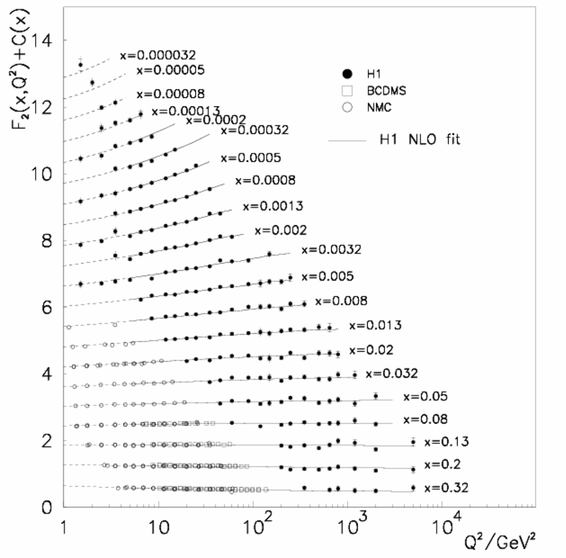
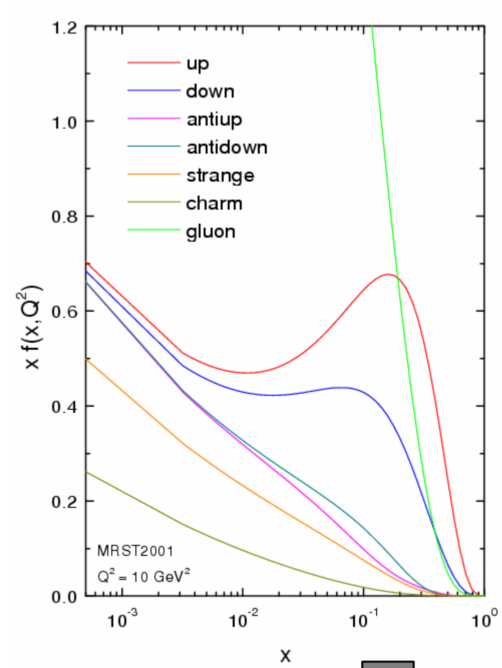
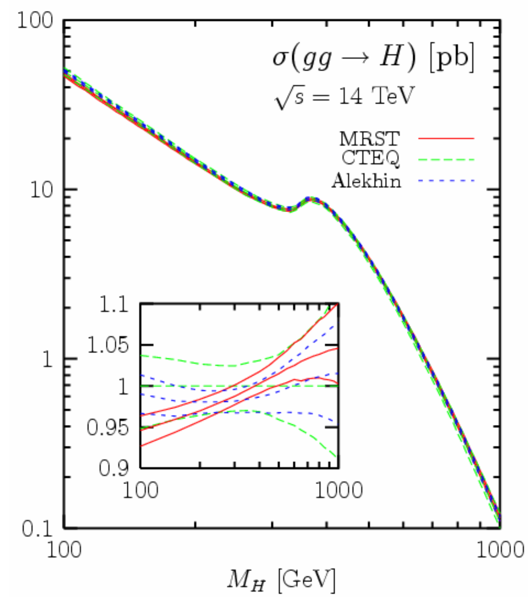
# Parton Distribution Functions: from HERA to LHC





James Stirling
(IPPP Durham)

CERN, 26 March 2004

# pdfs at LHC

- high precision (SM and BSM) cross section predictions require precision pdfs: $\delta\sigma_{th} = \delta\sigma_{pdf} + \ldots$
- 'standard candle' processes (e.g. $\sigma_Z$) to
  - check formalism
  - measure machine luminosity?
- learning more about pdfs from LHC measurements (e.g. high-$E_T$ jets $\rightarrow$ gluon, $W^+/W^-$ $\rightarrow$ sea quarks)

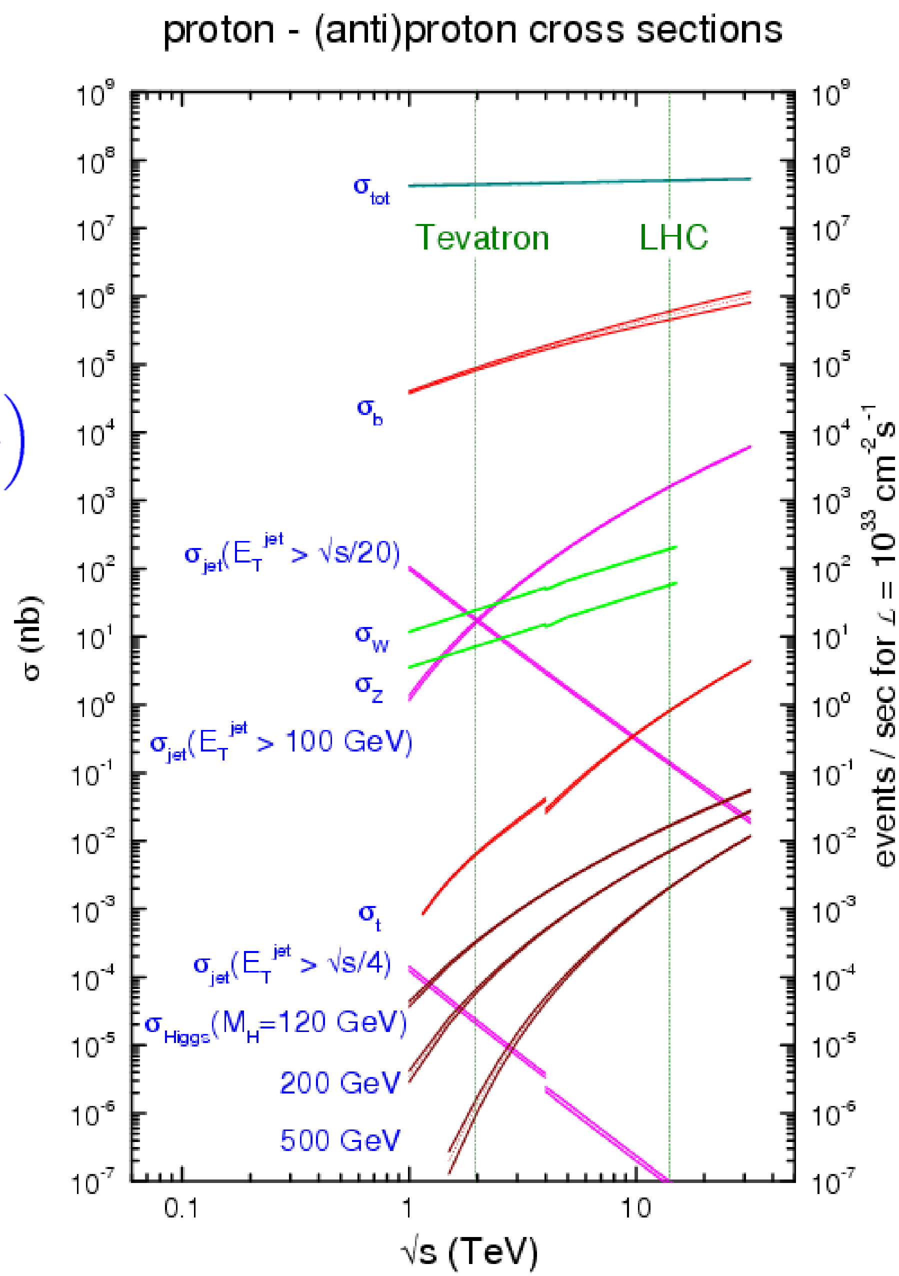
QCD factorization theorem for short-distance inclusive processes

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2)\, f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$

where X=W, Z, H, high-$E_T$ jets, … and $\hat{\sigma}$ known

- to some fixed order in pQCD and EW
- in some leading logarithm approximation (LL, NLL, …) to all orders via resummation

