Supersymmetry and naturalness

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Hierarchy problem

- The SM is an effective theory valid below Λ
- The SM parameters are determined at Λ in terms of more fundamental physics

$$(174\,\text{GeV})^2 = |\langle H \rangle|^2 \simeq -\frac{m_H^2(174\,\text{GeV})}{\lambda_H(174\,\text{GeV})}$$

$$(200\,\text{GeV})^2 \gtrsim |m_H^2(174\,\text{GeV})| \simeq \left| m_H^2(\Lambda) - \frac{3}{4\pi^2} h_t^2 \Lambda^2 \right|$$

$$\simeq \left| m_H^2(\Lambda) - \left(200\,\text{GeV} \frac{\Lambda}{0.7\,\text{TeV}}\right)^2 \right|$$

For $m_H \sim 200$ GeV and $\Lambda > 0.7$ TeV

$$\text{FT} \sim \left(\frac{\Lambda}{0.7\,\text{TeV}}\right)^2$$

FT + EWPT + gauge coupling unification = SUSY ?

$\Lambda_{\text{strong}} \gtrsim 5\text{-}10$ TeV

little hierarchy problem

a 2% accident ?

No naturalness problem for a generic SUSY extension of the SM defined at the scale Λ

However, a serious problem arises if both

- the SUSY extension is in turn an effective theory valid below $M \sim 10^{16}-10^{18}$ GeV (gauge unification, ν's, p-decay, inflation, baryogenesis)

$$m_H^2(\Lambda) = m_H^2(M) - 6\frac{h_t^2}{(4\pi)^2} m_{\tilde{t}}^2 \log\frac{M^2}{\Lambda^2}$$

- the SUSY extension is minimal (MSSM)

$$\lambda_H = 2\cdot\frac{g^2+g'^2}{4} \sim 0.25$$

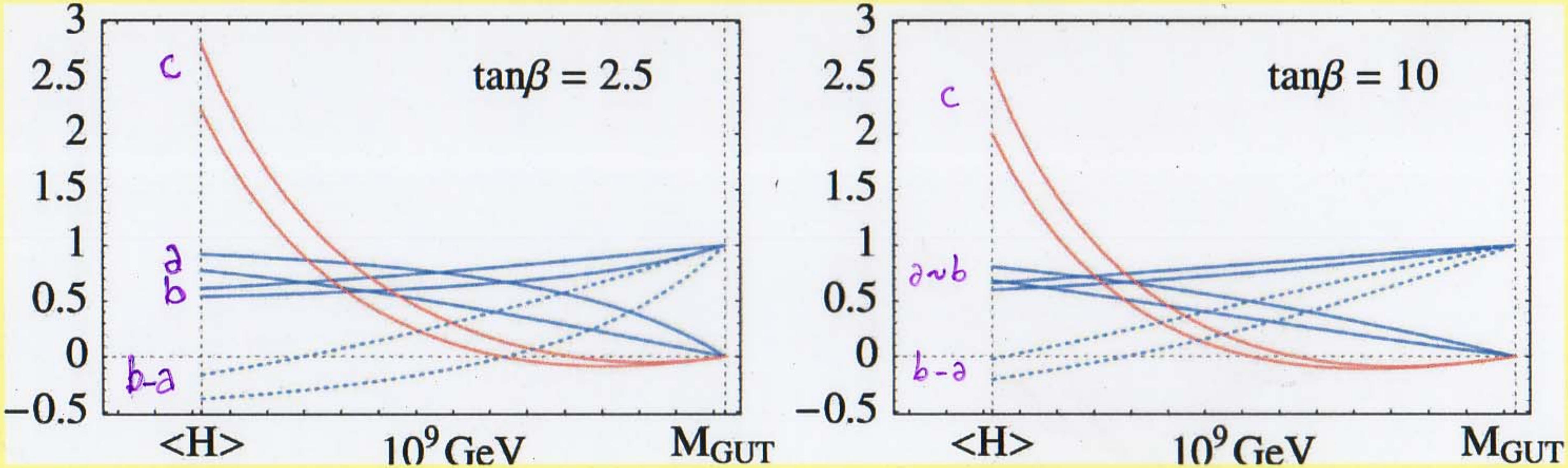
$$m_H^2(174\,\text{GeV}) \sim -\lambda_H (174\,\text{GeV})^2 \sim -(90\,\text{GeV})^2$$

