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Soft-Collinear Effective Theory: overview and applications

Enrico Lunghi
(University of Zurich)

E.L., D. Pirjol, D. Wyler, Nucl. Phys. B649 (2003) 349, [hep-ph/0210091]
A. Hardmeier, E.L., D. Pirjol, D. Wyler, Nucl. Phys. B, [hep-ph/0307171]

★ $B\to\gamma e\nu$, $B\to\gamma\gamma$, $B\to\gamma ee$, $B\to(\rho,K^*)\gamma$, $B\to K^{(*)}e^+e^-$
$B\to D^{(*)}\pi$, $B\to\pi\pi$, $B\to K\pi$, $B\to(\pi,\rho)e\nu$, ...

★ DIS, Drell-Yan, Jet production, ...

energetic particles
($Q\gg\Lambda_{QCD}$)

QCD-improved factorisation (BBNS)

[Beneke, Buchalla, Neubert, Sachrajda]

Proof of factorisation requires the analysis of IR divergencies of Feynman diagrams order by order in perturbation theory

Tools: Identification of the Regions & Threshold Expansion

Assuming tools are correct, one can systemise BBNS using an EFT approach:

Soft Collinear Effective Theory (SCET)

[Bauer, Fleming, Luke, Pirjol, Stewart;
Beneke, Chapovsky, Diehl, Feldmann; Hill, Neubert]

Basic idea

The relevant scales are: m_W^2 m_b^2 $\Lambda_{QCD} m_b$ perturbative (integrate out) — Λ_{QCD}^2 non-perturbative

- If we supplement the integration of perturbative modes with an expansion in Λ_{QCD}/m_b (built in in T.E.) we obtain (not always) amplitudes factorised in terms of simple objects (Form Factors and Light-cone wave functions)

- $m_W \gg m_b$: $$A(B\to X)=\langle X|H_{\text{eff}}|B\rangle=\sum_i \underbrace{C_i(\mu_b)}_{\text{SD}}\underbrace{\langle X|O_i(\mu_b)|B\rangle}_{\text{LD}}$$

- $m_b \gg \Lambda_{QCD}$: scales are of the same order of the external momenta

$$m_b \longrightarrow \sqrt{\Lambda_{QCD} m_b} \longrightarrow \Lambda_{QCD}$$

Wilson coefficients — Jet functions — Form Factors, Wave functions

- $m_b \gg \sqrt{\Lambda_{QCD} m_b} \gg \Lambda_{QCD}$

  - The scales m_b^2 and $\Lambda_{QCD} m_b$ are perturbative:

$$\langle X | O(\mu_b) | B \rangle = C(\sqrt{\Lambda_{QCD} m_b})\; J(\Lambda_{QCD}) * \langle X | \overline{O}(\Lambda_{QCD}) | B \rangle + O(\frac{\Lambda_{QCD}}{m_b})$$

WC's	Jet functions	Form Factors Wave functions	Power Corrections !!

- The effective theory apporach (SCET) is as rigorous as the standard effective hamiltonian approach

- The Λ_{QCD}/m_b expansion is necessary in order to be able to write the effective theory at all

- For some decays the matrix elements $\langle X | \overline{O}(\Lambda_{QCD}) | B \rangle$ are expressed in terms of light-cone wave functions and/or obey symmetry relations

Basic kinematics

Energetic light particles in B decays move close to the light cone:

- the pion in $B \to D\pi$
- the mesons in $B \to (K^{(*)}, \rho)\gamma$
- the X_s system close to the photon energy endpoint in $B \to X_s \gamma$
- the spectator quark struck by the photon in $B \to \gamma e \bar{\nu}$

We use light-cone coordinates: $n = (1,0,0,1),\ \bar{n} = (1,0,0,-1)$

$$p^\mu = \bar{n}\cdot p \frac{n^\mu}{2} + n\cdot p \frac{\bar{n}^\mu}{2} + p_\perp^\mu \equiv p^- \frac{n^\mu}{2} + p^+ \frac{\bar{n}^\mu}{2} + p_\perp^\mu = (p^-, p^+, p_\perp)$$

$$p^2 = p^- p^+ - p_\perp^2$$

$$p = Q(1, \lambda^2, \lambda) \ \text{ with } \ Q \sim O(m_b) \ \text{ and } \ \lambda \ll 1$$

soft quarks struck by photons: $p = (Q, \Lambda_{QCD}, \sqrt{\Lambda_{QCD} Q}) \Rightarrow \lambda = \sqrt{\frac{\Lambda_{QCD}}{m_b}}$

quarks inside light mesons: $p = (Q, \frac{\Lambda_{QCD}^2}{Q}, \Lambda_{QCD}) \Rightarrow \lambda = \frac{\Lambda_{QCD}}{m_b}$

