

Correzioni elettrodeboli

ad alte energie

- Review -

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EW interactions at $E \gg \pi = \text{weak scale} \sim 100 \text{ GeV}$

- EW rad. corr: $\frac{\Delta\sigma}{\sigma} \sim \alpha_w \log^2 \frac{E^2}{\pi^2} + \dots$
 $\sim 7\% @ \text{TeV}$

Kuroda et al '91
 Degross et al '92
 Beenekker et al '93

- Tied to IR structure of the theory: P.C.

coll $\log \frac{E}{\pi}$ IR $\log \frac{E}{\pi}$, divergent if $\pi \rightarrow 0$ D. Comelli '98



- Possible to address higher orders/resummation of leading subleading subsubleading ... terms

L^{2n} L^{2n-1} L^{2n-2}

more than 50 works P.C., M.C., Comelli, Faden, Nelles, Lipatov, Kühn, Pozzorini, Denner, Vertegnas, Beccaria....

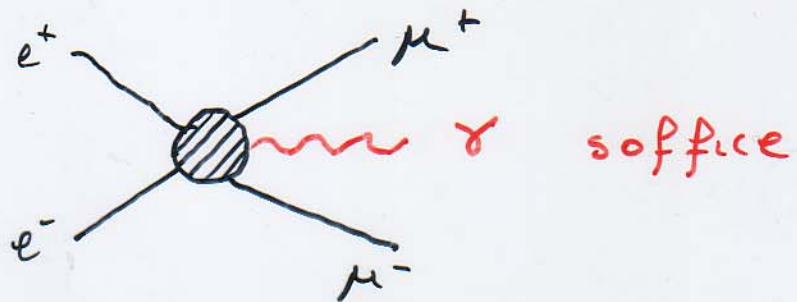
99 ÷ 03

- In contrast with the QED, QCD cases, even fully inclusive (W, Z, γ radiation) quantities are affected by IR "divergences" $\log^2 \frac{E^2}{\pi^2}$:

Block-Nordsieck violation P.C.
 M.C. 99 ÷ 02
 D. Comelli

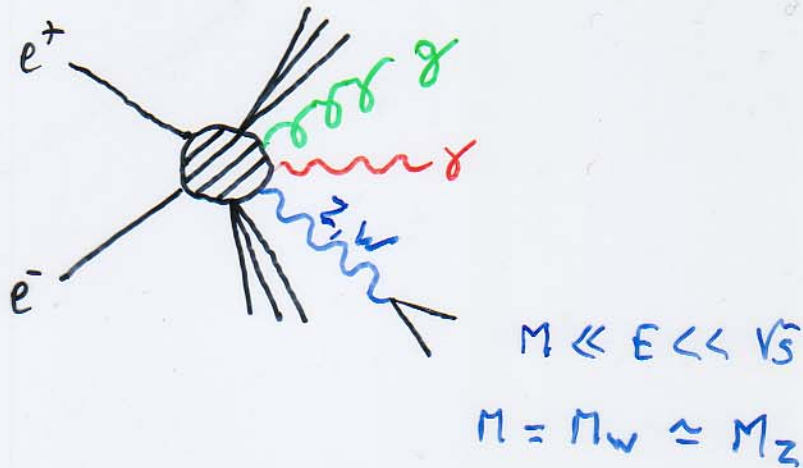
Osservabili "esclusive"

$$e^+e^- \rightarrow \mu^+\mu^- \gamma$$



Osservabili "inclusive"

$$e^+e^- \rightarrow q\bar{q} X$$



- Large angle scattering

$$|s| \sim |t| \sim |u| \gg M$$

- Standard Model

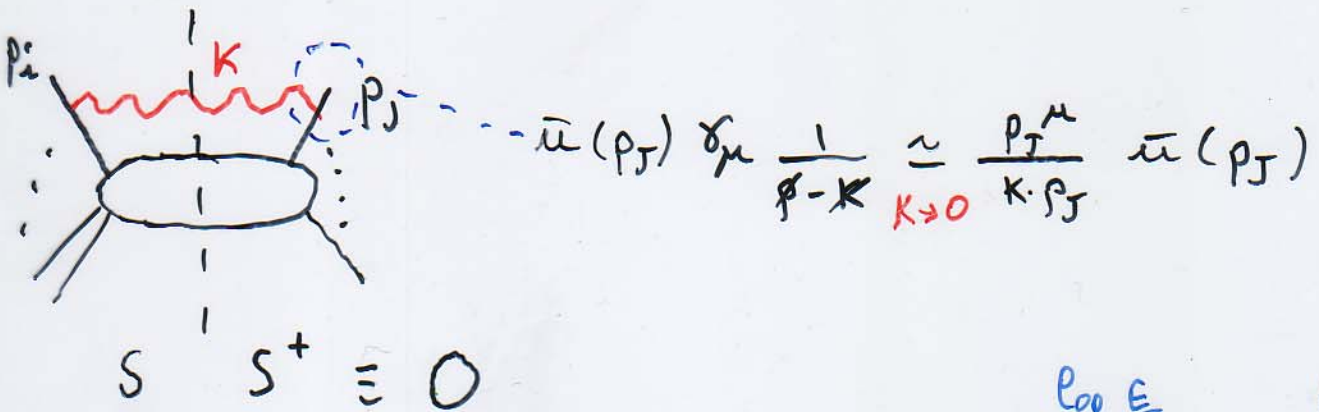
- ~~BN~~: virtuali e reali non si cancellano nelle quantità inclusive

Correzioni EW e TeV - 1 Loop

$$\left(\frac{\Delta\sigma}{\sigma}\right)^2 \xrightarrow{\sqrt{s} \gg M} \alpha \left\{ a \log^2 \frac{s}{M^2} + b \log \frac{s}{M^2} + c + o\left(\frac{M^2}{s}\right) \right\}$$