| | |
|---|---|
| now: | NLO pQCD |
| ‘soon’: | NNLO |

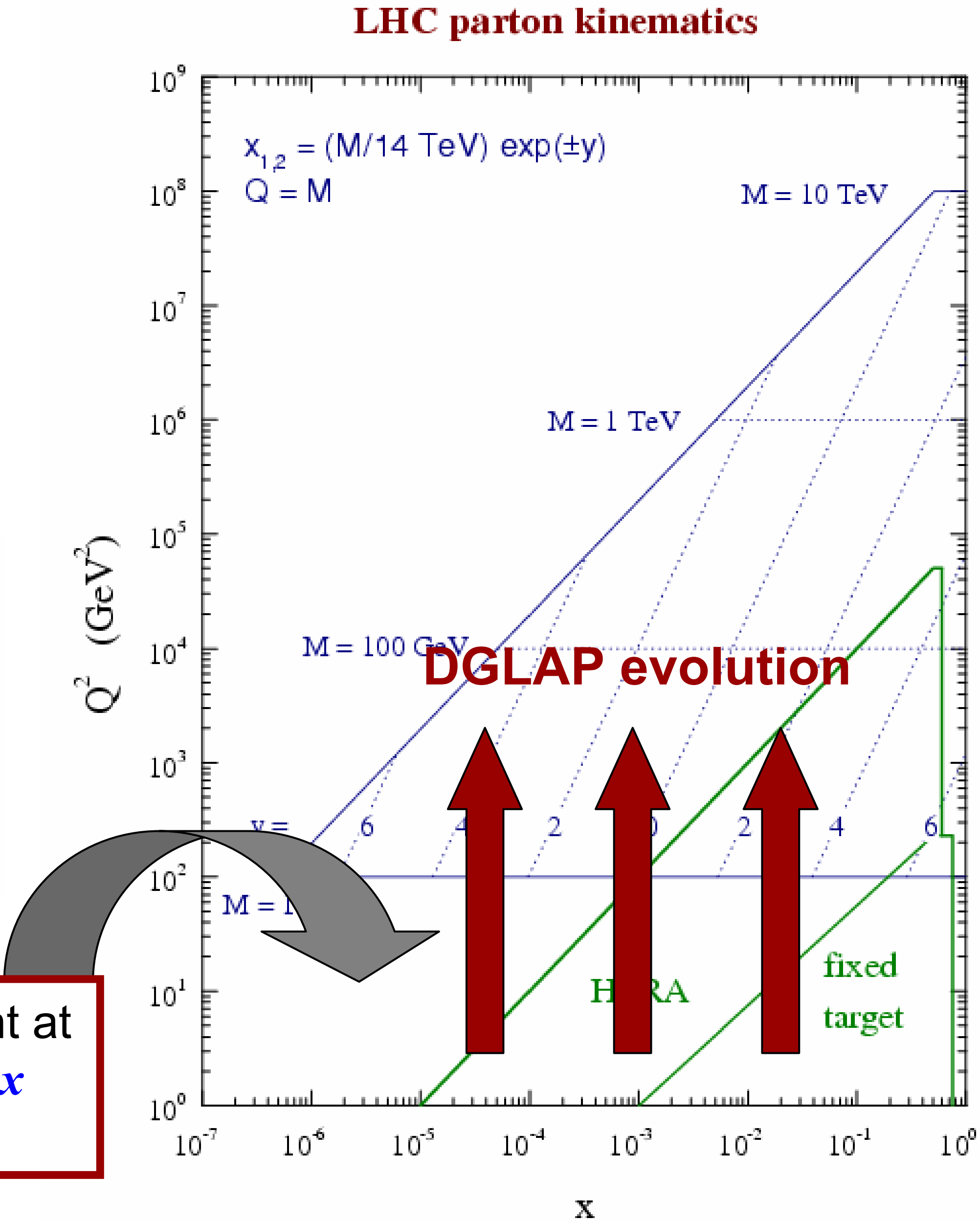


momentum fractions $x_1$ and $x_2$ determined by mass and rapidity of **X**

$x$ dependence of $f(x,Q^2)$ determined by fit to data, $Q^2$ dependence determined by **DGLAP** equations:

$$\frac{\partial q_i(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\Big\{P_{q_iq_j}(y,\alpha_S)\, q_j(\frac{x}{y},Q^2) + P_{q_ig}(y,\alpha_S)\, g(\frac{x}{y},Q^2)\Big\}$$

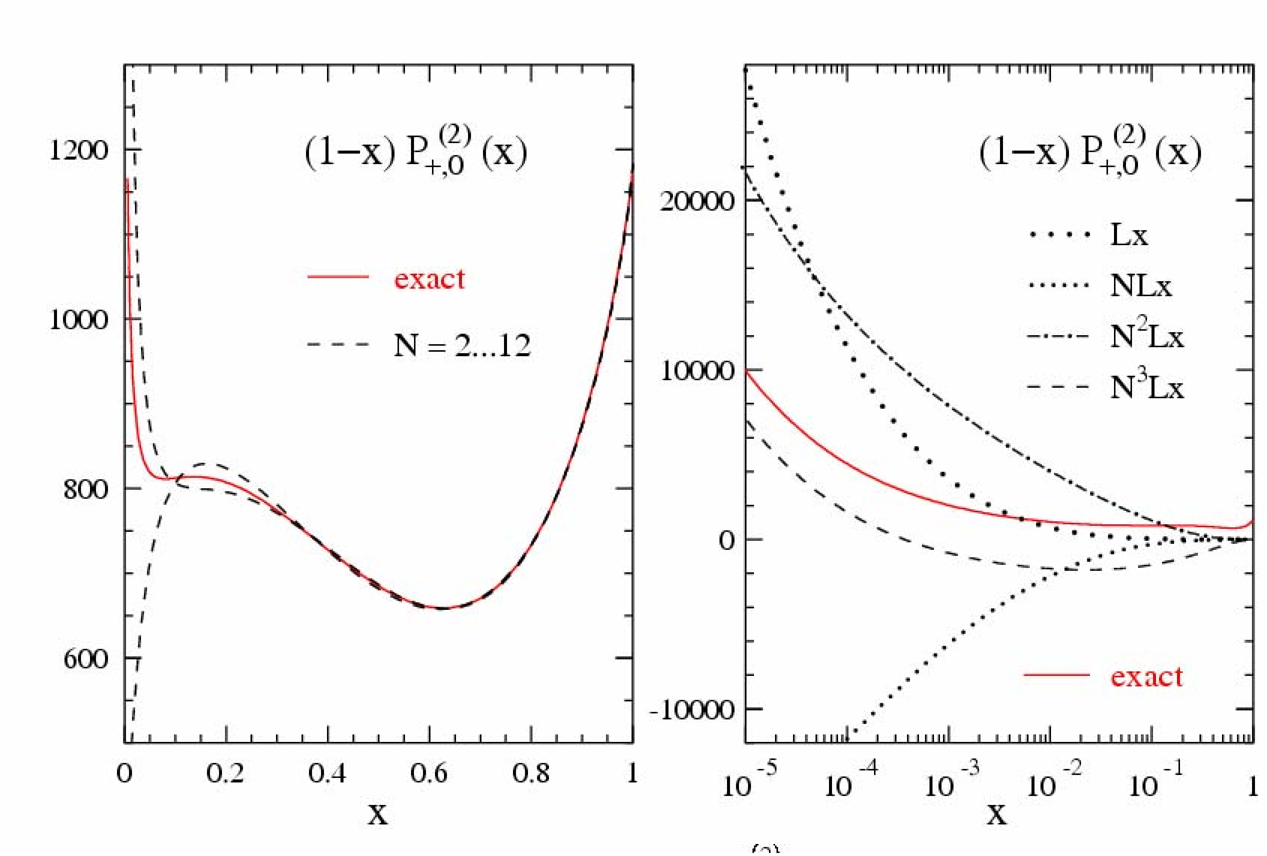
$$\frac{\partial g(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\Big\{P_{gq_j}(y,\alpha_S)\, q_j(\frac{x}{y},Q^2) + P_{gg}(y,\alpha_S)\, g(\frac{x}{y},Q^2)\Big\}$$




