Heavy Quarkonium

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Quarkonium Working Group

Motivations

- Competitive source of some SM parameters:

  m_t, m_b, m_c, α_{s}, ...

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- Competitive source of some SM parameters:
  m_t, m_b, m_c, $\alpha_{\rm s}$, ...

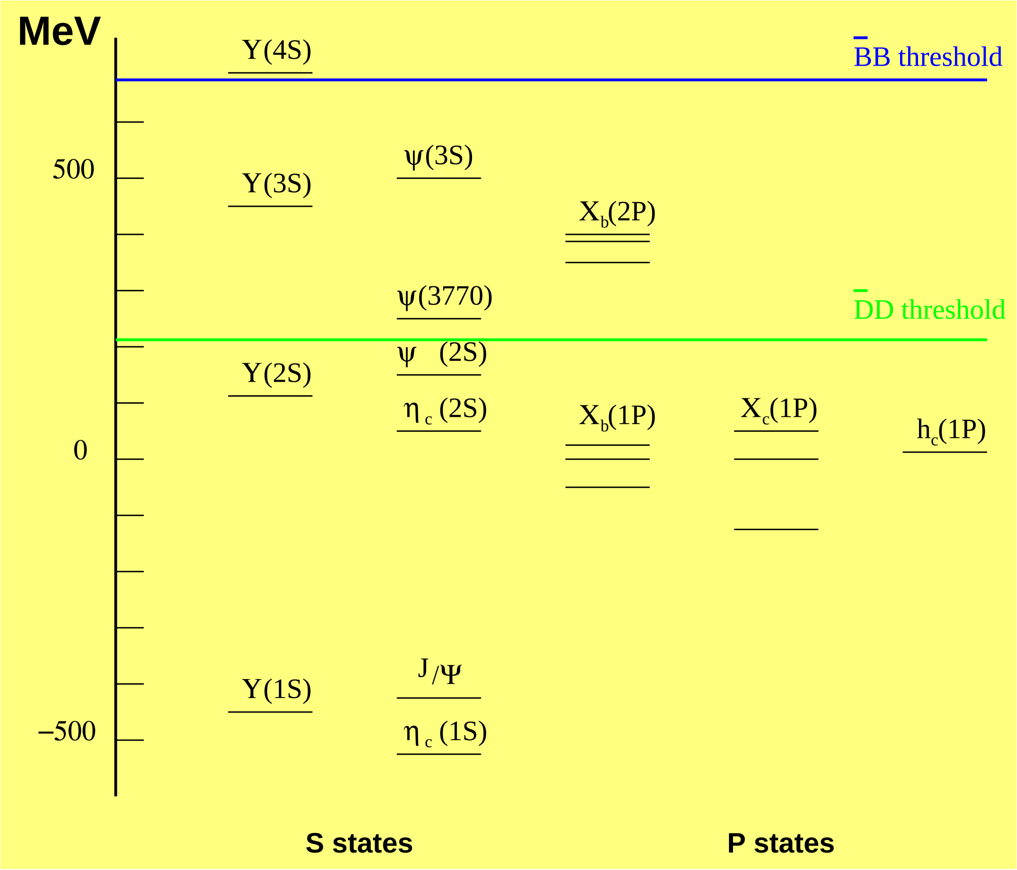
- Privileged system to study the interplay of perturbative and non-perturbative QCD.

  The modern framework is provided by Non-Relativistic EFTs. They enable a systematic study, combining perturbative and lattice QCD.

Quarkonium at Present & Future Colliders

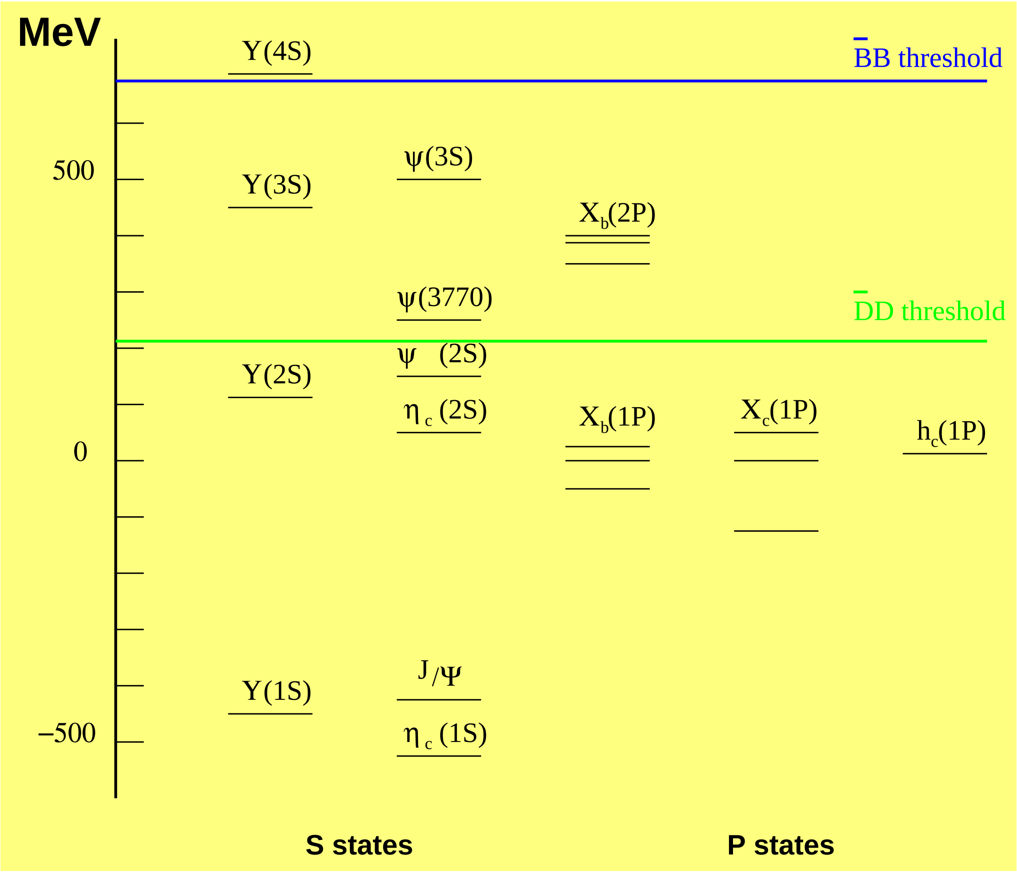
- $c\bar{c}$ BES, E835, KEDR, CLEO-c; $b\bar{b}$ CLEO-III
- hybrids, glueballs (BES, CEBAF, ...)
- Production at Fermilab (CDF, D0)
- Production at Hera (Zeus, H1)
- Production at B factories (BaBar, Belle)
- Quark-gluon plasma
  (NA60 at CERN, Star and Phenix at RHIC)
- Physics at NLC (TESLA,CLIC)
- LHC at CERN, Panda at GSI

Quarkonium Scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

Quarkonium Scales



The mass scale is perturbative:
$m_b \simeq 5$ GeV, $m_c \simeq 1.5$ GeV

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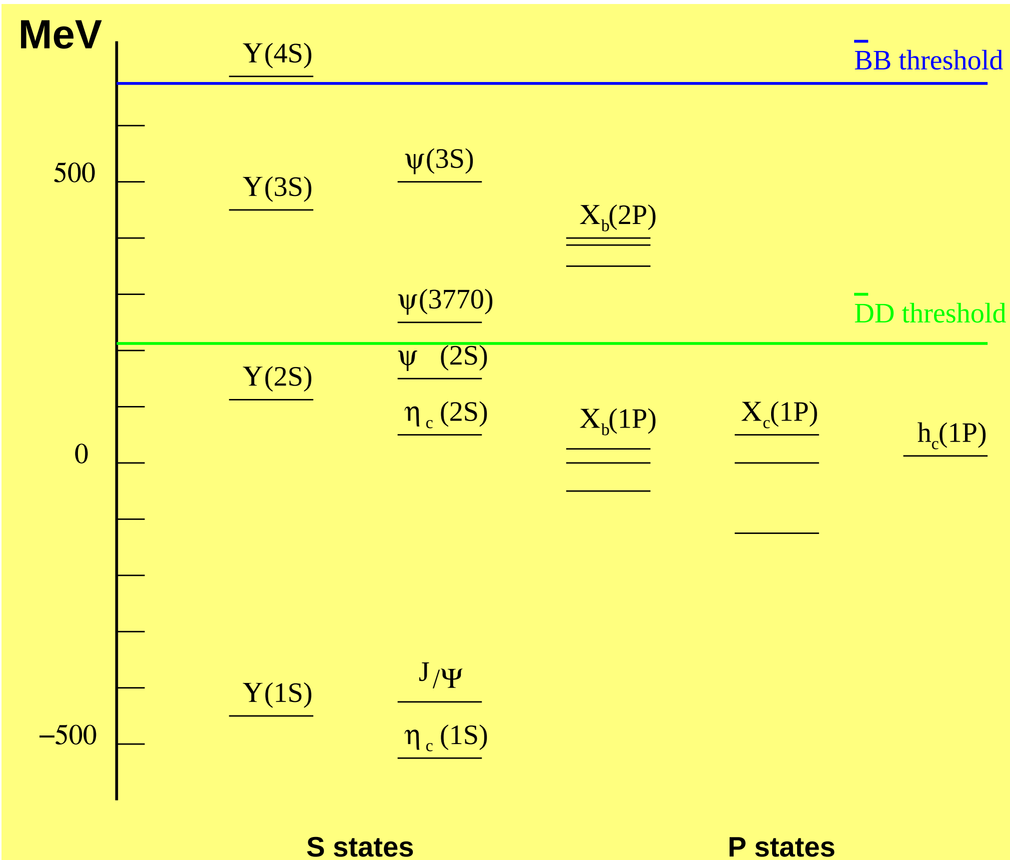
Quarkonium Scales