The modes

- Coleman-Norton theorem: singularities correspond to propagation of modes on mass-shell
- Independent fields describe soft and collinear fluctuations of quarks and gluons

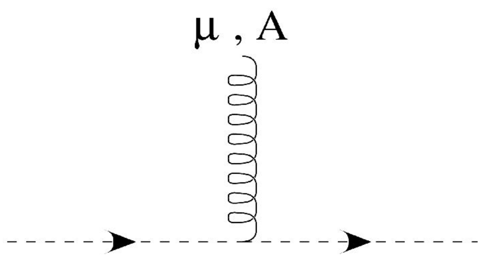
hard	$p_h = (Q, Q, Q)$	$p_h^2 = Q^2$	perturbative
hard-collinear (Ξ, A_{hc})	$p_{hc} = (Q, \Lambda, \sqrt{\Lambda Q})$	$p_{hc}^2 = \Lambda Q$	perturbative
soft (q_s, A_s)	$p_s = (\Lambda, \Lambda, \Lambda)$	$p_s^2 = \Lambda^2$	non-perturbative
collinear (ξ, A_c)	$p_c = (Q, \Lambda^2/Q, \Lambda)$	$p_c^2 = \Lambda^2$	non-perturbative

$(Q \sim m_b, \Lambda = \Lambda_{QCD})$

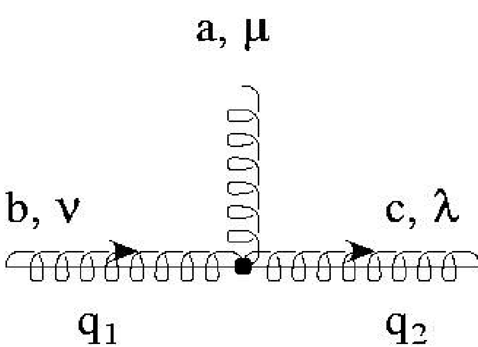
- hard-collinear modes can and, therefore, have to be integrated out
- this cannot be done exactly and one needs to use perturbation theory: they produce the so-called jet-functions
- in inclusive processes they are present as external states (e.g. $B \to X_s \gamma$) but, using dispersion relations, they appear again as internal lines.

Decoupling of soft gluons

- Coupling of soft gluons to collinear quarks and gluons vanish in the light cone gauge



$$= -i\, g\, T^A n_\mu \frac{\hat{\bar{n}}}{2}$$



$$= g\, f^{abc} n_\mu \bar{n}\cdot q_1\, g_{\nu\lambda}$$

- It is possible to remove them from the leading order lagrangian by field redefinitions:

$$\xi_{n,p} \to Y[n\cdot A_s]\, \xi^{(0)}_{n,p}$$
$$A_{n,q} \to Y[n\cdot A_s]\, A^{(0)}_{n,q}\, Y^{+}[n\cdot A_s]$$

with $Y[n\cdot A_s] = P \exp\left[i\, g \int_{-\infty}^{x} d\lambda\, n\cdot A_s(\lambda n_\mu) \right]$

- All ultrasoft effects are moved into external operators:

$$L_c^0[\xi_{n,p}, A_{n,q}, A_s] = L_c^0[\xi^{(0)}_{n,p}, A^{(0)}_{n,q}, 0]$$

Processes without mesons in the final state

- Since there are no mesons in the final state, we need only SCET$_I$

- Up to power corrections we have:

$$A(B\to f) = C_W \langle f|O_{QCD}|B\rangle \to C_W\, C_b \langle f|O_{SCET}|B\rangle = C_W\, C_b \langle f|O^{(0)}_{SCET}|B\rangle$$

decoupling of soft gluons

C_W and C_b are Wilson coefficients that encode contributions from the scales m_W^2 and m_b^2

- If the SCET$_I$ operator, after the decoupling of soft gluons, factorises:

$$A(B\to f) \to C_W\, C_b \langle f|O_C^{(0)}\, O_S^{(0)}|B\rangle = C_W\, C_b \langle f|O_C^{(0)}|0\rangle \langle 0|O_S^{(0)}|B\rangle$$

$$= C_W\, C_b\, J_C * \phi_B + O(\Lambda_{QCD}/m_b)$$

where J_C and ϕ_B encode contributions from the scales $\Lambda_{QCD} m_b$ and Λ^2_{QCD}

Processes with mesons in the final state

- Matching between $SCET_I$ and $SCET_{II}$: $O_{SCET_I} \to O_{SCET_{II}}$
- Off-shell external states and dimensional regularization do not regularize all the IR divergences. This results in the appearence of end-point singularities in the convolution of hard scattering kernels and light cone wave functions.