\uparrow \uparrow \uparrow \uparrow
 LL NLL NNLL mass-suppressed

Es: DL (Sudakov) approx iconele



$$\Delta\sigma^1 \approx \int \frac{d^3k}{\omega} \frac{p_i \cdot p_j}{(k \cdot p_i)(k \cdot p_j)} \quad \sigma^0 \approx \sigma^0 \int \frac{d\omega}{\omega} \int \frac{d\Omega^2}{\Omega^2}$$

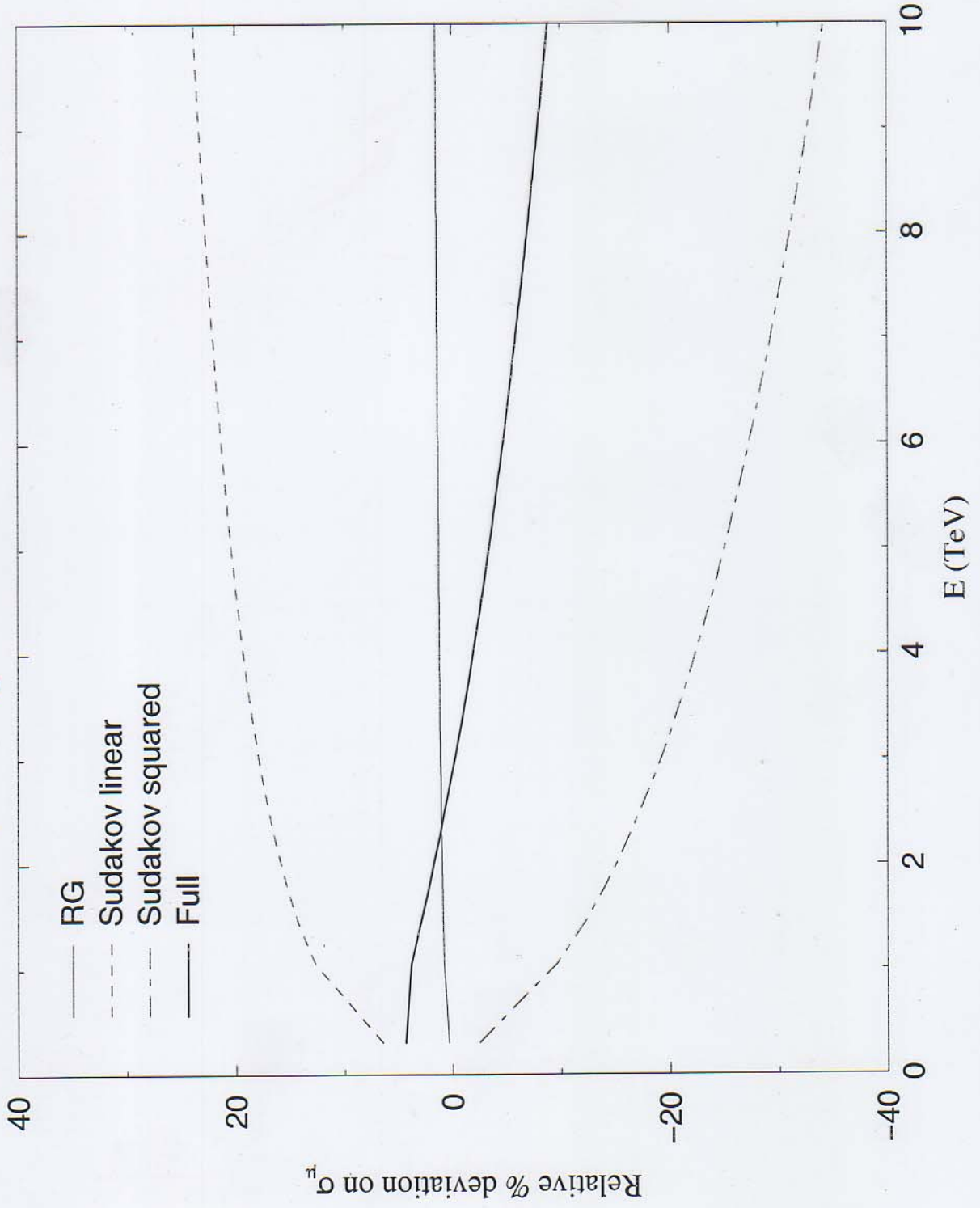
$\omega \gg M$ IR coll

$\nearrow \log \frac{E}{\pi}$ $\nearrow \log \frac{E}{\pi}$

- inserzioni su gambe esterne (calcolo semplice)
- Fattorizzazione dei DL $\sigma^1 \approx \sigma^0 (1 + \alpha \log^2 \frac{s}{M^2})$
- Strutture di isospin non banale: sorprese!

One Loop - NLLC

Beccaria
Ciccolini Comelli Vertegnaschi
Renard Leyssec Goumeris



- Dominante de IR (non UV)

- Cancellazioni fra LL e NLL

- Precisione a NLL $\lesssim 1\%$ \Rightarrow Necessità di higher order

One Loop - LHC

Giuseppe Gotti Ross '04

$$\begin{aligned} q\bar{q} &\rightarrow qV \\ qg &\rightarrow qV \end{aligned} \quad V = \gamma, Z \quad \left(\frac{\Delta\sigma}{\sigma}\right)^2 \sim 10 \div 20\%$$

Accomando Denner Pottorini '04

pp $\rightarrow l\nu_l l'\bar{\nu}'$				
$P_T^{\text{cut}}(l'\bar{\nu}')$ [GeV]	σ_{Born} [fb]	σ [fb]	Δ [%]	$1/\sqrt{2L\sigma_{\text{Born}}}$ [%]
250	1.716	1.595	-7.1	5.4
300	0.899	0.811	-9.8	7.5
350	0.503	0.441	-12.4	10
400	0.296	0.252	-14.9	13
450	0.181	0.150	-17.1	16.6
500	0.114	0.092	-19.3	20.9

