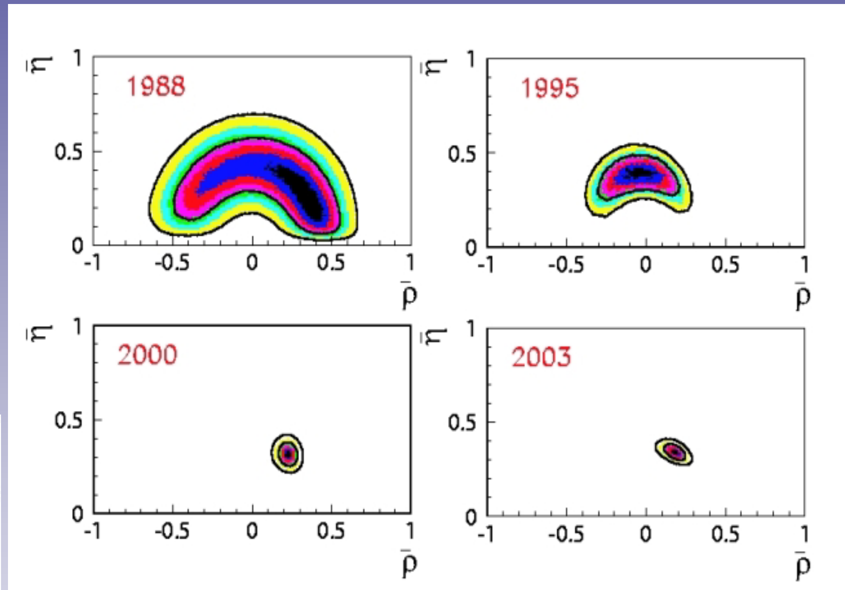
Decadimenti rari del B: risultati e sfide per il futuro

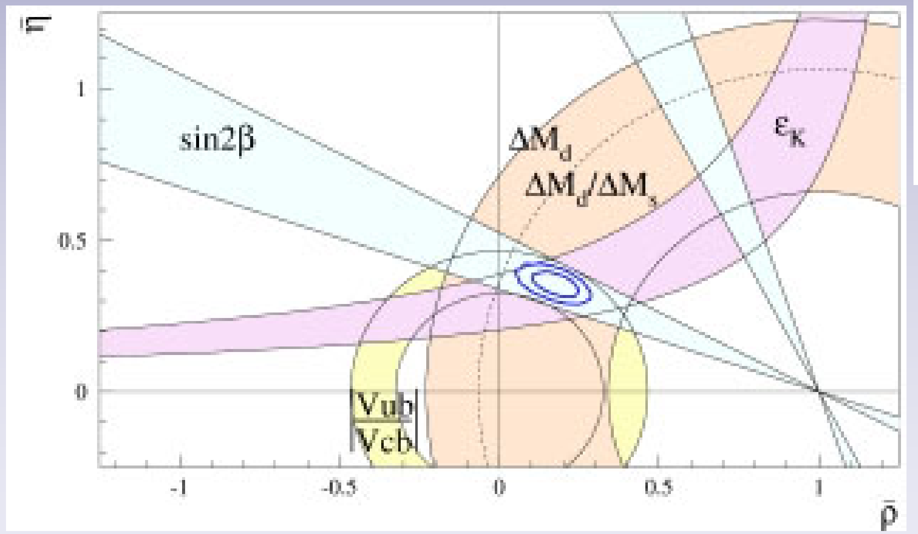
Paolo Gambino
INFN Torino

Any room left for New Physics in the flavor sector?

With advent of B factories B physics has entered a new precision era

Presently, only $\Delta F=2$ transitions are considered in the fits





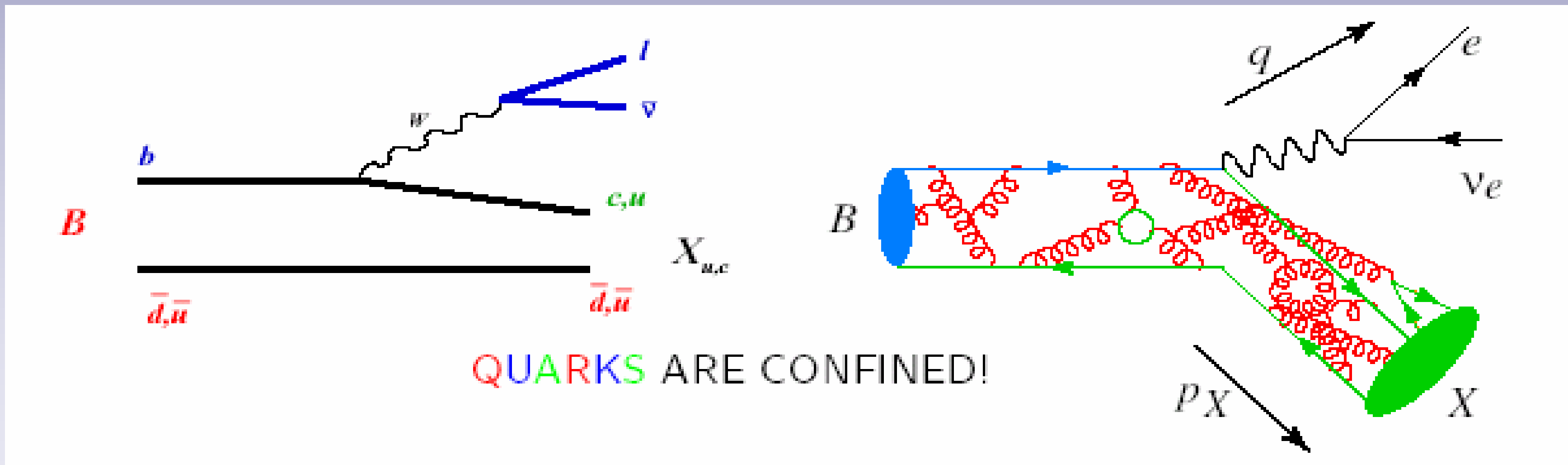
$\Delta F=1$ could be affected by new physics in a different way

$B \to \Phi K_s$?

Why inclusive?

$\Lambda_{QCD} \ll m_b$: inclusive decays admit systematic expansion in Λ_{QCD}/m_b
Non-pert corrections are generally small and can be controlled

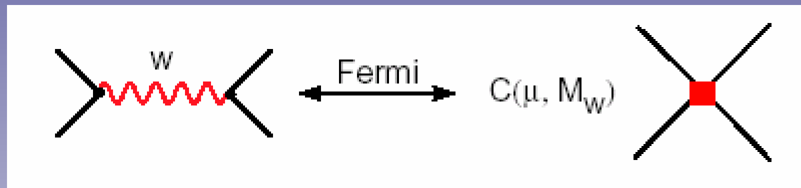
Hadronization probability =1 because we sum over all states
Approximately insensitive to details of meson structure as $\Lambda_{QCD} \ll m_b$



b -> s inclusive transitions

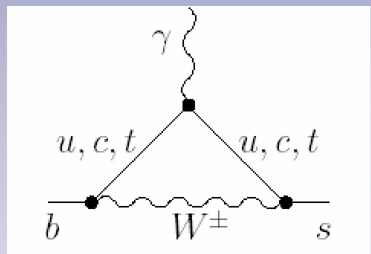
$\Lambda_{QCD} \ll m_b \ll M_W$

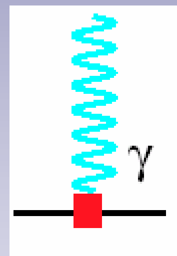
Large L=log m_b/M_W must be resumed.
LO: $\alpha_s^n L^n$, NLO: $\alpha_s^n L^{n-1}$



Tower of local ops
OPE

$m_b \ll M_W$





But many more operators appear adding gluons

$$\mathcal{L}_{QCD\times QED}(u,d,s,c,b,e,\mu,\tau) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{32} C_i(\mu)\, Q_i \,.$$