**Q.** is NLO (or NNLO) DGLAP sufficient at small x? Are higher-orders $\sim \alpha_S^n \log^m x$ important?

# Full 3-loop (NNLO) non-singlet DGLAP splitting function!

new




Moch, Vermaseren and Vogt, hep-ph/0403192

# pdfs from global fits

**Formalism**
NLO DGLAP
MSbar factorisation
$Q_0^2$
functional form @ $Q_0^2$
sea quark (a)symmetry
etc.

$f_i(x,Q^2) \pm \delta f_i(x,Q^2)$

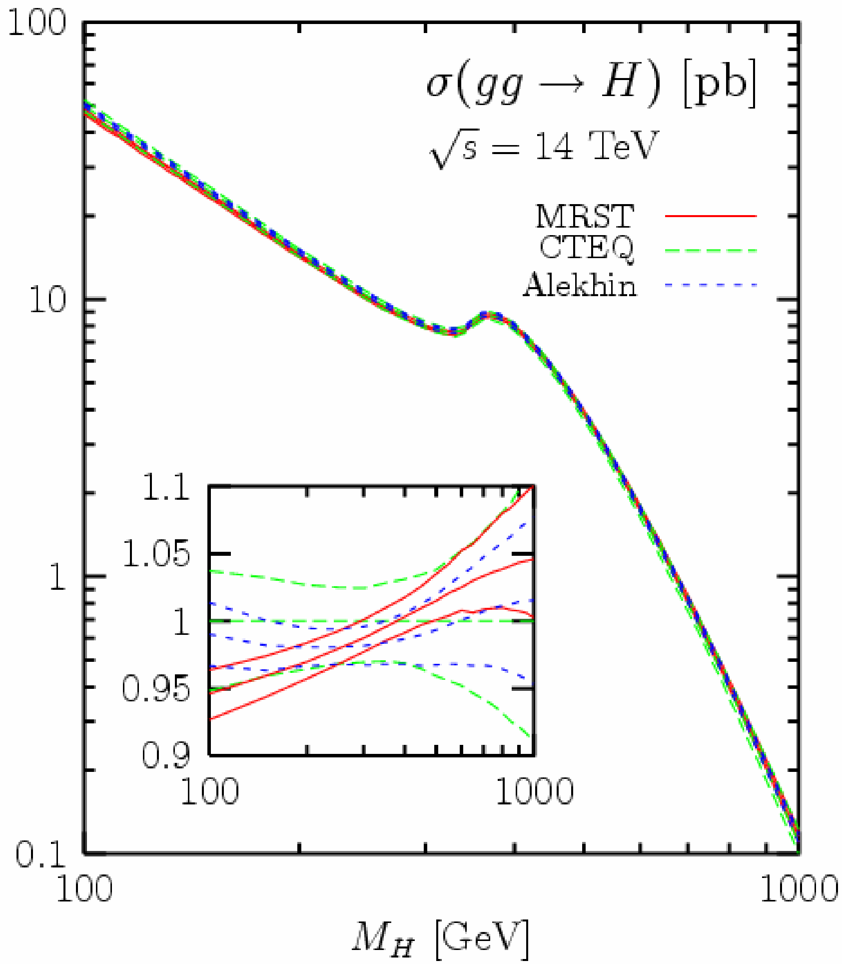
$\alpha_S(M_Z)$

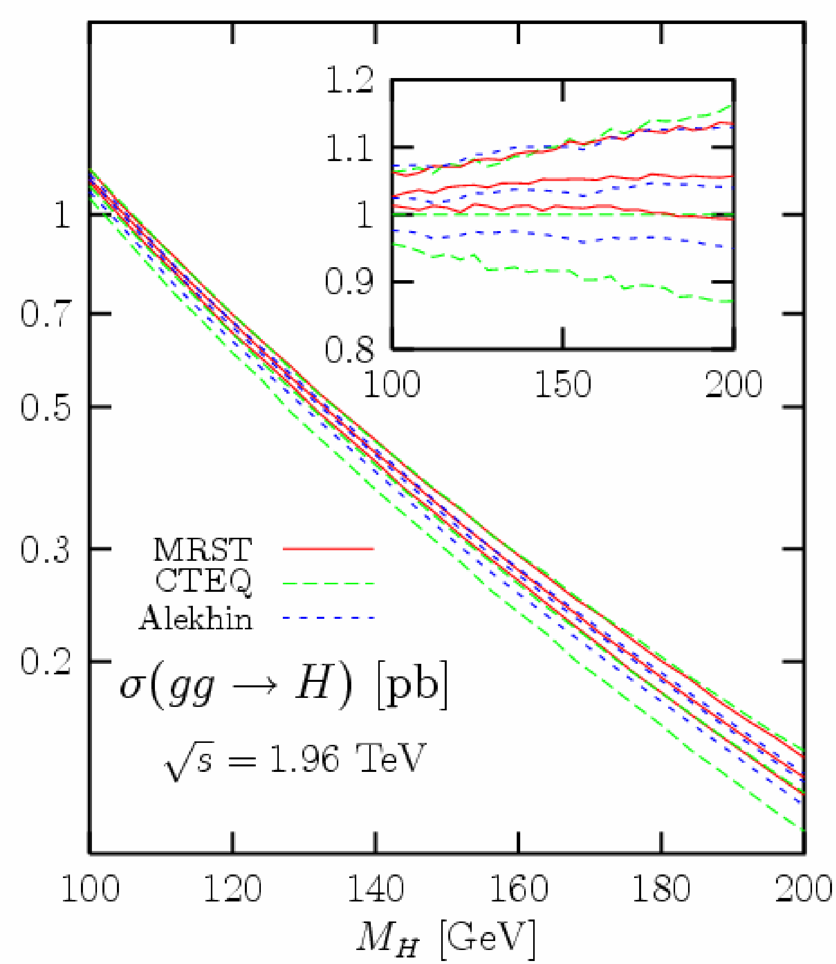
**Data**
DIS (SLAC, BCDMS, NMC, E665, CCFR, H1, ZEUS, … )
Drell-Yan (E605, E772, E866, …)
High $E_T$ jets (CDF, D0)
W rapidity asymmetry (CDF)
$\nu$N dimuon (CCFR, NuTeV)
etc.

**Who?**
Alekhin, CTEQ, MRST, GGK, Botje, H1, ZEUS, GRV, BFP, …

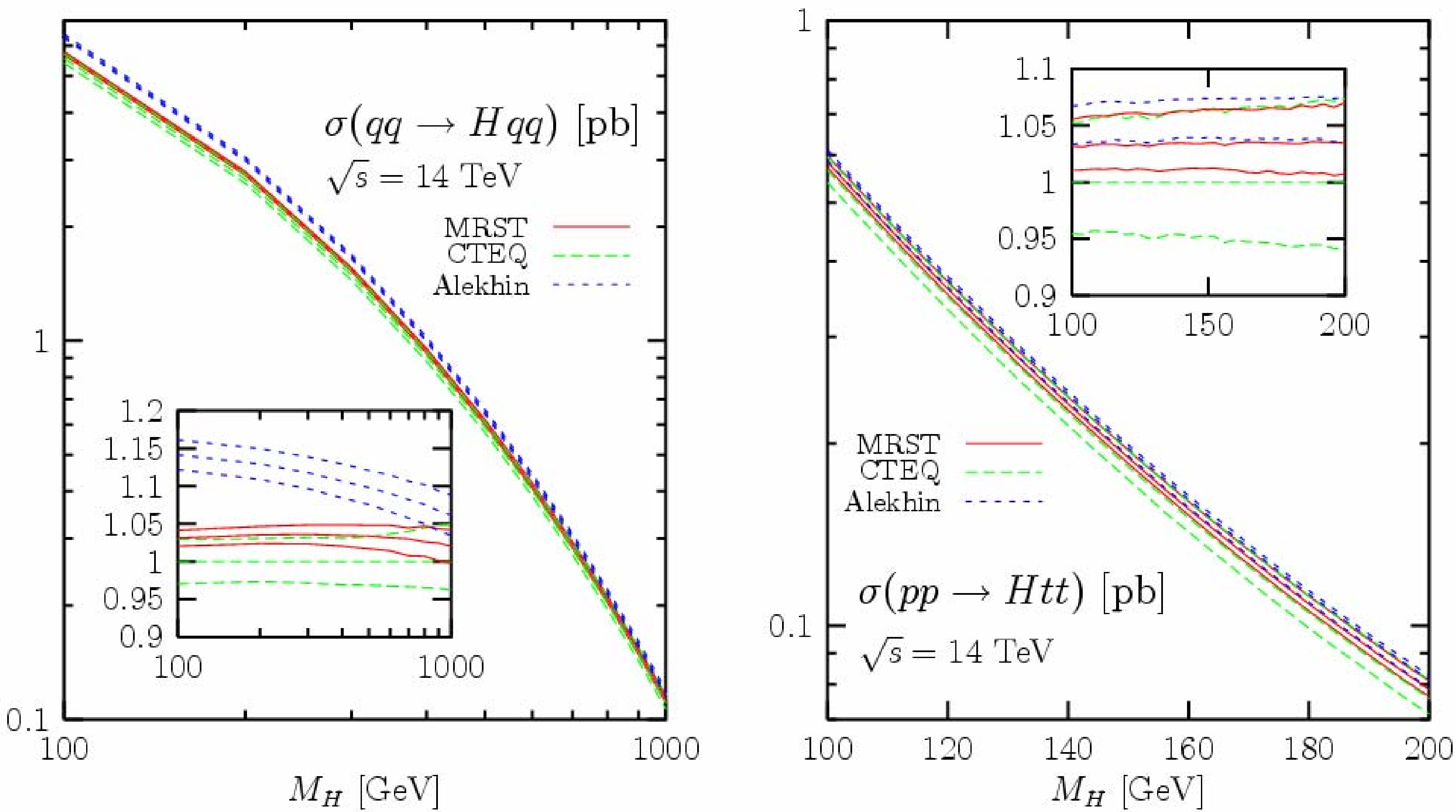
http://durpdg.dur.ac.uk/hepdata/pdf.html