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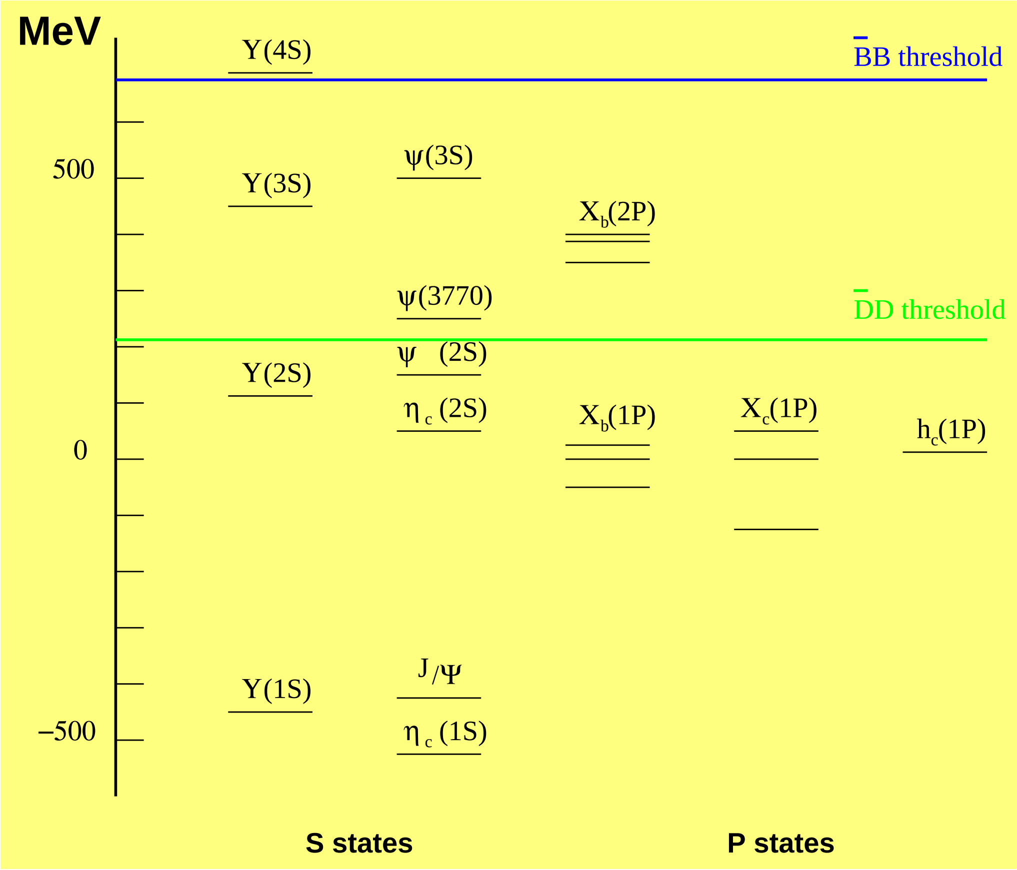
The system is non-relativistic:
$\Delta_n E \sim mv^2$, $\Delta_{\rm fs} E \sim mv^4$
$v_b^2 \simeq 0.1$, $v_c^2 \simeq 0.3$

Quarkonium Scales



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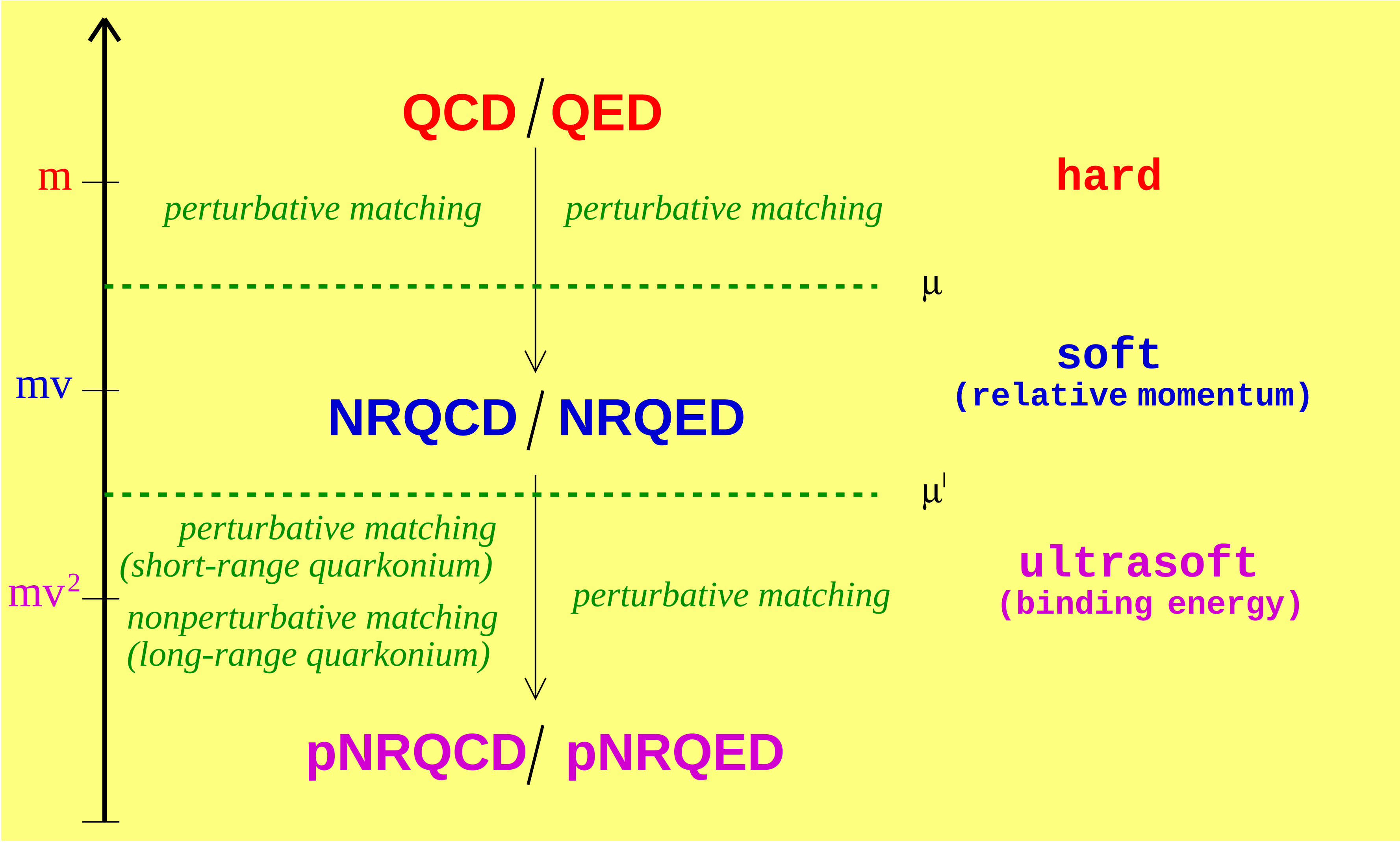
The dynamical scales are:
$r \sim 1/mv$, $E \sim mv^2 \qquad v \ll 1$

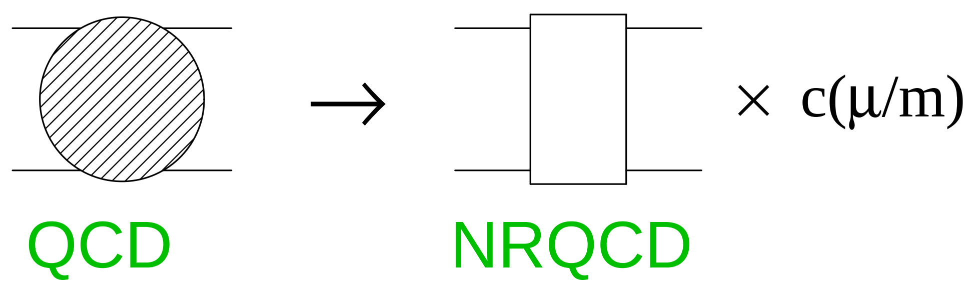
Non-Relativistic EFT



In QCD another scale is relevant: $\Lambda_{\rm QCD}$

NRQCD

Degrees of freedom that scale like m are integrated out:

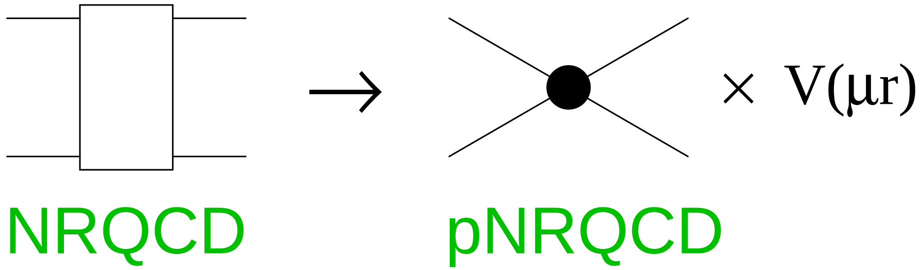


- The matching is perturbative.
- The Lagrangian is organized as an expansion in v and $\alpha_s(m)$.

