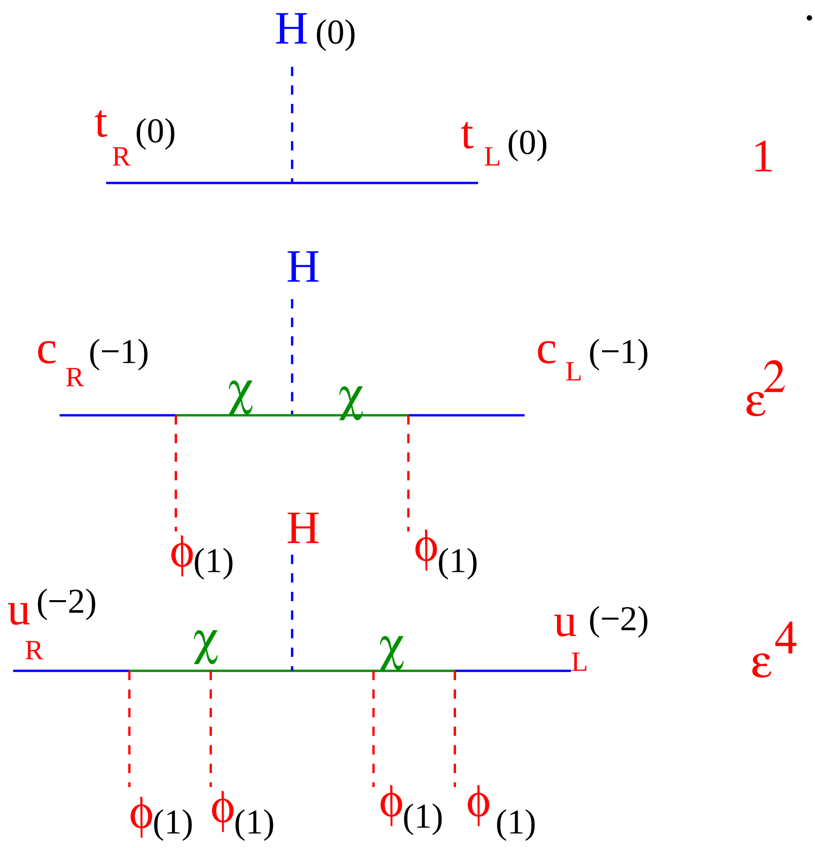
THE FROGGATT-NIELSEN SCENARIO

- The idea goes as follows: extend the SM (or a GUT) with:
  - A new gauged $U(1)$ symmetry .
  - One (or more) scalar singlet fields ϕ_{FN} charged under the $U(1)$. A vev $<\phi_{FN}>\neq 0$ breaks the $U(1)$ symmetry slightly below the GUT scale.
  - Extra fermions χ_i with masses of order the GUT scale .
- The idea is to assume that the $U(1)$ charges of quarks, leptons and Higgs are such that all standard renormalizable Yukawa couplings are forbidden except for those of the heaviest quarks and leptons.
- The lightest generations get suppressed effective Yukawa couplings from non-renormalizable couplings involving the scalar singlet field ϕ_{FN}:

$$\frac{h}{(M_X)^{n_{ij}}}\overline{\psi}_i\psi_j H_{WS}\phi_{FN}^{n_{ij}} \;\; ; \;\; i,j=1,2,3$$

- Once ϕ_{FN} gets a vev $<\phi_{FN}>=\epsilon\, M_X$, $\epsilon\simeq 0.1$, one gets effective Yukawa couplings Y_{ij}:

$$Y_{ij}\;\simeq\; h\;\epsilon^{n_{ij}}$$



- A number of models of this class have been constructed which are able to reproduce most qualitative patterns of observed masses and mixings by appropriately choosing $U(1)$ charges of SM particles.
- The $U(1)$'s are typically anomalous. Those anomalies may be canceled in the string context under certain conditions , giving rise to gauge coupling unification without a GUT symmetry.[a].

[a] Ibanez,Ross; Binetruy,Ramond (1994)

THE FINE-TUNING PUZZLES

1) The Cosmological Constant Problem

- We know that many physical processes exist contributing the the vacuum energy.
- Consider for example the tree level Higgs scalar potential in the SM:

$$V(H) = -\mu^2|H|^2 + \lambda|H|^4$$

- The value of the potential at the minimum is given by

$$V_{min} = -\frac{|\mu|^4}{4\lambda} = \neq 0$$

- On the other hand Einstein's gravity equations state:

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = -8\pi G_N < T^{\mu\nu} > = -8\pi G_N V_{min} g^{\mu\nu}$$

- where $R^{\mu\nu}$ is the Ricci tensor, $R = R^\mu_\mu$ and $T^{\mu\nu}$ is the energy-momentum tensor.
- V_{min} act as a cosmological constant: vacuum energy has a dynamical effect on space-time curvature [a].

