Multi-jet topologies and multi-scale QCD Theoretical Thoughts

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Fixed order QCD

QCD

Well understood for scattering processes with one hard scale.

Total cross sections well described by:

- Fixed (NL) order perturbative calculations
- DGLAP evolution of PDFs

Examples:

- Dijet production (gg scattering) at large transverse momentum ($\hat{s} \sim p_\perp^2$) at hadron colliders (also $p_\perp$-distribution)
- Single inclusive jets (also angular correllation) at HERA

Fixed order QCD

Fixed order perturbative expansion of observable R

$$R = r_0 + r_1\alpha_s(Q^2) + r_2\alpha^2(Q^2) + r_3\alpha^3(Q^2) + r_4\alpha^4(Q^2) + \cdots$$

One universal, running coupling

Perturbative QCD works!

Multi-Jet and Multi-Scale QCD: attempt to extend our understanding of perturbative QCD away from the study of total cross sections with one, super perturbative scale.

Excitement/Drawback: *not* just 5-10% effects!

Surprises may lurk round the corner

Resummation

Consider the perturbative expansion of an observable

$$R = r_0 + r_1\alpha_s + r_2\alpha^2 + r_3\alpha^3 + r_4\alpha^4 + \cdots$$

For multi-jet and multi-scale QCD, r_n will contain large logarithms so that $\alpha_s \ln(\cdots)$ is large

$$R = r_0 + \left(r_1^{LL}\ln(\cdots) + r_1^{NLL}\right)\alpha_s + \left(r_2^{LL}\ln^2(\cdots) + r_2^{NLL}\ln(\cdots) + r_2^{SL}\right)\alpha_s^2 +$$
$$= r_0 + \sum_n r_n^{LL}(\alpha_s\ln(\cdots))^n + \sum_n r_n^{NLL}\alpha_s(\alpha_s\ln(\cdots))^n + \text{sub-leading terms}$$

Replace the perturbative parameter α_s with $\alpha_s \ln(\cdots)$.

Non-global logarithms

Event shapes variables v: distill complicated multi-parton topology into one number.
Perturbative series often dominated by (LL) terms

$$\alpha_s^n \ln^{2n-1} v/v$$

for $v << 1$.

Recently realised: LL in. NLL out OR NLL in.

M. Dasgupta and G.P. Salam

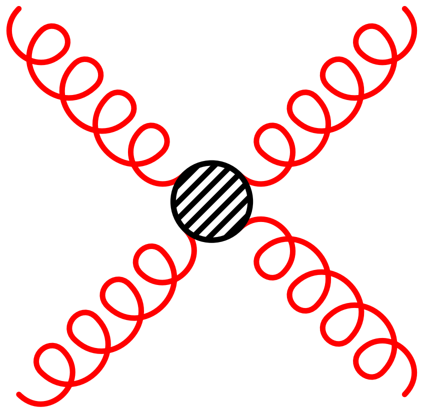
Multi-scale QCD

- BFKL (**B**alitskii, **F**adin, **K**uraev, **L**ipatov): resummation of large logarithms in the perturbation series for processes with two large (perturbative) and disparate energy scales ($\hat{s} \gg |\hat{t}|$) (forward scattering, small x DIS...)
- The cross section for the process $A + B \rightarrow A' + B'$ factorises as

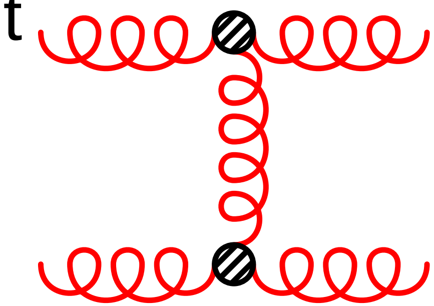
$$\sigma(s) = \int \frac{d^2\mathbf{k}_a}{2\pi\mathbf{k}_a^2} \int \frac{d^2\mathbf{k}_b}{2\pi\mathbf{k}_b^2} \, \Phi_A(\mathbf{k}_a) \, f\left(\mathbf{k}_a, \mathbf{k}_b, \Delta = \ln\frac{s}{s_0}\right) \Phi_B(\mathbf{k}_b)$$

- $\Phi_A(\mathbf{k}_a), \Phi_B(\mathbf{k}_b)$ process dependent *impact factors* (calculated for many process at LL and for e.g. gg and $\gamma^*\gamma^*$ scattering at NLL)
- $f\left(\mathbf{k}_a, \mathbf{k}_b, \Delta\right)$ process independent *Gluon Green's function*

Dijet Production

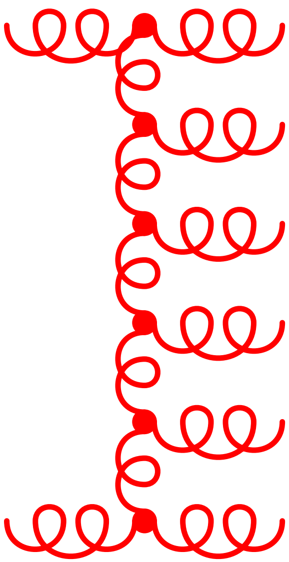


High Energy Limit $\xrightarrow{\hat{s}/|\hat{t}| \to \infty}$



BFKL evolution of the t-channel gluon

$P_{Ta}, \Delta y$



$P_{Tb}, 0$

$$\hat{s} \sim p_T^2 e^{\Delta y}$$

$$|\hat{t}| \sim p_T^2$$

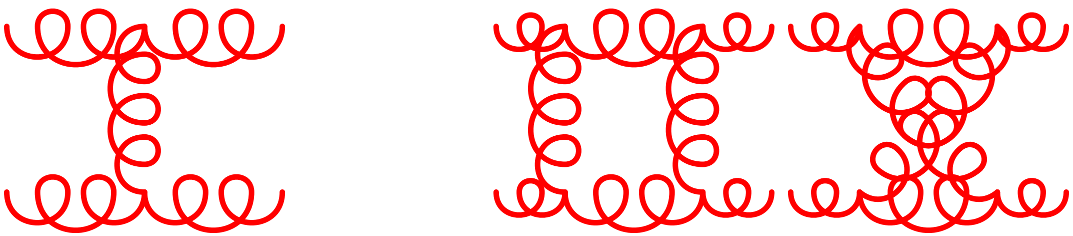
$$\ln \frac{\hat{s}}{|\hat{t}|} \sim \Delta y$$