Caswell Lepage 86, Bodwin Braaten Lepage 95, Manohar 97

pNRQCD for $mv \gg \Lambda_{\rm QCD}$

Degrees of freedom that scale like mv are integrated out:



pNRQCD for $mv \gg \Lambda_{\mathrm{QCD}}$

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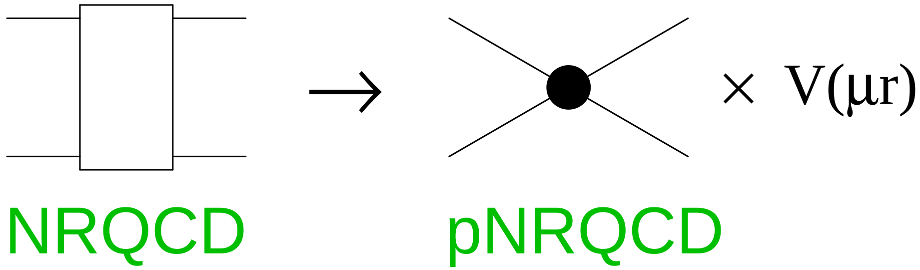

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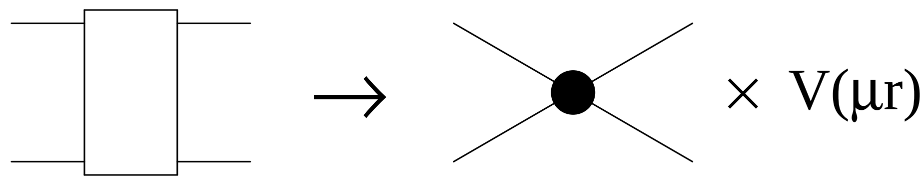


NRQCD   pNRQCD

- Degrees of freedom: quarks and gluons

  Q-$\bar{Q}$ states, with energy $\sim \Lambda_{\mathrm{QCD}}, mv^2$

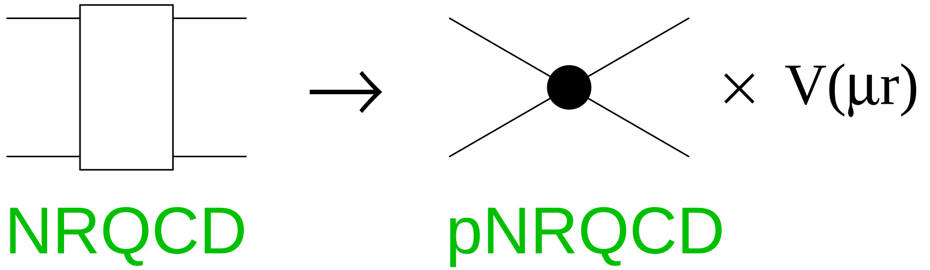
  momentum $\lesssim mv$

  $\Rightarrow$ i) singlet S ii) octet O

  Gluons with energy and momentum $\sim \Lambda_{\mathrm{QCD}}, mv^2$

pNRQCD for $mv \gg \Lambda_{\rm QCD}$

Degrees of freedom that scale like mv are integrated out:



- Power counting: $r \sim \frac{1}{mv}$ and $t, R \sim \frac{1}{mv^2}, \frac{1}{\Lambda_{\rm QCD}}$

  The gauge fields are multipole expanded:

  $$A(R,r,t) = A(R,t) + \mathbf{r} \cdot \boldsymbol{\nabla} A(R,t) + \ldots$$

  Non-analytic behaviour in $r \to$ matching coefficients V

pNRQCD for $mv \gg \Lambda_{\rm QCD}$

$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) \mathrm{O} \right\}$$

LO in r

$$\theta(T)\, e^{-iTH_s}$$

$$\theta(T)\, e^{-iTH_o} \left(e^{-i\int dt\, A^{\rm adj}} \right)$$

pNRQCD for $mv \gg \Lambda_{\rm QCD}$

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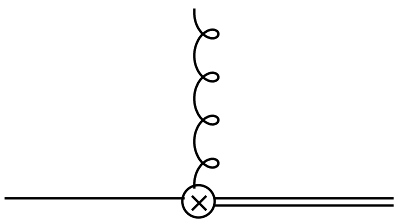
LO in r

$$+V_A \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,\mathrm{S} + \mathrm{S}^\dagger \mathbf{r}\cdot g\mathbf{E}\,\mathrm{O} \right\}$$
$$+\frac{V_B}{2} \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,\mathrm{O} + \mathrm{O}^\dagger \mathrm{O}\,\mathbf{r}\cdot g\mathbf{E} \right\}$$
$$-\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a}$$

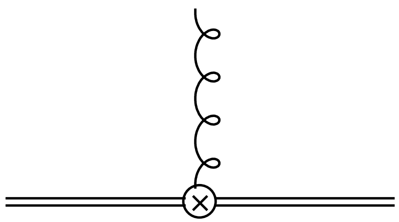
NLO in r

Pineda Soto 97, Brambilla Pineda Soto Vairo 99, 00, 03

pNRQCD for $mv \gg \Lambda_{\mathrm{QCD}}$



$O^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S}$

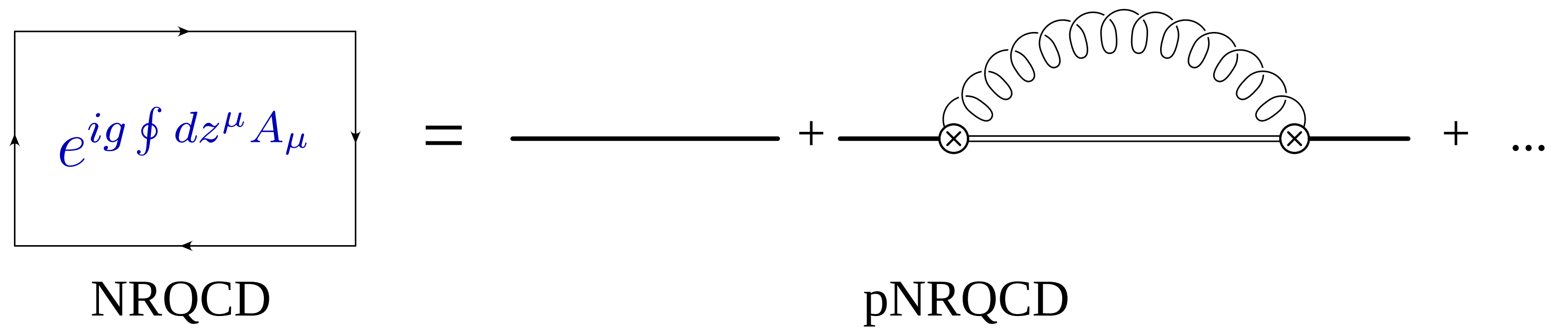


$O^\dagger \{\mathbf{r} \cdot g\mathbf{E},\, \mathrm{O}\}$

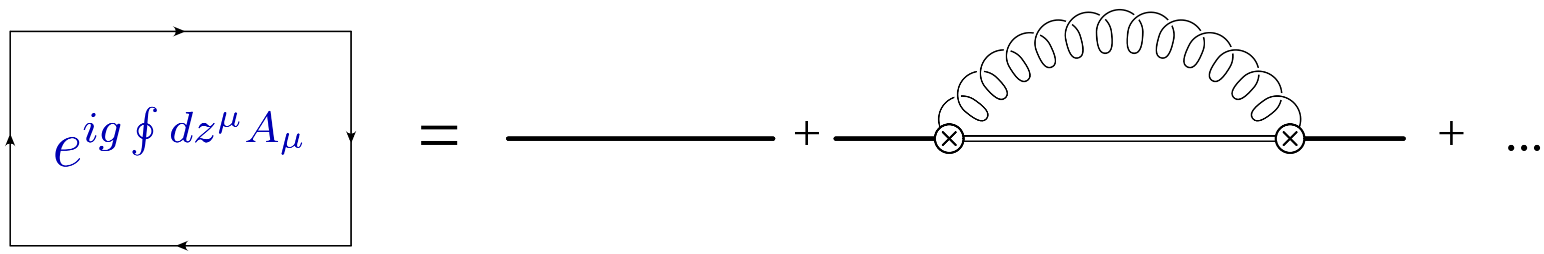
$$
\begin{aligned}
&+V_A \mathrm{Tr}\left\{\mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S} + \mathrm{S}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O}\right\} \\
&+\frac{V_B}{2} \mathrm{Tr}\left\{\mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \mathbf{r} \cdot g\mathbf{E}\right\} \\
&-\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a}
\end{aligned}
$$