[a] Linde (1974);Veltman (1975)

- Cosmological data from supernovae and WMAP seem to indicate that there is a positive very small cosmological constant Λ_{cc} of order:

$$\Lambda_{cc} \simeq \left(10^{-3}\, eV\right)^4$$

- This is many, many orders of magnitude smaller than the expected contribution from EW physics, of order M_W^4.
- Loop contributions to the vacuum energy diverge quartically sofine-tuning of $V_{min} = 0$ would be a extremely unnatural possibility.
- furthermore there will be anyhow very large contributions to Λ_{cc} from other processes like e.g., QCD condensation.
- The cosmological constant problem is still to be solved. Some directions considered are:
  - Modifications of Einstein's gravity at long distances
  - Quintessence scenarios
  - Use the anthropic principle.

2) The Strong-CP Problem

- QCD has (due to instantons) a CP-violating piece in its Lagrangian:

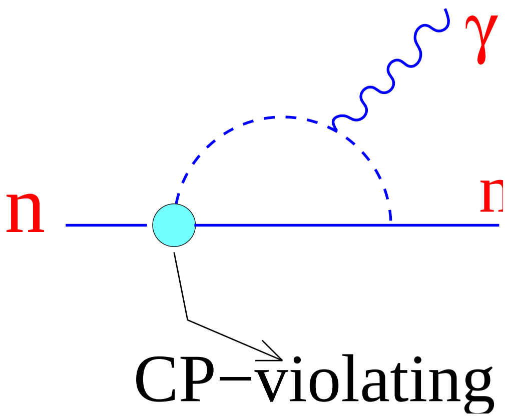
$$L_\theta = \frac{\theta}{32\pi^2}\tilde{F}_{\mu\nu}F^{\mu\nu}$$

- (This is proportional to the chromomagnetic product $\vec{E}.\vec{B}$, which is explicitely CP-violating)

- Naively one would expect $\theta \simeq g_3 \simeq 1$. However such term induces very large contributions to the electric dipole moment of the neutron:

$$d_n\,(\bar{n}\gamma_5\sigma_{\mu\nu}F^{\mu\nu}n)$$

- From graphs like:



- Experimentally one has [a]

$$|d_n| \; < \; 6.3 \times 10^{-26} \, e - cm.(90\% c.l.)$$

- This implies a bound on the θ-parameter:

$$\theta \; \leq \; 2 \times 10^{-10}$$

- This is at least 10 orders of magnitude smaller than expected!.
- Unlike the case of the cosmological constant, in this case there are a few promising proposals to solve it.

The axion solution

- The idea is to introduce a new very light pseudoscalar field $a(x)$, the AXION with a coupling identical to that of the θ-parameter:

$$L_a \; = \; \frac{a(x)}{f_a 32\pi^2} \tilde{F}_{\mu\nu} F^{\mu\nu}$$

- Such type of field a(x) and couplings appear if the axion is the goldstone boson of a spontaneously broken global $U(1)_{PQ}$ symmetry [b].
- The axionic coupling appears if there is a $U(1)_{PQ} - SU(3)_c - SU(3)_c$ anomaly.

[a] Smith et al. (1990);Altarev et al.(1992)

[b] Peccei,Quinn (1977);Weinberg;Wilczek (1978).

- The SM alone does not allow for such a symmetry. One has to extend the Higgs sector by adding e.g. a scalar singlet to the SM.
- The nice point is that QCD instanton effects create a scalar potential of the form:

$$V(a) \simeq (1 - cos(\theta + a/f_a))$$

- This potential has its minimum at $\theta + a/f_a = 0$, which corresponds to vanishing effective $\bar{\theta} = \theta + a/f_a$.
- So in the axion scheme the dynamics choses the effective θ-parameter to vanish.
- The axion has mass $m_a \simeq m_\pi f_\pi / f_a$, where f_a is the scale of $U(1)_{PQ}$ breaking and $f_\pi = 93$ MeV
- There are strong astrophysical and cosmological bounds on f_a:

$$4 \times 10^9 GeV(astro.) \leq f_a \leq 10^{12} GeV(cosmol.)$$

- Thus the axion should be extremely light and very weakly coupled, couplings $\simeq m_q/f_a$. The axion is said to be 'invisible' [c].
- The axion is a candidate for dark matter.

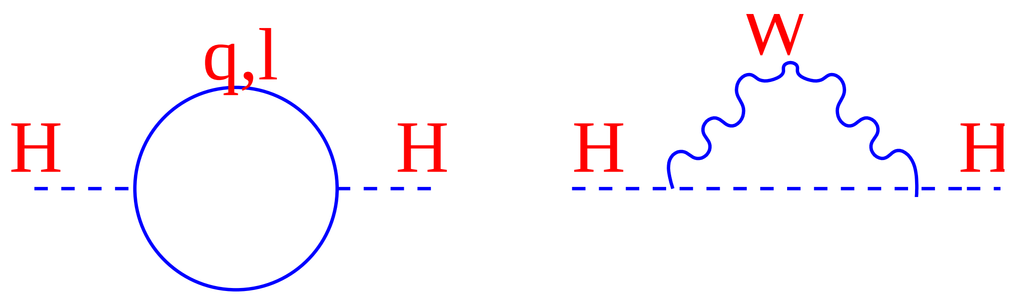
[c] Kim (1979);Shifman et al.;Dine et al. (1980)