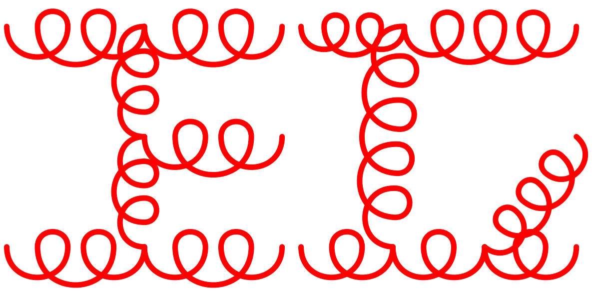
BFKL resums to all orders terms in the perturbative expansion of the form

$$\left(\alpha_s \ln \frac{\hat{s}}{|\hat{t}|}\right)^n \sim (\alpha_s \Delta y)^n$$

BFKL at LLA

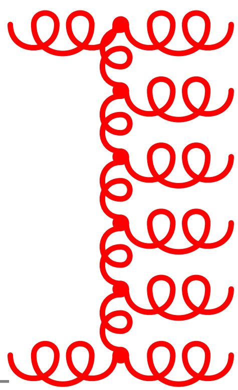
Exactly which diagrams contribute?





etc.

All these contributions can be calculated using effective vertices and propagators for the “reggeized gluon”.



The BFKL Equation

- Expressed in terms of the Mellin transformed Gluon Green's function

$$f\left(\mathbf{k}_a, \mathbf{k}_b, \Delta\right) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} d\omega\, e^{\omega \Delta} f_\omega\left(\mathbf{k}_a, \mathbf{k}_b\right)$$

- The Gluon Green's function fulfil (to LLA and NLLA) the BFKL equation (in dim. regularisation $(D = 4 + 2\epsilon)$):

$$\omega f_\omega\left(\mathbf{k}_a, \mathbf{k}_b\right) = \delta^{(2+2\epsilon)}\left(\mathbf{k}_a - \mathbf{k}_b\right) + \int d^{2+2\epsilon}\mathbf{k}' \mathcal{K}\left(\mathbf{k}_a, \mathbf{k}'\right) f_\omega\left(\mathbf{k}', \mathbf{k}_b\right)$$

where the BFKL kernel $\mathcal{K}\left(\mathbf{k}_a, \mathbf{k}'\right)$ is calculated to LLA or NLLA respectively

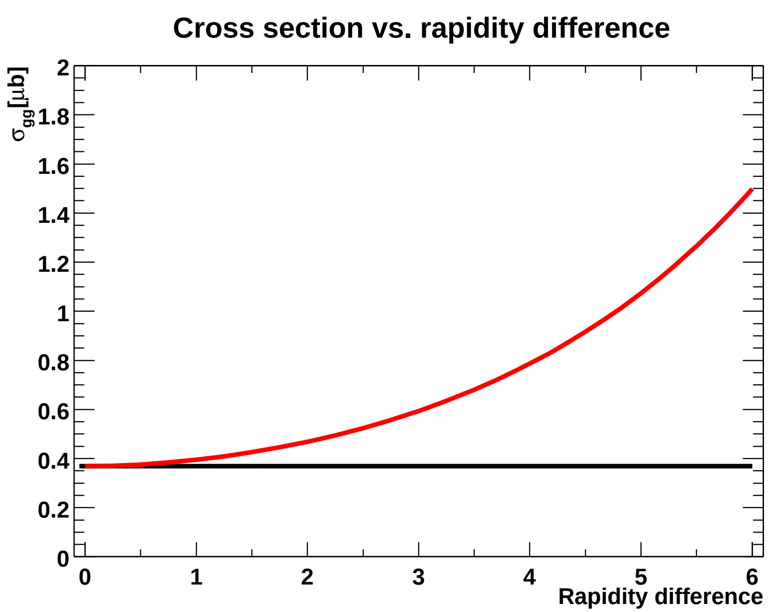
The BFKL Equation at LLA

Analytic solution for angular averaged gluon Green's function

$$\bar{f}\left(k_a, k_b, \Delta\right) = \frac{4}{k_a k_b} \int_0^\infty d\nu \left(\frac{k_a^2}{k_b^2}\right)^{i\nu} e^{\bar{\alpha}_s \Delta \chi_0(\nu)}$$

with the LL eigenvalue

$$\chi_0(\nu) = -2\,\mathrm{Re}\left\{\psi\left(\frac{1}{2} + i\nu\right) - \psi(1)\right\}.$$

