SEESAW, SUSY AND SO(10)

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SEESAW

LEFTHANDED NEUTRINO

$(B-L)(\nu) = -1$ $\qquad \nu \in$ DOUBLET OF $SU(2)_L$

$\Downarrow$ IF LR SYMMETRY ASSUMED

$\exists$ RIGHTHANDED NEUTRINO

$(B-L)(\nu^c) = +1$ $\qquad \nu^c \in$ DOUBLET OF $SU(2)_R$ (SM SINGLET)

$\Downarrow$ $M_{\nu_R} \propto \langle \Delta_R \rangle$ — TRIPLET OF $SU(2)_R$

LARGE (MAJORANA MASS)

DIRAC

$M_{\nu_D} \propto \langle \phi \rangle$

BIDOUBLET OF $SU(2)_L \times SU(2)_R$

MAJORANA

$$\nu^c M_{\nu_R} \nu^c + \nu^c M_{\nu_D} \nu$$

$\Downarrow$ INTEGRATE OUT HEAVY ν^c

$$M_\nu = - M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D}$$

SMALL MASS !

- YANAGIDA
- GLASHOW
- GELL-MANN, RAMOND, SLANSKY
- MOHAPATRA, SENJANOVIĆ

ANOTHER CONTRIBUTION TO SEESAW

$\nu^c M_{\nu_R} \nu^c$ COMES FROM

$\nu^c \underbrace{\Delta_R}_{SU(2)_R \text{ TRIPLET}} \nu^c$

$\Downarrow$ LR SYMMETRY

$\nu \underbrace{\Delta_L}_{SU(2)_L \text{ TRIPLET}} \nu$

LR MODELS : $H, \bar{H} \longrightarrow \phi \; (2,2)$

$SU(2)_L \qquad SU(2)_R$

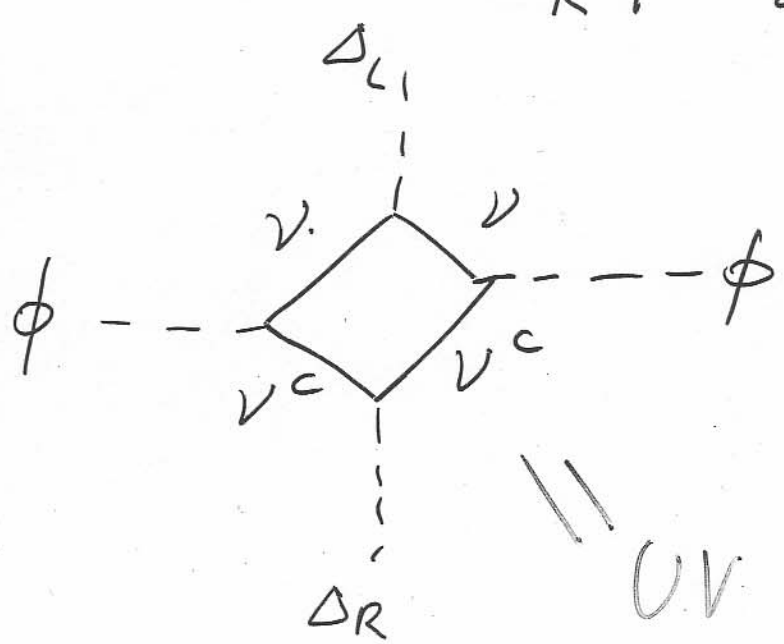
$\Rightarrow$ TERM $\Delta_R \phi^2 \Delta_L$ PRESENT

FROM

$- \nu^c \phi \nu$

$- \nu^c \Delta_R \nu^c$

$- \nu \Delta_L \nu$



|| UV DIVERGENT !

$$V = -M^2(\Delta_R^2 + \Delta_L^2) + \Delta_R \phi^2 \Delta_L$$

LARGE MASS

$$\Rightarrow \langle \Delta_L \rangle \approx \frac{\langle \Delta_R \phi \rangle^2}{M^2} \sim \frac{M_W^2}{M}$$