**LHAPDFv2**




Djouadi & Ferrag, hep-ph/0310209

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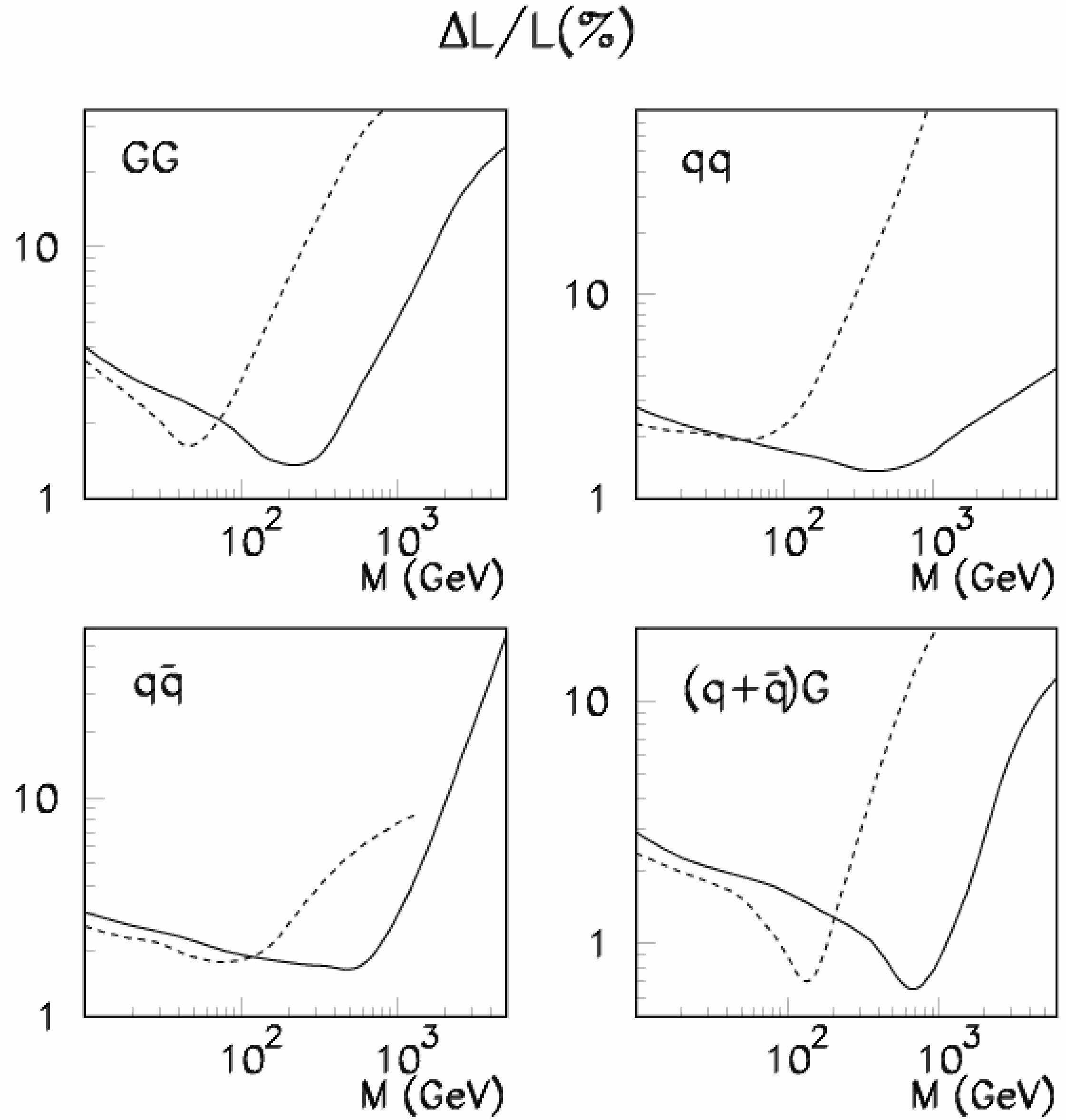
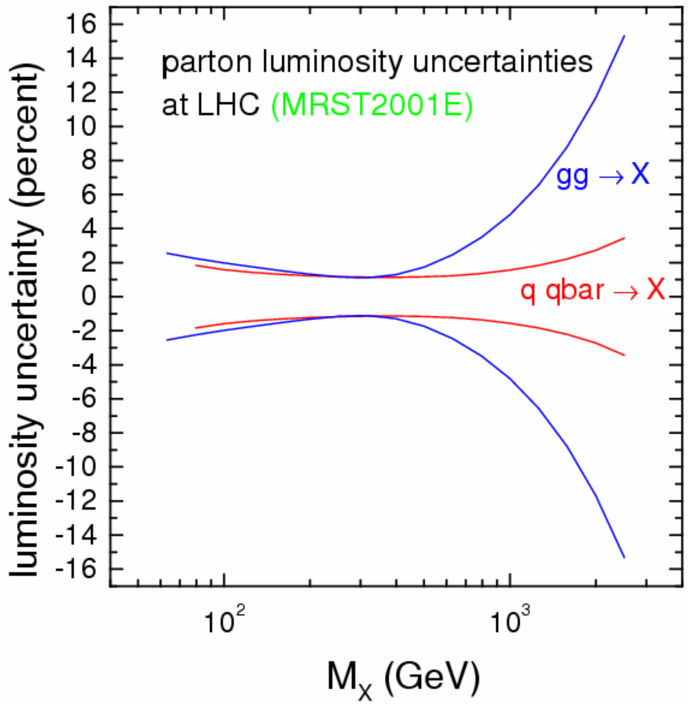



pdf uncertainties encoded in parton-parton *luminosity functions*

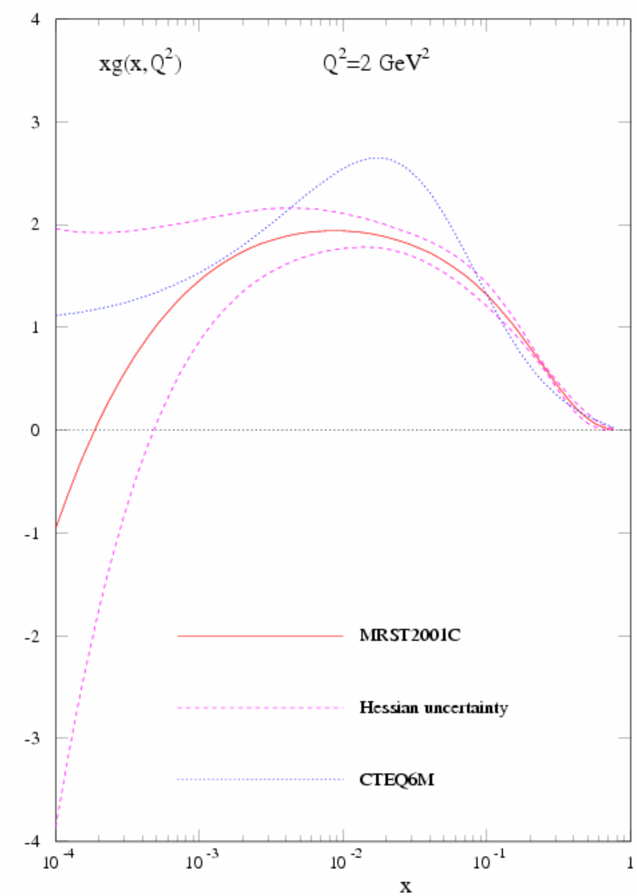
solid = LHC
dashed = Tevatron
Alekhin 2002




Note: high-x gluon should become better determined from Run 2 Tevatron data

Q. by how much?

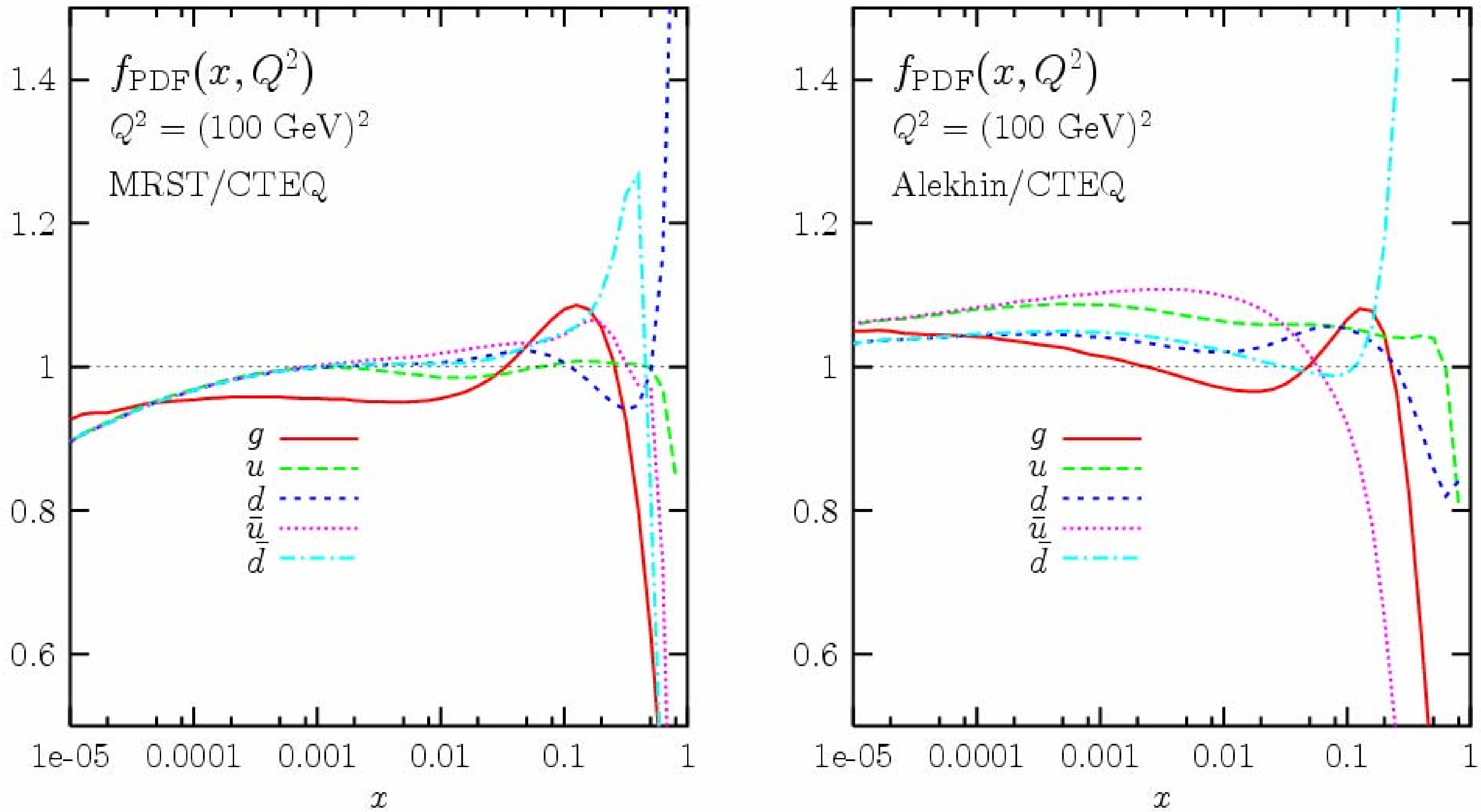
Note: CTEQ gluon 'more or less' consistent with MRST gluon