Table 1: Cross section for pp $\rightarrow l\nu_l l'\bar{\nu}'$ for various values of $P_T^{\text{cut}}(l'\bar{\nu}')$

$$\bar{d}u \rightarrow WZ, W\gamma$$

$$\left(\frac{\Delta\sigma}{\sigma}\right)^2 \sim 5 \div 20\%$$

- Higher order probabilmente irrilevanti e LHC

Risommazione - tecniche di QCD ma differenze importanti

• Fattorizzazione

$$\sigma = \int dx f(x, \mu^2) \hat{\sigma}_H(xs)$$

↑ scale soft
↑ scale dure


- Fattorizzazione \leftrightarrow C.W.I. Amati Perrotti Venetiano '78

$$K_\mu \text{ wavy } \textcircled{\text{hatched}} = \begin{cases} 0 & \text{unbroken (QCD)} \\ \pi \text{ --- } \textcircled{\text{hatched}} & \text{broken (EW)} \end{cases}$$

- accoppiamenti $\propto \pi$ riscalabili? (vedi avanti)


- Dimostrare α LL all orders
 NLL 1 loop Denner Pottorini '01

• Ordinamento



LL, NLL

 \cong



K_μ

$\mu \ll K_\perp^1 \ll K_\perp^i \quad i \neq 1$

$\mu^2 \frac{\partial}{\partial \mu^2} f(x, \mu^2) = \alpha [f \otimes P](x) \quad f \otimes P = \int_x^1 P(x) f\left(\frac{x}{z}\right) \frac{dz}{z}$

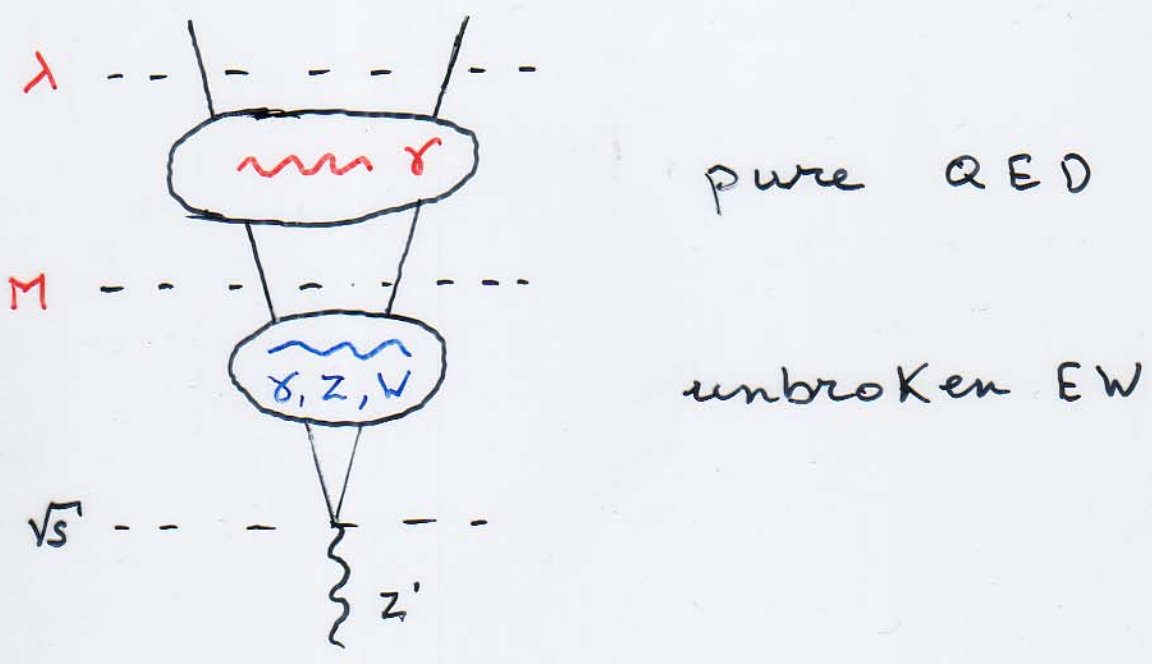
• Separazione di scale

$$\Delta \ll K_\perp^\delta \ll \pi \quad \pi \ll K_\perp^\delta, K_\perp^W, K_\perp^Z \ll \sqrt{s}$$

Osservabili esclusive e LL $d^m e^i L^j \quad i+j = 2m$

3 scale $\sqrt{s}, \Lambda, \lambda$

$l = \log \frac{\Lambda}{\lambda}$ $L = \log \frac{\sqrt{s}}{\Lambda}$
massa \uparrow δ



pure QED

unbroken EW

Cosa vuol dire "soffice" ?

ordinamenti in w Ciafe Comelli '00 Lenna
 K_{\perp} Fadun et al '00

risultati diversi. Calcoli a 2 loops con
 tecniche di approx Hoi et al '00 confermano
 Beenakker et al '00
 Denner et al '03

i risultati di Fadun et al. (Perche' ?)

Aglietti, Bonciani '03
 '04

2 loop esatto !