$$(90\,\mathrm{GeV})^2 \sim$$

$$\simeq \left| m_H^2(\Lambda) - \frac{3}{4\pi^2} h_t^2\, m_{\tilde{t}}^2 \log \frac{M^2}{\Lambda^2} - \frac{3}{4\pi^2} h_t^2 \Lambda^2 \right|$$

$$FT \sim \left(\frac{m_{\tilde{t}}}{90\,\mathrm{GeV}} \right)^2$$

Relevant parameters at M: $m_Q^2 , m_H^2 , M_3 \; \mu$ (A = 0)



$$\frac{M_Z^2}{2} \simeq -m_u^2(\langle H\rangle) = a\, m_Q^2 - b\, m_H^2 + c\, M_3^2 - d\,\mu^2 \simeq (a-b)\, m_0^2 + c\, M_3^2 - d\,\mu^2$$

(tree level + RGE, moderately large $\tan\beta$)

$$1 \simeq -\frac{m_u^2}{M_Z^2/2} = \left(\frac{m_Q}{75\,\text{GeV}}\right)^2 - \left(\frac{m_H}{80\,\text{GeV}}\right)^2 + \left(\frac{M_3}{45\,\text{GeV}}\right)^2 - \left(\frac{\mu}{65\text{-}90\,\text{GeV}}\right)^2 \simeq \left(\frac{m_0}{200\,\text{GeV}}\right)^2 + \left(\frac{M_3}{45\,\text{GeV}}\right)^2 + \dots$$

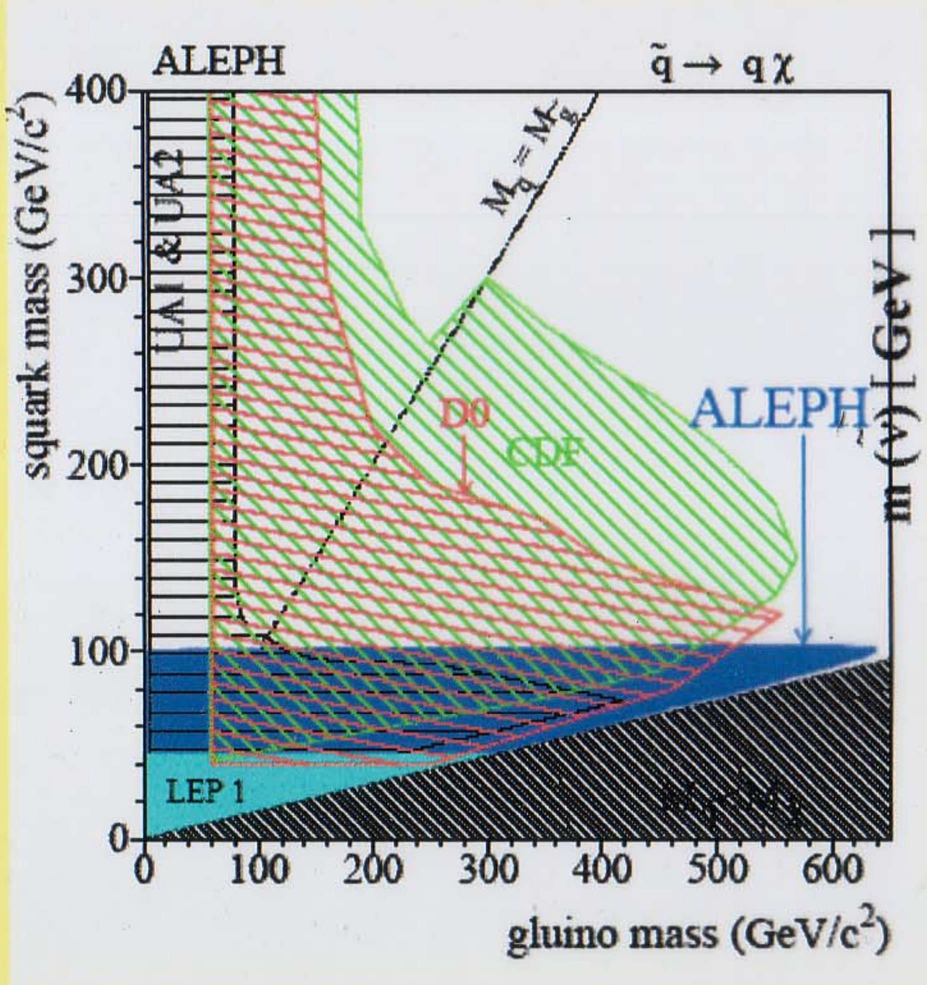
↑ Universality

Lower limits on $M_3, m_{Q,H}, \mu$ from experimental limits on
- $M_{\tilde g}$
- $m^2_{\tilde q}$
- $M_{\chi^\pm}$
- m_h

