# Invisible Higgs in the ADD model at LHC and LC

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- **Review of the ADD model**
- **Invisible Higgs**
- **Conclusions**

**Based on**
**Battaglia, DD, Gunion, Wells hep-ph/0402062**

# ADD model: a model with large ED

A geometrical reformulation of the hierarchy problem, combining braneworld and Kaluza Klein ideas.
Gravity in $D = 4 + \delta$ dimensions, SM particles in 4 dimensions.
(Arkani-Hamed, Dimopoulos, Dvali, Antoniadis)

$$\overline{M}_P^2 = M_D^{\delta+2} R^\delta \sim (\mathrm{TeV})^{\delta+2} R^\delta$$

$R$ is the radius of the compactified space, a $\delta$-torus.
$\overline{M}_P$ ($\overline{M}_P = (8\pi G_N)^{-1/2}$) is not fundamental, $\overline{M}_P$ large because $R$ is large.
Fenomenological implications:
light KK states (KK gravitons and graviscalars)

$$m_{\vec{n}}^2 = \vec{n}^2 R^2 \quad \vec{n} = (n_1, \ldots, n_\delta)$$

$$\Delta m_{\vec{n}} \sim 10^{-3}\mathrm{eV} - 10\,\mathrm{MeV}, \ \delta = 2 - 6$$

and very long lived ($\sim 10^{10}$yr).
Interactions with SM fields

$$-\frac{1}{\overline{M}_P} G^{(\vec{n})\mu\nu} T_{\mu\nu} + \frac{1}{\overline{M}_P}\sqrt{\frac{\delta - 1}{3(\delta + 2)}} H^{(\vec{n})} T_\mu^{\ \mu}$$

## 95% CL Limits on $M_D$ (TeV)

| $\delta$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| **Collider bounds** | | | | | |
| LEP 2/Tevatron (Giudice, Strumia) | 1.45 | 1.09 | 0.87 | 0.72 | 0.65 |
| **Present non-collider bounds** | | | | | |
| SN1987A (Hannestead, Raffelt) | 22 | 2 | | | |
| Diffuse $\gamma$ rays from SN/NS (H, R) | 97 | 8 | 1.5 | | |
| Excess heat from $\gamma$ hitting the NS (H, R) | 1800 | 77 | 9.4 | 2.1 | |

Collider bounds: from graviton emission process at LEP 2 ($e^+e^- \to \gamma \not{E}_T$ , $e^+e^- \to Z \not{E}_T$ ) and Tevatron ($p\bar{p} \to \gamma \not{E}_T$ ,$p\bar{p} \to jets\ \not{E}_T$ ).

The presence of an interaction between the Higgs $H$ and the Ricci scalar curvature of the induced 4-dimensional metric $g_{ind}$,

$$S = -\xi \int d^4x \sqrt{g_{ind}} R(g_{ind}) H^\dagger H$$

generates, after the shift $H = (\frac{v+h}{\sqrt{2}}, 0)$, a mixing term (Giudice, Rattazzi and Wells) $(H^{\vec{n}} = \frac{1}{\sqrt{2}}(s_{\vec{n}} + i a_{\vec{n}}))$

$$\mathcal{L}_{\rm mix} = \epsilon h \sum_{\vec{n}>0} s_{\vec{n}} \tag{1}$$

with

$$\epsilon = -\frac{2\sqrt{2}}{\overline{M}_P} \xi v m_h^2 \sqrt{\frac{3(\delta-1)}{\delta+2}}\,.$$

$\xi$ is a dimensionless parameter and $s_{\vec{n}}$ is a graviscalar KK excitation with mass $m_{\vec{n}}^2 = 4\pi^2 \vec{n}^2 / L^2$, $L = 2\pi R$.
Effective mixing is generated (Antoniadis, Sturani) when the Higgs leaves on brane intersection

$$\xi = \frac{1}{4}\sqrt{\frac{\delta+2}{6\delta(\delta-1)}}$$

This mixing generates an oscillation of the Higgs itself into the closest KK graviscalar levels which are invisible since they are weakly interacting and mainly reside in the extra dimensions.
The mixing invisible width $\Gamma_{h\to\mathrm{graviscalar}}$ calculated by extracting the imaginary part of the mixing contribution to the Higgs self energy. (Giudice et al, Wells)

The mixing term can be eliminated with the transformation

$$h \sim N\left[h' + \sum_{\vec{m}>0} \frac{\epsilon}{m_h^2 - m_{\vec{m}}^2} s'_{\vec{m}}\right] \quad N \sim \left[1 + \sum_{\vec{n}>0} \frac{\epsilon^2}{(m_h^2 - m_{\vec{m}}^2)^2}\right]^{-1/2}$$

and

$$s_n = N_n\left[s'_{\vec{n}} - \frac{\epsilon}{m_h^2 - m_{\vec{n}}^2} h'\right] \quad N_n = \left[1 + \frac{\epsilon^2}{(m_h^2 - m_{\vec{n}}^2)^2}\right]^{-1/2}$$