NLO in r

The Static Potential



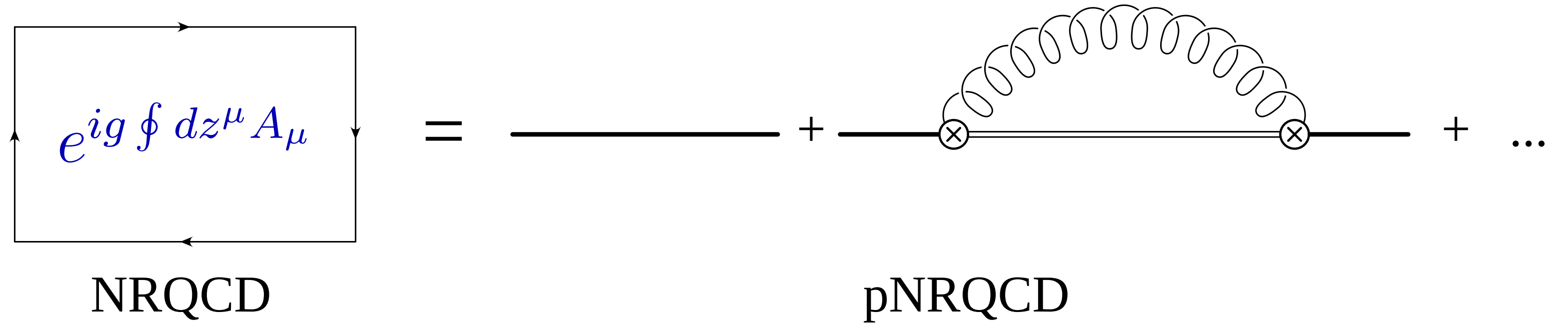
The Static Potential



NRQCD pNRQCD

$$V_s(r) = \lim_{T\to\infty} \frac{i}{T} \ln \langle \Box \rangle + \int dt\, e^{it(V_0 - V_s)} \langle r\cdot E(t)\, r\cdot E(0)\rangle + \ldots$$

The Static Potential



$$V_s(r,\mu) = -C_F \frac{\alpha_{V_s}(r,\mu)}{r}$$

$$\alpha_{V_s}(r,\mu) = \alpha_s(r)\left[1 + \tilde{a}_1 \alpha_s(r) + \tilde{a}_2 (\alpha_s(r))^2 + \frac{\alpha_s^3}{\pi}\frac{C_A^3}{12}\ln \mu r\right]$$

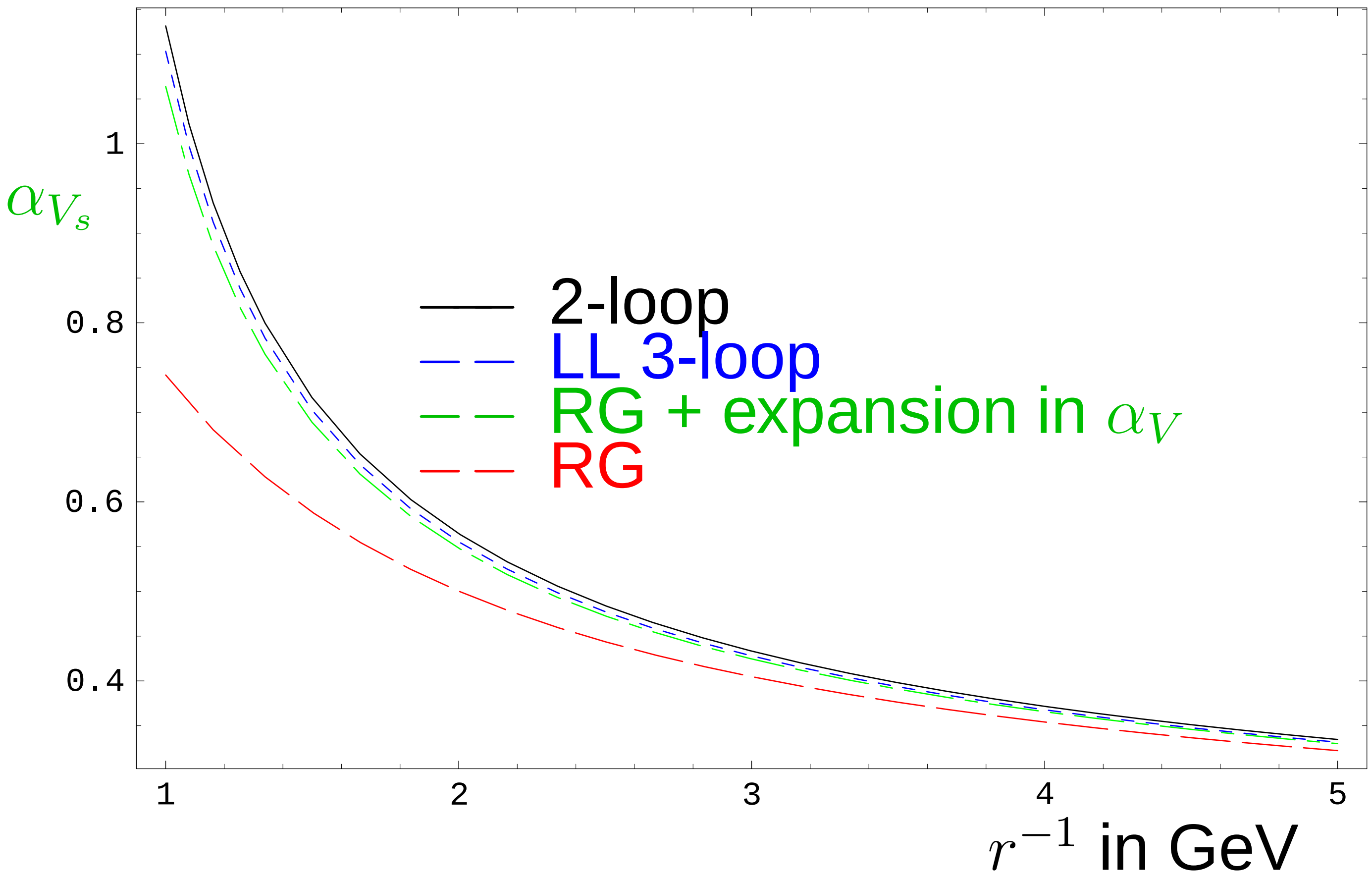
$\tilde{a}_1$ Billoire 80, $\tilde{a}_2$ Schröder 99, Peter 97,

3 loop LL Brambilla Pineda Soto Vairo 99

Summing Logs

$$\begin{cases} \mu \dfrac{d}{d\mu} \alpha_{V_s} = \dfrac{2}{3} \dfrac{\alpha_s}{\pi} V_A^2 \left[\left(\dfrac{C_A}{2} - C_F \right) \alpha_{V_o} + C_F \alpha_{V_s} \right]^3 \\ \mu \dfrac{d}{d\mu} \alpha_{V_o} = \dfrac{2}{3} \dfrac{\alpha_s}{\pi} V_A^2 \left[\left(\dfrac{C_A}{2} - C_F \right) \alpha_{V_o} + C_F \alpha_{V_s} \right]^3 \\ \mu \dfrac{d}{d\mu} \alpha_s = \alpha_s \beta(\alpha_s) \\ \mu \dfrac{d}{d\mu} V_A = 0 \\ \mu \dfrac{d}{d\mu} V_B = 0 \end{cases}$$

Summing Logs



$(\mu = \alpha_s(r)/r;\ N_f = 4)$

Pineda Soto 00

Summing $(\alpha_{\mathrm{s}}\beta_0)^n$

V_s is affected by renormalons: $V_s(renormalon) = C_0 + C_2 r^2 + \ldots$

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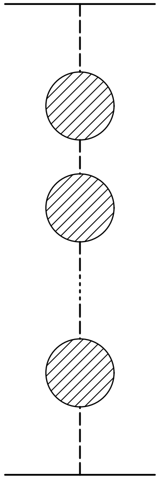
$$C_0 \simeq -2\frac{C_F\alpha_s(\mu)}{\pi}\mu\sum_{n=0}^{\infty} n!\left(\frac{\beta_0\alpha_s(\mu)}{2\pi}\right)^n$$

$$\Rightarrow \delta C_0 \sim \Lambda_{QCD}$$

$$C_2 \simeq \frac{1}{9}\frac{C_F\alpha_s(\mu)}{\pi}\mu^3\sum_{n=0}^{\infty} n!\left(\frac{\beta_0\alpha_s(\mu)}{6\pi}\right)^n$$

$$\Rightarrow \delta C_2 \sim \Lambda_{QCD}^3$$



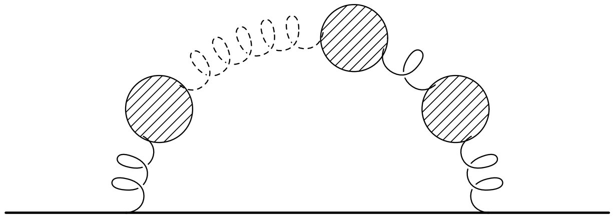
$$1/r \gg \mu \gg \Lambda_{QCD}$$

Aglietti Ligeti 95

Summing $(\alpha_s \beta_0)^n$

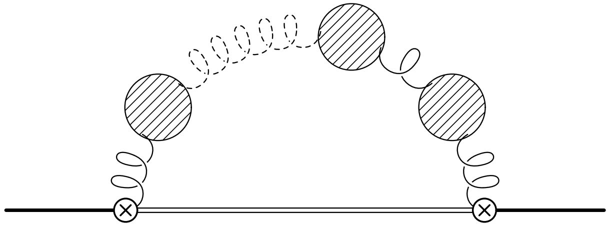
V_s is affected by renormalons: $V_s(renormalon) = C_0 + C_2 r^2 + \ldots$

The $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon cancels against the pole mass.