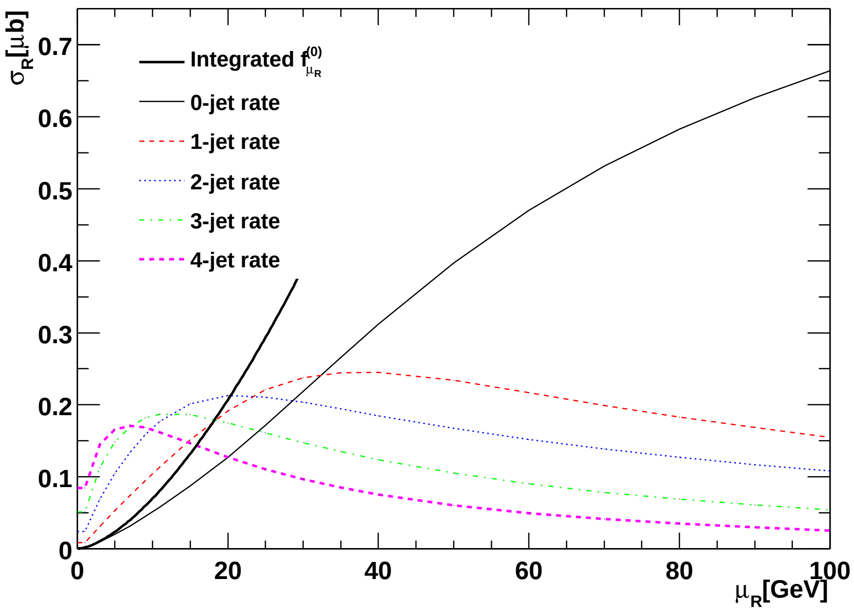


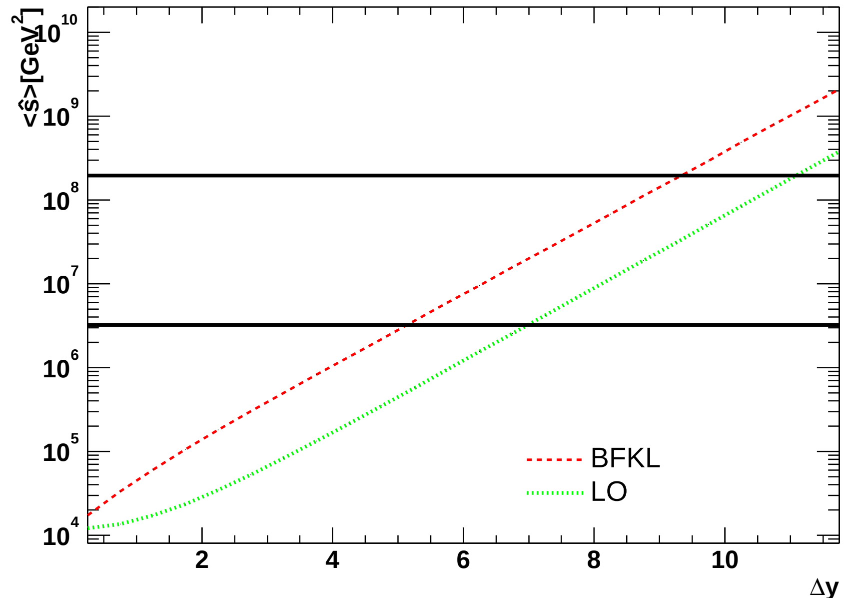
BFKL rise in cross section! Integrated over the full $\mathbf{k}$ phase space for gluon emission and allowing any number of gluons to radiate!!!

$$\hat{\sigma}_{gg} \to \frac{\pi C_A^2 \alpha_s^2}{2 P_{T,\min}^2} \frac{e^{\lambda \Delta y}}{\sqrt{\pi B \Delta y}}, B = 14\zeta(3)\bar{\alpha}_s, \quad \lambda = \frac{\alpha_s C_A}{\pi} 4 \ln 2 \approx 0.45$$

Recent results obtained in LLA

$\Delta y = 5$





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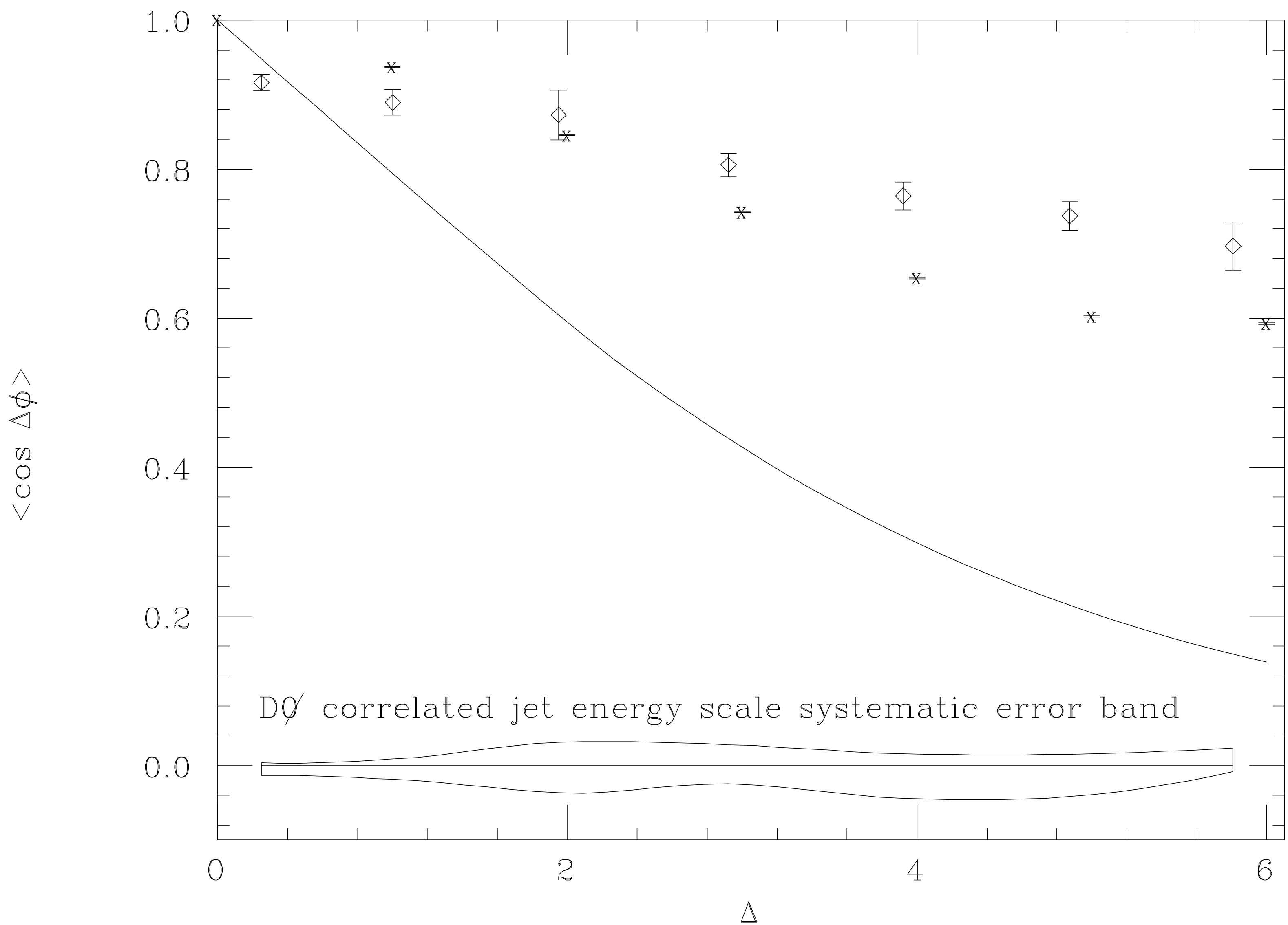
Recent results obtained in LLA