Inclusive decays admit systematic expansion in α_s and Λ_{QCD}/m_b
(except charm loop contributions)

The main ingredients

Process independent:

- The Wilson coefficients C_i (encode the short distance information, initial conditions)
- The Anomalous Dimension Matrix (the mixing among operators, the large logs, determines the evolution)

Process dependent: the matrix elemts

$B \to X_s\gamma$: NLO QCD calculation completed, all results checked, EW , power corrections

$B \to X_s ll$: NNLO & EW calculation just completed,power corrections

NLO ADM: a real tour de force

Physical operators

$$Q_1 = (\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L)\,,$$
$$Q_2 = (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L)\,,$$
$$Q_3 = (\bar{s}_L\gamma_\mu b_L)\sum_q(\bar{q}\gamma^\mu q)\,,$$
$$Q_4 = (\bar{s}_L\gamma_\mu T^a b_L)\sum_q(\bar{q}\gamma^\mu T^a q)\,,$$
$$Q_5 = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho b_L)\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho q)\,,$$
$$Q_6 = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho T^a b_L)\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho T^a q)$$
$$Q_7 = \frac{e}{g^2}m_b(\bar{s}_L\sigma^{\mu\nu}b_R)F_{\mu\nu}\,,$$
$$Q_8 = \frac{1}{g}m_b(\bar{s}_L\sigma^{\mu\nu}T^a b_R)G^a_{\mu\nu}\,,$$
$$Q_9 = \frac{e^2}{g^2}(\bar{s}_L\gamma_\mu b_L)\sum_\ell(\bar{\ell}\gamma^\mu\ell)\,,$$
$$Q_{10} = \frac{e^2}{g^2}(\bar{s}_L\gamma_\mu b_L)\sum_\ell(\bar{\ell}\gamma^\mu\gamma_5\ell)\,,$$

EOM vanishing ops

$$Q_{11} = \frac{e}{g^2}\bar{s}_L\gamma^\mu b_L\partial^\nu F_{\mu\nu} + \frac{e^2}{g^2}(\bar{s}_L\gamma_\mu b_L)\sum_f Q_f(\bar{f}\gamma^\mu f)\,,$$
$$Q_{12} = \frac{1}{g}\bar{s}_L\gamma^\mu T^a b_L D^\nu G^a_{\mu\nu} + Q_4\,,$$
$$Q_{13} = \frac{1}{g^2}m_b\bar{s}_L\not{D}\not{D}b_R\,,$$
$$Q_{14} = \frac{i}{g^2}\bar{s}_L\not{D}\not{D}\not{D}b_L\,,$$
$$Q_{15} = \frac{ie}{g^2}\left[\bar{s}_L\overleftarrow{\not{D}}\sigma^{\mu\nu}b_L F_{\mu\nu} - F_{\mu\nu}\bar{s}_L\sigma^{\mu\nu}\not{D}b_L\right] + Q_7\,,$$
$$Q_{16} = \frac{i}{g}\left[\bar{s}_L\overleftarrow{\not{D}}\sigma^{\mu\nu}T^a b_L G^a_{\mu\nu} - G^a_{\mu\nu}\bar{s}_L T^a\sigma^{\mu\nu}\not{D}b_L\right] + Q_8\,,$$

Gauge variant ops

$$Q_{17} = \frac{i}{g}m_b\bar{s}_L\left[\overleftarrow{\not{D}}\not{G} - \not{G}\not{D}\right]b_R\,,$$
$$Q_{18} = i\left[\bar{s}_L\left(\overleftarrow{\not{D}}\not{G}\not{G} - \not{G}\not{G}\not{D}\right)b_L - im_b\bar{s}_L\not{G}\not{G}b_R\right]\,,$$
$$Q_{19} = \frac{1}{g}\left[\bar{s}_L\left(\overleftarrow{\not{D}}\overleftarrow{\not{D}}\not{G} + \not{G}\not{D}\not{D}\right)b_L + im_b\bar{s}_L\not{G}\not{D}b_R\right]\,,$$
$$Q_{20} = i\left[\bar{s}_L\left(\overleftarrow{\not{D}}G^a_\mu G^{a\mu} - G^a_\mu G^{a\mu}\not{D}\right)b_L - im_b\bar{s}_L G^a_\mu G^{a\mu}b_R\right]\,,$$
$$Q_{21} = \frac{1}{g}\left[\bar{s}_L\left(\overleftarrow{\not{D}}\overleftarrow{D}_\mu G^\mu + G_\mu D^\mu\not{D}\right)b_L + im_b\bar{s}_L G_\mu D^\mu b_R\right]\,,$$
$$Q_{22} = \frac{1}{g}\left[\bar{s}_L\left(\overleftarrow{\not{D}}T^a + T^a\not{D}\right)b_L + im_b\bar{s}_L T^a b_R\right]\partial^\mu G^a_\mu\,,$$
$$Q_{23} = \frac{1}{g}\left[\bar{s}_L\overleftarrow{\not{D}}\not{G}\not{D}b_L + im_b\bar{s}_L\overleftarrow{\not{D}}\not{G}b_R\right]\,,$$
$$Q_{24} = d^{abc}\left[\bar{s}_L\left(\overleftarrow{\not{D}}T^a - T^a\not{D}\right)b_L - im_b\bar{s}_L T^a b_R\right]G^b_\mu G^{c\mu}\,,$$

Evanescent operators

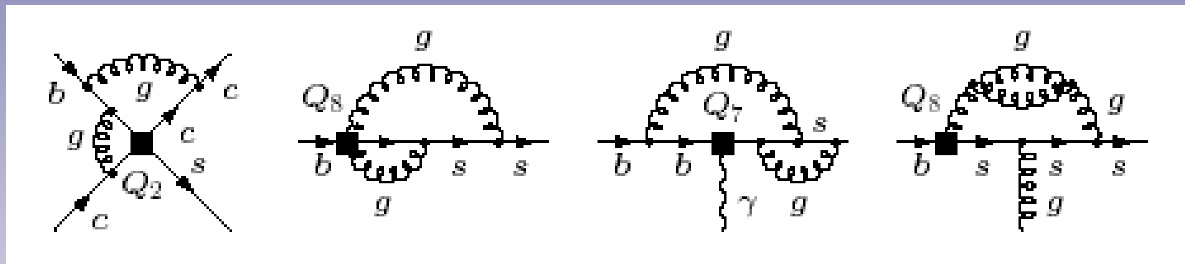
$$Q_{25} = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho T^a c_L)(\bar{c}_L\gamma^\mu\gamma^\nu\gamma^\rho T^a b_L) - 16Q_1\,,$$
$$Q_{26} = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho c_L)(\bar{c}_L\gamma^\mu\gamma^\nu\gamma^\rho b_L) - 16Q_2\,,$$
$$Q_{27} = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\tau b_L)\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\tau q) + 64Q_3 - 20Q_5\,,$$
$$Q_{28} = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\tau T^a b_L)\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\tau T^a q) + 64Q_4 - 20Q_6\,,$$
$$Q_{29} = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\tau T^a c_L)(\bar{c}_L\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\tau T^a b_L) - 256Q_1 - 20Q_{25}\,,$$
$$Q_{30} = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\tau c_L)(\bar{c}_L\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\tau b_L) - 256Q_2 - 20Q_{26}\,,$$
$$Q_{31} = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\tau\gamma_\upsilon\gamma_\omega b_L)\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\tau\gamma^\upsilon\gamma^\omega q) + 1280Q_3 - 336Q_5\,,$$
$$Q_{32} = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\tau\gamma_\upsilon\gamma_\omega T^a b_L)\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\tau\gamma^\upsilon\gamma^\omega T^a q) + 1280Q_4 - 336Q_6$$