In computing a process such as $WW \to h' + \sum_{\vec{m}>0} s'_{\vec{m}} \to F$, the full coherent sum over physical states must be performed. The result at the amplitude level is

$$\mathcal{A}(WW \to F)(p^2) \sim \frac{g_{WWh} g_{hF}}{p^2 - m_h^2 + i m_h \Gamma_h + iG(p^2) + F(p^2) + i\bar{\epsilon}}$$

where

$$F(p^2) \equiv -\epsilon^2 \mathrm{Re}\left[\sum_{\vec{m}>0} \frac{1}{p^2 - m_{\vec{m}}^2 + i\bar{\epsilon}}\right]$$

and

$$G(p^2) \equiv -\epsilon^2 \mathrm{Im}\left[\sum_{\vec{m}>0} \frac{1}{p^2 - m_{\vec{m}}^2 + i\bar{\epsilon}}\right]$$

The sum is replaced by an integral

$$\sum_{\vec{n}>0} \frac{1}{p^2 - m_{\vec{n}}^2 + i\bar{\epsilon}} \to \int dm^2 \rho_\delta(m) \frac{1}{p^2 - m^2 + i\bar{\epsilon}}$$

with $\rho_\delta(m) = L^\delta m^{\delta-2}/((4\pi)^{(\delta/2)}\Gamma(\delta/2))$ density of KK states ($dN = \rho_\delta(m) dm^2$).

Writing $F(p^2) = F(m_{h_{ren}}^2) + (p^2 - m_{h_{ren}}^2)F'(m_{h_{ren}}^2) + \ldots$, where $m_{h_{ren}}^2 - m_h^2 + F(m_{h_{ren}}^2) = 0$, we obtain the structure

$$\mathcal{A}(WW \to F)(p^2) \sim \frac{g_{WWh} g_{hF}}{(p^2 - m_{h_{ren}}^2)[1 + F'(m_{h_{ren}}^2)] + i m_h(\Gamma_h + \Gamma_{inv})}$$

with

$$m_h \Gamma_{inv} = G(p^2)|_{m_{h_{ren}}^2} = Im\Sigma(p^2)|_{m_{h_{ren}}^2}$$

$$\begin{aligned} Im\Sigma(p^2) \quad & \to -\epsilon^2 Im \frac{1}{2} \int dm^2 \rho_\delta(m) \frac{1}{p^2 - m^2 + i\bar{\epsilon}} \\ = \quad & -\epsilon^2 \frac{1}{4} \frac{\overline{M}_P^2}{M_D^{2+\delta}} S_{\delta-1}(-\pi)(p^2)^{(\delta-2)/2} \\ = \quad & 2\pi \frac{3(\delta-1)}{\delta+2} \xi^2 v^2 m_h^2 \frac{(p^2)^{(\delta-2)/2}}{M_D^{2+\delta}} S_{\delta-1} \end{aligned}$$

where $S_{\delta-1} = 2\pi^{\delta/2}/\Gamma(\delta/2)$. For a light Higgs boson both the wave function renormalization and the mass renormalization effects are small. A simple estimate of the quantity $F'(m_{h\,ren}^2)$, appearing in the wave function renormalization, suggests that it is of order $\xi^2 \frac{m_h^4}{\Lambda^4}$, where $\Lambda \sim M_D$, therefore quite small for the $m_h \ll M_D$.

$$\Gamma(h \to \text{graviscalar}) \equiv \Gamma(h \to \sum_{\vec{n}>0} s_{\vec{n}}) = 2\pi\xi^2 v^2 \frac{3(\delta-1)}{\delta+2} \frac{m_h^{1+\delta}}{M_D^{2+\delta}} S_{\delta-1}$$

$$\sim (16\,MeV) 20^{\delta-2} \xi^2 S_{\delta-1} \frac{3(\delta-1)}{\delta+2} \times \left(\frac{m_h}{150\,GeV}\right)^{1+\delta} \left(\frac{3\,TeV}{M_D}\right)^{2+\delta}$$

**Increasing $\delta$, $\Gamma(h \to \text{graviscalar})$ decreases: the density of states in which the Higgs can oscillate decreases.**

# Invisible width from direct two graviscalar decay

In addition to the Higgs invisible decay due to the oscillation in graviscalar by mixing, one expects also a contribution to the invisible width from the $H$ decays into two graviscalars. Several sources of the cubic interactions:

$$\begin{aligned}\mathcal{L}_{cubic} &= -\frac{1}{2v}m_h^2 h^3 - \frac{\epsilon}{3\xi v}h^2\sum_{\vec{n}>0}s_{\vec{n}} - \frac{\epsilon}{v m_h^2}\partial^\mu h\partial_\mu h\sum_{\vec{n}>0}s_{\vec{n}} \\ &+\frac{5}{2}\frac{\epsilon}{v}\sum_{\vec{n}>0}s_{\vec{n}}h^2 + \frac{2}{3}\frac{\epsilon^2}{\xi v m_h^2}h\sum_{\vec{n}>0}s_{\vec{n}}\sum_{\vec{m}>0}s_{\vec{m}} + 2\frac{\epsilon^2}{v m_h^4}h\sum_{\vec{n}>0}s_{\vec{n}}\sum_{\vec{m}>0}\Box s_{\vec{m}}\end{aligned}$$