# why do 'best fit' pdfs and errors differ?

- different data sets in fit
  - different subselection of data
  - different treatment of exp. sys. errors
- different choice of
  - tolerance to define $\pm\delta f_i$ (CTEQ: $\Delta\chi^2=100$, Alekhin: $\Delta\chi^2=1$)
  - factorisation/renormalisation scheme/scale
  - $Q_0^2$
  - parametric form $Ax^a(1-x)^b[..]$ etc
  - $\alpha_S$
  - treatment of heavy flavours
  - theoretical assumptions about $x\to 0,1$ behaviour
  - theoretical assumptions about sea flavour symmetry
  - evolution and cross section codes (removable differences!)

→ see talks in PDF Working Group!



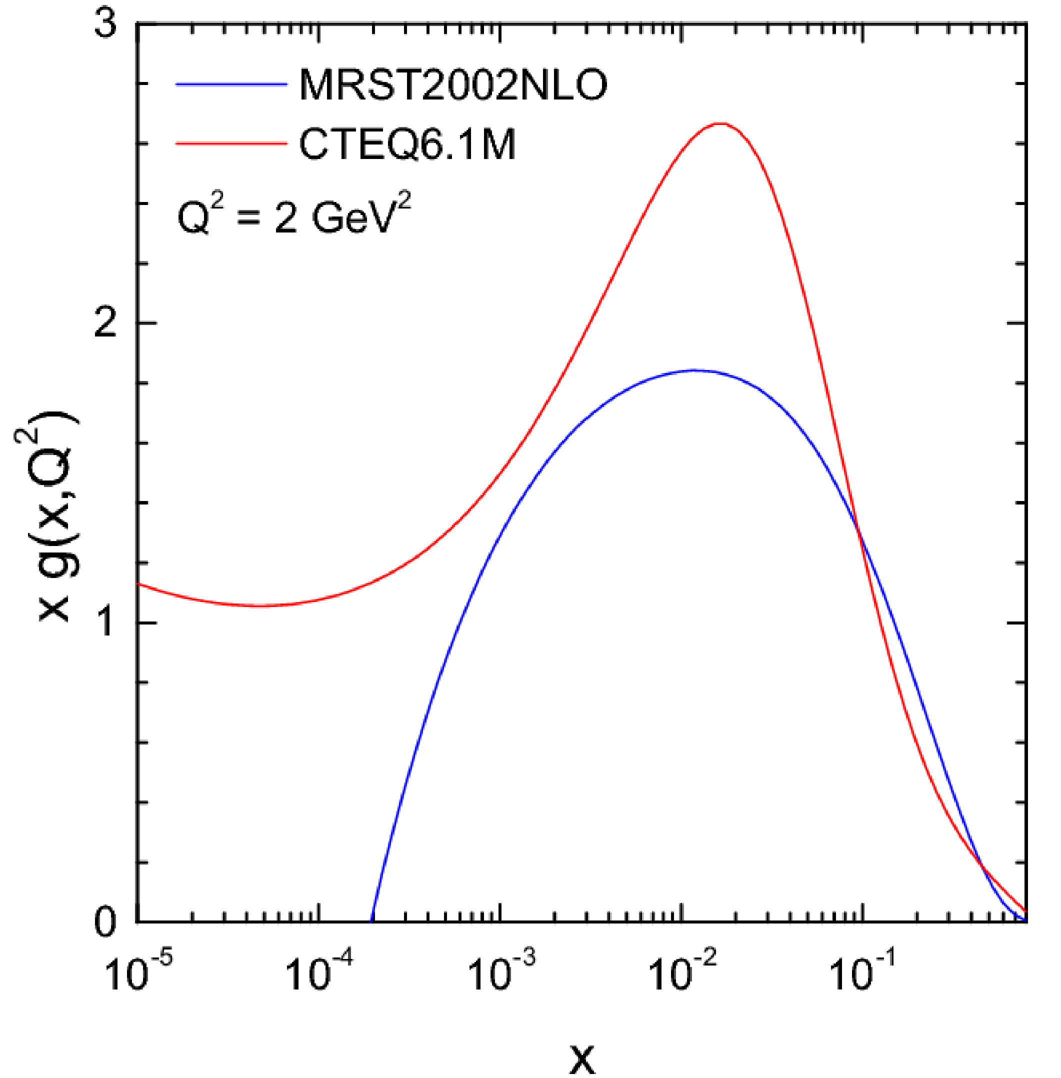

Djouadi & Ferrag, hep-ph/0310209

- MRST: $Q_0^2$ = 1 GeV$^2$, $Q_{cut}^2$ = 2 GeV$^2$

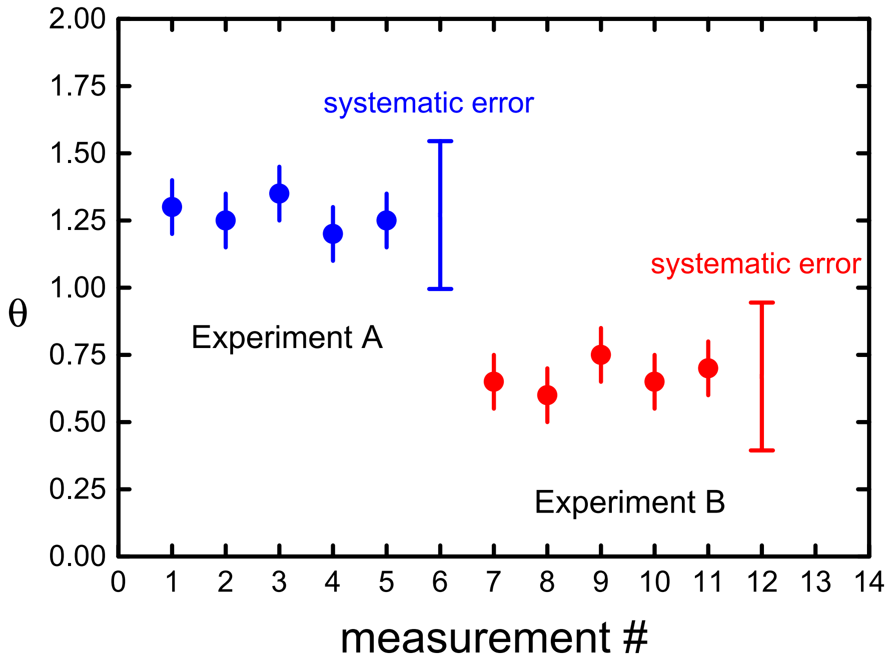
$$xg = Ax^a(1-x)^b(1+Cx^{0.5}+Dx) - Ex^c(1-x)^d$$