SINCE $\langle \Delta_R \rangle \approx M$

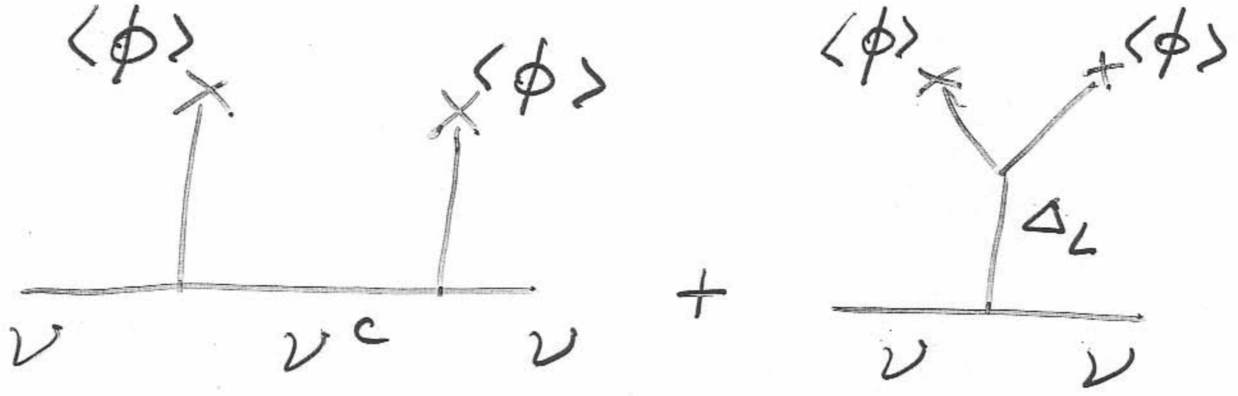
$\langle \phi \rangle \approx M_W$

$$\Rightarrow \underset{\displaystyle M_{\nu_L}}{\overset{\displaystyle \nu \langle \Delta_L \rangle \nu}{\shortparallel}}$$

ANOTHER CONTRIBUTION TO SEESAW

NEUTRINO MASS

$$M_N = -M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D} + M_{\nu_L}$$



$SU(2)_L$ SINGLET (ν^c)

TYPE I
OR
CANONICAL
SEESAW

$SU(2)_L$ TRIPLET (Δ_L)

TYPE II
OR
NONCANONICAL
SEESAW

IN GENERAL $M_{\nu_L}, M_{\nu_R}, M_{\nu_D}$ ARBITRARY

- IMPORTANT TO CONNECT TO CHARGED FERMION SECTOR
- A FRAMEWORK NEEDED

$\Rightarrow$ GRAND UNIFIED THEORY
(SO(10))

WHY GRANDUNIFICATION ?

(1) TOO MANY FORCES (3) IN SM.

(2) TOO MANY REPRESENTATIONS

(3 FAMILIES OF L, e^c, Q, u^c, d^c)

(3) QUANTIZATION OF ELECTRIC CHARGE

(IN SM POSSIBLE EXPLANATIONS

- MAGNETIC MONOPOLES
- ANOMALY CANCELLATION)

BOTH PRESENT IN GUT

1) FERMION MASSES AND MIXINGS

- GUTS ARE NOT THEORIES OF FLAVOUR.
- STILL SOME CONSTRAINTS

$\boxed{\text{MSSM}}$

$$W_y = H Q Y_U u^c + \bar{H} Q Y_D d^c + \bar{H} L Y_E e^c$$

BUT IN A GUT

$$Q, L, u^c, d^c, e^c \subset \Psi$$

$\Longrightarrow$ RELATIONS BETWEEN $Y_{U,D,E}$?