Feycht et al '03
 '04

$L^4, L^3, L^2, L, 1, \dots$

Osservabili esclusive e NLL

$$d^m L^i L^j \quad i+j \geq 2n-1$$

Melles '00

Kühn Penn Smirnov '00

Tecniche mutuete da QCD

Assuntioni:

- fattorizzazione e NLL all orders
- separationi di scale e NLL QED $\begin{matrix} | \\ | \\ | \\ M \end{matrix}$ unbroken EW

Recentemente confermato a 2 loops

Potzorini hep-ph/0401087

Note:

- non direttamente confrontabili perché diversi cutoff IR per δ
- manca calcolo "vero": reali + virtuali

Osservabili esclusive e NLL

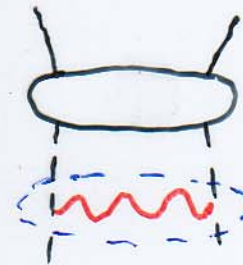
$$d^m l^i L^j \quad i+j \geq 2n-2$$

Kühn, Nach, Penin, Smirnov '01

- four-fermion processes
- Grosse cancellazioni fra LL, NLL, NNLL

$$\left(\frac{\Delta\sigma}{\sigma}\right)_{\text{NLL}}^{\text{2 loop}} \sim 1 \div 2 \%$$

Separazione "unbroken EW" + "pure QED"
dubbia per i longitudinali



$$\propto \int \frac{dz}{1-z} \int \frac{dK_{\perp}^2}{K_{\perp}^2} \sim \alpha \log^2 \frac{s}{\pi^2}$$



$$\propto \pi^2 \int \frac{dK_{\perp}^2}{K_{\perp}^4} \sim \alpha \text{ FINITO}$$



$$\propto \pi$$

$$\alpha^2 \log^2 \frac{s}{\pi^2}$$

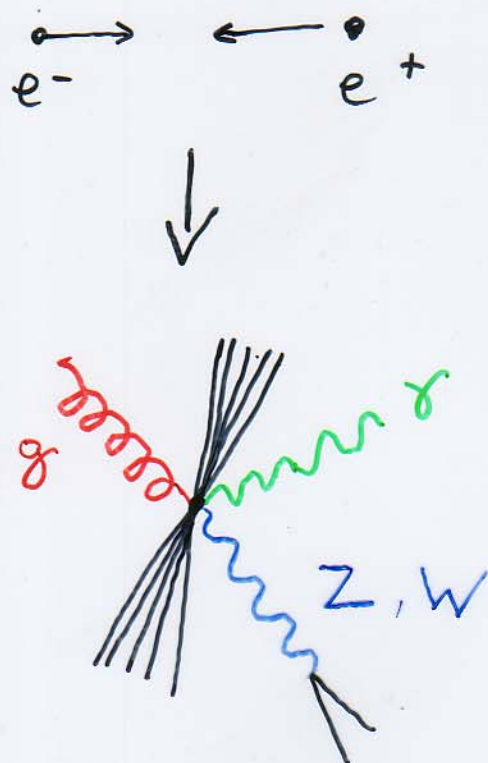
NNLL e 2 loops

fehlt Kühn, Penin, Smirnov

hep-ph/0404082

Fully inclusive observables

Ex : $e^+e^- \rightarrow \text{hadrons} + X$



~~BA~~

Large angle scattering:

$$|s| \sim |t| \sim |u| \gg m_w^2$$

P. Cefaloni, M. Cefaloni, D. Comelli

'88 \rightarrow '03

GENERAL IR THEOREMS

- KLN : f, i "hard", g "soft"

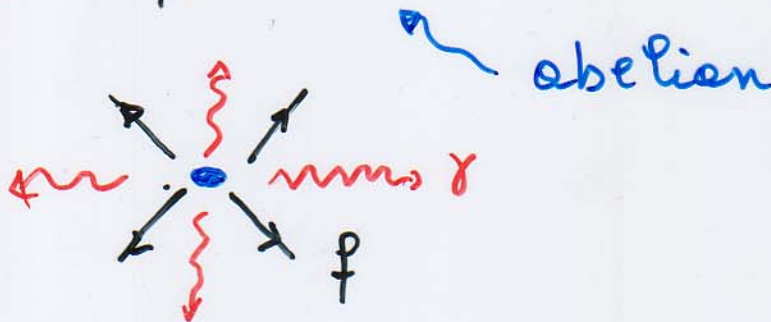
$$\sum_{i, f, g} |\langle f, g(\Delta) | S | i, g(\Delta) \rangle|^2 \text{ IR finite}$$



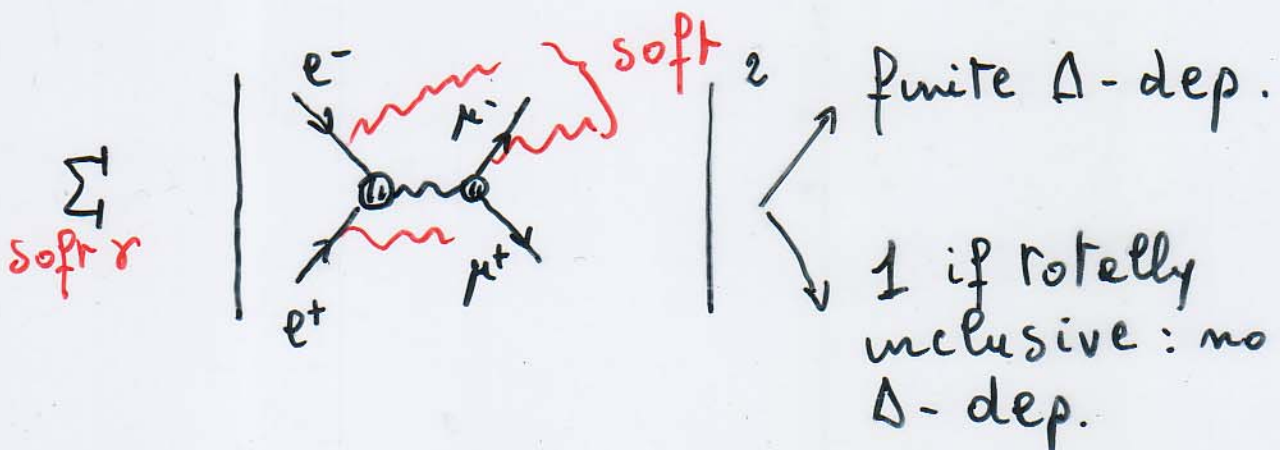
Unphysical ! NO general th. for phys. σ 's