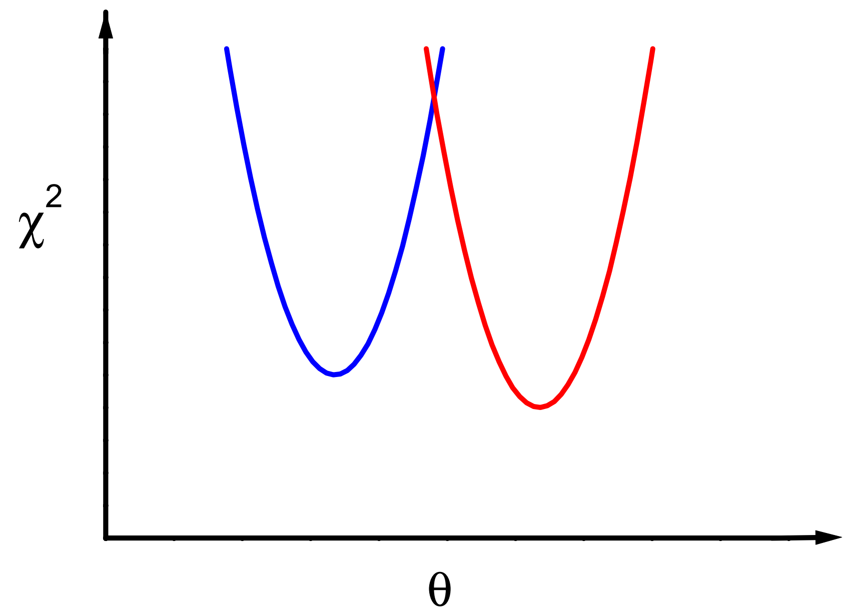
- CTEQ6: $Q_0^2$ = 1.69 GeV$^2$, $Q_{cut}^2$ = 4 GeV$^2$

$$xg = Ax^a(1-x)^b e^{cx}(1+Cx)^d$$



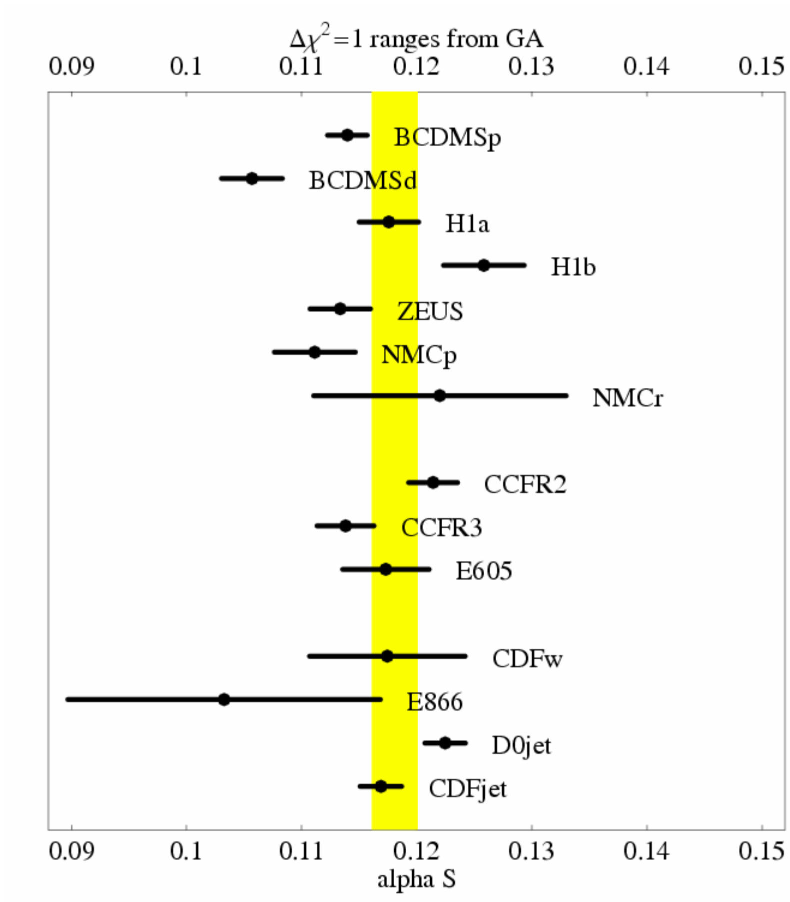
# tensions within the global fit?

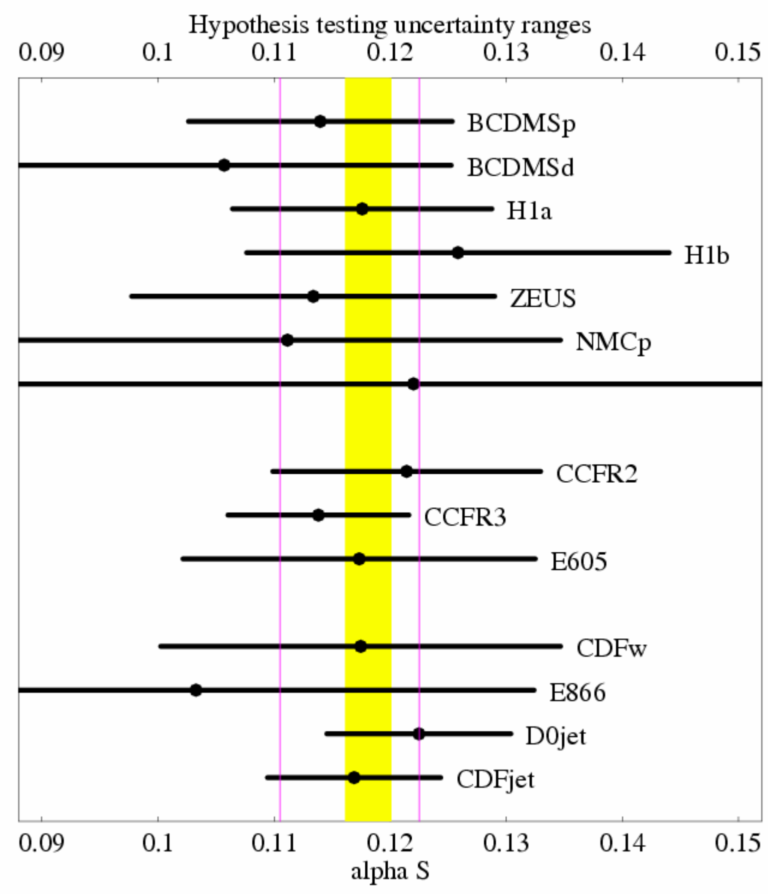



- with dataset A in fit, $\Delta\chi^2=1$ ; with A and B in fit, $\Delta\chi^2=?$
- 'tensions' between data sets arise, for example,
  - between DIS data sets (e.g. **μH** and **νN** data)
  - when jet and Drell-Yan data are combined with DIS data

# CTEQ $\alpha_S(M_Z)$ values from global analysis with $\Delta\chi^2$ = 1, 100



as small *x* data are systematically removed from the MRST global fit, the quality of the fit improves until stability is reached at around *x* ~ 0.005 (MRST hep-ph/0308087)

Q. Is fixed–order DGLAP insufficient for small-x DIS data?!

Δ = improvement in $\chi^2$ to remaining data / # of data points removed

| $x_{\rm cut}$ : | 0 | 0.0002 | 0.001 | 0.0025 | 0.005 | 0.01 |
|---|---|---|---|---|---|---|
| # data points | 2097 | 2050 | 1961 | 1898 | 1826 | 1762 |
| $\alpha_S(M_Z^2)$ | 0.1197 | 0.1200 | 0.1196 | 0.1185 | 0.1178 | 0.1180 |
| $\chi^2(x > 0)$ | 2267 | | | | | |
| $\chi^2(x > 0.0002)$ | 2212 | 2203 | | | | |
| $\chi^2(x > 0.001)$ | 2134 | 2128 | 2119 | | | |
| $\chi^2(x > 0.0025)$ | 2069 | 2064 | 2055 | 2040 | | |
| $\chi^2(x > 0.005)$ | 2024 | 2019 | 2012 | 1993 | 1973 | |
| $\chi^2(x > 0.01)$ | 1965 | 1961 | 1953 | 1934 | 1917 | 1916 |
| $\Delta_i^{i+1}$ | | 0.19 | 0.10 | 0.24 | 0.28 | 0.02 |

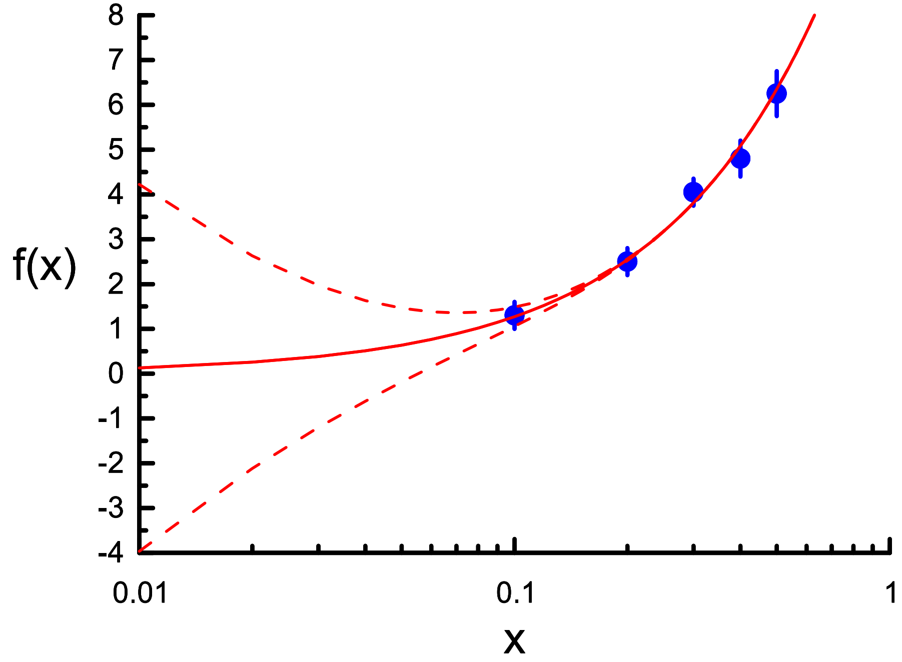
the stability of the small-*x* fit can be recovered by adding to the fit empirical contributions of the form

$$P_{gg} \to P_{gg}^{\mathrm{NLO}} + A\overline{\alpha}_S^4 \left( \frac{\ln^3 1/x}{3!} - \frac{\ln^2 1/x}{2!} \right)$$

$$P_{qg} \to P_{qg}^{\mathrm{NLO}} + B\alpha_S \frac{n_f}{3\pi} \overline{\alpha}_S^4 \left( \frac{\ln^3 1/x}{3!} - \frac{\ln^2 1/x}{2!} \right)$$