($\tan\beta$)

NOTE: FT neglected

Gluinos and squarks



$M_{\tilde{g}} \leftrightarrow M_3$

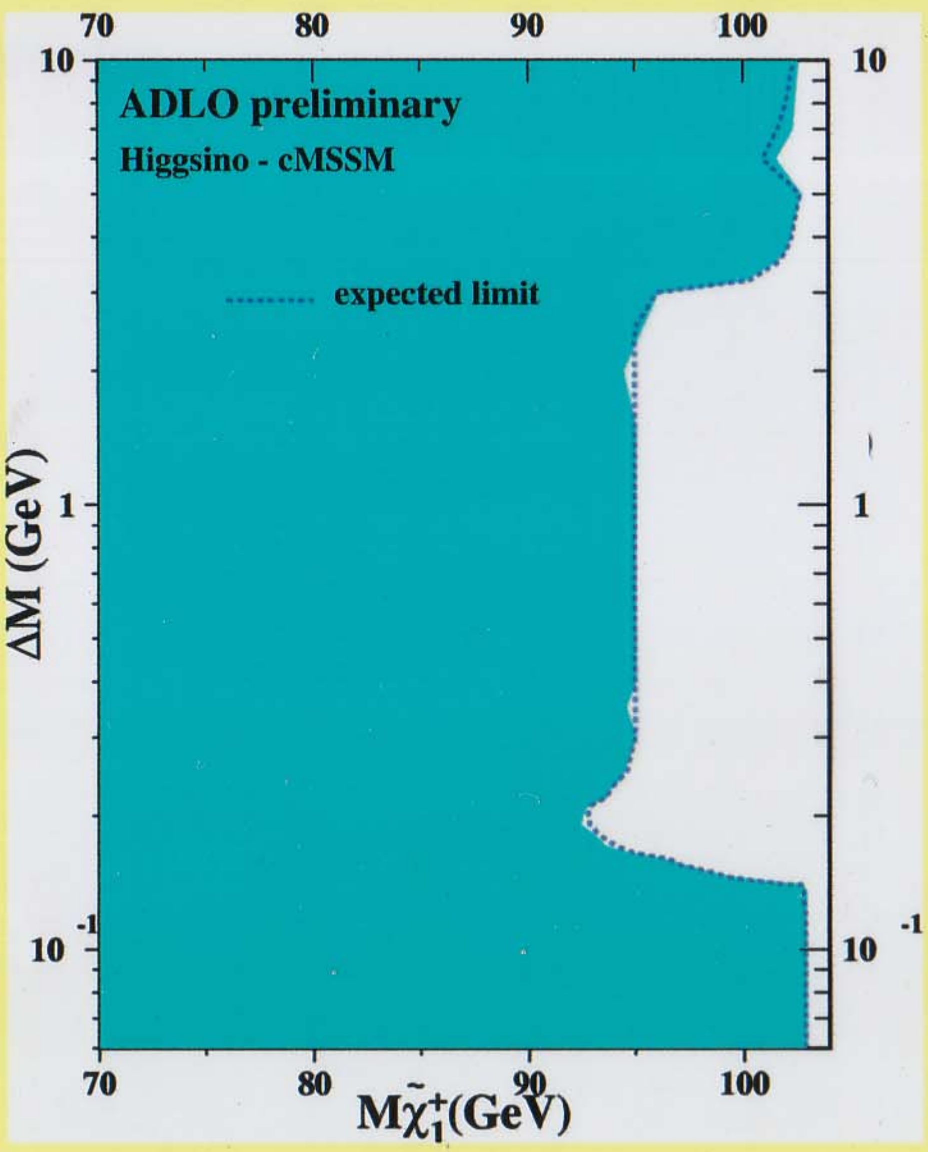
$M_{\tilde{g}} >$ 195 GeV forces $\Delta > 2.5$
260 GeV 5
500 GeV 17

$m^2_{\tilde{t}_R} \simeq 0.5\, m^2_Q - 0.25\, m^2_H + 4 M_3^2$

$m_{\tilde{t}_R} >$ 300 GeV forces $\Delta > 25$ (7)
260 GeV 10 (3)
100 GeV 40 (12)