3) The Hierarchy Problem

- A first way to present the problem is to understand why:

$$G_{Newton} \ll G_{Fermi} \ or \ M_{Planck} \gg M_W$$

- In the SM the scale of weak interactions is fixed by the mass of the Higgs field μ^2.
- Unlike fermion masses, scalar masses μ^2 recieve huge loop corrections from graphs like



- These corrections to the mass are quadratically divergent:

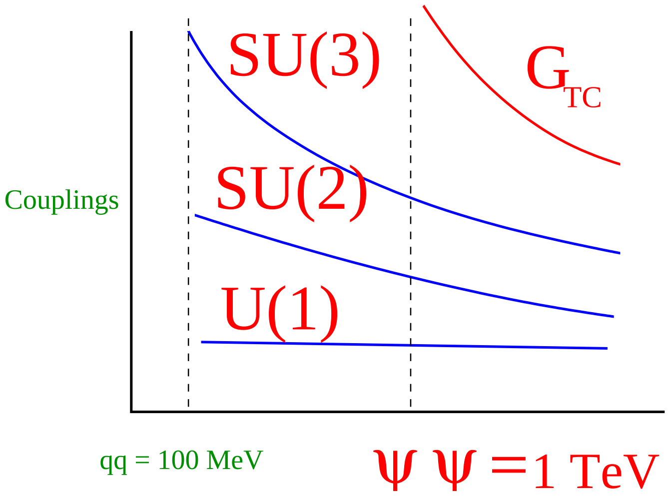
$$\mu^2_{Higgs} \simeq \frac{\alpha}{4\pi} \Lambda^2_{cut-off}$$

- Of course, one can always renormalize the value of μ^2 so that it is small of order M_W^2. It sounds rather artificial though.

- Morover if a physical cut-off like M_p or M_{string} exists, the loop corrections will be finite and huge.
- So the question of the hierarchy problem is how to mantain the Higgs field sufficiently light to trigger EW symmetry breaking, given that its natural value would be M_{GUT} or M_p or M_{string}.
- There are a few PROPOSSED SOLUTIONS
  - Technicolor or other strongly interacting solutions
  - Low energy Supersymmetry
  - Extra dimenssions at a TeV with:
  - String scale at a TeV
  - *Little Higgs*
- Unfortunatelly I will not have time to talk about the last one.

TECHNICOLOR

- In this scheme $SU(2)_L \times U(1)$ breaking is a rescaled version up to 1 TeV of QCD chiral condensate symmetry breaking[a]
- One assumes there is a new QCD-like extra gauge interaction which becomes strong at $\Lambda_{TC} \simeq 1$ TeV:

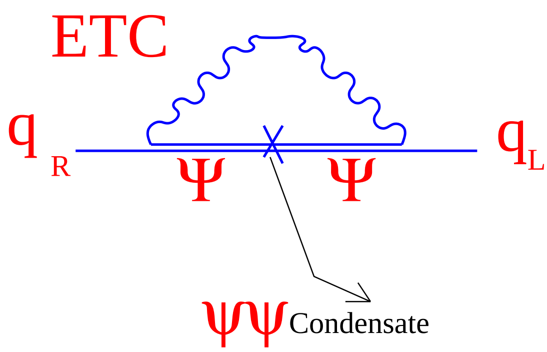


- Instead of an elemantary scalar Higgs, the gauge symmetry is broken by a condensate of 'techniquarks' Ψ:

$$<\bar{\Psi}_R \Psi_L> \neq 0$$

[a]Weinberg;Susskind (1975)

- This symmetry breaking gives rise to Goldstone bosons which are swallowed by the $W^{\pm}, Z^0$ to become massive.
- Since there are no elementary Higgs scalars there is no hierarchy problem. Furthermore, one can understand the hierarchy $M_p \gg M_W$ because of the running of technicolor interactions.
- Techniquarks are strongly bound by the technicolor interactions G_{TC} and form *technihadrons*. These should be produced copiously at colliders.
- The problem, as usual in BSM physics, appears in trying to give also masses to quarks and leptons.
- One has to further extend the theory with new gauge interactions connecting usual quark/leptons to techniquark/leptons,EXTENDED TECHNICOLOR (ETC):



- This gives fermion masses of order [a]:

$$m_q \simeq \frac{g^2_{ETC}}{(4\pi)^2} \frac{\Lambda^3_{TC}}{M^2_{ETC}}$$

- The problem is that in order to get quarks heavy enough the ETC scale cannot be too large.
- Since in order to get CKM mixing the ETC gauge bosons should change flavour, large FCNC above experimental bounds are typically obtained.
- In addition precision EW measurements seem to rule out too large symmetry breaking sectors, as is the case of TC and ETC .
- This makes the construction of viable technicolor models very difficult.
- Let us discuss now the most popular perturbative solution to the hierarchy problem: LOW ENERGY SUPERSYMMETRY.

[a] Dimopoulos,Susskind (1979)