New ADM calculation

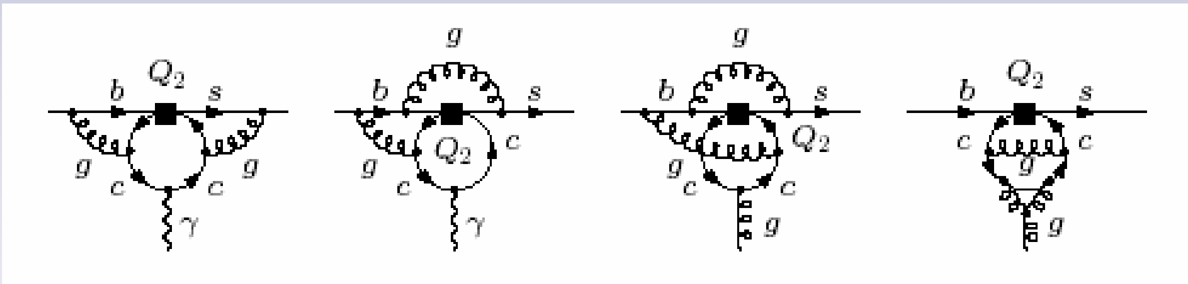
Check of Chetyrkin,Misiak,Munz & NEWADM for b->sll

Gorbahn,Haisch,PG

2loop



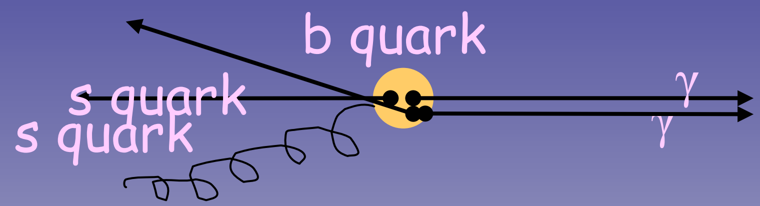
3loop



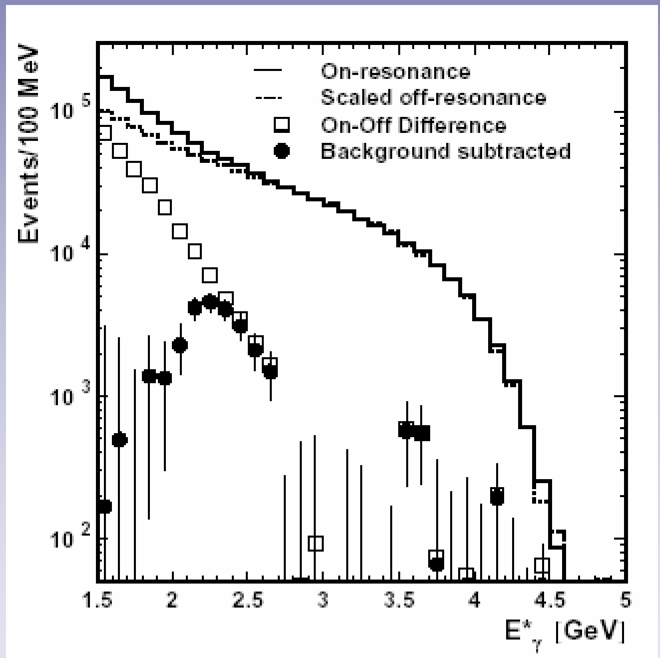
The ADM for b->s: method

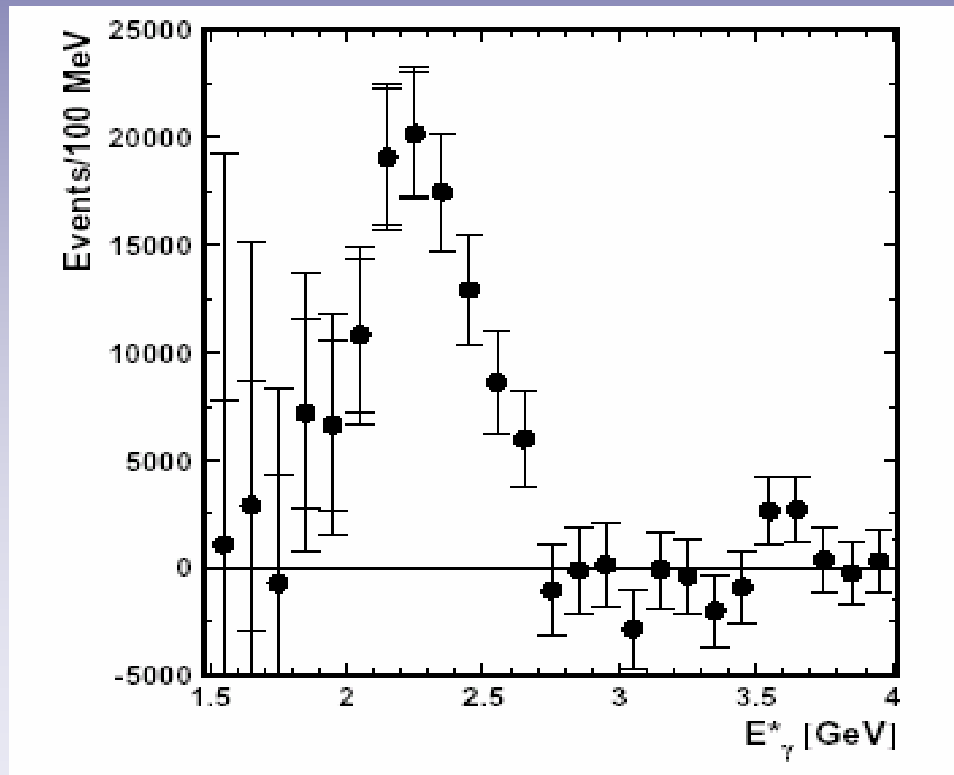
- R_ξ gauge
- Anticommuting γ_5 : choice of basis allows it
- Common mass M for all fields to distinguish UV from IR. After 1/M expansion, extract UV
- Pro: allows 3 loop calculation (only tadpoles,MATAD)
- Con: insertion of non-phys ops and c.t. M^2G^2
- **Checks:** locality,gauge inv,indep of external states, basis completeness, no mixing non-phys->phys

The photon spectrum and its uses



Motion of b quark inside B and gluon radiation smear the spike at $m_b/2$





Belle NEW: lower cut at 1.8GeV

Photon spectrum & its uses (II)

The photon spectrum is very insensitive to new physics, can be used to study the B meson structure

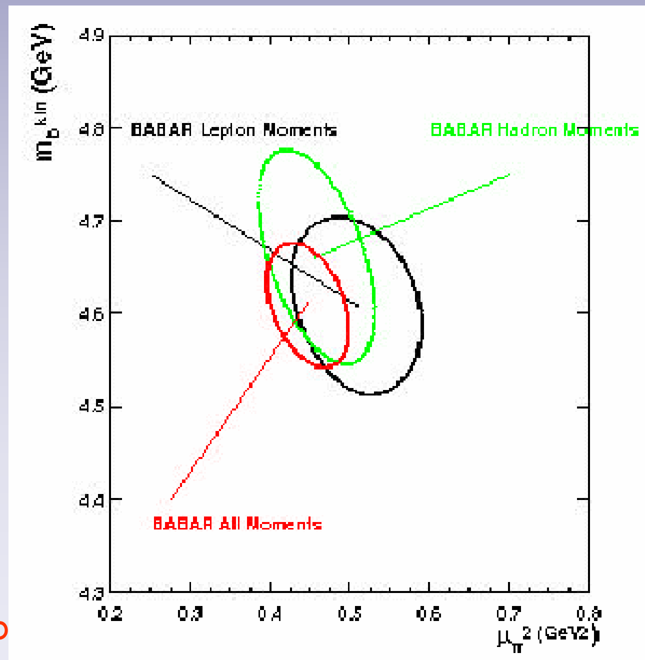
$\langle E_\gamma \rangle = m_b + \ldots$ $\quad$ $\mathrm{var}\langle E_\gamma \rangle = \mu_\pi^2/12 + \ldots$ $\quad$ $\mu_\pi^2 \sim \langle B|\bar{b}D^2 b|B\rangle$