Universality

Charginos



$$M_{\chi^\pm} = \begin{pmatrix} M_2 & \sqrt{2} M_Z c_W s_\beta \\ \sqrt{2} M_Z c_W c_\beta & \mu \end{pmatrix}$$

$|\mu|, M_2 \gtrsim 100$ GeV

if gaugino masses unify, $M_{\tilde{g}} > 350$ GeV, $\Delta > 10$

does not give a significant contribution to Δ

tanβ (moderately large)

$$\tan\beta^{-1} = \frac{\mu B}{m_u^2 + m_d^2}$$

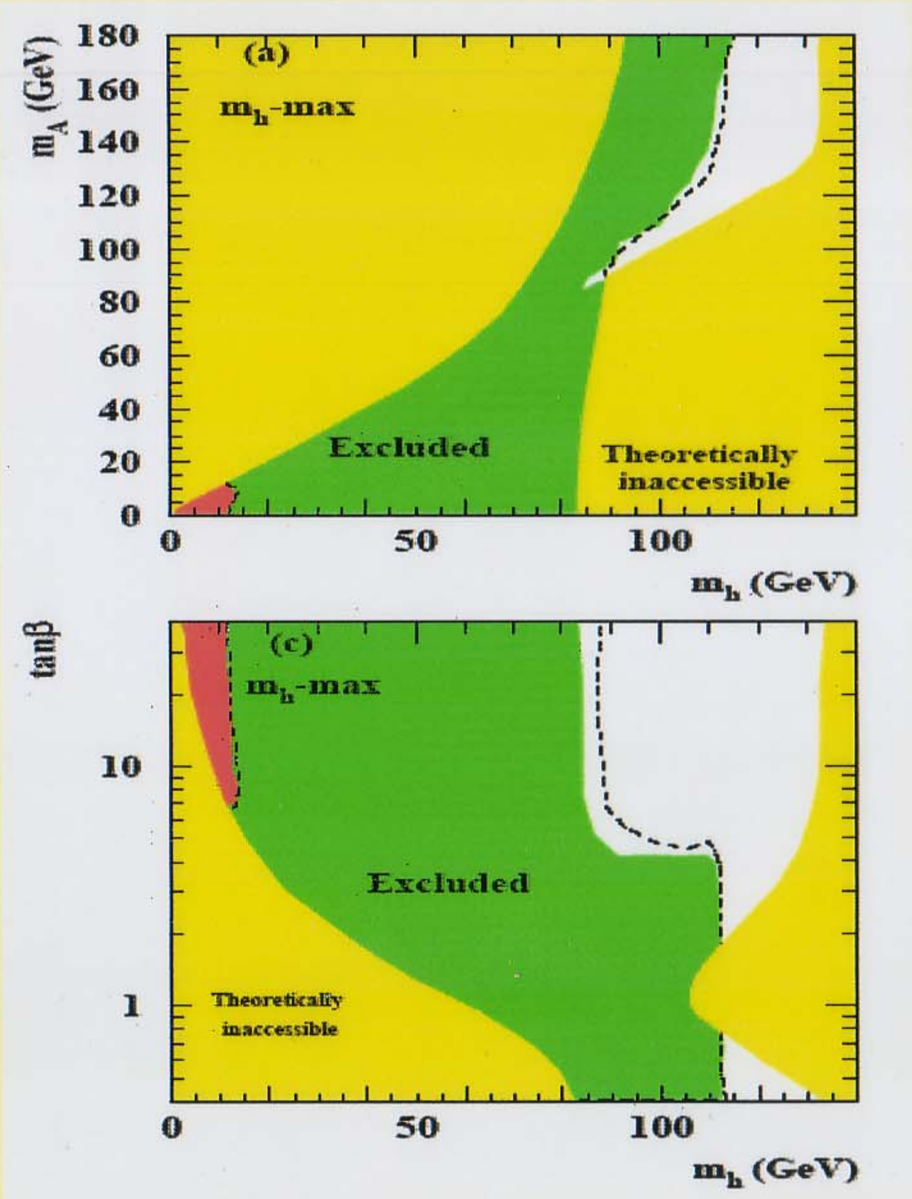
$$\mu B = 1.5\,\mu M_\chi + \ldots \qquad \Delta_{\mu B} \sim \left(\frac{130\,\text{GeV}}{\mu B}\right)^2$$

$$m_u^2 + m_d^2 \simeq \mu B \tan\beta \simeq \frac{\tan\beta}{\Delta_{\mu B}}\,(130\,\text{GeV})^2$$

