Overview of small x theory

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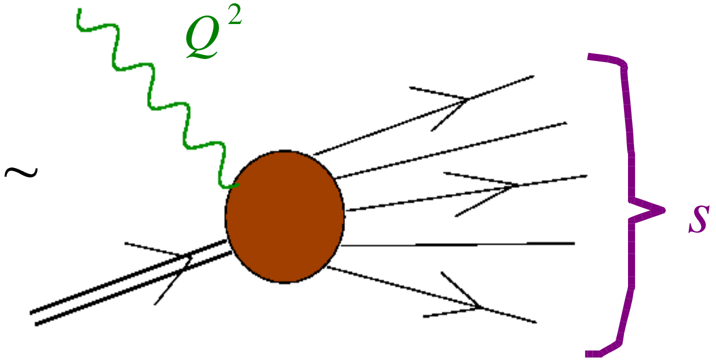
High energy kinematics

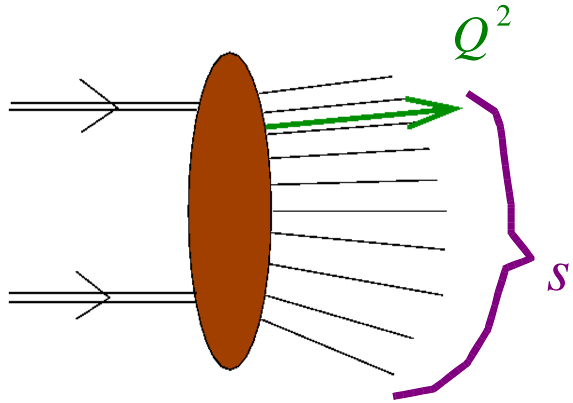
HERA

$F_2 \sim$



Tevatron / LHC

$\frac{d\sigma_{pp}}{d^2 Q} \sim$



$s \gg Q^2 \gg \Lambda^2$

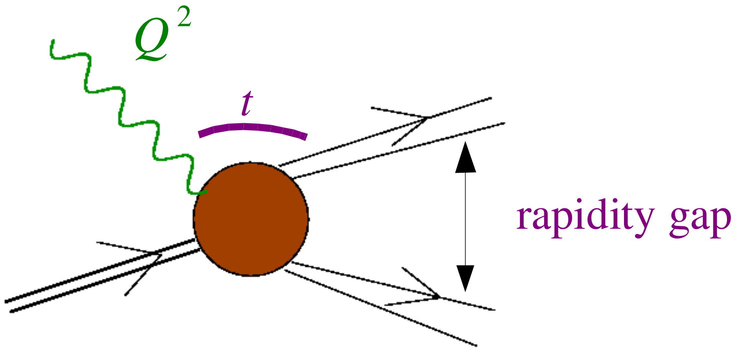
$x = \frac{Q^2}{Q^2 + s} \ll 1 \qquad [\ Y = \log(1/x)\ ]$

$\alpha_s(Q^2) \ll 1 \qquad \alpha_s \log(1/x) \sim 1$

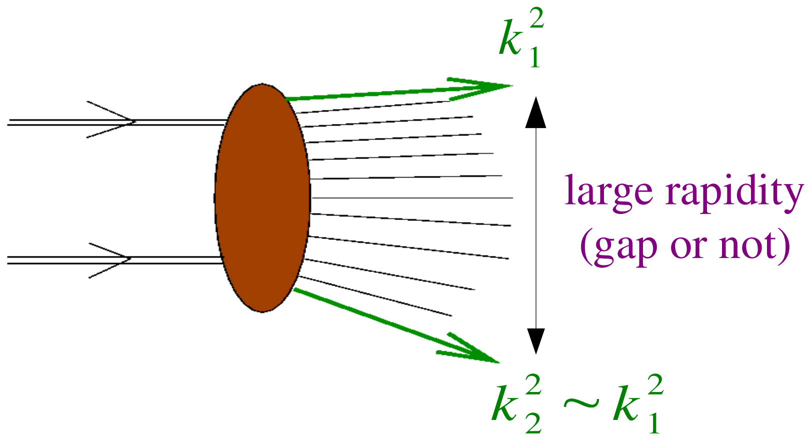
DGLAP works well for total cross sections in DIS at small x, **BUT**:

- ✗ not justified theoretically: one should resum $\sum c_n \left(\alpha_s \log(1/x)\right)^n$
- ✗ fails to describe some observables in DIS and pp
- ✗ some specific small-x phenomena have no natural formulation

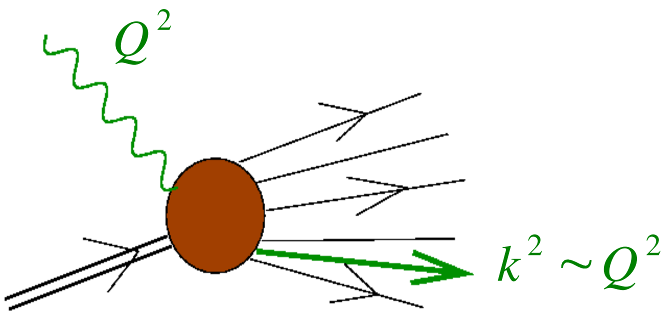
Specific small-x phenomena



Hard diffraction



Mueller-Navelet jets



Forward jet

A specific treatment of the small-x regime is needed!

Outline

- Collinear versus kt factorization. BFKL, CCFM
- The dipole factorization
- Non linear evolution: BK equation, color glass condensate
- Phenomenology of the dipole model

Main focus on HERA physics

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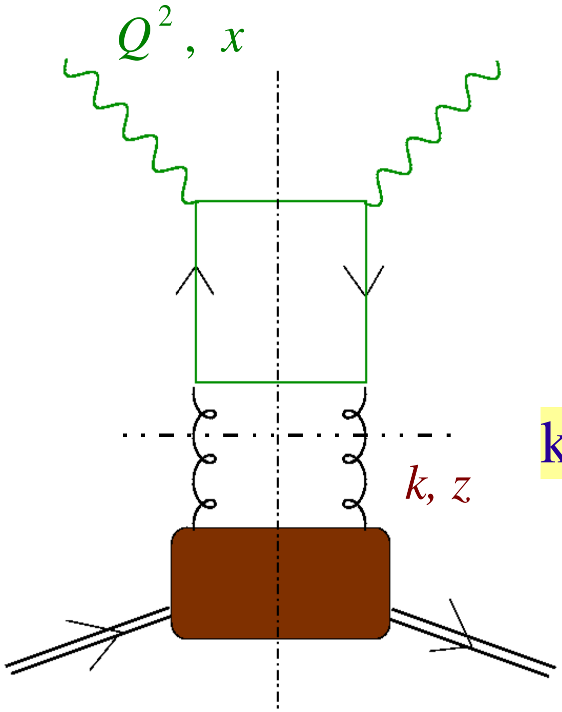
Collinear versus kt-factorization