Importance of extending to $E_\gamma \sim 1.8$ GeV or less
for the determination of both the BR AND the B parameters (Bigi Uraltsev)

Excellent agreement with NEW Babar fit to moments of s.l. distributions

$M_b(1\mathrm{GeV}) = 4.585 \pm 0.059$ GeV
$\mu_\pi^2(1\mathrm{GeV}) = 0.454 \pm 0.052$ GeV^2

$M_b(1\mathrm{GeV}) = 4.611 \pm 0.067$ GeV
$\mu_\pi^2(1\mathrm{GeV}) = 0.447 \pm 0.052$ GeV^2
without Belle photon moments

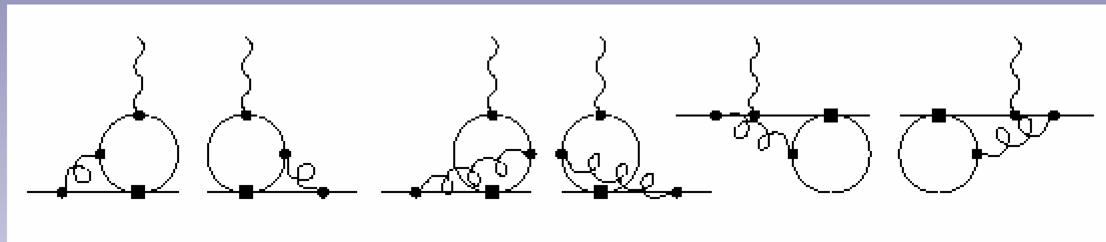


Extremely important for inclusive V_{cb}, V_{ub}

The charm mass problem

m_c enters the phase factor $C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B} \to X_c e\bar{\nu}]}{\Gamma[\bar{B} \to X_u e\bar{\nu}]}$ = 0.581±0.017

And the NLO matrix elements



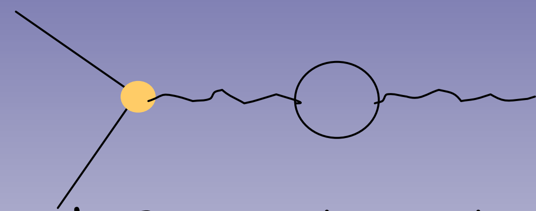
LO diagrams vanish: the definition of m_c is a NNLO issue.
Numerically important because these are large NLO contributions:
$m_c(m_c)$=1.25±0.10 GeV $m_c(m_b)$=0.85±0.11 GeV m_c(pole)~1.5GeV
But pole mass has nothing to do with these loops

Changing m_c/m_b from 0.29 (pole) to 0.22 (MSbar) increases BRγ by 11%
0.22 ±0.04 gives DOMINANT 6% theory error

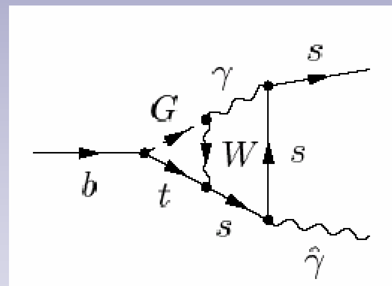
Electroweak effects in BRγ

Almost complete calculation of $O(\alpha\alpha_s^n L^n)$ effects
Haisch, PG

Normalization of α : Czarnecki Marciano



Other leading log effects small except in sl rate Baranowski, Misiak



NLO EW effects:

1. Two loop matching conditions ($1/\sin^2\theta_W$ etc)
2. QED-QCD evolution neglecting 3loop $O(\alpha\alpha_s)$ ADM
3. QED matrix elements

Total EW effects -3.6% in BRγ, of which -1.6% due to NLO
Dilution due to interplay with QCD, small M_{Higgs} dependence

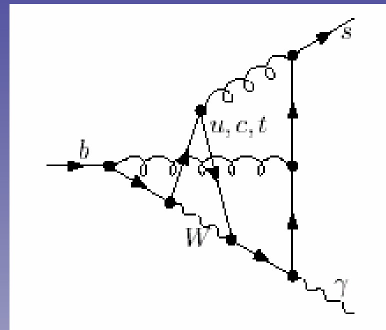
Error anatomy of BRγ

$$
\begin{aligned}
\mathrm{BR}\left[\bar{B} \to X_s\gamma\right]_{E_\gamma > 1.6\,\mathrm{Gev}} &= (3.61 \pm 0.30) \times 10^{-4}\,, \\
&= 3.61 \times 10^{-4}\,(1 \pm 0.06_{(m_c/m_b \text{ in } K_c)} \pm 0.04_{(\text{other NNLO})} \\
&\quad \pm 0.01_{(\text{pert C})} \pm 0.02_{\lambda_1} \pm 0.02_{\Delta} \\
&\quad \pm 0.02_{\alpha_s(M_Z)} \pm 0.02_{\mathrm{BR(semilept)_{exp}}} \pm 0.01_{m_t})
\end{aligned}
$$

Total error 8% dominated by charm mass
Can be partially resolved by NNLO
Update under way

First steps towards the NNLO...

- NNLO $C_{7,8}$ matching completed
  Misiak, Steinhauser   <2% in MSbar scheme, $M_W < \mu < M_t$



- NNLO ADM of the 4quark operators Gorbahn,Haisch
  used already for NNLO b->sll

- Soon: NNLO mixing of Q_7 Q_8 (Gorbahn, Haisch) and
  NNLO Matrix element of Q_2 in some approx (Misiak Steinhauser)

Most difficult part 4loop ADM & bremstrahlung

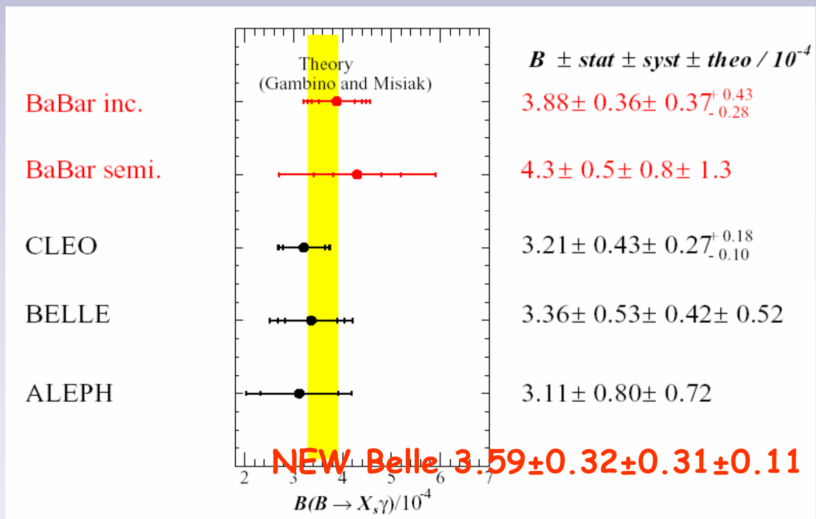
B->$X_s\gamma$: a new physics killer

It is the best measured rare decay
Good agreement with SM strongly constrains most new models
But don't expect surprises...