$2\times$  $= -C_0$

Beneke 98, Pineda 98, Hoang Smith Stelzer Willenbrock 99

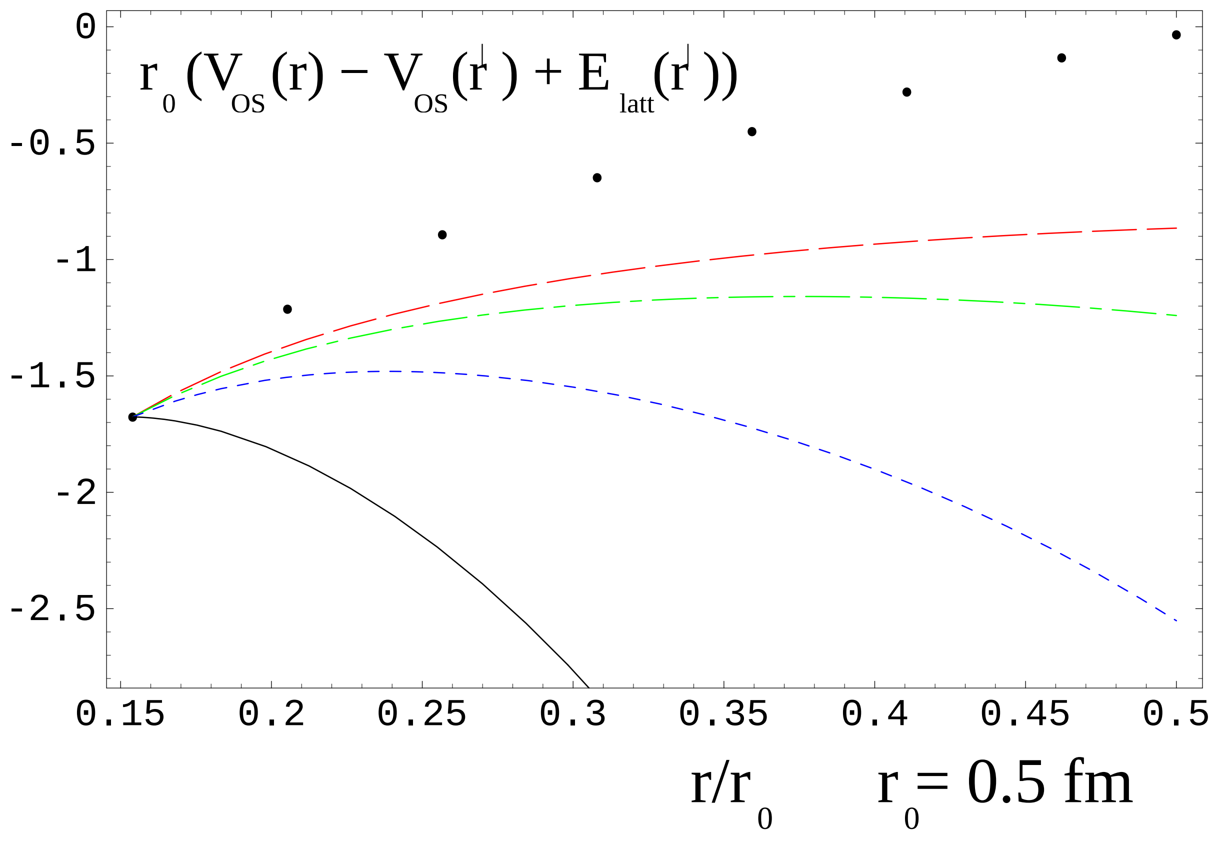
The $\mathcal{O}(\Lambda^3_{\rm QCD})$ renormalon cancels in pNRQCD.

 $= -C_2 r^2$

Brambilla Pineda Soto Vairo 99

Static potential vs lattice QCD

Renormalon subtraction (RS) is crucial in comparing the perturbative static potential with lattice data.



NNLL + 3 loop est.

NNLO

NLO

LO

$$\alpha_s = \alpha_s(1/r)$$
$$\nu_f = \nu_{us} = 2.5\, r_0^{-1}$$
$$r' = 0.15399\, r_0$$

Pineda 02

Static potential vs lattice QCD

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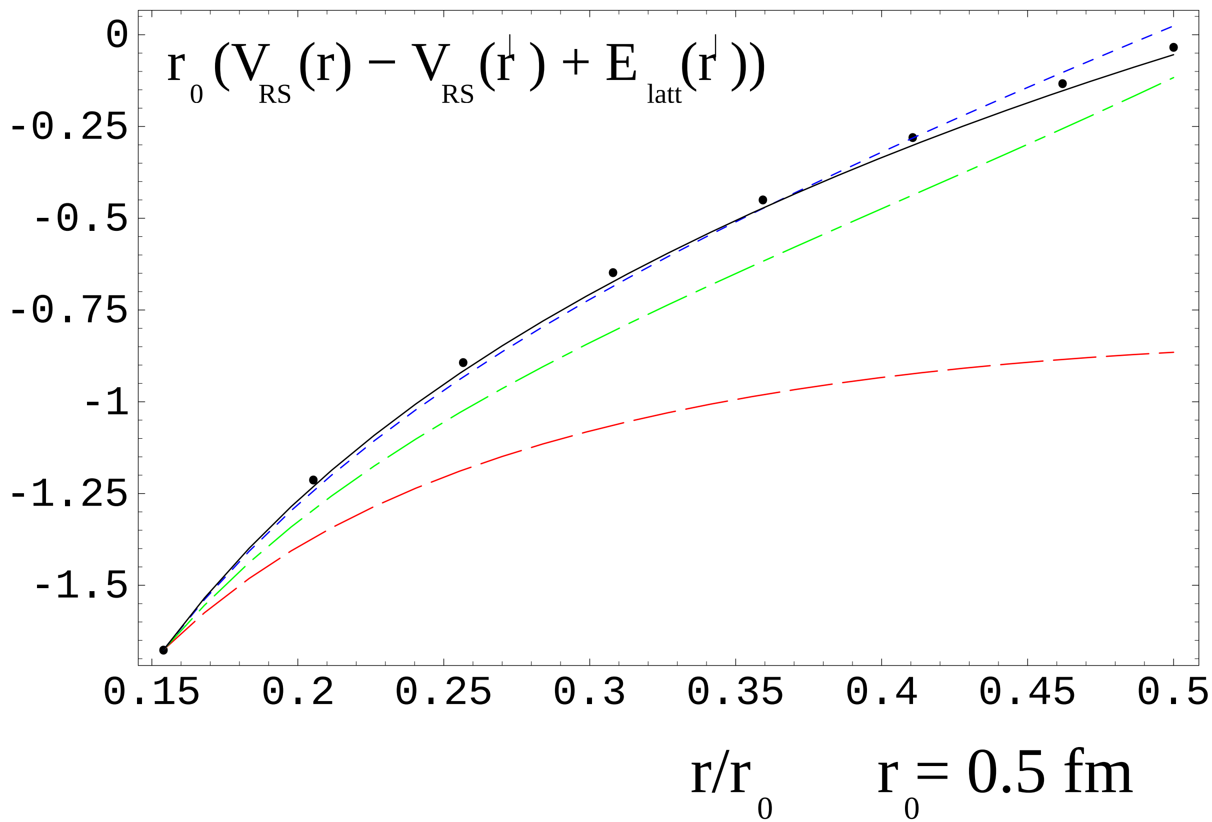

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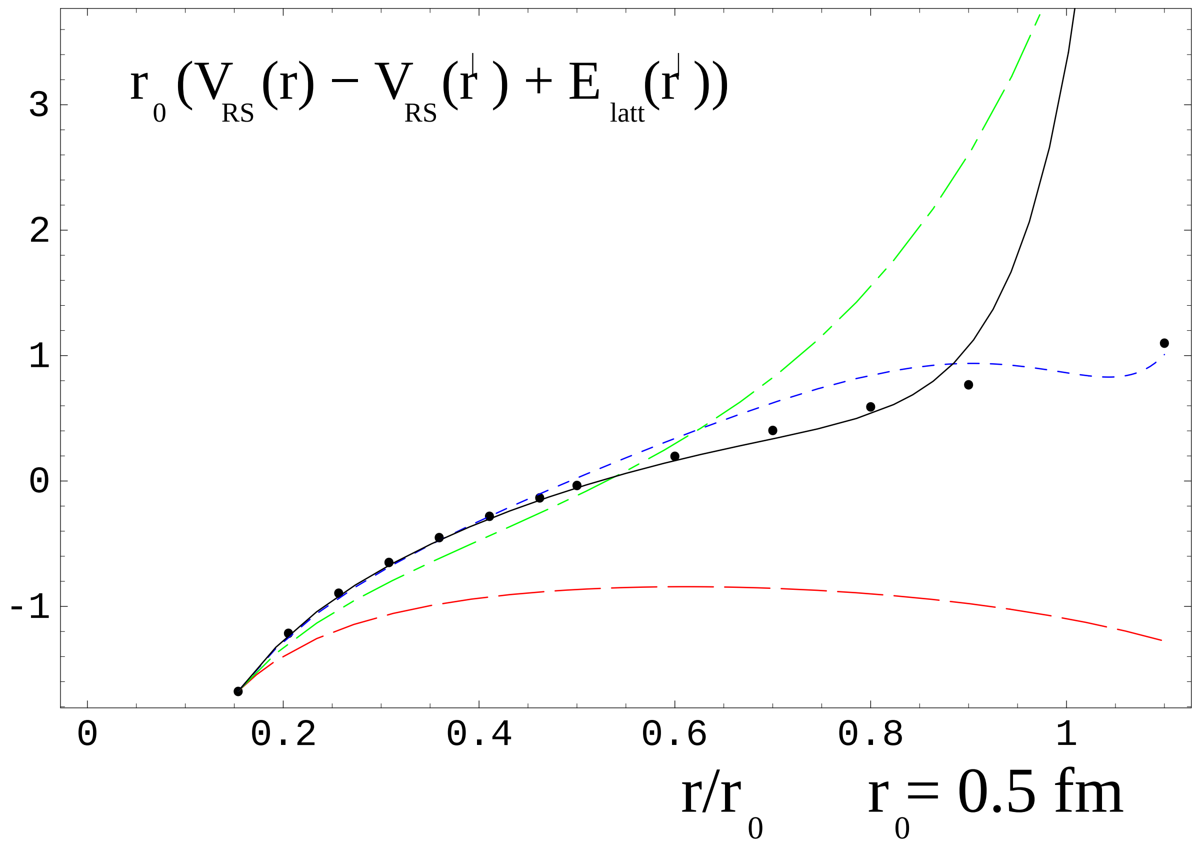
Pineda 02

Static potential vs lattice QCD

No signal of short range non-perturbative effects.