Collinear factorization

$Q^2 \gg k^2 \simeq 0$

$$\sigma(x, Q^2) \sim \int \frac{dz}{z} \, C\left(\frac{x}{z}\right) \, f(z, Q^2)$$

coefficient function

gluon density
obeys DGLAP

kt factorization

$Q^2 \sim k^2 \quad , \quad s \gg Q^2$

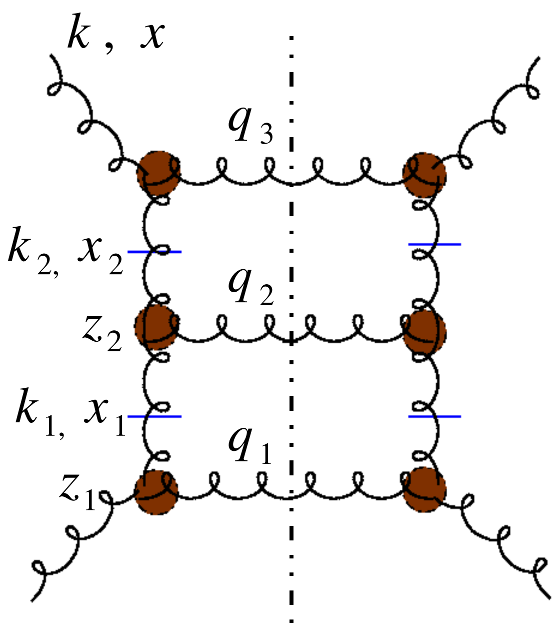
$$\sigma(x, Q^2) \sim \int \frac{dz}{z} \int^{Q^2} d^2 k \; \hat{\sigma}\left(\frac{x}{z}, k^2\right) \, \mathcal{F}(z, k^2)$$

off-shell photon-gluon
matrix element

unintegrated gluon density
obeys BFKL

Catani, Ciafaloni, Hautmann;
Collins, Ellis;
Levin, Ryskin, Shabelski, Shuvaev (1991)

The BFKL equation



Balitsky, Fadin, Kuraev, Lipatov (1977)

Resummation of the terms relevant at small x in the perturbation series

$$\mathcal{F}(x,k^2) = \sum a_n(k)\big(\alpha_s \log(1/x)\big)^n + \sum b_n(k)\alpha_s\big(\alpha_s \log(1/x)\big)^n + \ldots$$

leading order next-to-leading order

$$\bar{\alpha} = \frac{\alpha_s N_c}{\pi}$$

Result at leading order, exclusive form:

$$\mathcal{F}(x,k^2) = \sum_{n=0}^{\infty}\left(\prod_{i=1}^{n}\int_{\mu^2}^{k^2}\frac{d^2 q_i}{\pi\, q_i^2}\int\frac{\bar{\alpha}}{z_i}\ \exp\big(-\bar{\alpha}\log(1/z_i)\log(k_i^2/\mu^2)\big)\right)\delta^2\Big(k - \sum q_i\Big)$$

splitting function virtual corrections

Predictions:

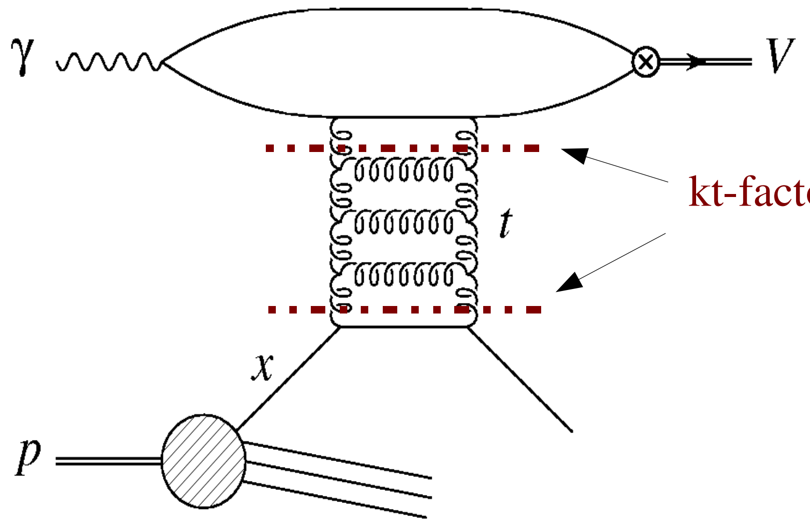
- $\mathcal{F}(x,k^2) \sim x^{-\bar{\alpha}\times 4\,\log 2} \sim x^{-0.5}$ if taken literally, ruled out by the HERA data
- number of final state gluons $\propto \bar{\alpha}\log(1/x)$

The non-forward BFKL equation

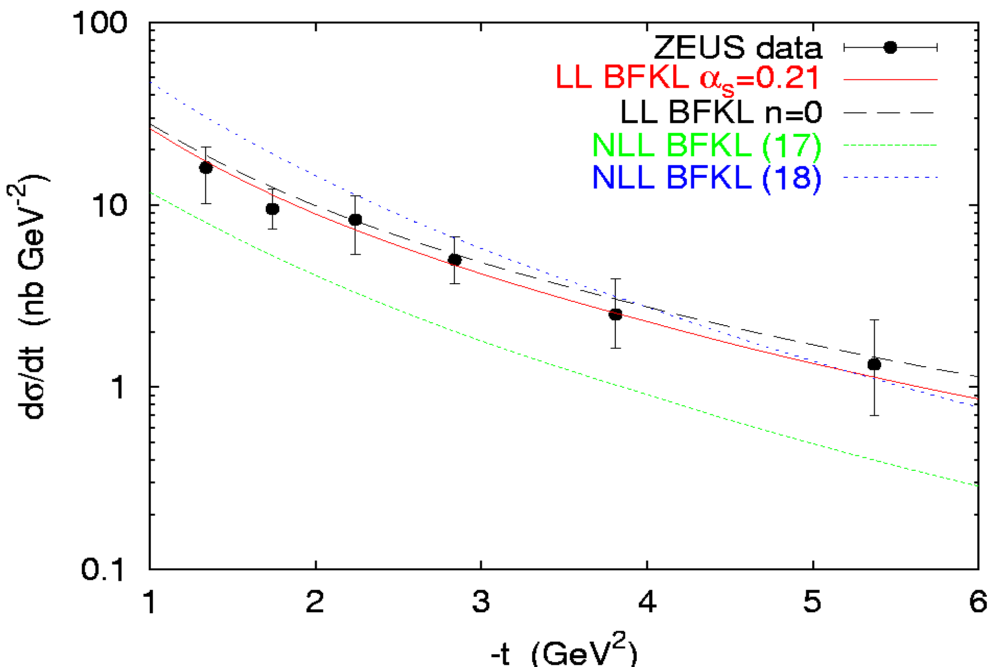
BFKL provides the elastic gluon-gluon scattering amplitude for any *t*