... with coefficients *A*, *B* found to be O(1) (and different for the NLO, NNLO fits); the starting gluon is still very negative at small *x* however

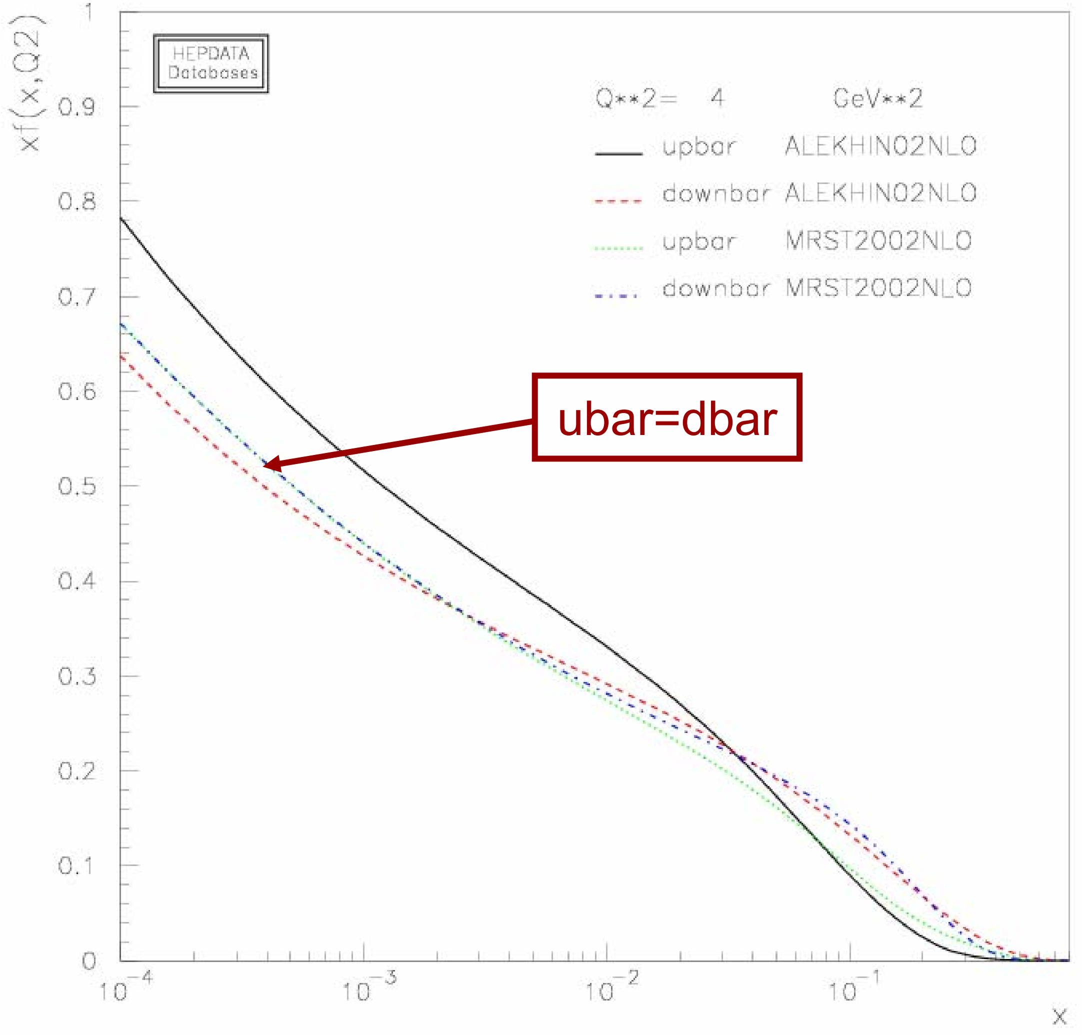
# extrapolation errors

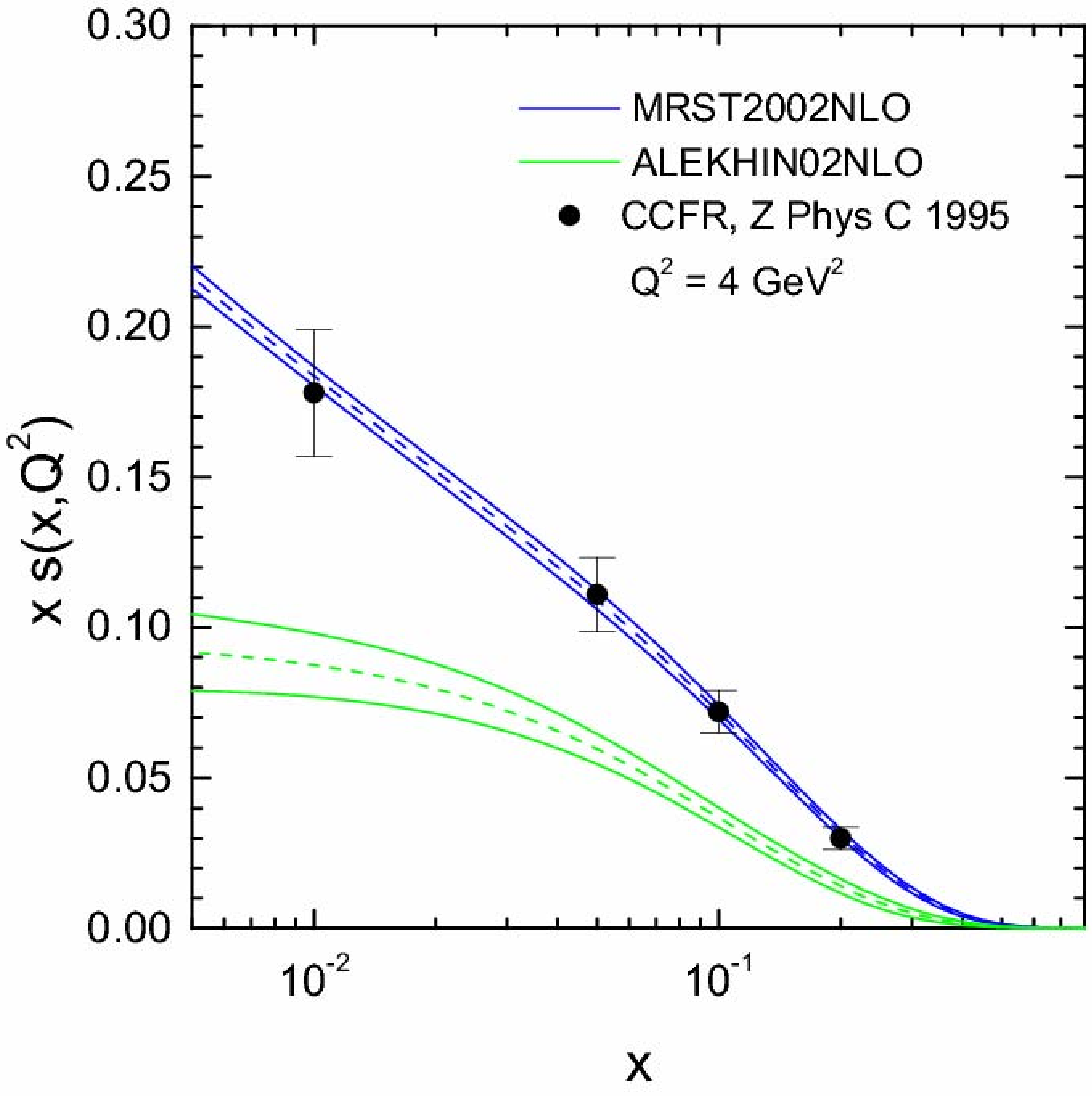



theoretical insight/guess: $f \sim A\,x$ as $x \to 0$

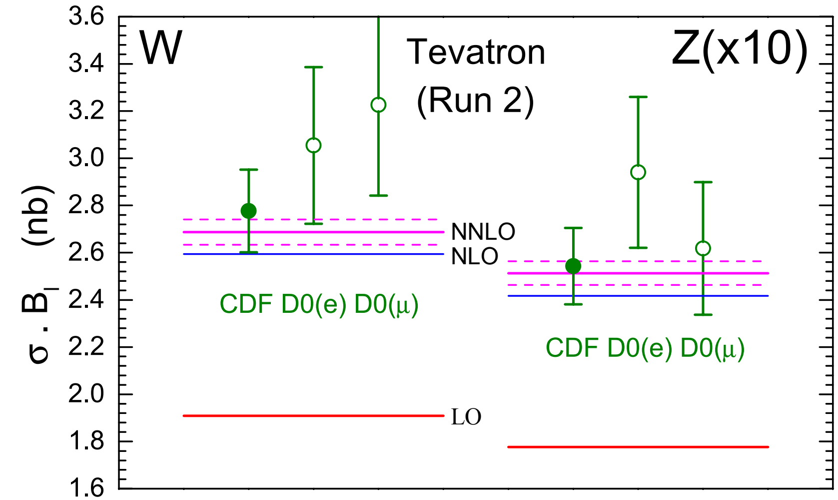
theoretical insight/guess: $f \sim \pm A\,x^{-0.5}$ as $x \to 0$

differences between the MRST and Alekhin *u* and *d* sea quarks near the starting scale