Exp: $B(B\to X_s\gamma)=(3.34\pm0.38)\times10^{-4}$

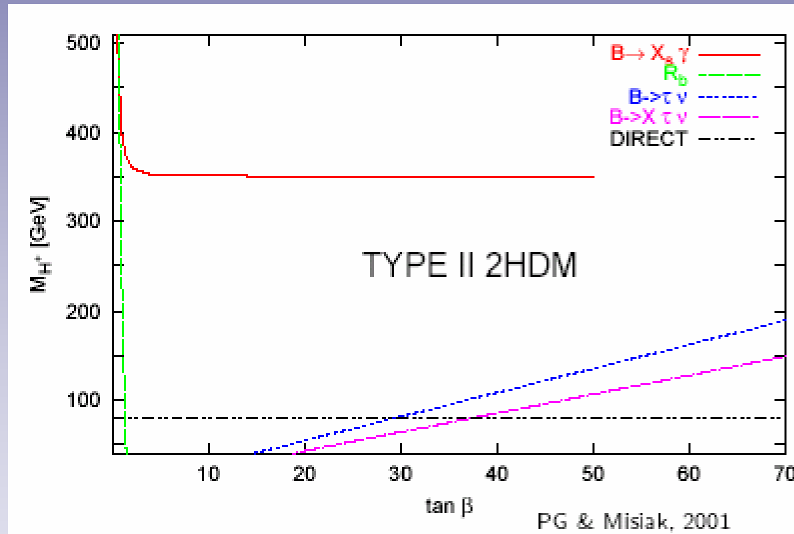
SM: $B(B\to X_s\gamma)=(3.70\pm0.30)\times10^{-4}$

Gambino-Misiak



charged Higgs mass bounds in typeII 2HDM

$$\mathcal{L} \sim V_{ud}\,\bar{u}\left(\frac{m_u}{\tan\beta}\,a_- + \tan\beta\, m_d\, a_+\right) dH^+$$



Does not carry over to MSSM!
But very strong bounds there too

Degrassi, Giudice,PG...

Main obstacle to explain $A_{CP}(B\to\Phi K_S)$ in MSSM

CP asymmetry

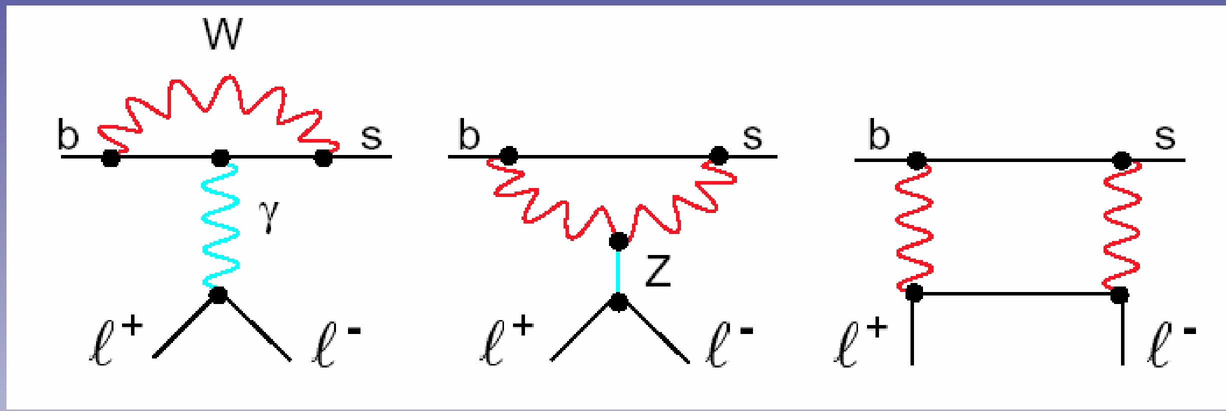
$$A_{CP}^{b\to q\gamma} \equiv \frac{\Gamma[\bar{B}\to X_q\gamma] - \Gamma[B\to X_{\bar{q}}\gamma]}{\Gamma[\bar{B}\to X_q\gamma] + \Gamma[B\to X_{\bar{q}}\gamma]}.$$

Belle: $A_{CP}(X_s\gamma)=-0.004\pm0.051\pm0.038$
$A_{CP}(K^*\gamma)=-0.015\pm0.044\pm0.012$

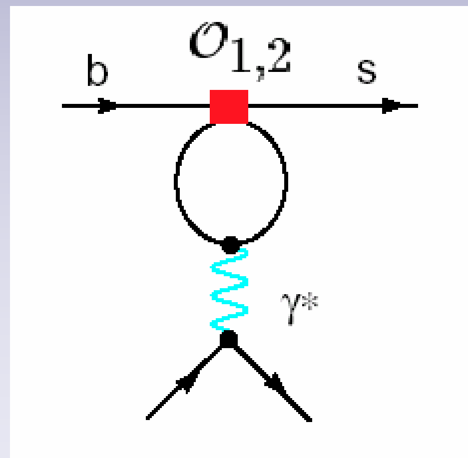
$A^{b\to s\gamma}$ strongly suppressed in SM (<1%), $A^{b\to d\gamma}$ less so
$A_{CP}(X_s\gamma+X_d\gamma)=0$ (unitarity!)

Very clean SM test Hurth,Lunghi,Porod

$b \to s l^+ l^-$: a more complicated case



This decay mode is sensitive to different operators, hence to different new physics



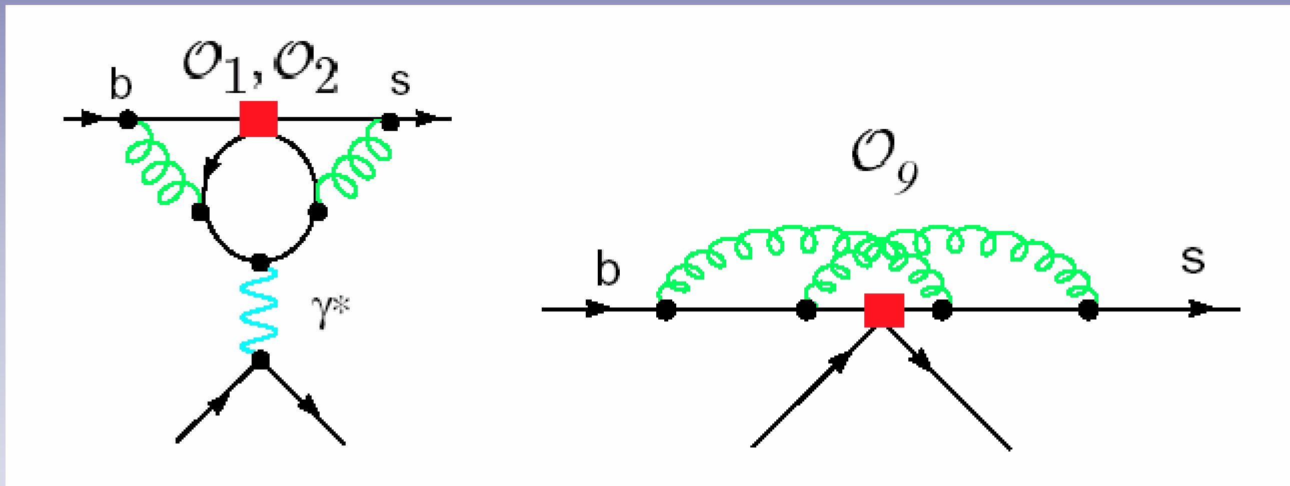
Here large logs are generated even without QCD: LO $\alpha_s^n L^{n+1}$, NLO $\alpha_s^n L^n$,...