Photoproduction of vector mesons

Enberg, Motyka, Poludniowski (2002)



kt-factorized impact factors



t provides the hard scale

Beyond leading log BFKL

Several phenomenological improvements:

✔ Exact kinematics of gluon emission Kwiecinski, Martin, Stasto (1997)

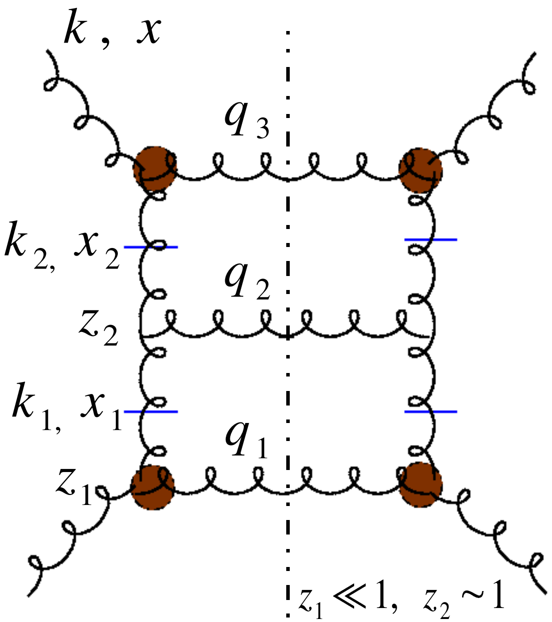
✔ Running coupling effects Collins, Kwiecinski (1989); Mueller, Kovchegov (1998)...

Full next-to-leading order BFKL kernel computed Fadin, Lipatov; Camici, Ciafaloni (1998)

- Corrections are **HUGE**!!! First, looked inconsistent [oscillating cross sections]
- now better understanding Salam; Ciafaloni, Colferai, Salam; Ball, Forte; Brodsky, Lipatov et al (1999)...
- predicted x-dependence agrees with the HERA data $\mathcal{F}(x, k^2) \sim x^{-0.2}$
- sound phenomenology under way Ciafaloni, Colferai, Salam, Stasto; Altarelli, Ball, Forte (1999-...)

The CCFM approach

Ciafaloni; Catani, Fiorani, Marchesini (1987-1990)



Interpolation between BFKL and DGLAP

Includes *consistently*:

- emission of soft gluons
- angular ordering of final state gluons

Sudakov form factor
Angular ordering of emitted gluons

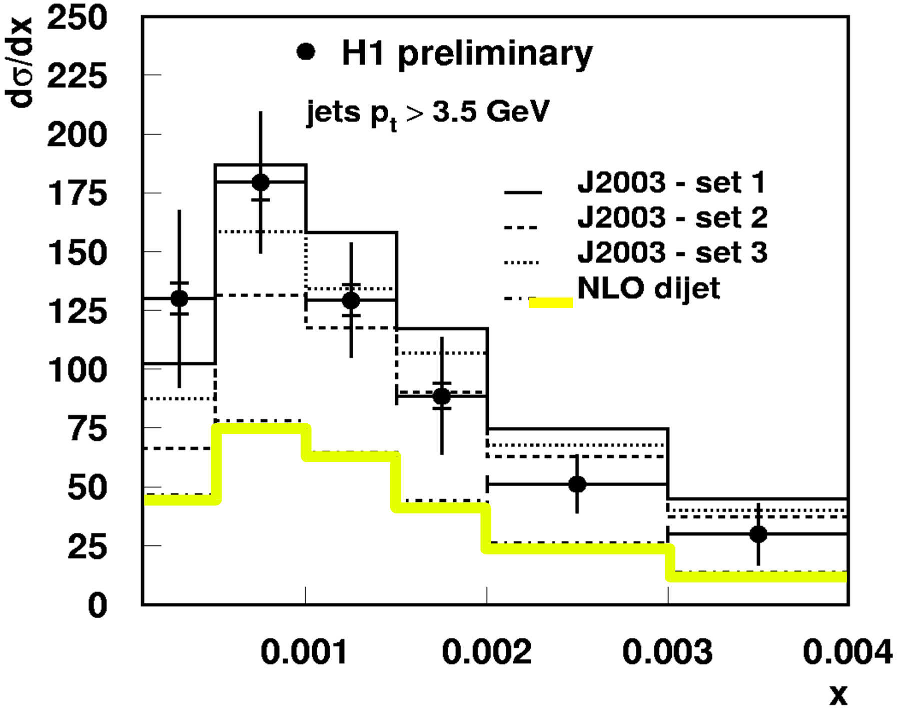
improved splitting function: $\bar{\alpha}\left(\frac{1}{z_i}+\frac{1}{1-z_i}\right)$

$$\mathcal{F}(x,k^2)=\sum_{n=0}^{\infty}\left(\prod_{i=1}^{n}\int_{\mu^2}^{k^2}\frac{d^2q_i}{\pi q_i^2}\int \cancel{\frac{\bar{\alpha}}{z_i}}\ \exp\left(-\bar{\alpha}\log(1/z_i)\log(k_i^2/\mu^2)\right)\right)\delta^2\left(k-\sum q_i\right)$$

Forward jets at HERA

Jung, Salam (2001)

Jung (2003)



Monte-carlo event generator **CASCADE**:

CCFM + off-shell matrix element
+ hadronization

CCFM works well while collinear factorization fails!

Reasonable description also of *c*, *b* production at the Tevatron

➡ Promising tool to study the hadronic final state!