  ⬇

  - soft-collinear (messanger) mode [Becher, Hill, Neubert]
  - analytic IR regulator [Beneke, Feldmann]
  - IR regulator at the lagrangian level [Bauer, Dorsten, Salem]

- Matching of the SM onto $SCET_I$
- Naive matching of $SCET_I$ onto $SCET_{II}$
- If, for a given operator, end-point singularities appear, the second step of the matching is affected by the details of the IR regulators and the operator leads to a non-factorizable contribution (e.g. soft form factor)
- Predictivity is preserved if one can show that the matrix elements of the non-factorizable $SCET_I$ operator obey symmetry relations.

- Care is required if on-shell charm quarks appear! (e.g. charming penguins)

$$\langle M | O^{(0)}_{SCET_{II}} | B \rangle = \langle M | O^{(0)}_{C} \, O^{(0)}_{S} | B \rangle = \langle M | O^{(0)}_{C} | 0 \rangle \, \langle 0 | O^{(0)}_{S} | B \rangle$$

Need not to be Light-Cone Wave Functions!

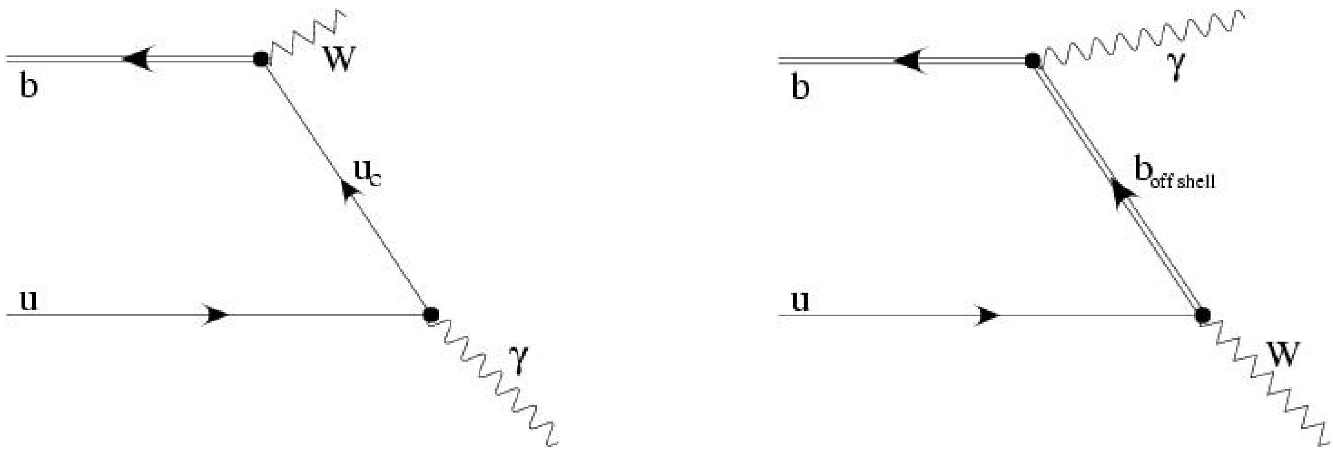
Let us assume that $O^{(0)}_{C}$ contains only 2 collinear quarks

- Write down the most general operators involving 2 collinear quarks at leading and subleading order in Λ_{QCD}/m_b
- Their matrix element between the vacuum and a pseudo-scalar or vector meson are given in terms of Light-Cone Wave Functions of twist-2 and twist-3.
- Whenever twist-3 wave functions are present, it essential to include contributions from higher fock states:
  - numerical impact ~ 10% - 20%
  - important for exact cancellations (Ward identity checks, etc.)

$B \to \gamma e \bar{\nu}$

- The simpler decay $B \to e \bar{\nu}$ is chiral suppressed
- This chiral suppression can be avoided allowing for a final state photon
- The effective Hamiltonian arises at tree level in the SM:

$$H_{\rm eff} = 4 G_F / \sqrt{2}\ V_{ub}\ (\bar{u}_L \gamma^\mu b_L)\ (\bar{e}_L \gamma_\mu \nu_L)$$



- Sensitive to V_{ub}
- Not expected to receive large New Physics contributions (e.g. from SUSY)
- Gives valuable pieces of information on the B meson wave function