However, numerically the leading log is subdominant, yielding an awkward series:
in BR $1 + 0.7\,(\alpha_s) + 5.5\,(\alpha_s^2) + \ldots$

Completing the NNLO calculation

Two missing ingredients:

3 loop ADM mixing O_2 into O_9 Gorbahn,Haisch,PG
3 loop ADM 4q operators Gorbahn,Haisch

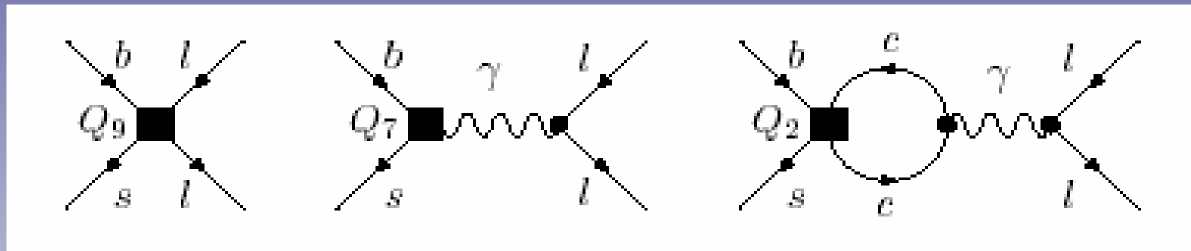


2loop Matrix element of O_9
Using calculations for sl
Czarnecki,Melnikov,Steinhauser

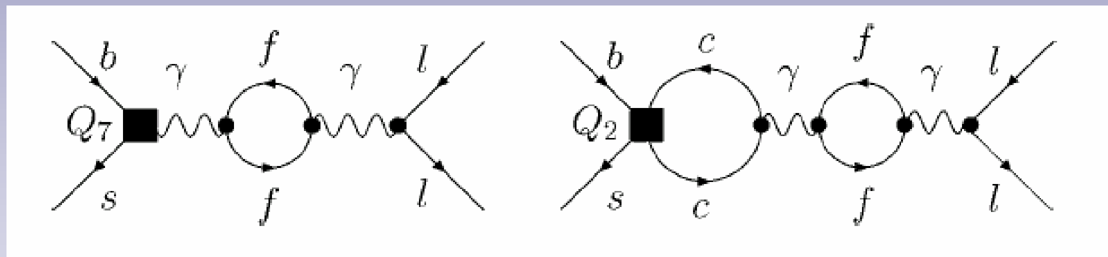
Electroweak corrections to $b \to s l^+ l^-$

This decay is suppressed by two e.w. couplings wrt $bs\gamma$:
$BR \sim \alpha(\mu)^2$ $\mu = m_b$ or M_W ? Difference is 8%!

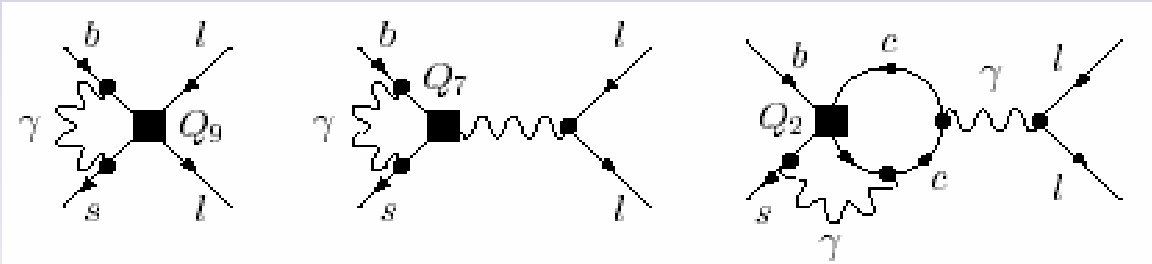
Lowest order diagrams



Two sources of large QED logs: running of α

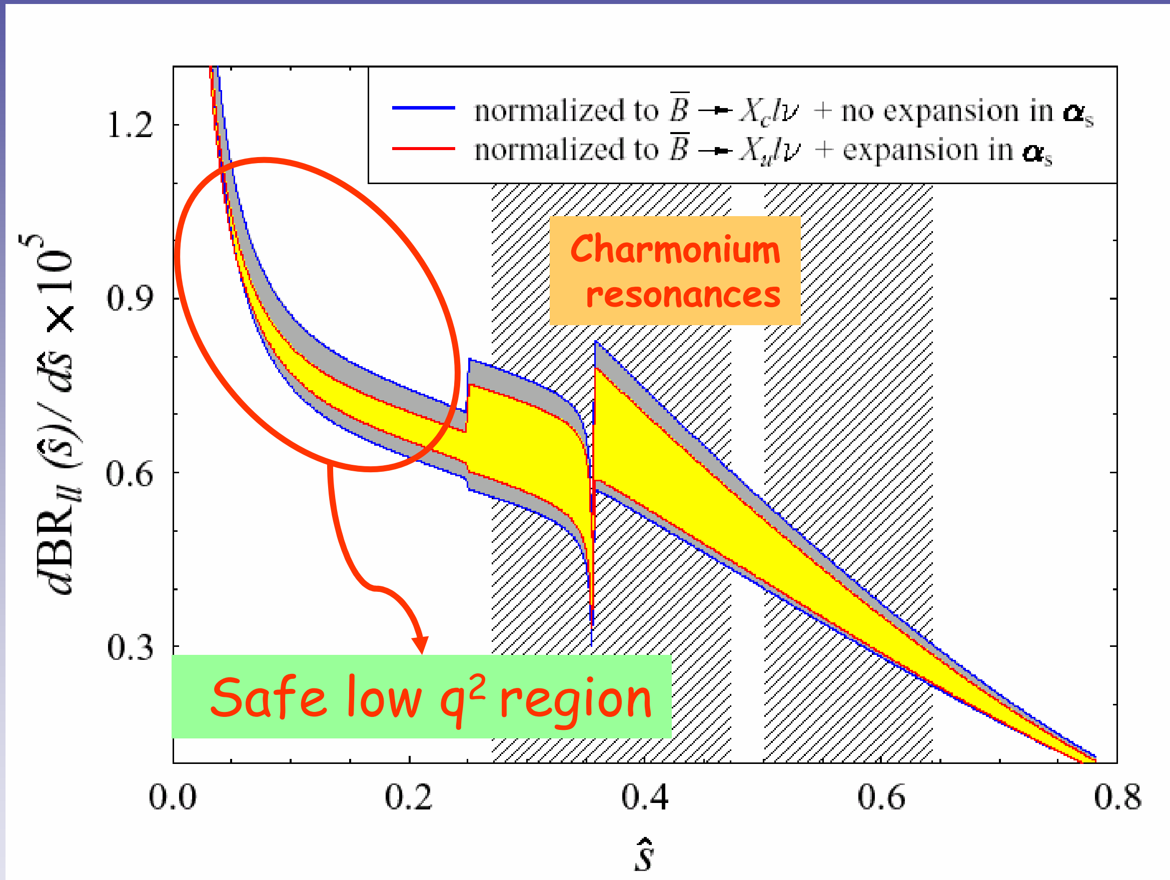


and of operators



LO and NLO EW effects New $O(\alpha)$,$O(\alpha\alpha_s)$ and $O(\alpha_s)$ ADM required

Results for the low q^2 region

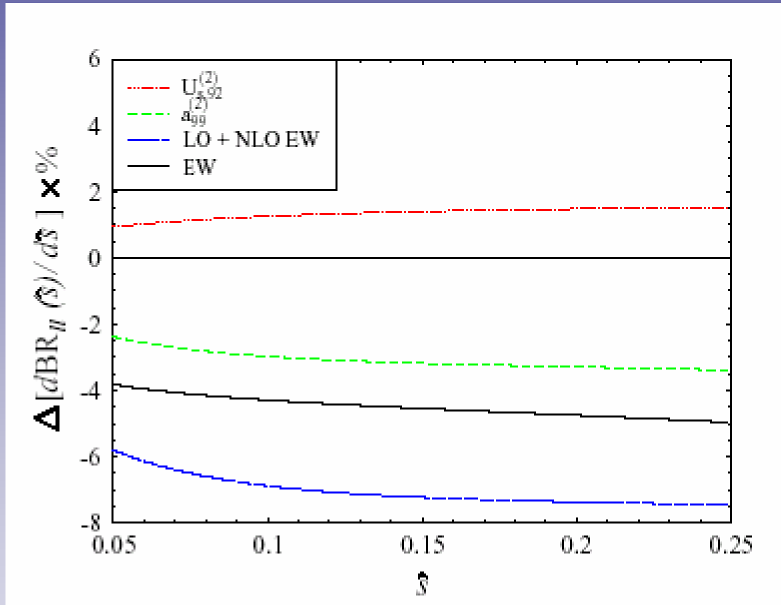


Bobeth, Gorbahn, Haisch, PG

Normalization to sl rate introduces 8% uncertainty from m_c

We use b->u and evaluate C apart, expand BR in α_s to minimize higher orders (pole mass), and reduce considerably error