More on BFKL, CCFM phenomenology in

« Small x phenomenology: summary and status »

- Andersson, Baranov, Bartels, Ciafaloni, Collins, Davidsson, Gustafson, Jung, Jonsson, Karlsson, Kimber, Kotikov, Kwiecinski, Lonnblad, Miu, Salam, Seymour, Sjostrand, Zotov (2002)

- Andersen, Baranov, Collins, Dokshitzer, Goerlich, Grindhammer, Gustafson, Jonsson, Jung, Kwiecinski, Levin, A. Lipatov, Lonnblad, Lublinsky, Maul, Milcewicz, Miu, G. Nowak, Sjostrand, Stasto, Timneanu, Turnau, Zotov (2003)

Outline

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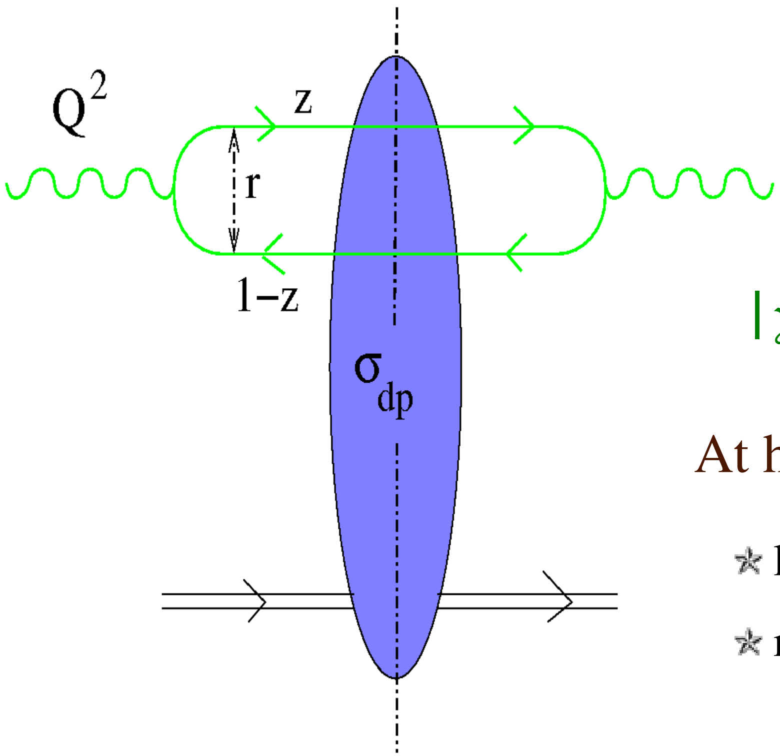
The dipole factorization

Nikolaev, Zakharov;
Mueller (1994)



longitudinal momentum fraction

$$|\gamma> = |\gamma>_{bare} + \int d^2 r \, dz \;\; \psi(z,r) |z, r> + \ldots$$

At high energy:

- lifetime of quark pair much larger than interaction time
- motion in transverse plane frozen

$$\boxed{\sigma(Y, Q^2) = \int d^2 r \int dz |\psi(z,r)|^2 \sigma_d(Y, r, \cancel{b})}$$

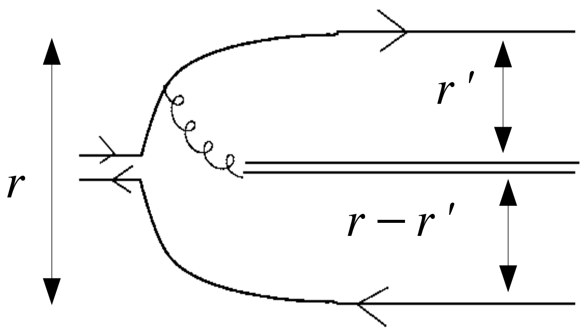
For a dipole target, leading order cross section: $\sigma_d(r_1, r_2) = 2\pi \alpha_s^2 r_<^2 \left(1 + \log \frac{r_>}{r_<}\right)$

hadronic target of size R: $= 2\pi R^2 N(Y, r)$

$N \leq 1$ from unitarity

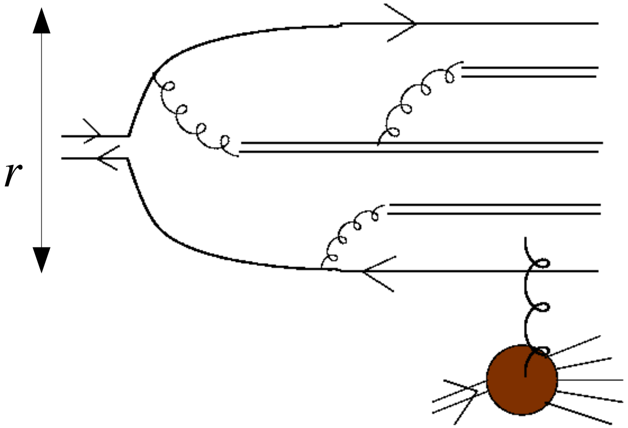
QCD evolution

Mueller (1994)



- Go to the rest frame of the target
- With increasing energy, higher Fock states are accessible
- Large $N_c \Rightarrow$ Fock state expanded on dipole basis

$$|r\rangle = |r\rangle_{bare} + \int d^2 r \;\; \psi_1(r')\,|r-r', r'\rangle + \ldots$$



- Each dipole becomes the seed of a new *independent* evolution

[leading log(1/*x*)]

$\Rightarrow$ dipole number density $\;\; n(Y) \sim e^{\lambda Y}$

amplitude: $\;\; N(Y, r) = \int d^2 r' \;\; n(Y, r') \;\; \sigma_d(r')$

- Detailed calculation: n and N obey the BFKL equation

$$\partial_Y N(Y,r) = \underbrace{\frac{\bar{\alpha}}{2\pi} \int d^2 r' \frac{r^2}{r'^2 (r-r')^2}}_{\text{branching probability}} \Big(\underbrace{N(Y,r') + N(Y,r-r')}_{\text{interaction of the newly created dipoles}} - \underbrace{N(Y,r)}_{\text{destroyed dipoles}} \Big)$$