Factorisation in $B \to \gamma e \nu$: results

$$A[B^- \to \gamma e \bar{\nu}] = \frac{4G_F}{\sqrt{2}} V_{ub} \langle \gamma | \bar{u} \gamma_\mu P_L b | B^- \rangle (\bar{e} \gamma^\mu P_L \nu)$$

$$\frac{1}{e} \langle \gamma(q,\varepsilon) | \bar{u} \gamma_\mu b | B(\mathrm{v}) \rangle = i \epsilon_{\mu\alpha\beta\delta} \varepsilon^\alpha \mathrm{v}^\beta q^\delta \, f_V(E_\gamma)$$

contact term

$$\frac{1}{e} \langle \gamma(q,\epsilon) | \bar{u} \gamma_\mu \gamma_5 b | B(\mathrm{v}) \rangle = [q_\mu (\mathrm{v}\cdot\varepsilon) - \varepsilon_\mu (\mathrm{v}\cdot q)] \, f_A(E_\gamma) + (\mathrm{v}\cdot\varepsilon) \mathrm{v}_\mu \frac{1}{\mathrm{v}\cdot q} f_B m_B$$

Analysis at all orders in α_s and at leading order in Λ_{QCD}/E_γ

$$\Lambda_{QCD} \ll E_\gamma^c < E_\gamma < \frac{m_B}{2}$$

- Equality of the form factors: $f_V(E_\gamma) = f_A(E_\gamma)$
- Factorization: form factors as a 1-dimensional convolution of perturbative hard scattering kernels and the B meson light cone wave function:

$$f(E_\gamma) = \int d\xi \, T(E_\gamma, \xi) \, \phi_B(\xi) = C(E_\gamma) \int d\xi \, J(E_\gamma, \xi) \, \phi_B(\xi)$$

[Descotes-Genon, Sachrajda: proof at the 1-loop level]
[Lunghi, Pirjol, Wyler; Bosch, Hill, Lange, Neubert: all order proof]

Relevance for QCD

- Use other V_{ub} determinations to extract informations on the B meson wave function

- The tree-level amplitude is proportional to $\lambda_b^{-1} = \int \phi_B(\xi)/\xi$
  The same parameter enter many other B decays:
  $B \to (K^* e\nu,\ \rho e\nu,\ K^*\gamma,\ \rho\gamma,\ \pi\pi,\ KK,\ K\pi, ...)$

- At 1-loop order the convolution integral involve other logarithmic moments of the B meson wave function: the size of the effect depends on the shape of the wave function itself

- The decays $B \to \gamma\gamma$ and $B \to \gamma e e$ depend on exactly the same convolution integral at all order in perturbation theory

The ratios $\Gamma(B \to \gamma\gamma)/\Gamma(B \to \gamma e\nu)$ and $\Gamma(B \to \gamma e e)/\Gamma(B \to \gamma e \nu)$ are free of hadronic uncertainties up to Λ_{QCD}/E_γ corrections

$B \to \gamma\gamma$ & $B \to \gamma e e$

- FCNC processes: the effective Hamiltonian arises at the loop level in the SM:

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \left(V_{tb} V_{td}^* \sum_{i=1}^{10} C_i O_i + V_{ub} V_{ud}^* \sum_{i=1}^{2} C_i O_i^u \right)$$

$$O_2 = \bar{d}_L \gamma^\mu c_L \, \bar{c}_L \gamma_\mu b_L$$

$$O_7 = \frac{e}{16\pi^2} m_b \, \bar{d}_L \sigma^{\mu\nu} b_R \, F_{\mu\nu}$$

$$O_8 = \frac{g_s}{16\pi^2} m_b \, \bar{d}_L \sigma^{\mu\nu} b_r \, G^a_{\mu\nu}$$

$$O_9 = \bar{d}_L \gamma^\mu b_L \, \bar{e} \gamma_\mu e$$

$$O_{10} = \bar{d}_L \gamma^\mu b_L \, \bar{e} \gamma_\mu \gamma_5 e$$

- Same effective Hamiltonian as for $b \to d\gamma$ and $b \to d e e$

- Strong sensitivity to new physics

- Traditional approach: only O_7

$$\langle \gamma\gamma | O_7 | B \rangle \propto \langle \gamma | \bar{d}\,\sigma^{\mu\nu} b | B \rangle \to g_+(E_\gamma),\; g_-(E_\gamma),\; g_0(E_\gamma)$$

- Both photons are energetic and we can apply the effective theory approach
  - the matrix elements of O_2 and O_8 are proportional to the O_7 one up to power corrections:
    [Descotes-Genon, Sachrajda: proof at order α_s]

$$\langle \gamma\gamma | O_{2,8} | B \rangle = (**)\,\langle \gamma\gamma | O_7 | B \rangle + O\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

m_b^2 fluctuations (perturbative)