The BFKL Equation at NLLA

- Both the *trajectory* $\omega(-\mathbf{k}_a^2)$ and the *real emission kernel* $\mathcal{K}_r$ are significantly more complicated than at LL
- Takes into account fermions and running coupling effects
- Furthermore, the impact factors at NLL are similarly complicated, and a fully analytic approach for cross sections seems almost hopeless
- We will propose a generalisation of the LL iterative solution that will solve the BFKL equation at NLL accuracy.

Iterative Solution at NLL

We propose an iterative approach to the BFKL equation at NLLA that solves the equation with *no approximations*

- Directly in energy/transverse momentum space
- The right language for use of impact factors (physics predictions!)
- Hopeful in extending the approach to final state studies like at LL

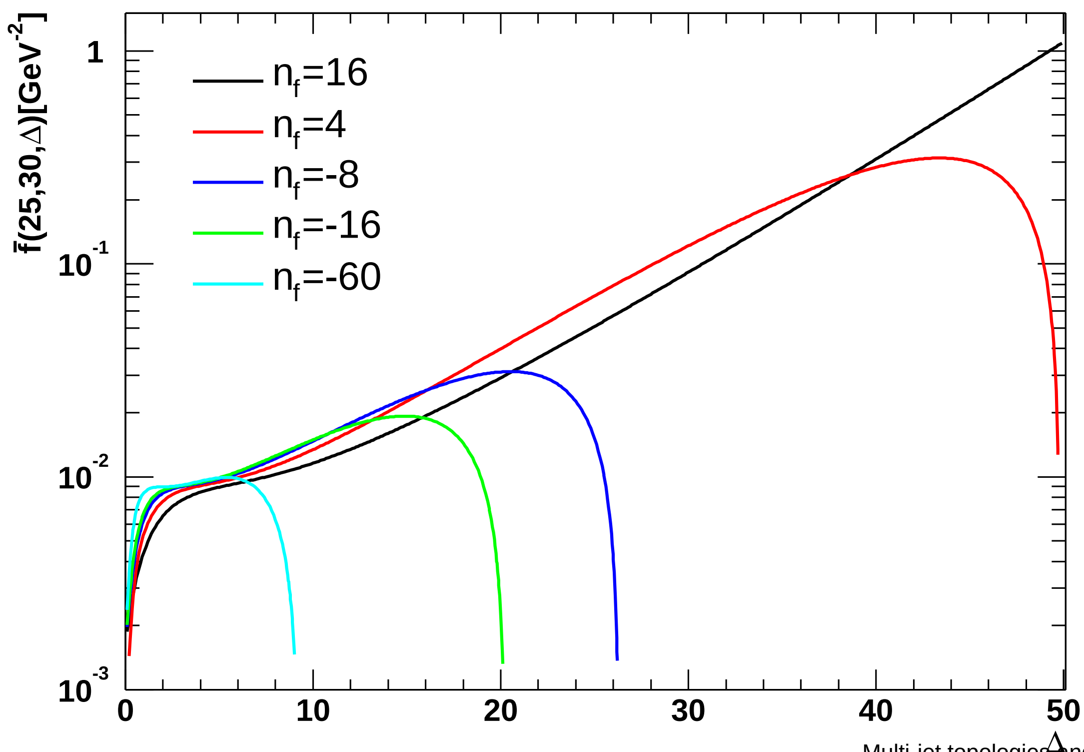
Understanding of NLL BFKL is still work in progress. Any extra light shed on this problem can be valuable help!
NLL BFKL breaks conformal invairance.

Standard approach exhibits pathological behaviour

Mellin analysis

Much recent research from many groups has concentrated on the treatment of the running coupling effects in the solution of the NLL BFKL equation.

G. Altarelli, R. D. Ball, S. Forte
M. Ciafaloni, D. Colferai, G. P. Salam, A.M. Stasto
R.S. Thorne



Iteration at NLL

Start from the BFKL equation

$$\omega f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right)=\delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}_b\right)+\int d^{2+2\epsilon}\mathbf{k}'\,\mathcal{K}\left(\mathbf{k}_a,\mathbf{k}'\right)f_\omega\left(\mathbf{k}',\mathbf{k}_b\right)$$

$$\mathcal{K}\left(\mathbf{k}_a,\mathbf{k}\right)=2\,\omega^{(\epsilon)}\left(\mathbf{k}_a^2\right)\delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}\right)+\mathcal{K}_r\left(\mathbf{k}_a,\mathbf{k}\right)$$

Need all terms (IR) finite to be able to iterate: split the kernel $\mathcal{K}_r$ into two parts: a ϵ–dependent, $\mathcal{K}_r^{(\epsilon)}$, and a ϵ–independent, $\widetilde{\mathcal{K}}_r$

$$\omega f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right)=\delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}_b\right)+\int d^{2+2\epsilon}\mathbf{k}\,2\,\omega^{(\epsilon)}\left(\mathbf{k}_a^2\right)\delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}\right)f_\omega\left(\mathbf{k},\mathbf{k}_b\right)$$

$$+\int d^{2+2\epsilon}\mathbf{k}\,\mathcal{K}_r^{(\epsilon)}\left(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}\right)f_\omega\left(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b\right)+\int d^{2+2\epsilon}\mathbf{k}\,\widetilde{\mathcal{K}}_r\left(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}\right)f_\omega\left(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b\right).$$