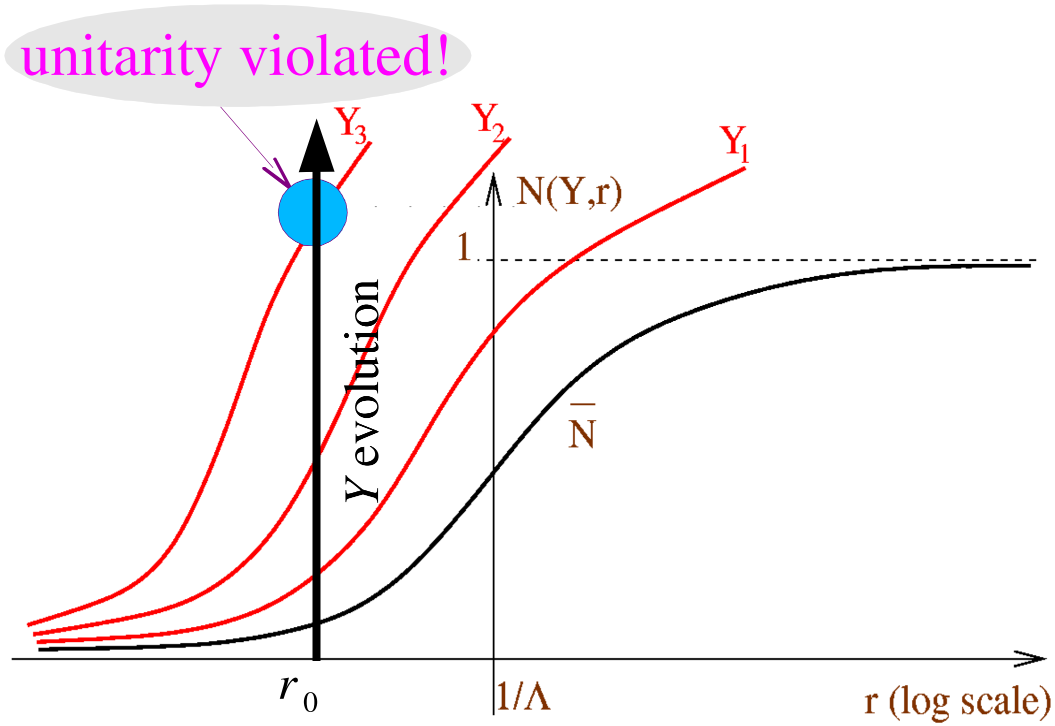
At this level:

$\boxed{\text{dipole model} \quad \Leftrightarrow \quad \text{kt factorization}}$

Navelet, Wallon (1998)

Unitarity violations

$$\text{Interaction probability} = |1 - N(Y,r)|^2 \;\Rightarrow\; N(Y,r) \leq 1$$



What's wrong?

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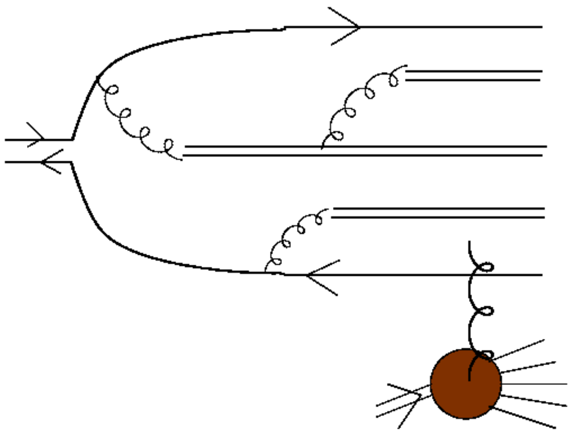
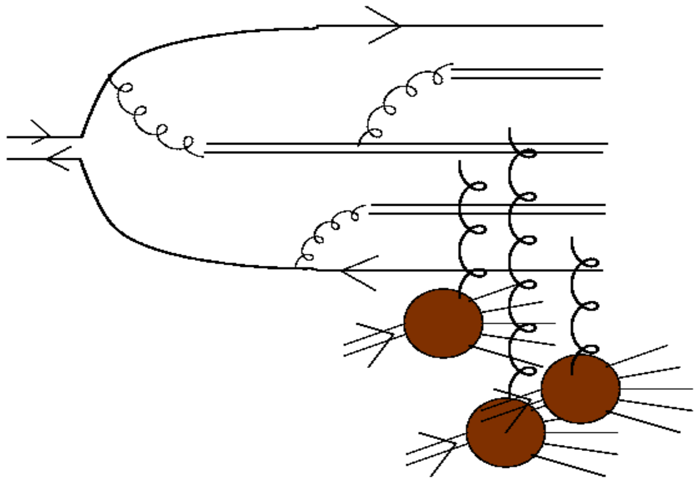
Going to higher energies: the BK equation

Gribov, Levin, Ryskin (1981); Mueller, Tang (1986)

Balitsky (1996); Kovchegov (1999)



BFKL: a single dipole interacts with the target



At higher energy:
all dipoles may interact simultaneously

Now the amplitude obeys a *nonlinear* evolution equation:

$$\partial_Y N(Y,r) = \frac{\bar{\alpha}}{2\pi}\int d^2r' \frac{r^2}{r'^2(r-r')^2}\Big(\underbrace{N(Y,r') + N(Y,r-r') - N(Y,r)}_{\text{BFKL}} \underbrace{- N(Y,r')N(Y,r-r')}_{\text{tames the growth when } N \text{ of order 1}}\Big)$$

Established only when the dipole interactions are ***independent!***

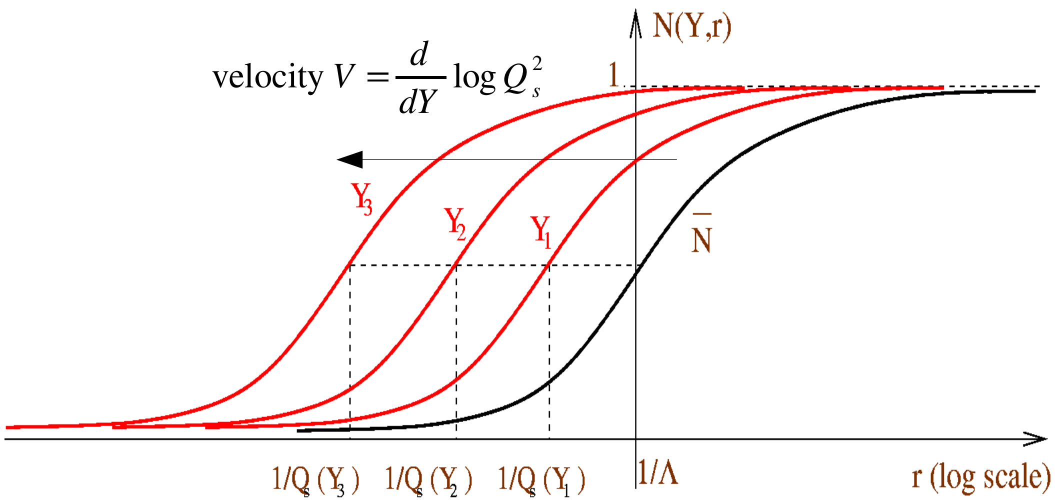
Solving BK (I)