- Factorization of the form factors
  [Lunghi, Pirjol, Wyler; Descotes-Genon, Sachrajda]

$$g_A(E_\gamma) = \int d\xi\, T_A(E_\gamma,\xi)\,\phi_B(\xi) = C_A(E_\gamma) \underbrace{\int d\xi\, J(E_\gamma,\xi)\,\phi_B(\xi)}$$

same convolution as in $b\to\gamma e\nu$

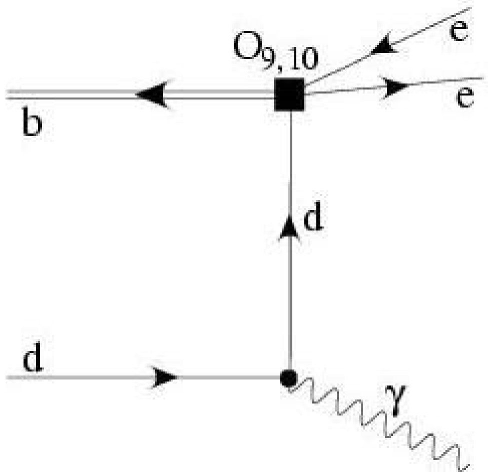
- Explicit form of the symmetry breaking corrections at order α_s

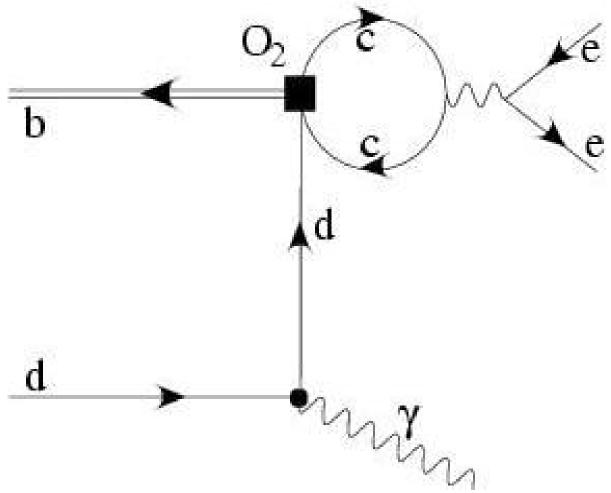
$$\frac{g_+(E_\gamma)}{f_V(E_\gamma)} = \frac{1}{2}\frac{Q_d}{Q_u}\left(1-\frac{\alpha_s C_F}{4\pi}\frac{E_\gamma}{E_\gamma-m_b/2}\log\frac{2E_\gamma}{m_b}\right)+o(\alpha_s^2)$$

$$g_-(E_\gamma)=-g_+(E_\gamma)+o(\alpha_s^2)$$

$$g_0(E_\gamma)=0+o(\alpha_s^2)$$

$B \to \gamma e e$





- ★ New feature: leading order long distance $c\bar{c}$ rescattering $(J/\psi, \psi', ...)$

  Cuts in the dilepton mass spectrum (same as in $b \to s e e$)

- low-s region: the photon energy is large and the SCET approach is feasible

- high-s region: the photon is soft and other methods have to be used
  [heavy quark symmetry, ...]

$$A(B\to\gamma e e) = \frac{G_F}{\sqrt{2}}\frac{\alpha_e}{\pi} V_{tb}V_{td}^* \Big[\; (C_9^{\rm eff}\,\bar{e}\gamma^\mu e + C_{10}\bar{e}\gamma^\mu\gamma_5 e)\,\overbrace{\langle\gamma|\bar{d}_L\gamma_\mu b_L|B\rangle}^{f_V,\, f_A}$$

$$-2\,C_7^{\rm eff}\frac{m_b}{q^2} q^\nu \underbrace{\langle\gamma|\bar{d}_L\sigma_{\mu\nu} b_L|B\rangle}_{g_+,\, g_-,\, g_0}\,\bar{e}\gamma^\mu e \; \Big]$$

- The ratios $\gamma e e/\gamma e \nu$ and $\gamma e e/\gamma\gamma$ are free of hadronic uncertainties up to power corrections

Conclusions and outlook

- New approach to the analysis of infrared divergences in QCD

- It allows to beyond the BBNS approach:
  - color suppressed decays
  - somewhat different factorization formulae

- Many proofs of factorization are already complete
  $(B \to D\pi\ ,\ B \to \gamma e \nu\ ,\ B \to X_s \gamma\ ,\ B \to \pi\pi\ ,\ B \to K\pi\ ...)$

- Phenomenological analysis has been worked out for $B \to \gamma$ transitions

- Will have strong impact on jet physics