Iteration at NLL, 2

Introduce a slice in the phase space (no approximation)

$$\begin{aligned}\omega f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right) &= \delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}_b\right)+\int d^{2+2\epsilon}\mathbf{k}\,2\,\omega^{(\epsilon)}\left(\mathbf{k}_a^2\right)\delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}\right)f_\omega\left(\mathbf{k},\mathbf{k}_b\right)\\ &+\int d^{2+2\epsilon}\mathbf{k}\,\mathcal{K}_r^{(\epsilon)}\left(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}\right)\left(\theta\left(\mathbf{k}^2-\lambda^2\right)+\theta\left(\lambda^2-\mathbf{k}^2\right)\right)f_\omega\left(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b\right)\\ &+\int d^{2+2\epsilon}\mathbf{k}\,\widetilde{\mathcal{K}}_r\left(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}\right)f_\omega\left(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b\right)\end{aligned}$$

approximate $f_\omega\left(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b\right)\simeq f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right)$ for $|\mathbf{k}|<\lambda$

$$\begin{aligned}\omega f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right) &= \delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}_b\right)\\ &+\left\{2\,\omega^{(\epsilon)}\left(\mathbf{k}_a^2\right)+\int d^{2+2\epsilon}\mathbf{k}\,\mathcal{K}_r^{(\epsilon)}\left(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}\right)\theta\left(\lambda^2-\mathbf{k}^2\right)\right\}f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right)\\ &+\int d^{2+2\epsilon}\mathbf{k}\,\left\{\mathcal{K}_r^{(\epsilon)}\left(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}\right)\theta\left(\mathbf{k}^2-\lambda^2\right)+\widetilde{\mathcal{K}}_r\left(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}\right)\right\}f_\omega\left(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b\right).\end{aligned}$$

($\lambda\to 0$ limit can be obtained)

Iteration at NLL, 3

$$
\begin{aligned}
\left(\omega - \omega_0\left(\mathbf{k}_a^2, \lambda^2\right)\right) f_\omega\left(\mathbf{k}_a, \mathbf{k}_b\right) &= \delta^{(2)}\left(\mathbf{k}_a - \mathbf{k}_b\right) \\
&+ \int d^2\mathbf{k} \left(\frac{1}{\pi \mathbf{k}^2} \xi\left(\mathbf{k}^2\right) \theta\left(\mathbf{k}^2 - \lambda^2\right) + \widetilde{\mathcal{K}}_r\left(\mathbf{k}_a, \mathbf{k}_a + \mathbf{k}\right) \right) f_\omega\left(\mathbf{k}_a + \mathbf{k}, \mathbf{k}_b\right)
\end{aligned}
$$

$$
\begin{aligned}
\omega_0\left(\mathbf{q}^2, \lambda^2\right) &\equiv -\xi\left(|\mathbf{q}|\,\lambda\right) \ln \frac{\mathbf{q}^2}{\lambda^2} + \eta \\
\xi\left(\mathrm{X}\right) &\equiv \bar{\alpha}_s + \frac{\bar{\alpha}_s^2}{4} \left[\frac{4}{3} - \frac{\pi^2}{3} + \frac{5}{3} \frac{\beta_0}{N_c} - \frac{\beta_0}{N_c} \ln \frac{\mathrm{X}}{\mu^2} \right] \\
\eta &\equiv \bar{\alpha}_s^2 \frac{3}{2} \zeta(3).
\end{aligned}
$$

$$
\widetilde{\mathcal{K}}_r\left(\mathbf{q}, \mathbf{q}'\right) = \frac{\bar{\alpha}_s^2}{4\pi} \left\{ \text{6 lines of equations...} \right\}.
$$

Iteration at NLL, 4

Iterate and take the inverse Mellin transform to find

$$
\begin{aligned}
f(\mathbf{k}_a, \mathbf{k}_b, \Delta) \quad &= \quad \exp\left(\omega_0\left(\mathbf{k}_a^2, \lambda^2, \mu\right)\Delta\right)\delta^{(2)}(\mathbf{k}_a - \mathbf{k}_b) \\
&+ \sum_{n=1}^{\infty} \prod_{i=1}^{n} \int d^2\mathbf{k}_i \left[\frac{\theta\left(\mathbf{k}_i^2 - \lambda^2\right)}{\pi \mathbf{k}_i^2} \xi\left(\mathbf{k}_i^2, \mu\right) \widetilde{\mathcal{K}}_r \left(\mathbf{k}_a + \sum_{l=0}^{i-1} \mathbf{k}_l, \mathbf{k}_a + \sum_{l=1}^{i} \mathbf{k}_l, \mu\right)\right] \\
&\times \int_0^{y_{i-1}} dy_i \, \exp\left[\omega_0\left(\left(\mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l\right)^2, \lambda^2, \mu\right)(y_{i-1} - y_i)\right] \\
&\qquad \times \exp\left[\omega_0\left(\left(\mathbf{k}_a + \sum_{l=1}^{n} \mathbf{k}_l\right)^2, \lambda^2, \mu\right)(y_n - 0)\right] \delta^{(2)}\left(\sum_{l=1}^{n} \mathbf{k}_l + \mathbf{k}_a - \mathbf{k}_b\right)
\end{aligned}
$$

May *seem* complicated, but all integrals are finite and can be found numerically.