Asymptotic solution: traveling waves

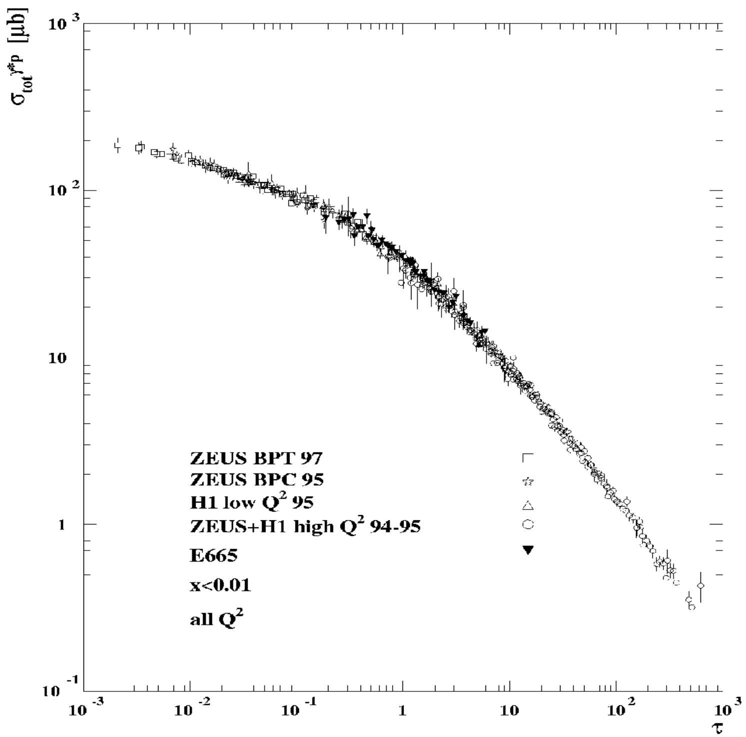
S.M., Peschanski (2003)

$$N(Y,r) = N\left(\log r^2 - \log 1/Q_s^2(Y)\right)$$

Also seen on numerical solutions:
Braun (2000); Golec-Biernat, Motyka, Stasto (2002);
Albacete, Armesto, Kovner, Salgado, Wiedemann (2003)



$Q_s(Y) =$ saturation scale



Geometric scaling

$$\Rightarrow \quad \sigma^{\gamma p}\left(Q^2/Q_s^2\right)$$

Stasto, Golec-Biernat, Kwiecinski (2000)

Solving BK (II)

Mueller, Triantafyllopoulos (2002)
S.M., Peschanski (2004)

- Wave selected at large Y has velocity

$$V = \frac{d}{dY} \log Q_s^2 = \bar{\alpha} \frac{\chi(\gamma_0)}{\gamma_0} - \frac{3}{2\gamma_0} \frac{1}{Y} + \frac{3}{\gamma_0^2} \sqrt{\frac{\pi}{2\,\bar{\alpha}\,\chi''(\gamma_0)}} \frac{1}{Y^{3/2}} + \ldots$$

$\chi(\gamma) =$ Mellin transform of BFKL kernel

$\gamma_0 \simeq 0.63$ solves $\frac{\chi(\gamma_0)}{\gamma_0} = \chi'(\gamma_0)$

Gribov, Levin, Ryskin (1981)

- Deep in the saturation region: Levin-Tuchin law

Levin, Tuchin (2000)

$$N(Y, r) = 1 - \exp\left(-c \log^2(r^2 Q_s^2(Y))\right)$$

equivalent to saturation of the gluon density

$$\mathcal{F}(x, k) \sim 2\pi R^2 \frac{1}{\alpha_s} \log \frac{Q_s^2}{k^2}$$

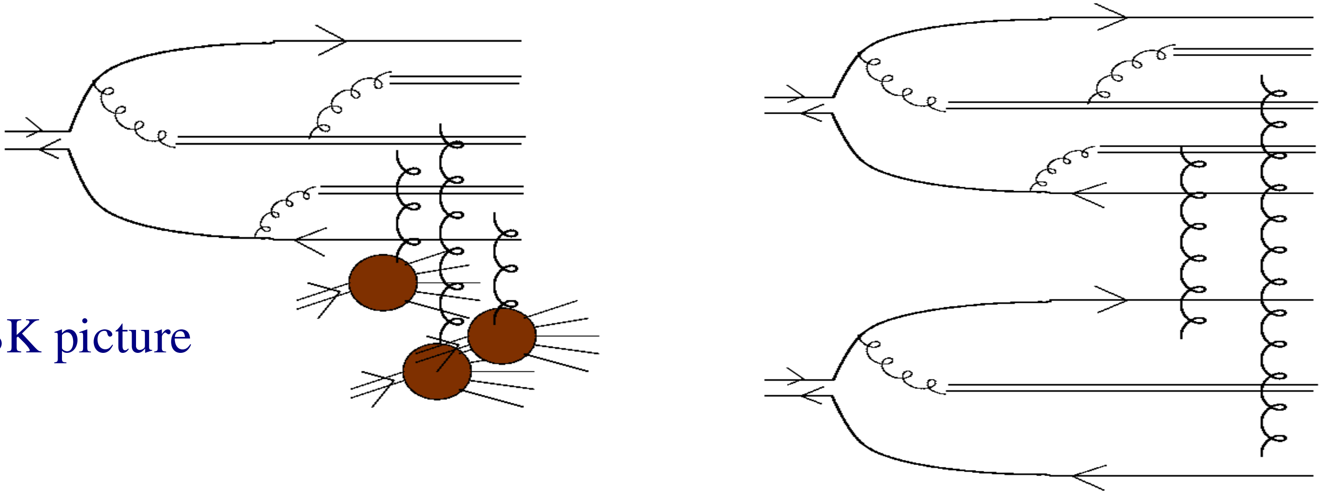
- Finite Y, near the saturation scale:

$$N(Y, r) = N_0 \times \underbrace{\left(r^2 Q_s^2(Y)\right)^{\gamma_0} \log\left(\frac{1}{r^2 Q_s^2}\right)}_{\text{geometric scaling}} \underbrace{\exp\left(\frac{-\log^2(r^2 Q_s^2)}{2\,\bar{\alpha}\,\chi''(\gamma_0) Y}\right)}_{\text{diffusion-type violations}} + \ldots$$

Color glass condensate

Balitsky (1996)

« JIMWLK »:Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1994-2002-...)