$\cdot\,(70\,\text{GeV})^2$

- If $m^2_{H_u}, m^2_{H_d}$ are independent $\quad m^2_{H_d} \sim 4\,\frac{\tan\beta}{\Delta_{\mu B}}\,m^2_{H_u}$
- If $m^2_{H_u} = m^2_{H_d}$ at $M \qquad \Delta_{M_2} \cdot \Delta_{\mu B} \gtrsim 3\tan\beta$

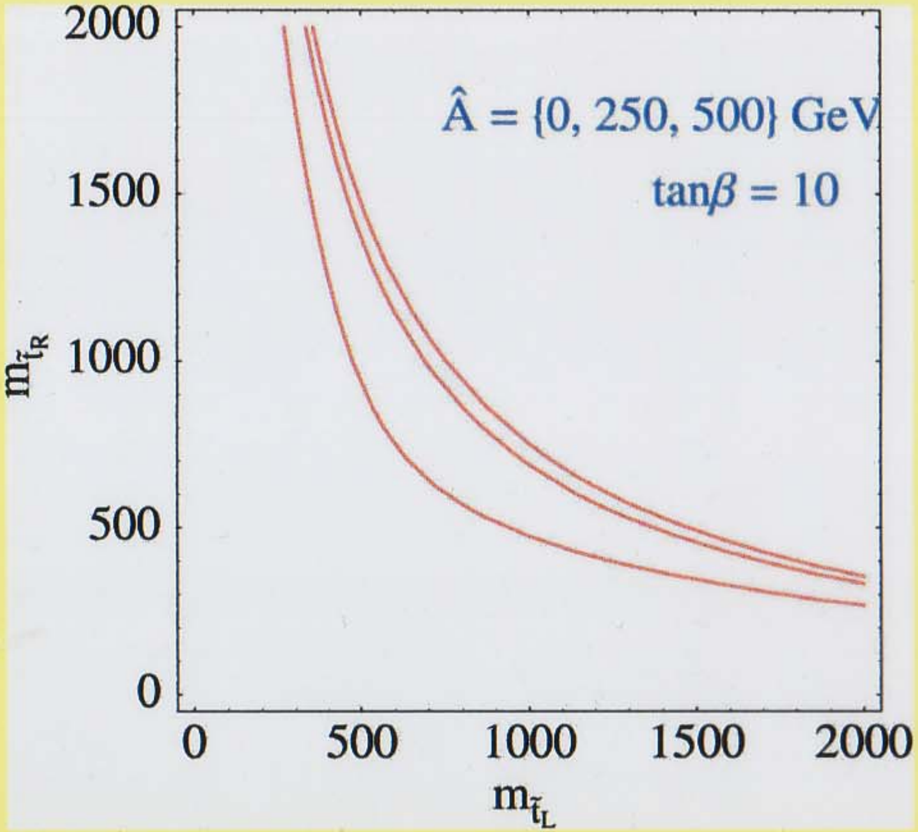
Lightest Higgs



$$m_h^2 \leq M_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} h_t^2 m_t^2 \log \frac{m_{\tilde{t}}^2}{m_t^2}$$