- QED : Bloch - Nordsieck

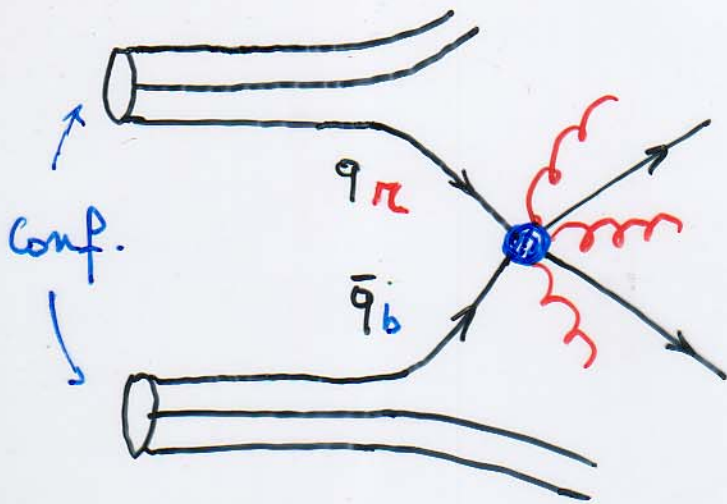
$$\sum_{\gamma} |\langle f, \gamma(\Delta) | S | i \rangle|^2 \text{ IR finite}$$



Physical : $\Delta = \text{exp. res.}$



QCD: no BN, only KLN BUT



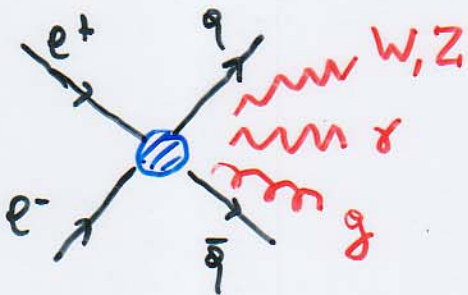
$\sum_{r,b}$: color average
over initial colors of
the hard "i" states

Then, Leading DL's cancel out
(subleading: Brodsky
Taylor
Bodwin
⋮)

EW:

FIXED nonabelian initial charges	}	P.C.
↓		P.C.
Full straight DL's uncancelled:		D.C.

Ex: $e^+e^- \rightarrow \text{had.}$, fully inclusive



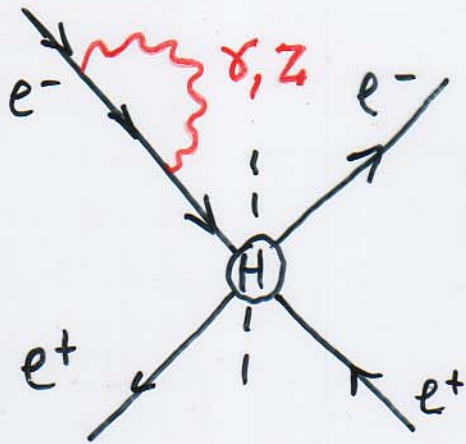
$$\sum_{q_i, \gamma, W, Z, g} \Rightarrow \Delta\sigma \sim d\omega \log^2 \frac{s}{\omega^2}$$

IR cutoff

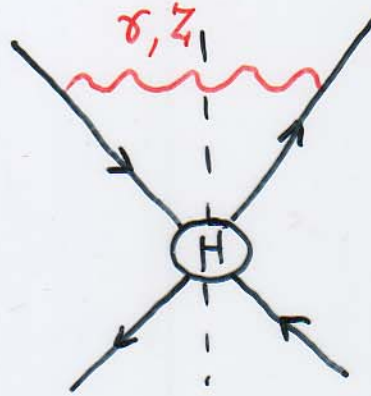
BN theorem maximally violated

$e^+ e^- \rightarrow \text{hadrons}$ at 1 loop DL level
 P. Cefaloni M. Cefaloni O. Comelli '00

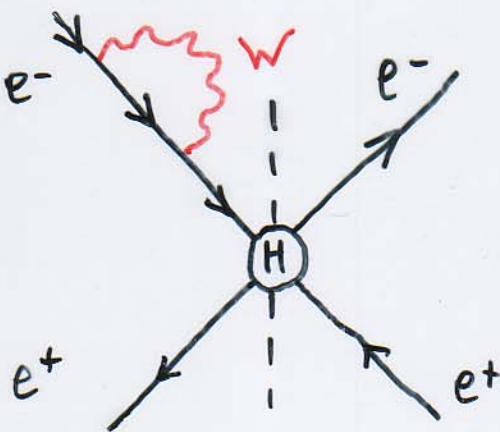
Virtual



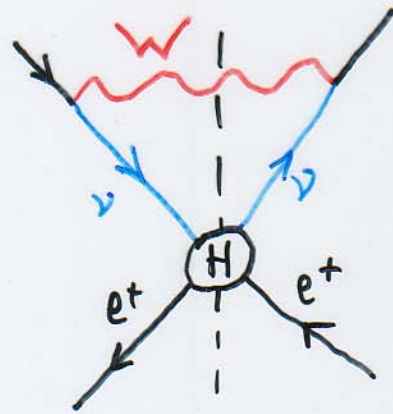
Real



$$-\alpha \log^2 \frac{\sqrt{s}}{M} \sigma_{e^+e^-}^H + \alpha \log^2 \frac{\sqrt{s}}{M} \sigma_{e^+e^-}^H = 0$$