Brambilla Sumino Vairo 01, Necco Sommer 01

Sumino 02, Pineda 02, Lee 02 03



$r_0 = 0.5$ fm

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Quarkonium Spectrum at LL $m\alpha_s^5$

$$E_n = \langle n|V_s(\mu)|n\rangle - i\frac{g^2}{3N_c}\int_0^\infty dt\, \langle n|\mathbf{r}e^{it(E_n^{(0)}-H_o)}\mathbf{r}|n\rangle\ \langle \mathbf{E}(t)\,\mathbf{E}(0)\rangle(\mu)$$

Brambilla Pineda Soto Vairo 99

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$$\sim e^{i\Lambda_{\rm QCD}t}$$

b mass from the Υ system

Ref.	Method	Order	$\overline{m}_b(\overline{m}_b)$ (MeV)
MY99	nonrelativistic Υ sum rules*	NNLO	4200 ± 100
BS99	"	"	4250 ± 80
H00	"	"	4170 ± 50
KS01	low moments sum rules	"	4209 ± 50
BSV01	spectrum, $\Upsilon(1S)$ resonance*	"	$4190 \pm 20 \pm 25 \pm 3$
P01	"	LL N^{3}LO*	$4210 \pm 90 \pm 25$
PS02	"	N^{3}LO*	4346 ± 70

* pole mass vs static potential renormal cancellation

* $\Upsilon(1S)$ mass at N^{3}LO:

$$\frac{\delta E_{\Upsilon(1S)}}{E_{\rm Bohr}} = \alpha_{\rm s}^3(104.819 + 15.297\log\alpha_{\rm s} + 0.001a_3)$$

$$\delta m_b(m_b) \simeq 25 \text{ MeV}$$

Melnikov Yelkhovsky 99, Beneke Signer 99, Hoang 00, Kuhn Steinhauser 01, Brambilla Sumino Vairo 01, Pineda 01, Penin Steinhauser 02

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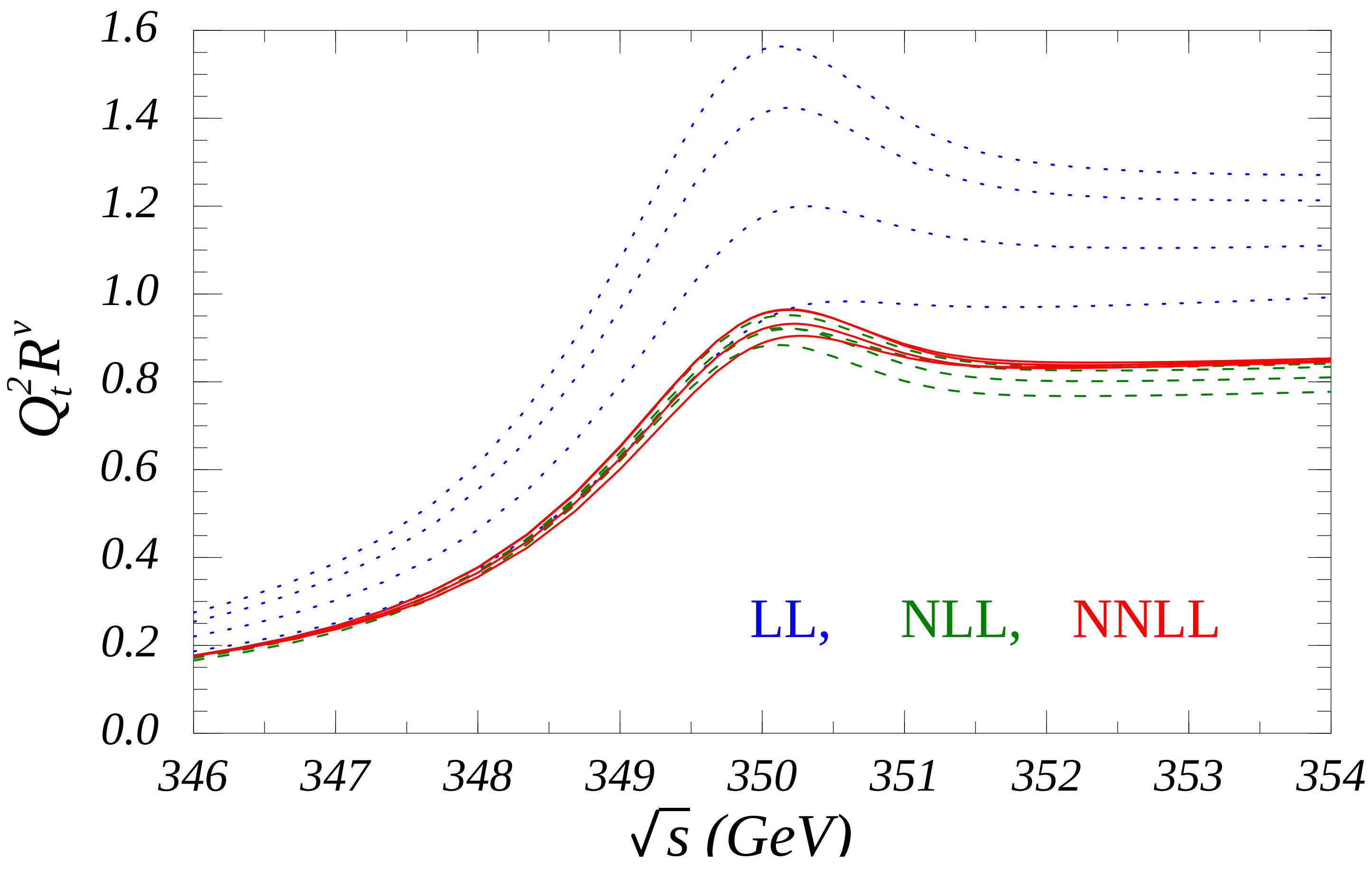
	Method	$\overline{m}_b(\overline{m}_b)$ (MeV)
B system	Inclusive moments	
	(i) lepton spectrum	4310 ± 130
	(ii) γ energy/ had. inv. mass	4220 ± 90
	lattice QCD (stat. limit)	4260 ± 90