Results for the low q^2 region



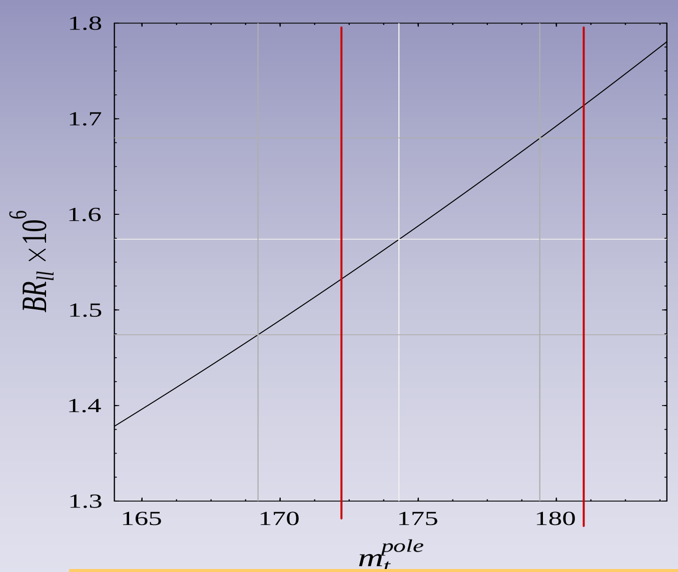
what is the best scale for $\alpha(\mu)^2$? due to cancellations, close to m_b

New $O(\alpha)$, $O(\alpha\alpha_s)$ and $O(\alpha_s)$ ADM required

Total shift due to new corrections (EW and NNLO): -4.5%

Error Anatomy for BR_{ll}

$$\mathrm{BR}_{\ell\ell}\left(1\,\mathrm{GeV}^2 \le q^2 \le 6\,\mathrm{GeV}^2\right) = \left[1.574 \pm^{0.106}_{0.100}\,|_{M_t} \pm^{0.059}_{0.075}\,|_{\mathrm{scale}} \pm 0.045_C \pm 0.035_{\mathrm{BR}_{sl}} \pm^{0.072}_{0.067}\,|_{m_b} \pm^{0.001}_{0.013}\,|_{m_c}\right] \times 10^{-6}$$



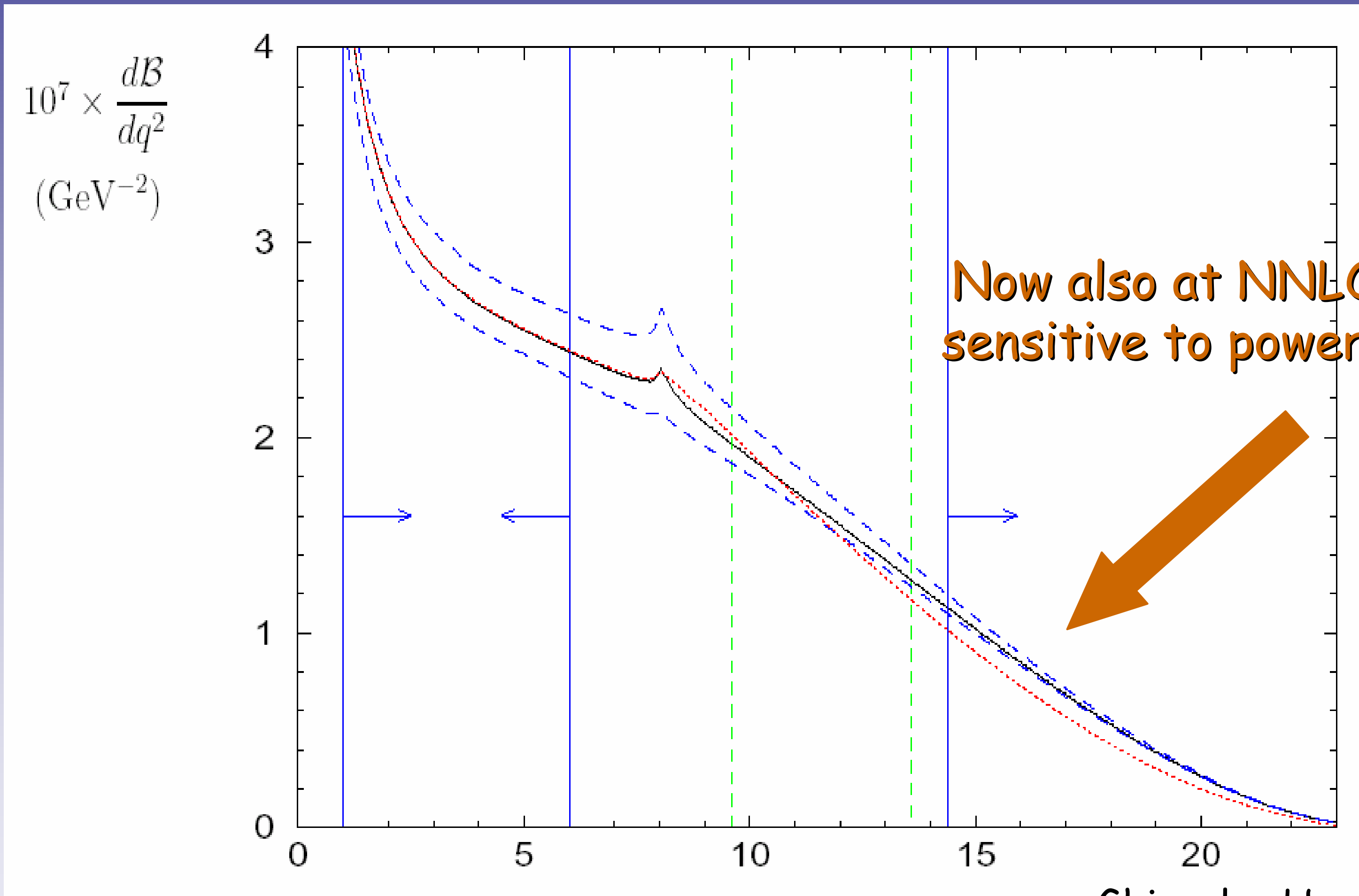
- M_{top} dominant error 7%
- scale uncertainty 5%
- m_b^{pole} = 4.80±0.15 GeV $\rightarrow$5%
- phase space factor 3%
- No m_c issue as charm enters at LO

TOTAL ERROR ~10%

BUT: m_b uncertainty is not a fundamental limitation
$\delta m_b^{\text{short distance}} \approx$ 30-50 MeV
simply change scheme!

EXP: only inclusive rate,
WA: $(6.2\pm1.1\pm1.6)\times10^{-6}$
Ali et al $(6.9\pm1.0)\times10^{-6}$,
Ghinculov et al $(4.8\pm0.8)\times10^{-6}$