BK picture

The assumption that dipoles interact independently is in general **not justified**

$$\partial_Y N(Y,r)=\frac{\bar{\alpha}}{2\pi}\int d^2 r' \frac{r^2}{r'^2 (r-r')^2}\left(N(Y,r')+N(Y,r-r')-N(Y,r)-N_2(Y,r',r-r')\right)$$

$$\neq N(Y,r')N(Y,r-r')$$

$\partial_Y N_2(Y, r_1, r_2) = \ \ldots \ N_3(Y, r_1, r_2, r_3) \ \ldots \ +$ more complicated color structures beyond dipoles

... infinite hierarchy of coupled equations

Numerical solutions: Salam (1995); Weigert, Rummukainen (2003)
Analytical approach: Iancu, Mueller (2003); Mueller, Shoshi (2004)

BK is only a mean field approximation; could be very bad!

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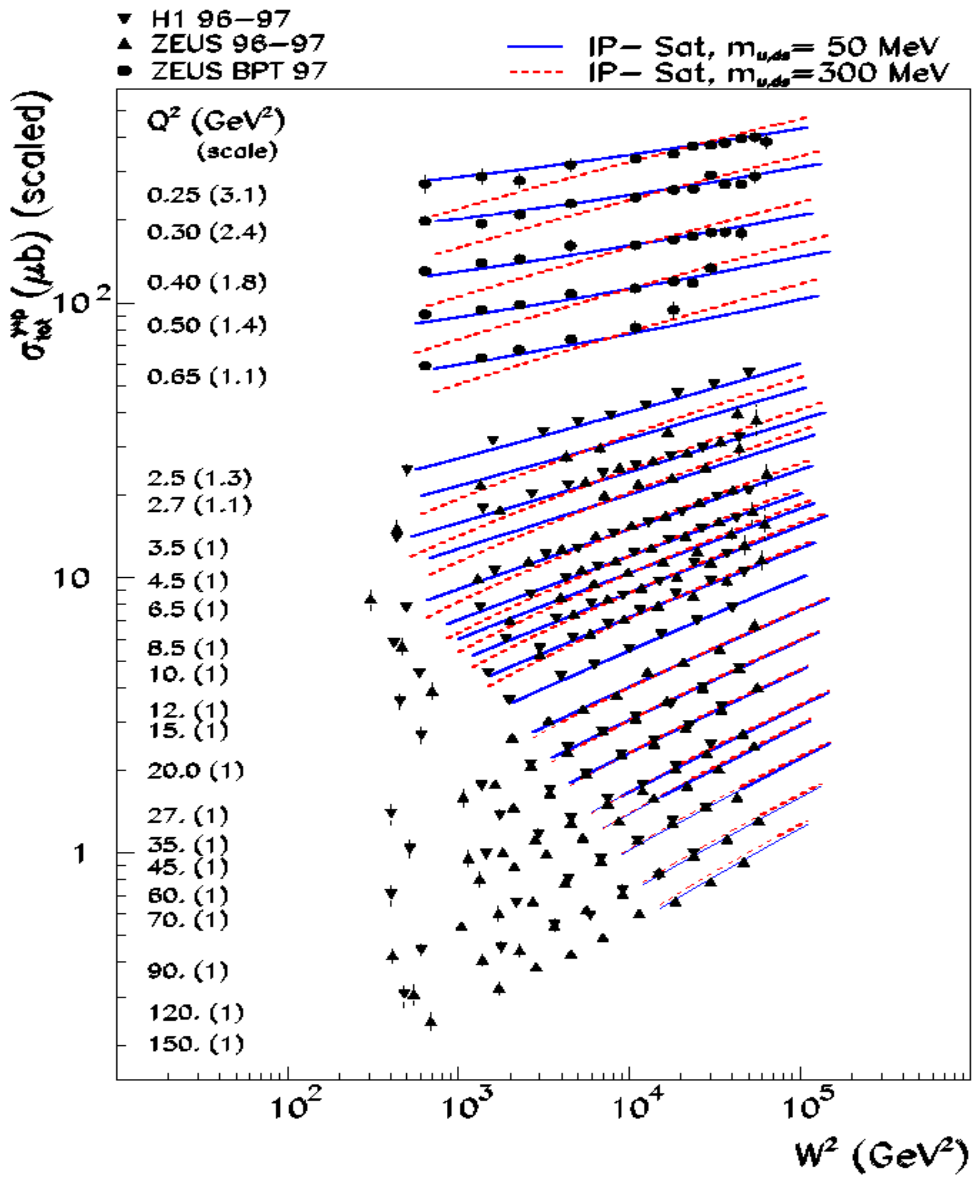
Phenomenology of the dipole model (I)

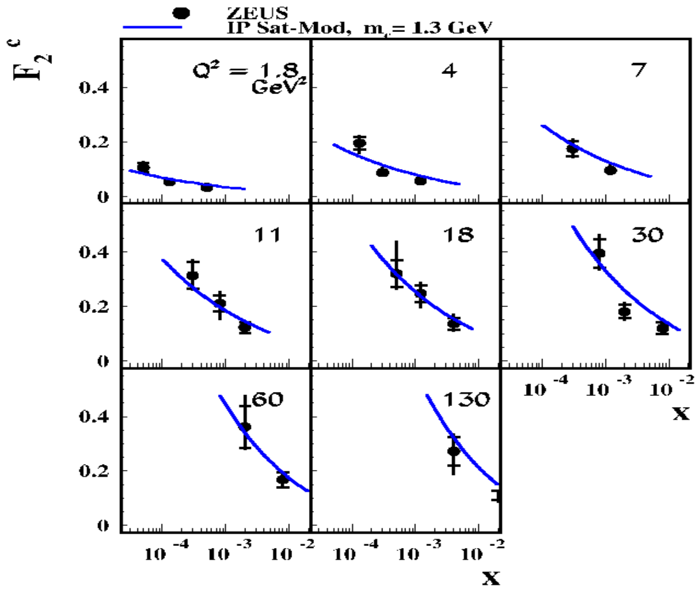
Golec-Biernat-Wüsthoff model: Golec-Biernat, Wusthoff (1999)