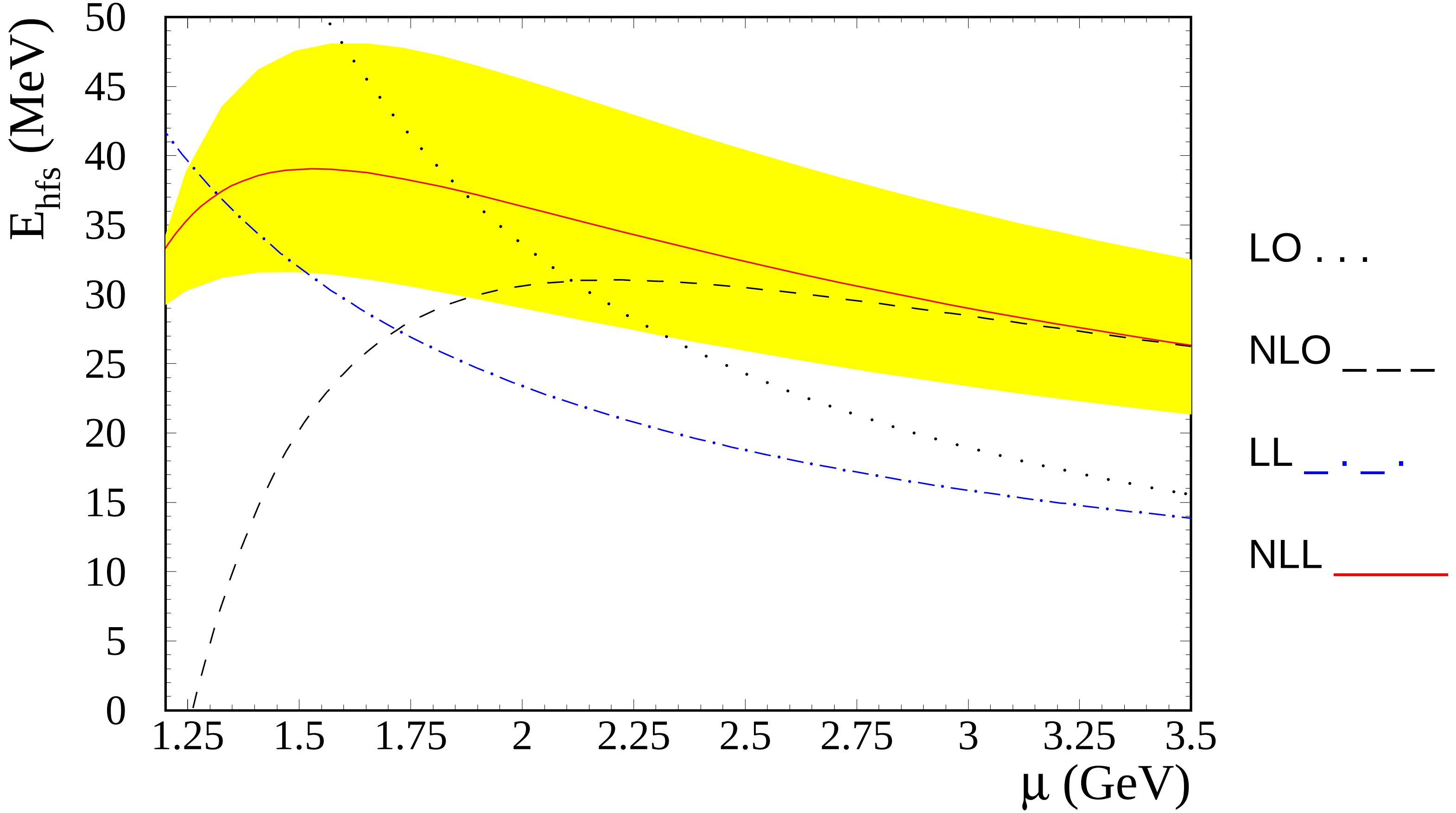
El-Khadra Luke 02

Threshold $t\bar{t}$ cross section



Hoang Teubner 01

η_b mass



$$M(\eta_b) = 9421 \pm 10\,(\text{th})\,{}^{+9}_{-8}\,(\delta\alpha_s)\ \text{MeV}$$

Kniehl Penin Pineda Smirnov Steinhauser 03

Hybrids and Gluelumps

- We consider gluonic excitations between static quarks.

Hybrids and Gluelumps

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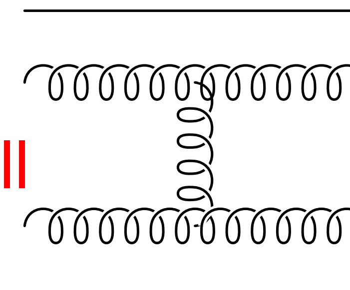
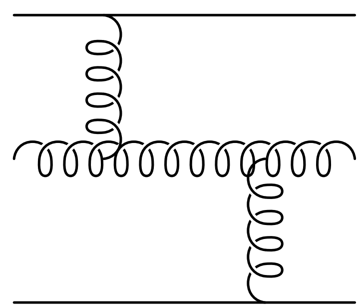
- These are of three types:

$(Q\bar{Q})_1$

$(Q\bar{Q})_1 +$ Glueball



Hybrid
$(Q\bar{Q})_8 G$



Hybrids and Gluelumps

- We consider gluonic excitations between static quarks.
- At short distance, $1/r \gg \Lambda_{\rm QCD}$, and at lowest order in the multipole expansion, the singlet decouples while the octet is still coupled to gluons.

Hybrids and Gluelumps

- We consider gluonic excitations between static quarks.
- Static hybrids at short distance are called gluelumps and are described by:

$$\mathrm{H}(R, r, t) = \mathrm{Tr}\{\mathrm{O}H\}$$

i.e. a static adjoint source (O) in the presence of a gluonic field (H).

- Depending on the glue operator H and its symmetries, the operator $\mathrm{Tr}\{\mathrm{O}H\}$ describes a speci£c gluelump of energy E_H.

Hybrids and Gluelumps

Symmetries of a diatomic molecule + C.C.

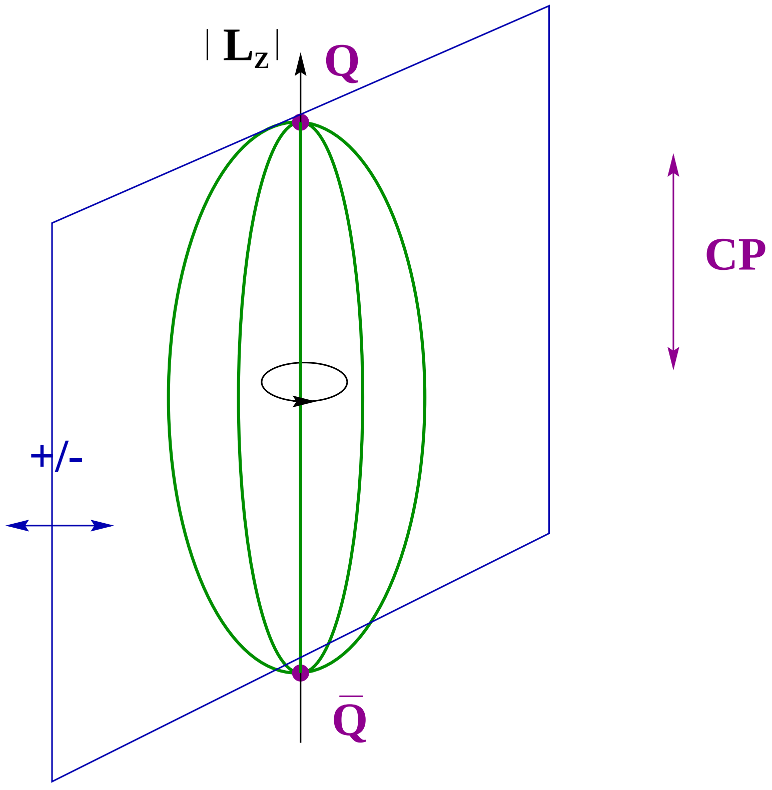
a) $|L_z| = 0, 1, 2, \ldots$

$= \Sigma, \Pi, \Delta \ldots$

b) CP (u/g)

c) Reflection ($+/-$)

(for Σ only)



Hybrids and Gluelumps

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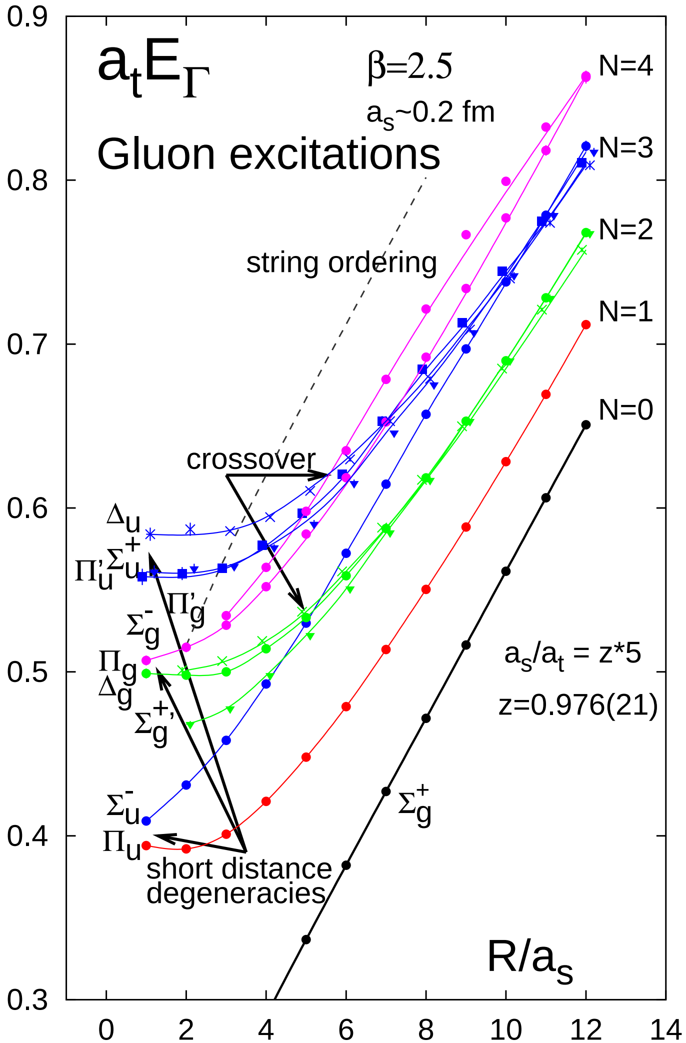
c) Reflection ($+/-$)
(for Σ only)

	$L=1$	$L=2$
$\Sigma_g^{+\,\prime}$	$\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$	
Σ_g^-		$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g	$\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$	
Π_g'		$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g		$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+$ $+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$

Σ_u^+		$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Σ_u^-	$\mathbf{r}\cdot\mathbf{B}$	
Π_u	$\mathbf{r}\times\mathbf{B}$	
Π_u'		$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
Δ_u		$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{E})^j+$ $+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{E})^i$

Brambilla Pineda Soto Vairo 99

Hybrids and Gluelumps



	$L=1$	$L=2$
$\Sigma_g^{+\prime}$	$\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$	
Σ_g^-		$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g	$\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$	
Π'_g		$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g		$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+$ $+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+		$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Σ_u^-	$\mathbf{r}\cdot\mathbf{B}$	
Π_u	$\mathbf{r}\times\mathbf{B}$	
Π'_u		$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
Δ_u		$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{E})^j+$ $+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{E})^i$