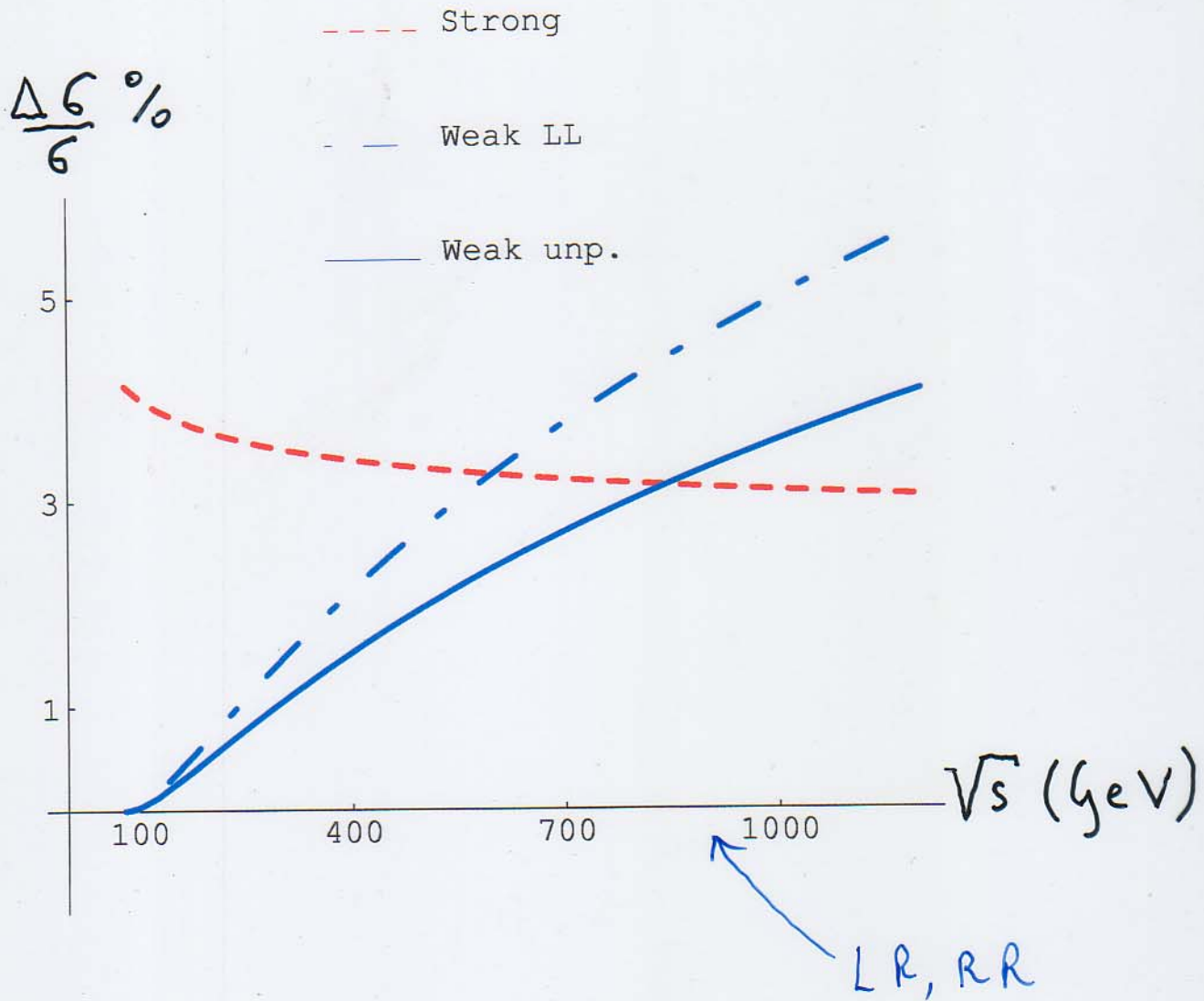
+



$$-\alpha \log^2 \frac{\sqrt{s}}{M} \sigma_{e^+e^-}^H + \alpha \log^2 \frac{\sqrt{s}}{M} \sigma_{e^+\nu}^H \neq 0 !!$$

- BN violation: strong dependence on the IR cutoff M ! $\Delta \sigma \sim \log^2 \frac{\sqrt{s}}{M}$
- W exchange affects only L particles in a universal way

$e^+ e^- \rightarrow \text{hadrons}$



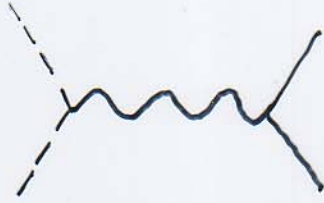
• Strong EW effect !

$$\sigma_{e^+e^-} = \sigma^H \left(1 + \underbrace{O\left(\frac{\alpha_s}{\pi}\right)}_{\sim 3\%} + \underbrace{O\left(\frac{\alpha_w}{4\pi} \log^2 \frac{s}{\pi^2}\right)}_{\sim 5\%} \right)$$

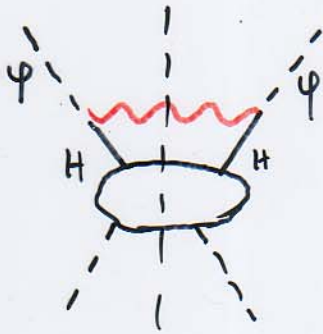
• Polarization dependence

"Move force"

- $W_L W_L \rightarrow f \bar{f}$



$$\sigma_H \sim \frac{\alpha^2}{s} \quad \Pi_H - \text{indep.}$$



$$\Delta \sigma \left(\alpha \log^2 \frac{\sqrt{s}}{\Lambda}, \alpha \log^2 \frac{M_H}{M_W} \right)$$

- ~~BN~~ in Theorie abeliane $W_L W_L$ scatt.