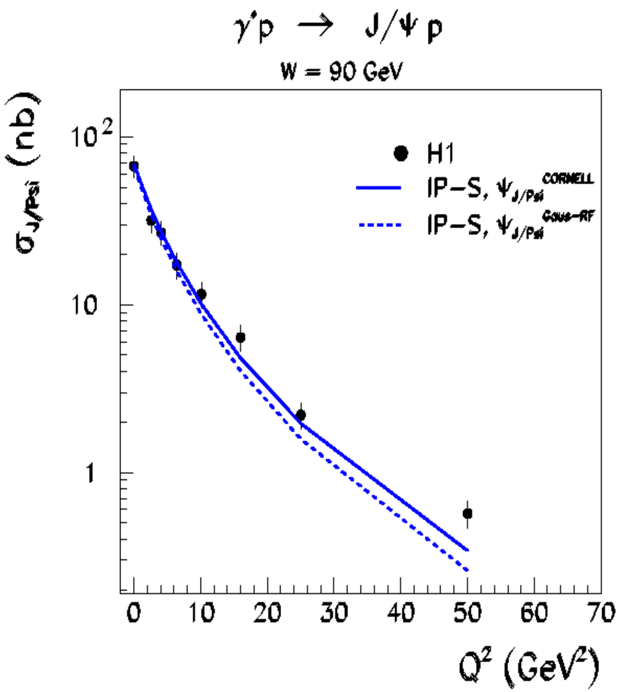
$$N(Y,r) = 1 - \exp\left(-\frac{r^2 Q_s^2(Y)}{4}\right) \qquad Q_s^2(Y) = \left(\frac{x}{x_0}\right)^{-\lambda} \text{GeV}^2$$

- Economical 3-parameter description of small x data, inclusive and diffractive
- Saturation scale at 1 GeV at HERA: a perturbative scale?
- Many recent improvements
  - ✔ DGLAP corrections Bartels, Golec-Biernat, Kowalski (2002)
  - ✔ Impact parameter dependence Kowalski, Teaney (2002)
  - ✔ Other similar proposals, better rooted in QCD Iancu, Itakura, S.M. (2003)
  - ✔ Phenomenology from numerical solutions of BK Levin, Lublinsky et al (2002)

The Kowalski-Teaney model (2003)

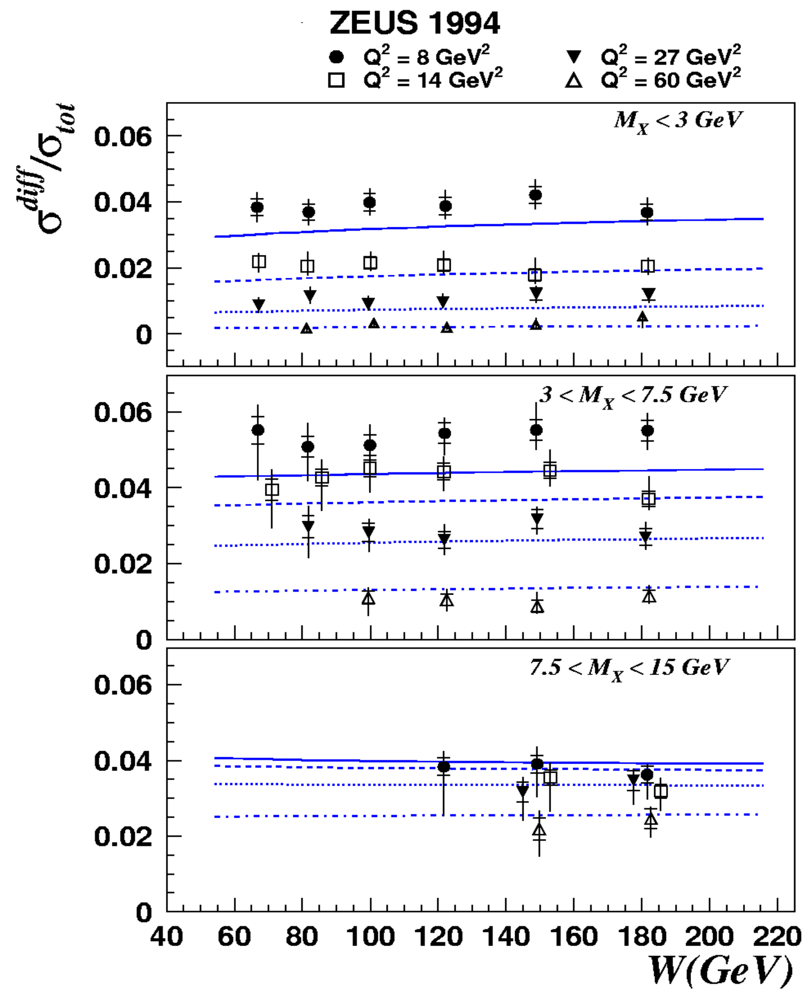






Phenomenology of the dipole model (II)

What is it good for?



Provides a natural formulation of diffraction [Good-Walker picture]

See e.g. S.M., Shoshi (2004)

First model to predict a constant diffractive/total ratio

See also Kovchegov, Levin (1999); Levin, Lublinsky (2001)

Also: dipole cross section is **universal**

- inclusive DIS, diffraction
- forward jets in pp...
- sufficiently inclusive observables

Summary

Various approaches to small x physics with different applications:

BFKL:

- ✔ total cross sections, when energy not too high.
- ✔ phenomenology of NLL in progress!
- ✗ but unitarity corrections have no « natural » formulation

CCFM:

- ✔ hadronic final state, both HERA and Tevatron/LHC

Dipoles:

- ✔ inclusive, diffractive, semi-inclusive observables
- ✔ unitarity corrections are incorporated in a natural way
- ✔ nice phenomenology
- ✗ but cannot describe exclusive final state!

Color glass condensate:

- ✔ more systematic treatment of unitarity corrections
- ✔ beyond large number of colors
- ✔ unifying approach to different processes: DIS, pp, pA, AA

<u>Still to be understood:</u>
impact parameter dependence, solution to the full JIMWLK equation...

Look for clear signatures of saturation at HERA, RHIC, LHC!