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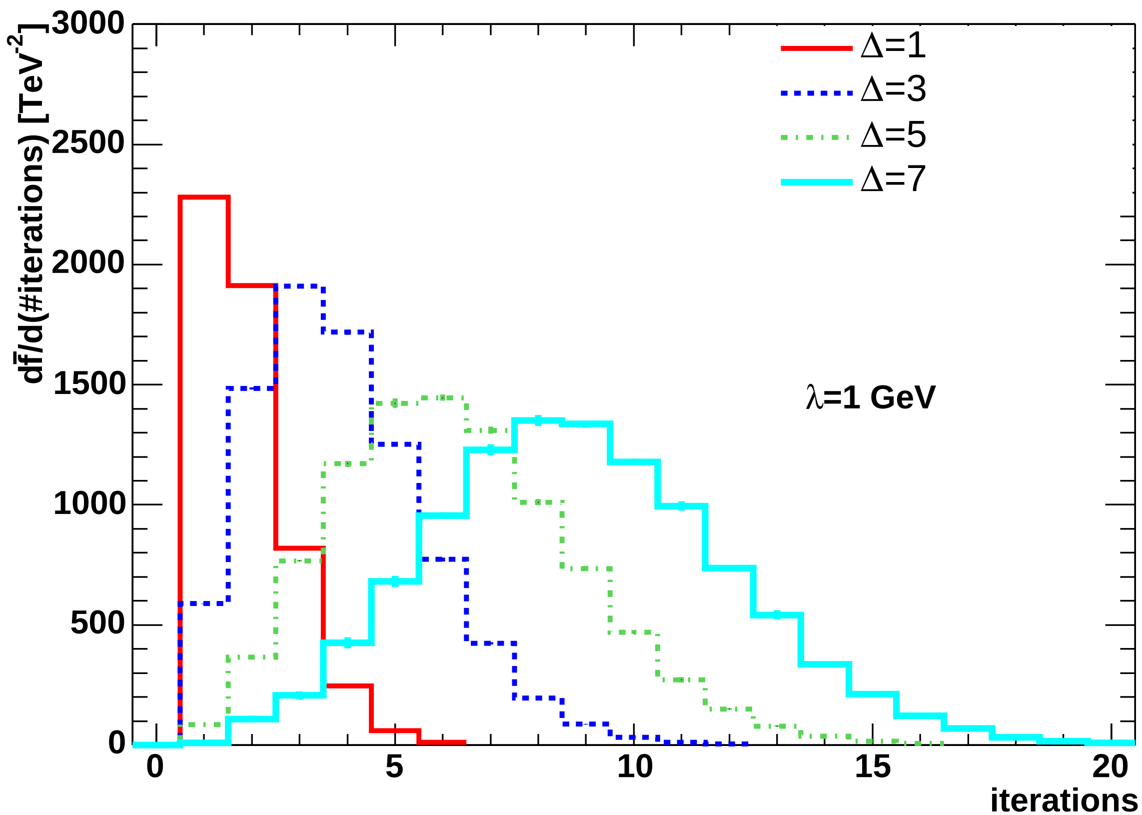
Convergence

$$\bar{f}\,(k_a, k_b, \Delta) \;\; = \;\; \int_0^{2\pi} d\theta\, f\,(k_a, k_b, \theta, \Delta)\,,$$