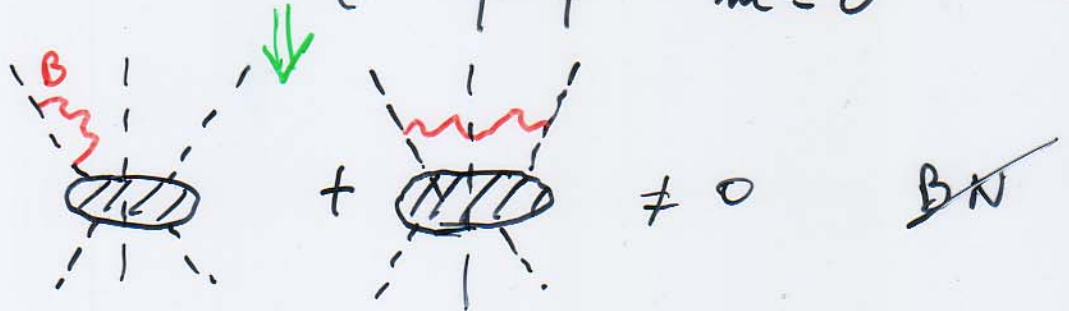
$$\phi = H + \epsilon \chi \xrightarrow{SB}$$

$$\begin{cases} H \sim \frac{1}{\sqrt{2}}(\phi + \phi^\dagger) \\ \chi \sim \frac{1}{\sqrt{2}}(\phi - \phi^\dagger) \end{cases}$$

$$m = \nu$$

mixing

$$m = 0$$

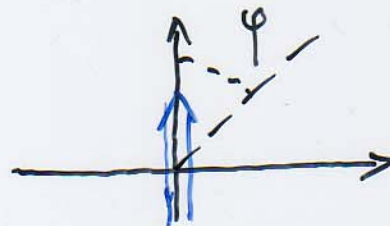


- BN in e^+e^- - effetti: $SU(2)$ e $U(1)$



$$e^+ = e^L + e^R$$

$$Y_L \neq Y_R$$



$$\frac{\Delta \sigma}{\sigma} \sim 10\%$$

P.C.