The experimental limit requires significant fine-tuning

Lightest Higgs - SM-like

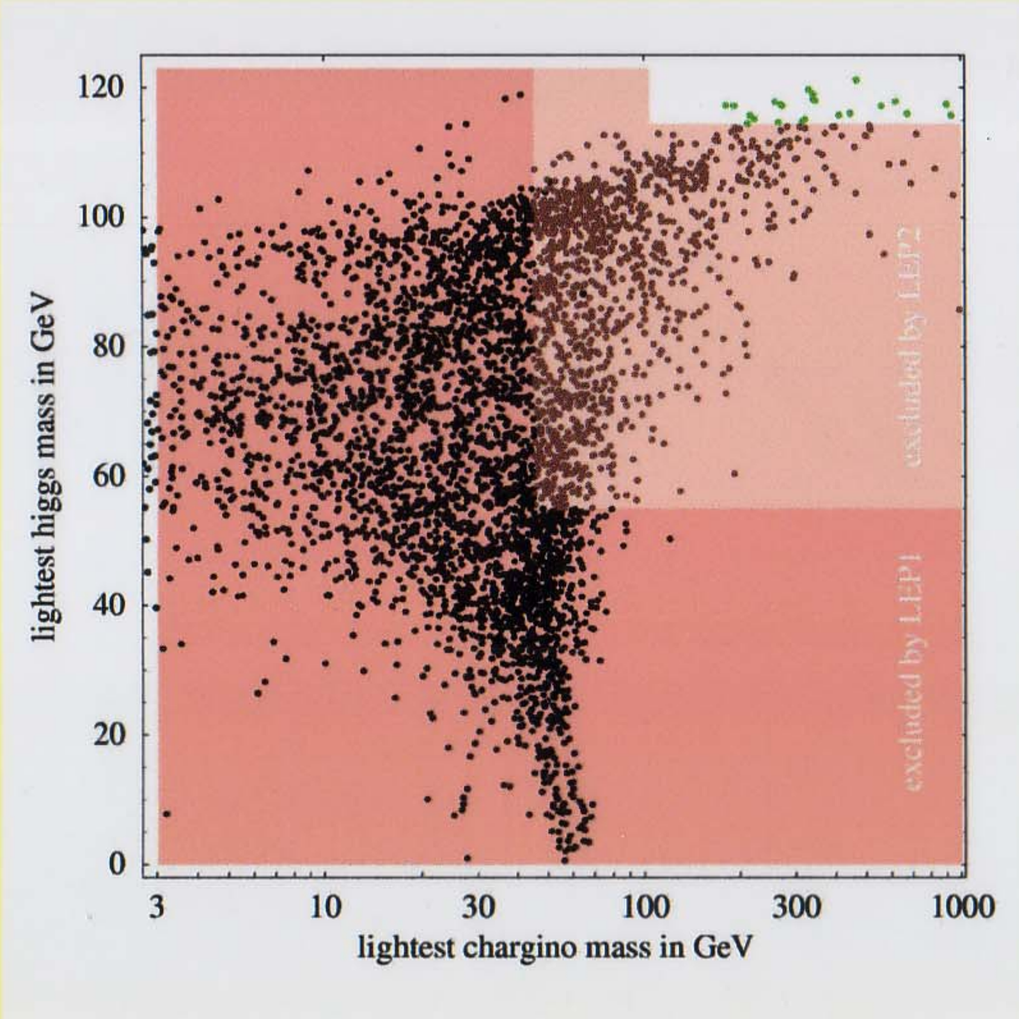


$m_{\tilde{t}} \gtrsim 600$ GeV

$\Delta \gtrsim 40$ (20)

What is left?

A quantitative measure of naturalness that automatically takes into account all considerations above



- Scan relative sizes of SUSY parameters
- Set overall scale by using $\langle H \rangle = 174$ GeV
- Calculate the SUSY spectrum and compare with experiment

~ 1% of points satisfy all constraints
(gaugino unification, universality, suppressed production neglected)

NMSSM: $\mu H_u H_d \to \lambda N H_u H_d$

Larger values of m_H^2 are in principle allowed

$$m_H^2 = -\lambda_H \langle H \rangle^2 \qquad \lambda_H = \frac{g^2+g'^2}{2}\cos^2 2\beta + \lambda^2 \sin^2 2\beta$$

m_H^2 still bounded by

- EWPT
- Landau pole of λ (depends on field content, $\tan\beta$, initial scale)
- Landau pole of λ_t (through $\sin 2\beta$)

3 benefits:

- $\lambda_H > \lambda_H^{MSSM}(max) = \dfrac{g^2+g'^2}{2}$, if $\lambda^2 \sin^2 2\beta \geq 0.25$

  ameliorates FT by a factor $\dfrac{\lambda^2 \sin^2 2\beta}{0.25}$ $\begin{cases} \sim 1-1.5 \text{ quite straightforward} \\ > 1.5 \text{ possible} \\ > 2, \text{ not worth} \end{cases}$

- $m_h^2 \leq M_Z^2 \left(\cos^2 2\beta + \dfrac{2\lambda^2}{g^2+g'^2} \sin^2 2\beta \right)$ tree level

  large radiative corrections not always needed

- the region with $m_h < 114$ GeV can be accessed more naturally ("A" does not need to be light)

Conclusions

- Low energy SUSY is still the easiest and most natural solution of the little hierarchy problem

  and the only one that accounts for gauge coupling unification and allows a connection with standard gravity, cosmology, neutrino physics

- The latter possibility, however, requires a significant fine-tuning

  unless non minimal models are considered