$\boxed{SO(10)}$

$$16 = (Q,\, u^c,\, d^c,\, L,\, e^c,\, \underline{\underline{\nu^c}})$$

RIGHTHANDED NEUTRINO

$$10_H = (H,\, T,\, \bar{H},\, \bar{T})$$

$$W_Y = 10_H\, 16\, Y_{10}\, 16$$

$$\Rightarrow \left\{ \begin{array}{ll} y_b = y_\tau = y_t & \text{GOOD!} \\ \left.\begin{array}{l} y_s = y_\mu = y_c \\ y_d = y_e = y_u \end{array}\right\} & \text{BAD} \\ \text{NO MIXING} & \text{BAD } (\theta_c = 0) \end{array} \right.$$

$$\underbrace{SU(2)_L \times SU(2)_R \times SU(4)_C}_{\text{PATI-SALAM}} \subset SO(10)$$

$$SU(4)_C \to SU(3)_C \times U(1)_{B-L}$$

$SU(3)_C$ ← QCD

$$\left.\begin{array}{l} SU(4)_C \to \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix} \\ \langle 10_H \rangle \text{ DOES NOT BREAK } SU(4)_C \end{array}\right\} \Rightarrow d \text{ AND } e \text{ SAME MASS}$$

HOW TO IMPROVE THE FIT?

(1) NEW HIGGS REPRESENTATIONS

$\boxed{SU(5)}$

$$45_H = \overline{H}^{p}_{qs} \left(= -\overline{H}^{p}_{sq} \right)$$

$$\sum_{p=1}^{5} \overline{H}^{p}_{ps} = 0$$

$$\Delta W_Y = \overline{45_H}\, \overline{5}\, Y_{45}\, 10 \quad \checkmark$$

$\boxed{SO(10)}$

- ANOTHER $10_H \Longrightarrow$ NOT ENOUGH
  - $\theta_c \neq 0$ ✓
  - $m_d = m_e$, $m_s = m_\mu$, $m_b = m_\tau$ — $SO(6) = SU(4)_c$ NOT BROKEN $\Downarrow$ BAD!

- BUT $\overline{126}_H$ OK!

$$\Delta W_Y = \overline{126}_H\, 16\, Y_{126}\, 16$$

$\Downarrow$

MUST GET A VEV IN $(2,2,15)_H$

$\hookrightarrow$ BREAKS $SU(4)_c$!

LAZARIDES, SHAFI, WETTERICH, 81
BABU, MOHAPATRA, 93

WITH 10_H AND $\overline{126}_H$:

$$W_Y = 10_H\, 16\, Y_{10}\, 16 + \overline{126}_H\, 16\, Y_{126}\, 16$$

UNDER PATI-SALAM $SU(2)_L \times SU(2)_R \times SU(4)_C$

$$10_H = \underline{(2,2,1)} + (1,1,6)$$

$$16 = (2,1,4) + (1,2,\bar{4})$$

ν_R

$$\overline{126}_H = \underline{(1,3,10)} + \underline{(3,1,\overline{10})} +$$

$$+ \underline{(2,2,15)} + (1,1,6)$$

UNDERLINED HAS NONZERO VEV

$$M_{\nu_R} = \underbrace{\langle(1,3,10)\rangle}\, Y_{126}$$

$= M_R$ (SCALE OF $SU(2)_R$ BREAKING)

LARGE, $\mathcal{O}(M_{GUT})$

(3) NO FLAVOUR SYMMETRY

ONLY GUT — SO(10)

TO ENSURE AUTOMATICALLY R-PARITY WE ARE FORCED <u>NOT</u> TO USE $16_H + \overline{16}_H$,

BUT INSTEAD

$$126_H + \overline{126}_H$$

5 INDEX ANTISYMMETRIC (ANTI) SELF-DUAL

$$R = (-1)^{3(B-L)+2S}$$

FOR BOSONS $(S=0)$

$$16 \xrightarrow{R} -16$$

$$126 \xrightarrow{R} +126$$

$$(B-L)_{\langle 16 \rangle} = 1$$

$$(B-L)_{\langle 126 \rangle} = 2$$

STABLE LSP !