Comelli

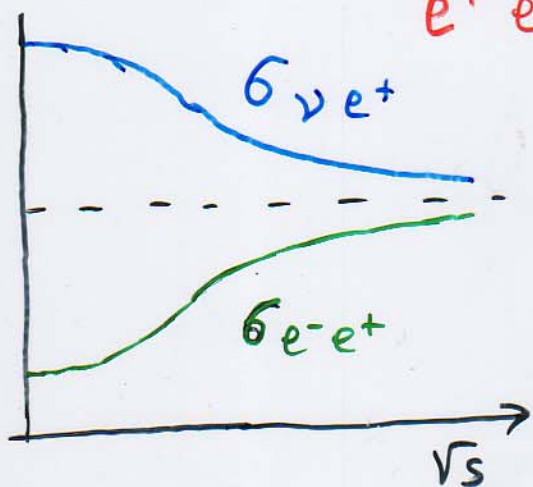
'03

Resummation of leading IR double logs

• No ordering ambiguities! $(\alpha_w \log^2 \frac{s}{\pi^2})^n$

• $s \sim tu \gg \pi_w^2 \sim \pi_e^2 = \pi^2$: recovered $SU(2) \otimes U(1)$

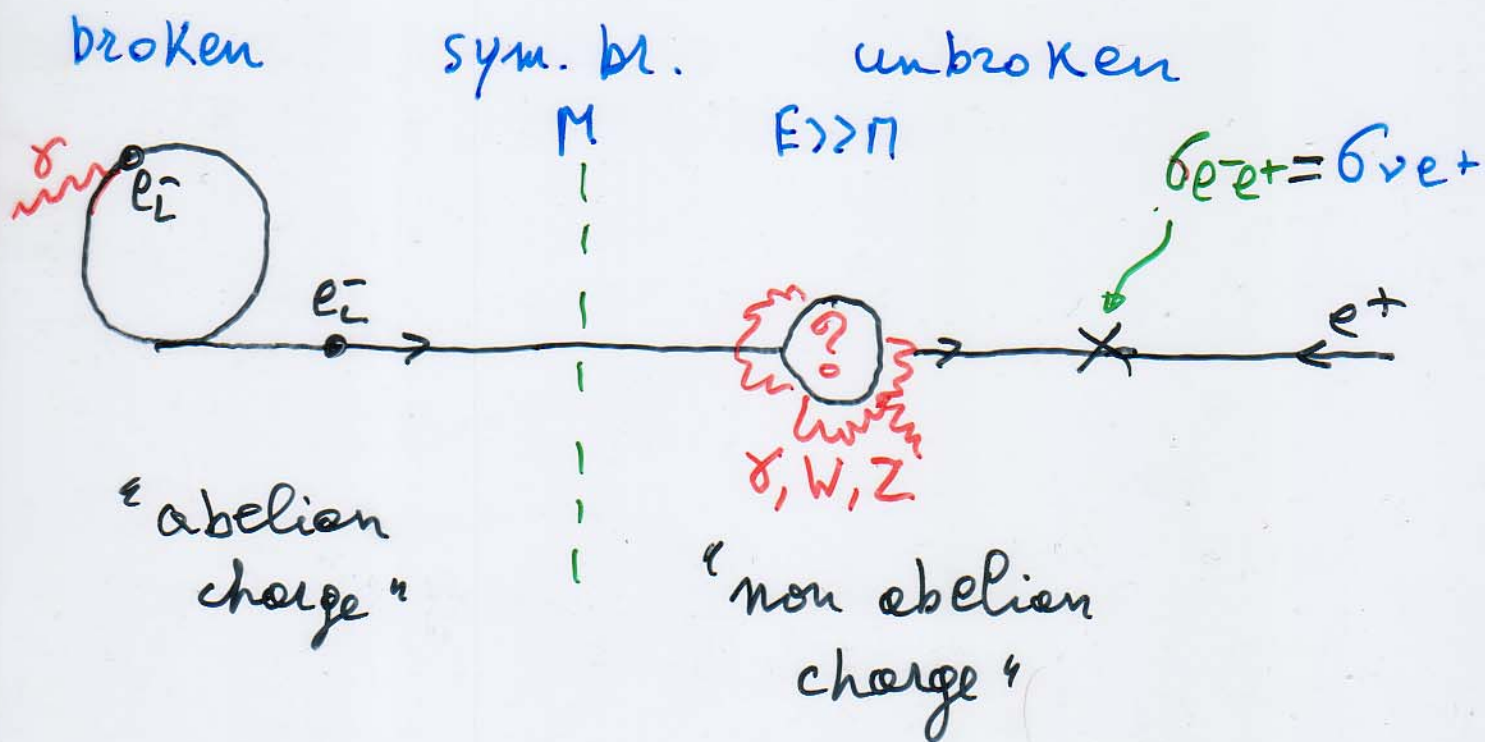
$e^+ e^- \rightarrow \text{hadrons}$



$$\sigma_{\nu e^+} - \sigma_{e^- e^+} = e^{-\alpha_w L^2} (\sigma_\nu - \sigma_e)_H$$

$$\sigma_{\nu e^+} + \sigma_{e^- e^+} = (\sigma_\nu + \sigma_e)_H$$

Can you tell an electron from a neutrino?



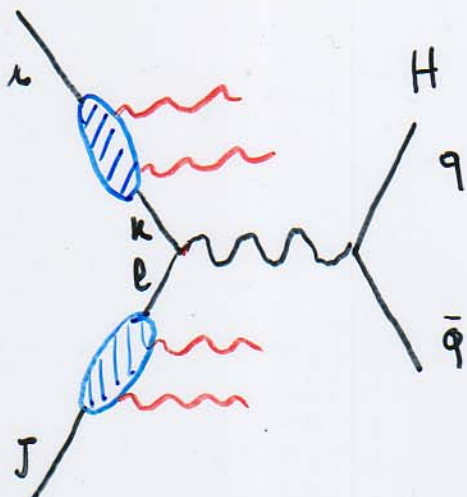
What about subleading logs?

EW evolution equations

Π. Liekele
P. C.

I-factorization

D. Comelli



probability of finding parton k inside parton i
 $i, k = \text{fermion, transv. bos}$

$$\sigma_{ij} = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} P_{ik}(x_1; s, \pi^2) \sigma_{kl}^H(x_1 p_1, x_2 p_2) P_{lj}(x_2; s, \pi^2)$$

describe all π -dep collinear/IR divergences

IR-free (π -indep.)

Differences with unbroken theories (QCD):

- factorization \leftrightarrow W.I., modified by G.B. insertion

$$k_\mu \text{ wavy } Q_i = \left[\pi \text{ --- } Q_i \right] \text{ not necessarily } \propto \frac{\pi^2}{s} \text{ (see later)}$$

- Both collinear and infrared logs are taken into account:

'EW DGLAP' resum

$$(\alpha L^2)^n, \dots, \alpha^n L^{n+1}, (\alpha L)^n$$

↑
QCD DGLAP

- written for trans. g.b., fermions in SU(2)
- Soon available for full S.B.

$$\frac{-d}{d \log \mu^2} \text{ (diagram)} = \text{ (diagram)} + \text{ (diagram)}$$

The diagram shows a vertex correction to a splitting function. On the left, a circle with a vertical dashed line through its center and four external lines (two on the left, two on the right). This is equal to the sum of two diagrams. The first diagram is the same as the left one. The second diagram is identical to the first but with a wavy line connecting the two bottom vertices of the circle.

$$\frac{-d}{d \log \mu^2} f_f^0(x, \mu^2) = \int_x^1 f_f^0\left(\frac{x}{z}, \mu^2\right) P_{f_f}^0(z) \equiv f_f^0 \otimes P_{f_f}^0$$

← QCD-like

$$\frac{-d}{d \log \mu^2} f_f^1(x, \mu^2) = (a \log \frac{s}{\mu^2} + b) f_f^1(x, \mu^2) + f_f^1 \otimes P_{f_f}^1$$

↑
NEW IR-singular splitting function

$$f_f^0 = \frac{f^v + f^e}{2} ; \quad f_f^1 = \frac{f^v - f^e}{2}$$

Redefinition:

$$f_f^1(x, \mu^2) = \exp\left[-\frac{d_W}{4\pi} \log^2 \frac{s}{\mu^2} - 3 \log \frac{s}{\mu^2}\right] \tilde{f}_f^1(x, \mu^2)$$

$$\frac{-d}{d \log \mu^2} \tilde{f}_f^1(x, \mu^2) = \tilde{f}_f^1 \otimes P_{f_f}^1$$

Conclusioni

- Correzioni EW a $\sqrt{s} \gtrsim 1 \text{ TeV}$ dominate
de dinamiche IR \Rightarrow effetti "grossi"
 $O(d_w \log^2 \frac{s}{\Lambda^2}) \sim 10\%$; higher orders per NLC
- Osservabili "esclusive"
 - 3 scale di massa $\sqrt{s}, \Lambda_w, \Lambda \gg m_f$
 - ambiguità di ordinamento
 - Disponibili LL, NLL, NNLL, 2 loops
- Osservabili "inclusive"
 - 2 scale $\sqrt{s}, \Lambda \Rightarrow$ ordinamento ok
 - $BW \Rightarrow \alpha d \log^2 \frac{\sqrt{s}}{\Lambda} \leftarrow \text{IR cutoff}$
- "Nuova fisica": BW in SVT, BW in
teorie abeliane, $e_L \dots$
- Equazioni di evoluzione collineari
coinvolgono nuove splitting functions
Presto disponibili in full SVT
- Se non si costruiscono NLC e TeV
"Much ado about nothing"