The high-q^2 tail



Now also at NNLO, but more sensitive to power corrections

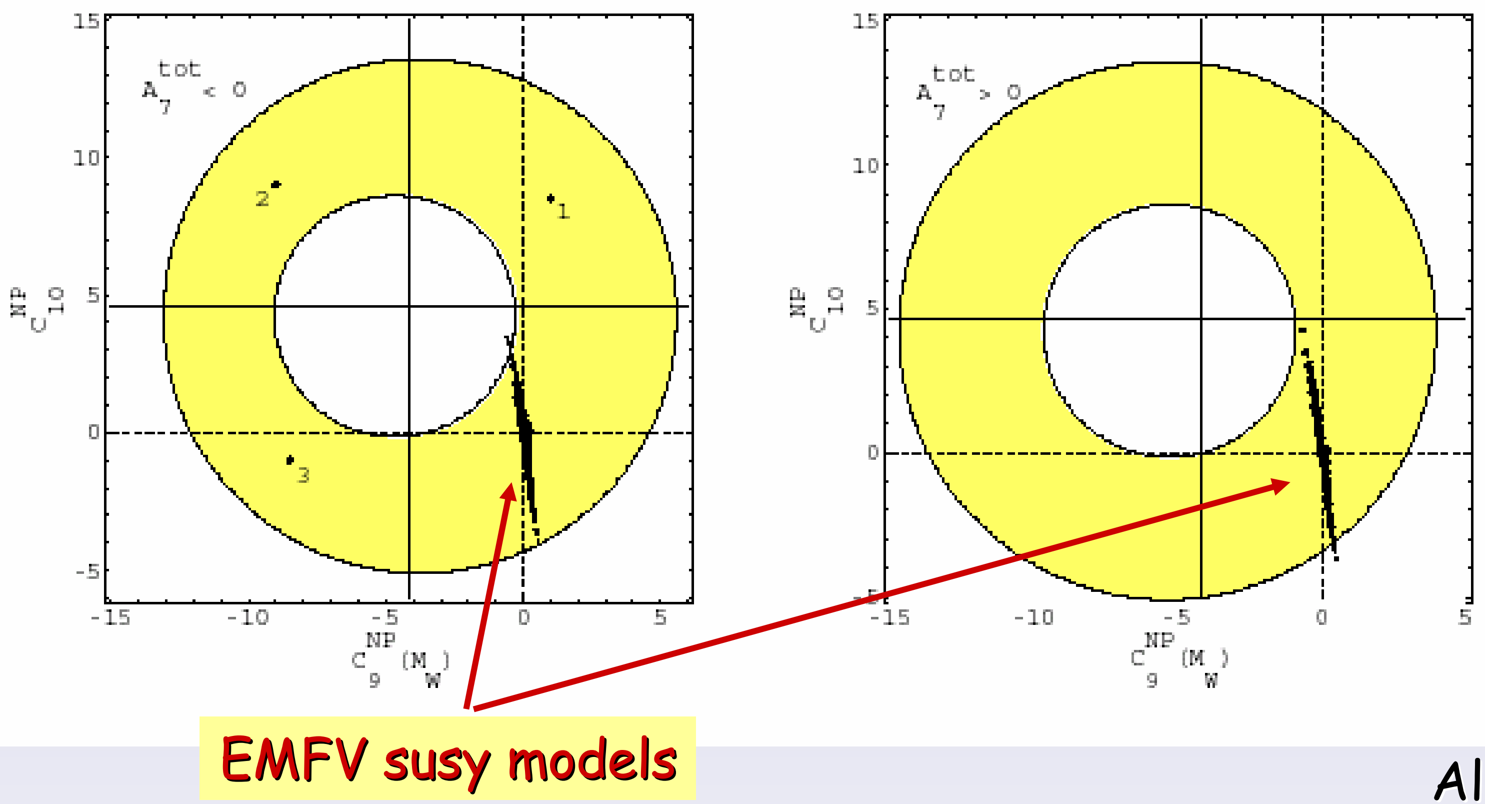
Ghinculov,Hurth,Isidori,Yao

$$\mathcal{B}(B \to X_s \ell^+\ell^-;\ q^2 > 14.4\ \mathrm{GeV}^2) \;=\; (4.04 \pm 0.78) \times 10^{-7}$$

Model independent contraints on new physics

From a combined analysis of inclusive and exclusive b->s transitions

90%CL

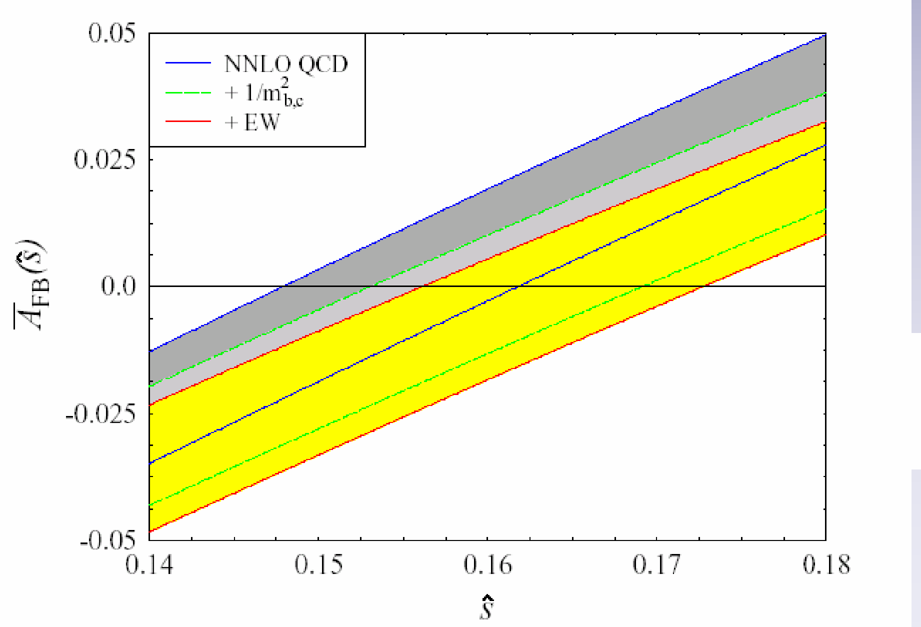


Ali et al

The FB asymmetry

$$\bar{A}_{\rm FB}(\hat{s}) = \frac{1}{d\Gamma[\bar{B}\to X_s\ell^+\ell^-]/d\hat{s}} \int_{-1}^{1} d\cos\theta_\ell \, \frac{d^2\Gamma[\bar{B}\to X_s\ell^+\ell^-]}{d\hat{s}\, d\cos\theta_\ell} \, {\rm sgn}(\cos\theta_\ell)$$

$$\approx -6\,{\rm Re}\left(\widetilde{C}^{\rm eff}_{7,{\rm FB}}(\hat{s})\widetilde{C}^{\rm eff}_{10,{\rm FB}}(\hat{s})^*\right) - 3\hat{s}\,{\rm Re}\left(\widetilde{C}^{\rm eff}_{9,{\rm FB}}(\hat{s})\widetilde{C}^{\rm eff}_{10,{\rm FB}}(\hat{s})^*\right) + A^{\rm Brems}_{\rm FB}(\hat{s})$$



Very sensitive to the way one treats formally NNNLO effects

$$\hat{s}_0 = 0.163 \pm 0.010_{\rm theory}$$

EW effects +3%

$$q_0^2 = (3.76 \pm 0.22_{\rm theory} \pm 0.24_{m_b})\ {\rm GeV}^2\,,$$

Summary

- The study of the flavor problem requires precise tests of the SM flavor structure in rare decays: inclusive modes guarantee the best theory control
- Radiative decays have reached maturity. NLO QCD, EW, and non-perturbative effects are routinely included. Theoretically, the challenge is NNLO, which seems to be needed because of charm mass problem. At last we have a lower cut on the photon energy.
- Rare semileptonic decays are just developing. More complicated, richer structure. We have completed NNLO and introduced dominant EW effects,reducing th error as in radiative modes. Now we need progress in experiment
- Exclusive modes: interesting too, especially $\rho\gamma/K^*\gamma$

Why are QCD corr. so large?

QCD corrections enhance $BR\gamma = BR(B \to X_s \gamma)$ by almost factor 4!
NLO adds ~30% to LO!

$$BR_\gamma = BR_{SL} \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha}{\pi f(\frac{m_c}{m_b})} |C_7(\mu_b)|^2$$

Split charm and top contributions to C_7: $C_7(M_W)=K_c-K_t$

$K_c= 0.64$; $K_t=0.45$ $\to BR\gamma \approx 1\times10^{-4}$ NO QCD

Resumming Leading Logs makes K_c and K_t run $K_t(m_b)=0.32$

The b mass associated to top loops is a high mass scale
The b mass associated to charm loops is low virtuality
Once $r=m_b(M_W)/m_b$ is factored out in front of K_t
QCD corrections are small, convergence good