$k_a = 25$ GeV, $k_b = 30$ GeV, $\lambda = 1$ GeV



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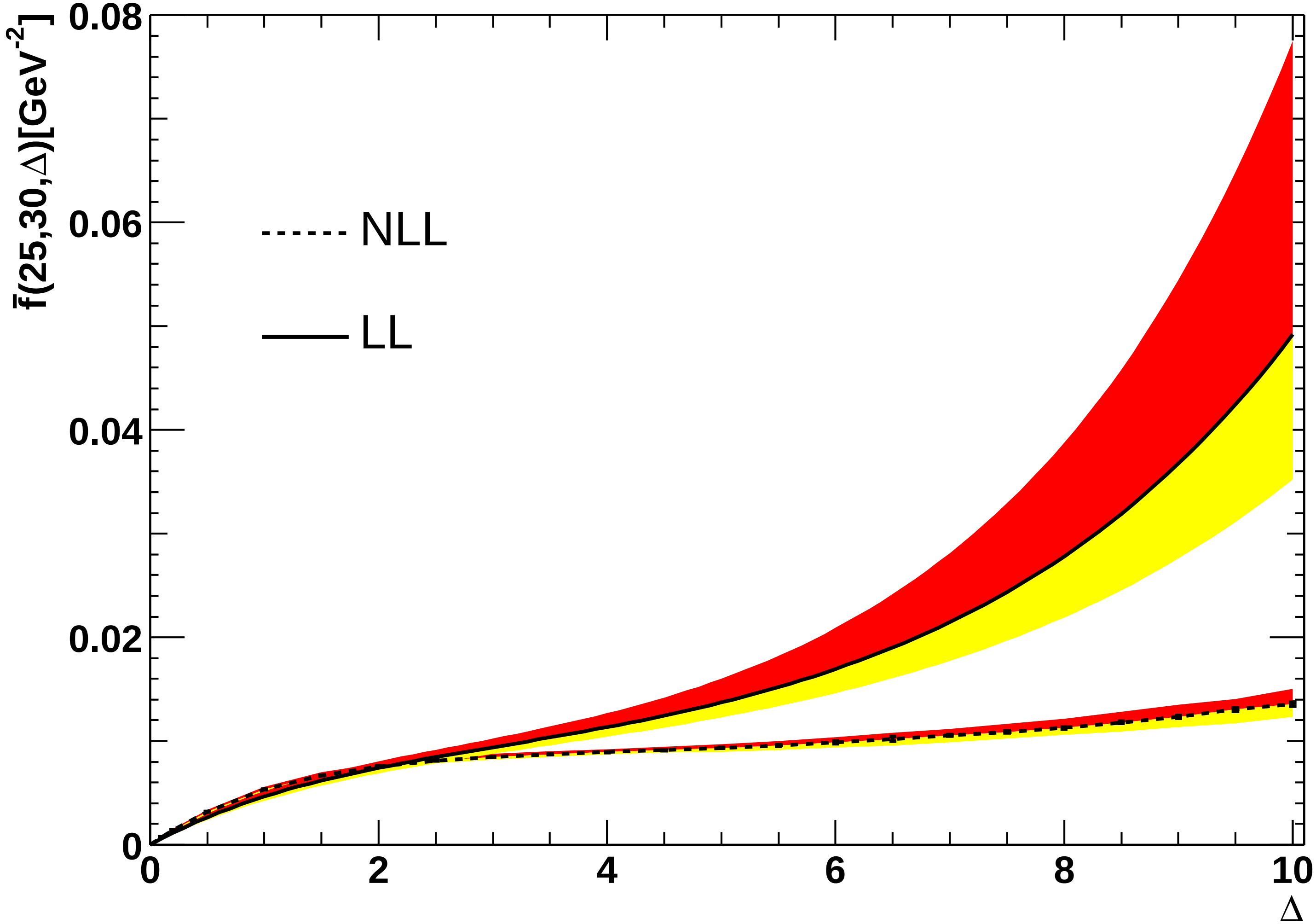
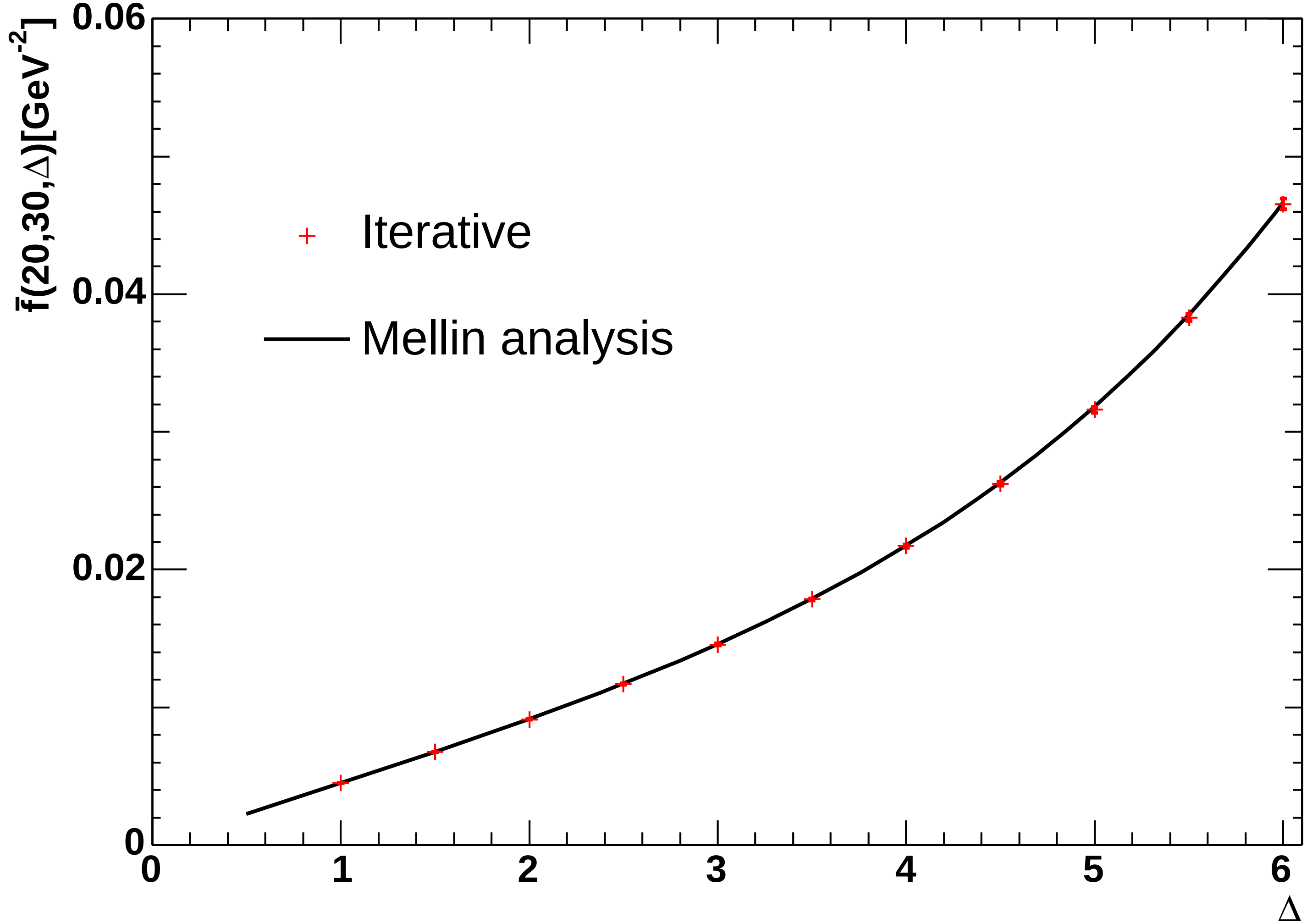
Dependence of f on Δ