WE NEED BOTH

$$V_{10} = \langle (2,2,1)_{10} \rangle \neq 0 \qquad \begin{pmatrix} V_{10}^u & 0 \\ 0 & V_{10}^d \end{pmatrix}$$

$$V_{126} = \langle (2,2,15)_{\overline{126}} \rangle \neq 0 \qquad \begin{pmatrix} V_{126}^u & 0 \\ 0 & V_{126}^d \end{pmatrix}$$

MODEL DEPENDENT

$$M_U = V_{10}^u Y_{10} + V_{126}^u Y_{126}$$

$$M_d = V_{10}^d Y_{10} + V_{126}^d Y_{126}$$

$$M_{\nu_D} = V_{10}^u Y_{10} - 3 V_{126}^u Y_{126}$$

$$M_e = V_{10}^d Y_{10} - 3 V_{126}^d Y_{126}$$

$$M_{\nu_R} = \langle (1,3,10)_{\overline{126}} \rangle Y_{126} = v_R Y_{126}$$

$$M_{\nu_L} = \langle (3,1,\overline{10})_{\overline{126}} \rangle Y_{126} = v_L Y_{126}$$

$$\langle (2,2,15) \rangle \hookrightarrow \langle 15 \rangle = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix} \propto (B-L)$$

SU(3) SINGLET

$$M_N = -M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D} + M_{\nu_L}$$

TYPE I SEESAW

TYPE II SEESAW

$\to$ FROM $\left(\frac{v_{10}^d}{v_{10}^u}\right), \left(\frac{v_{126}^d}{v_{126}^u}\right), \left(\frac{v_L}{v_{126}^u}\right), \left(\frac{v_R}{v_{126}^u}\right)$

DEFINE

$$x = \left(\frac{v_{10}^d}{v_{10}^u}\right) \Big/ \left(\frac{v_{126}^d}{v_{126}^u}\right)$$

$$y = \left(\frac{v_{10}^d}{v_{10}^u}\right)$$

$$\alpha = \frac{16}{x^2 \left(\frac{v_L}{v_{126}^u}\right)\left(\frac{v_R}{v_{126}^u}\right)}$$

$$\beta = \frac{4}{\left(\frac{v_R}{v_{126}^u}\right)\left(\frac{v_{126}^u}{v_{126}^d}\right)}$$

TO GET

① $\to$ $$(1-x) M_E = 4y\, M_U - (3+x)\, M_D$$

② $\to$ $$\beta M_N = -\alpha \left[\frac{3(1-x) M_D + (1+3x) M_E}{4}\right] * (M_D - M_E)^{-1} * \left[\frac{3(1-x) M_D + (1+3x) M_E}{4}\right] + (M_D - M_E)$$

→ SIMPLIFIED ANALYSIS

- 2^{ND} AND 3^{RD} GENERATIONS ONLY
- REAL PARAMETERS

① $$(1-x)M_E = 4y\,M_U - (3+x)M_D$$

KNOWN: $m_{\tau,\mu}$, $m_{t,c}$, $m_{b,s}$

+ ANGLE BETWEEN U AND D (θ_{cb})

UNKNOWN: x , y

+ ANGLE BETWEEN E AND D (θ_D)

3 EQUATIONS FOR 3 UNKNOWNS

EQUATION

② $$\beta M_N = -\alpha\left[\frac{3(1-x)M_D + (1+3x)M_E}{4}\right] * (M_D - M_E)^{-1} * \left[\frac{3(1-x)M_D + (1+3x)M_E}{4}\right] + (M_D - M_E)$$

DETERMINED UP TO α AND β:

$\alpha \to \infty$: TYPE I SEESAW

$\alpha \to 0$: TYPE II SEESAW

MIXED TYPE I + TYPE II CASE:

- ASSUME $\overset{\approx 0}{m_2} \ll m_3$ $\begin{pmatrix} \text{-2 GENERATIONS} \\ \text{- REAL} \end{pmatrix}$
- FINITE θ_q

$$\tan 2\theta_\ell = \frac{\sin(2\theta_q)}{2\sin^2\theta_q - \Delta}$$

$$\Delta = \frac{1}{1-9\alpha}\left[-5\alpha + (1-4\alpha)\,\epsilon\right]$$

$$\epsilon = \frac{m_b - m_\tau}{m_b}$$ - EXPERIMENTALLY SMALL

TYPE I $(\alpha = \infty)$: $\Delta = \frac{5+4\epsilon}{9}$, θ_ℓ SMALL

TYPE II $(\alpha = 0)$: $\Delta = \epsilon$, θ_ℓ LARGE

TYPE II SEESAW FAVOURED

IF M_2 RESTORED ~~[illegible]~~

$\Rightarrow$ 2 SOLUTIONS

$$\epsilon_u = \frac{m_c}{m_t}$$

$$\epsilon_d = \frac{m_s}{m_b}$$

$$\epsilon_e = \frac{m_\mu}{m_\tau}$$

$$\epsilon_i, \theta_q \sim 10^{-2}$$

$$\epsilon_i, \theta_q \sim \mathcal{O}(\delta)$$

1^{ST} SOLUTION (SMALL θ_D)

$$M_N \sim -\alpha \begin{pmatrix} \delta & \delta/\epsilon \\ \delta/\epsilon & 1/\epsilon \end{pmatrix} + \begin{pmatrix} \delta & \delta \\ \delta & \epsilon \end{pmatrix}$$

TYPE I TYPE II

$$\tan 2\theta_e \simeq \frac{2\delta\left(1+\frac{\alpha}{\epsilon}\right)}{\delta(1+\alpha)+\epsilon+\frac{\alpha}{\epsilon}}$$

- DUTTA, MIMURA, MOHAPATRA, 04
- BAJC, SENJANOVIĆ, VISSANI, 04

$\epsilon \sim \mathcal{O}(\delta)$ b-τ (EXP)

$$\boxed{\theta_e \simeq \mathcal{O}(1) \Leftrightarrow \alpha \lesssim \mathcal{O}(\delta^2)}$$ (TYPE II)

TYPE I ($\alpha \to \infty$) EXCLUDED $\Rightarrow \theta_e \simeq \mathcal{O}(\delta)$

2^{ND} SOLUTION (LARGE θ_D)

$$M_N \sim -\alpha \begin{pmatrix} \delta^2 & \delta \\ \delta & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\tan 2\theta_e \simeq \frac{1+\alpha\delta}{1+\alpha}$$

$$\boxed{\theta_e \simeq \mathcal{O}(1) \Longleftrightarrow \alpha \lesssim \mathcal{O}(1)}$$

TYPE II OR MIXED

IN NO CASE $\alpha \to \infty$ (TYPE I)
CAN GIVE LARGE $\theta_e = \theta_{ATM}$

BAJC, SENJANOVIĆ, VISSANI, 04

POSSIBLE TO UNDERSTAND

WHY $\theta_{atm} \approx$ LARGE IN

TYPE $\underline{II}$ SEE-SAW:

$$M_{\nu_L} \propto Y_{126}$$

$$\begin{cases} M_d = v_{10}^d Y_{10} + v_{126}^d Y_{126} \\ M_e = v_{10}^d Y_{10} - 3\, v_{126}^d Y_{126} \end{cases}$$

$$\Rightarrow \quad Y_{126} \propto M_d - M_e$$

$$\Rightarrow \quad M_{\nu_L} \propto M_d - M_e$$

BRAHMACHARI, MOHAPATRA, 98

APPROX.:

- 2-3 GENERATION CASE
- NEGLECT $m_{s,\mu} \ll m_{b,\tau}$
- M_d AND M_e HAVE SMALL MIXINGS

$$M_{\nu_L} \propto \begin{pmatrix} 0 & 0 \\ 0 & m_b - m_\tau \end{pmatrix}$$

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$$\boxed{\text{LARGE } \theta_{atm} \Longleftrightarrow b-\tau \text{ UNIFICATION}}$$

IN MINIMAL SUSY SO(10) :

- LARGE θ_{ATM} GOOD ARGUMENT FOR TYPE II SEESAW DOMINATION
- $b-\tau$ UNIFICATION COULD BE EXPLAINED
- ANOTHER SOLUTION (LARGE θ_D) WITH POSSIBLE MIXED TYPE I + II SEESAW