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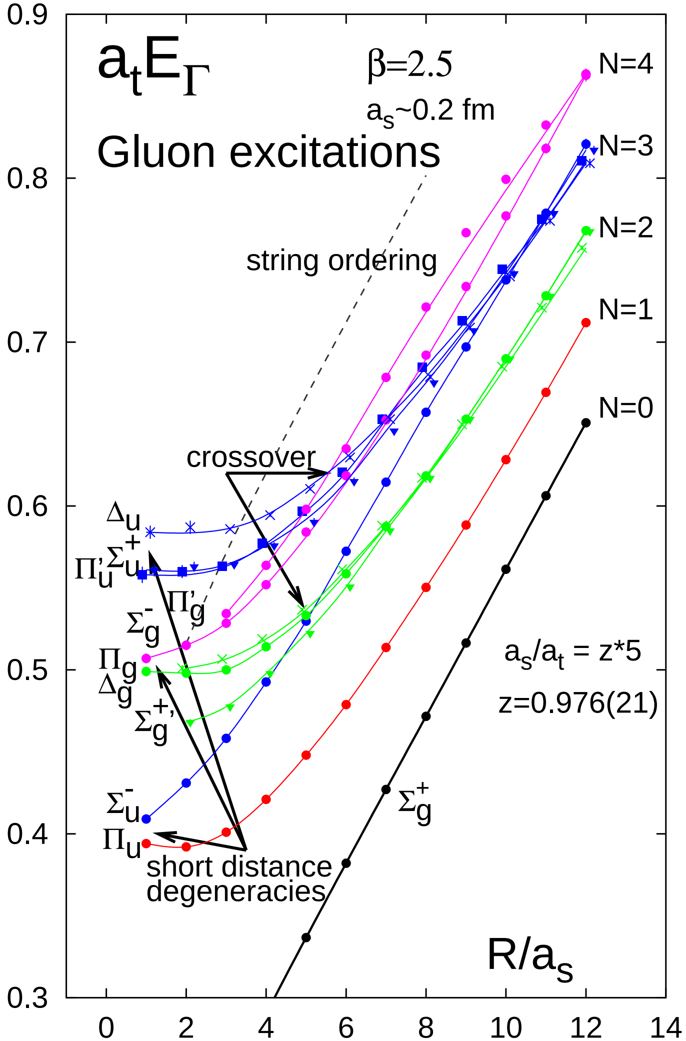
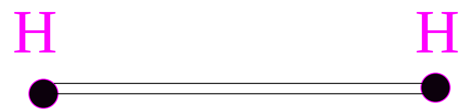
Brambilla Pineda Soto Vairo 99

Hybrids and Gluelumps



Juge Kuti Morningstar 98, 02

At LO in the multipole expansion



$$= e^{-iTE_H}$$

$$E_H = V_o + \frac{i}{T} \ln \langle H^a(\frac{T}{2}) \phi^{\rm adj}_{ab} H^b(-\frac{T}{2}) \rangle$$

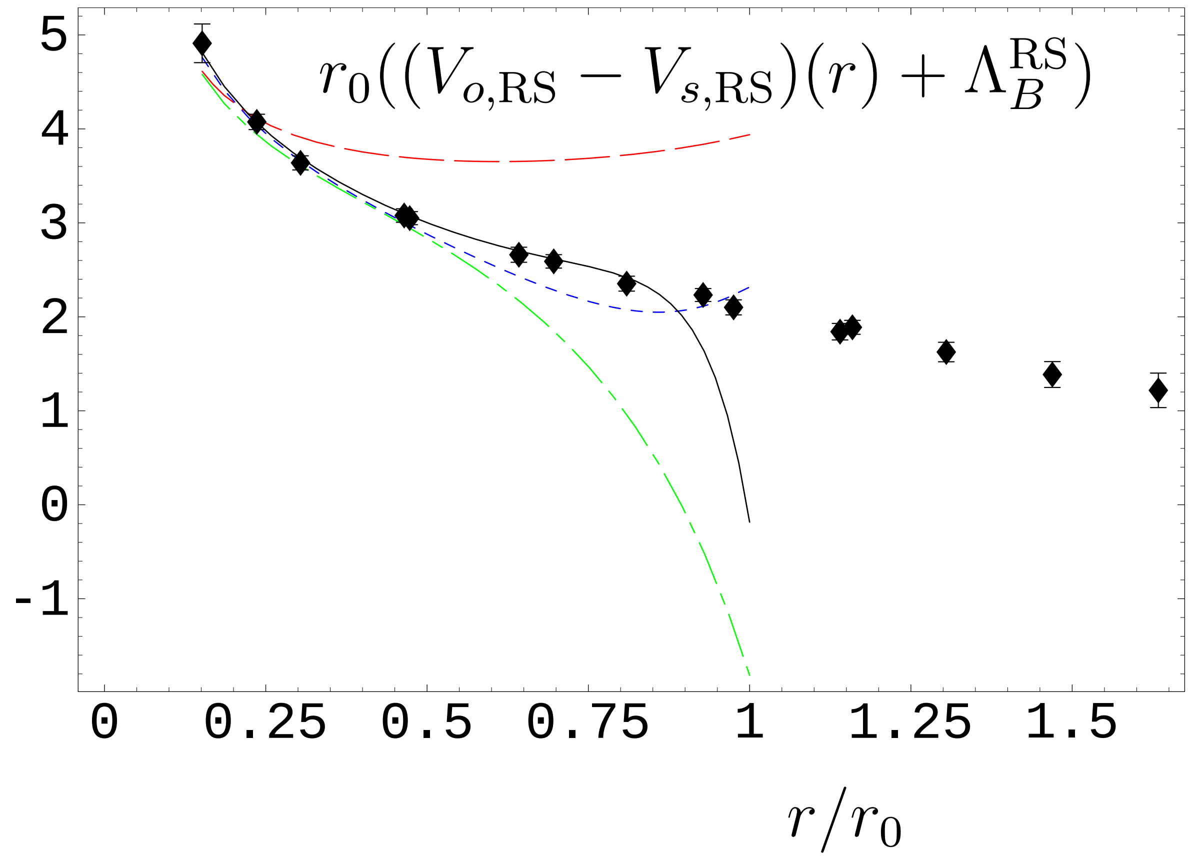
From

$$\langle H^a(\frac{T}{2}) \phi^{\rm adj}_{ab} H^b(-\frac{T}{2}) \rangle^{\rm np} \sim h\, e^{-iT\Lambda_H}$$

$$E_H(r) = V_o(r) + \Lambda_H$$

Octet potential vs lattice QCD

Renormalon subtraction (RS) is crucial in comparing the perturbative static octet potential with lattice data.



NNLL + 3 loop est.

NNLO

NLO

LO

$\alpha_{\mathrm{s}} = \alpha_{\mathrm{s}}(1/r)$

$\nu_f = \nu_{us} = 2.5\, r_0^{-1}$

Lattice data of $E_{\Pi_u} - E_{\Sigma_g^+}$

Bali Pineda 03

Octet potential vs lattice QCD

Λ_B correlation length

$$\Lambda_B^{\rm RS}(\nu_f = 2.5\, r_0^{-1}) = [2.25 \pm 0.10(\text{latt.}) \pm 0.21(\text{th.}) \pm 0.08(\Lambda_{\overline{\rm MS}})]\ r_0^{-1}$$

$$\text{for} \quad \nu_f = 2.5\, r_0^{-1} \approx 1 \ \text{ GeV}$$

$$\Lambda_B^{\rm RS}(1\,\text{GeV}) = [0.887 \pm 0.039(\text{latt.}) \pm 0.083(\text{th.}) \pm 0.032(\Lambda_{\overline{\rm MS}})]\,\text{GeV}$$

Octet potential vs lattice QCD

Higher Gluelump excitations

J^{PC}	H	$\Lambda_H^{\mathrm{RS}} r_0$	Λ_H^{RS}/GeV
1^{+-}	B_i	2.25(39)	0.87(15)
1^{--}	E_i	3.18(41)	1.25(16)
2^{--}	$D_{\{i}B_{j\}}$	3.69(42)	1.45(17)
2^{+-}	$D_{\{i}E_{j\}}$	4.72(48)	1.86(19)
3^{+-}	$D_{\{i}D_j B_{k\}}$	4.72(45)	1.86(18)
0^{++}	$\mathbf{B}^2$	5.02(46)	1.98(18)
4^{--}	$D_{\{i}D_j D_k B_{l\}}$	5.41(46)	2.13(18)
1^{-+}	$(\mathbf{B} \wedge \mathbf{E})_i$	5.45(51)	2.15(20)

Foster Michael 99, Bali Pineda 03

Conclusion

Heavy quarkonium is

- a competitive source for some of the SM parameters:

  m_t, m_b, m_c, $\alpha_{\rm s}$, ...

- a privileged system to study the interplay of perturbative and non-perturbative QCD.

  $\longrightarrow$ *large order perturbation theory vs lattice QCD*

  $\longrightarrow$ *precision physics from lattice QCD* http://www.qwg.to.infn.it

QWG III workshop: 12-15 October 2004 IHEP Beijing

$\longrightarrow$ *Yellow Report 2004*