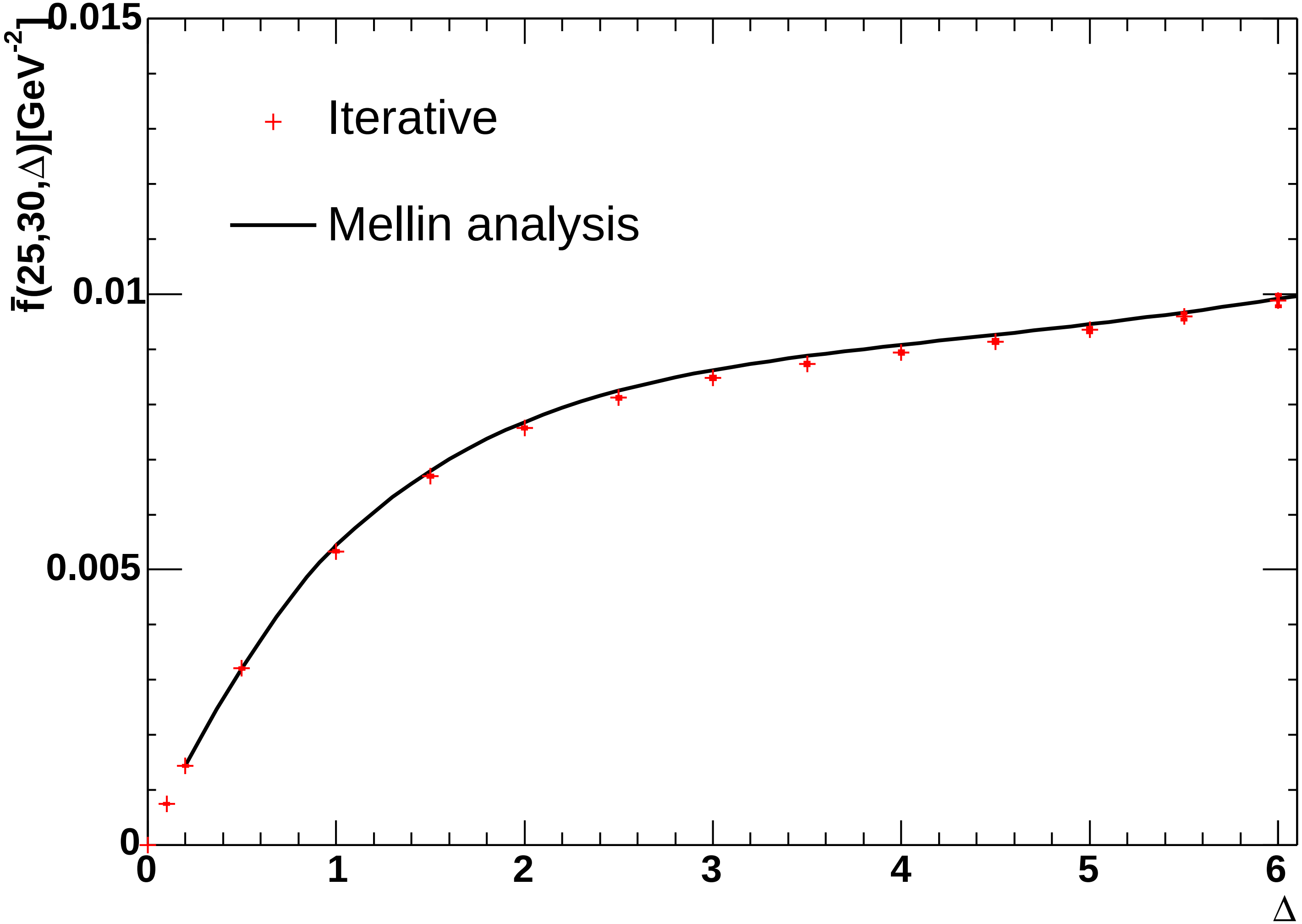


$N = 4$ SYM

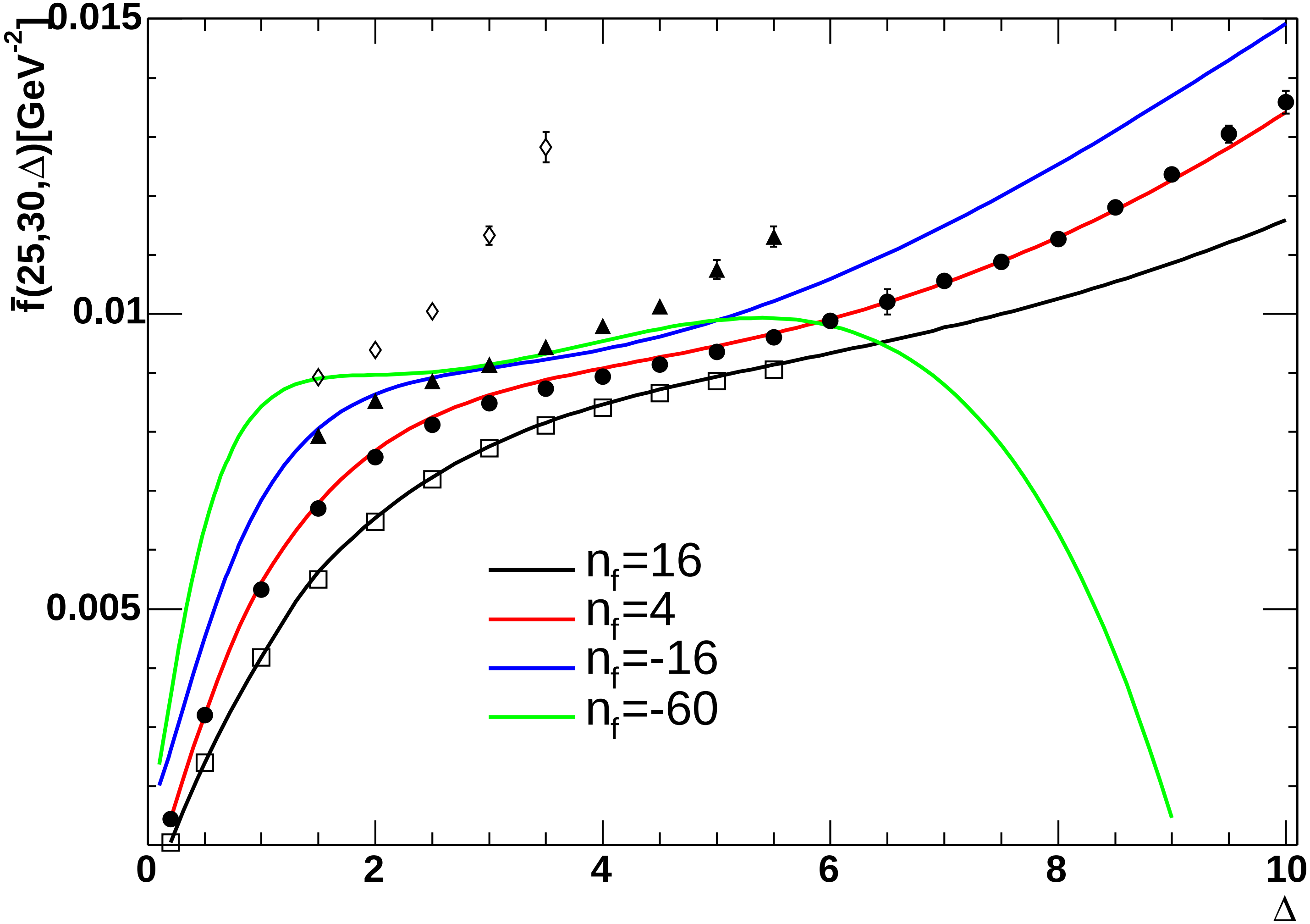


$\bar{f}(20,30,\Delta)$[GeV^{-2}]
0.06
0.04
0.02
0
0
1
2
3
4
5
6
Δ
Iterative
Mellin analysis

Comparison with Mellin Analysis



Comparison with Mellin Analysis



Conclusion

- Theoretical understanding still work in progress (to be discussed during this workshop)
- Formalism general enough to incorporate later changes/resummations etc.
- Full event information at NLL (to be done)
- Predictions for experiments . . .