MRST2002NLO
ALEKHIN02NLO
CCFR, Z Phys C 1995
Q² = 4 GeV²
0.30
0.25
0.20
0.15
0.10
0.05
0.00
x s(x,Q²)
10⁻²
10⁻¹
x

# σ(W) and σ(Z) : precision predictions and measurements at the LHC

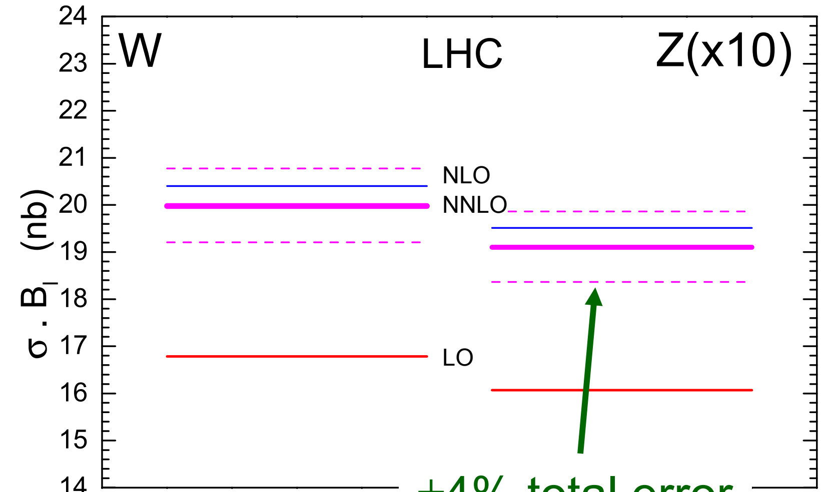



| LHC | $\sigma_{NLO}$(W) (nb) |
|---|---|
| MRST2002 | 204 ± 4 (expt) |
| CTEQ6 | 205 ± 8 (expt) |
| Alekhin02 | 215 ± 6 (tot) |

similar partons

different $\Delta\chi^2$

different partons



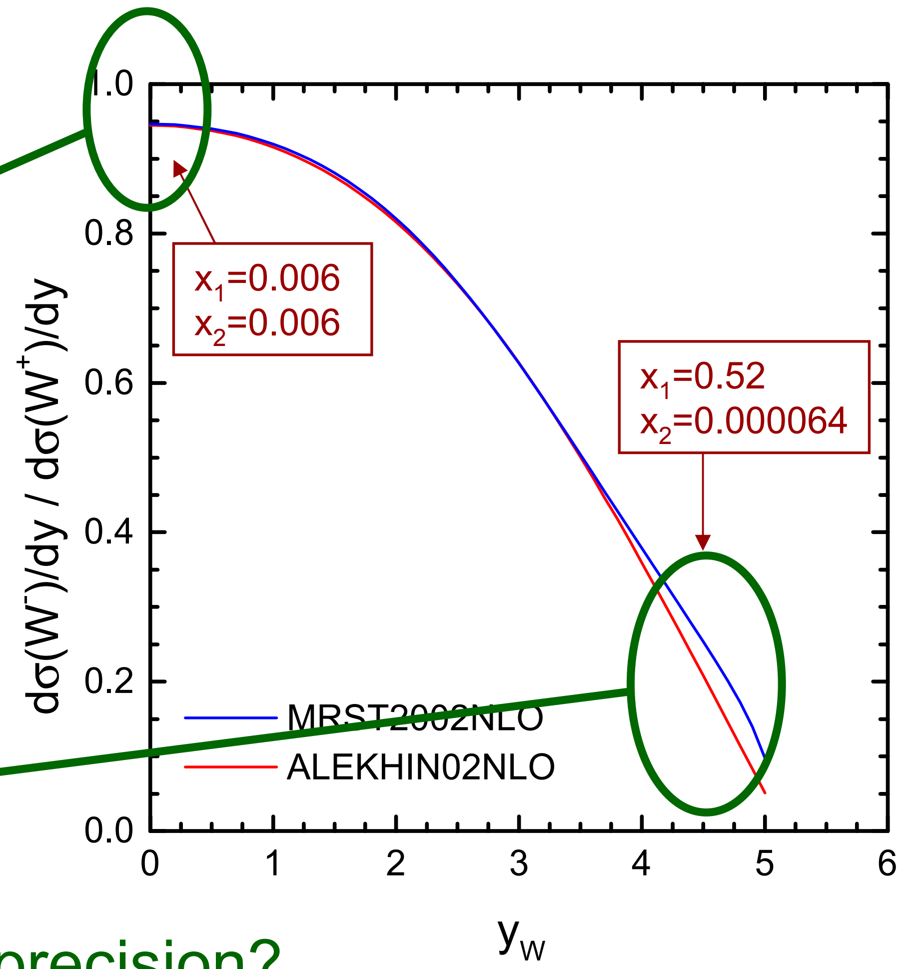

partons: MRST2002

NNLO evolution: van Neerven, Vogt approximation to Vermaseren et al. moments

NNLO W,Z corrections: van Neerven et al. with Harlander, Kilgore corrections

# ratio of W⁻ and W⁺ rapidity distributions

$$\frac{d\sigma(W^-)}{d\sigma(W^+)} = \frac{d(x_1)\bar{u}(x_2)+\cdots}{u(x_1)\bar{d}(x_2)+\cdots}$$



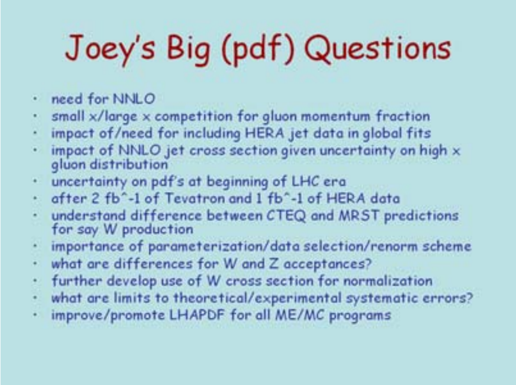

ratio close to 1 because $u \approx \bar{u}$ etc.
(note: MRST error = ±1½%)

sensitive to large-x d/u and small x $\bar{u}/\bar{d}$ ratios

Q. What is the experimental precision?

# summary and outlook

- need a better understanding of differences between pdf sets (central values and errors): not just ‘experimental errors’ (easier) but theoretical errors too (harder).
- are apparent ‘tensions’ between data sets caused by experiment or theory?
- is high-precision $F_2$ data from HERA revealing breakdown of fixed-order DGLAP? If so, what are implications for LHC?
- $F_L$ ?!
- the impact of *full* NNLO DGLAP?



## Joey's Big (pdf) Questions

- need for NNLO
- small x/large x competition for gluon momentum fraction
- impact of/need for including HERA jet data in global fits
- impact of NNLO jet cross section given uncertainty on high x gluon distribution
- uncertainty on pdf's at beginning of LHC era
- after 2 fb^-1 of Tevatron and 1 fb^-1 of HERA data
- understand difference between CTEQ and MRST predictions for say W production
- importance of parameterization/data selection/renorm scheme
- what are differences for W and Z acceptances?
- further develop use of W cross section for normalization
- what are limits to theoretical/experimental systematic errors?
- improve/promote LHAPDF for all ME/MC programs

KITP Collider Physics Conference

# summary and outlook contd.

- impact on global fits of HERA jet data? (e.g. $\gamma^* g \rightarrow jets$)
- current interest in flavour structure of sea: e.g. $ubar \neq dbar$ and $s \neq sbar$ (NuTeV). Can LHC provide information? (e.g. $s\,g \rightarrow W\,c$ )
- moving beyond inclusive regime – the role of unintegrated (in $k_T$) parton distributions?
- improvements expected by LHC start-up? (i.e. from theory + new HERA, Tevatron data)