In the various terms we substitute

$$\begin{aligned} h &\rightarrow h' \\ h^2 &\rightarrow 2h'\sum_{\vec{m}>0}\frac{\epsilon}{m_h^2 - m_{\vec{m}}^2}s_{\vec{m}} \\ \partial^\mu h\partial_\mu h &\rightarrow 2\partial^\mu h'\sum_{\vec{m}>0}\frac{\epsilon}{m_h^2 - m_{\vec{m}}^2}\partial_\mu s_{\vec{m}} \\ h^3 &\rightarrow 3h'\sum_{\vec{m}>0}\sum_{\vec{n}>0}\frac{\epsilon}{m_h^2 - m_{\vec{m}}^2}\frac{\epsilon}{m_h^2 - m_{\vec{n}}^2}s_{\vec{m}}s_{\vec{n}}\end{aligned}$$

We have

$$\Gamma(h \to graviscalar\ pairs) = \frac{1}{2} \sum_{\vec{l}>0,\vec{k}>0} \frac{1}{16\pi m_h} |g_{\vec{l},\vec{k}}|^2 \lambda(m_h^2, m_{\vec{k}}^2, m_{\vec{l}}^2)\,,$$

where

$$\lambda(x,y,z) = \left[1 + (y/x)^2 + (z/x)^2 - 2y/x - 2z/x - 2yz/x^2\right]^{1/2}$$

and vertices given by

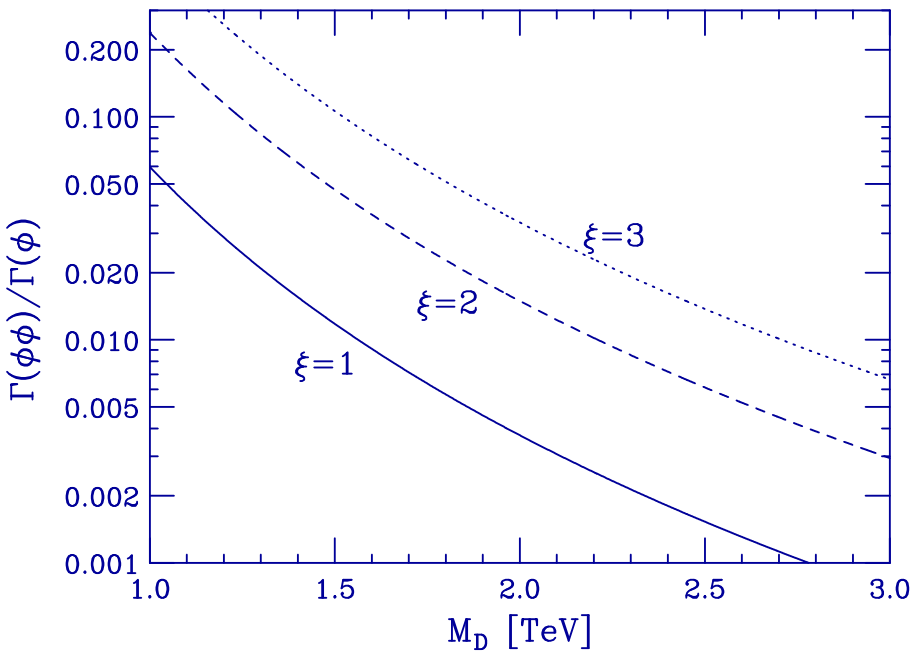
$$\begin{aligned}
\frac{g_{\vec{l}\vec{k}}}{\epsilon^2} \quad = \quad & -3\frac{m_h^2}{v}\frac{1}{(m_h^2 - m_{\vec{l}}^2)(m_h^2 - m_{\vec{k}}^2)} + \left(\frac{5}{v} - \frac{2}{3\xi v}\right)\left(\frac{1}{m_h^2 - m_{\vec{l}}^2} + \frac{1}{m_h^2 - m_{\vec{k}}^2}\right) \\
& -\frac{1}{v m_h^2}\left[\frac{m_h^2 + m_{\vec{k}}^2 - m_{\vec{l}}^2}{m_h^2 - m_{\vec{k}}^2} + \frac{m_h^2 + m_{\vec{l}}^2 - m_{\vec{k}}^2}{m_h^2 - m_{\vec{l}}^2}\right] + \frac{4}{3}\frac{1}{\xi v m_h^2} \\
& -\frac{2}{v m_h^4}(m_{\vec{m}}^2 + m_{\vec{l}}^2)
\end{aligned}$$

$$\Gamma(h' \to \text{graviscalar pairs}) = \frac{18}{\pi} \frac{m_h^{3+2\delta} v^2}{M_D^{4+2\delta}} \xi^4 \left(\frac{\delta - 1}{\delta + 2}\right)^2 \left[\frac{\pi^{\delta/2}}{\Gamma(\delta/2)}\right]^2 I \,,$$

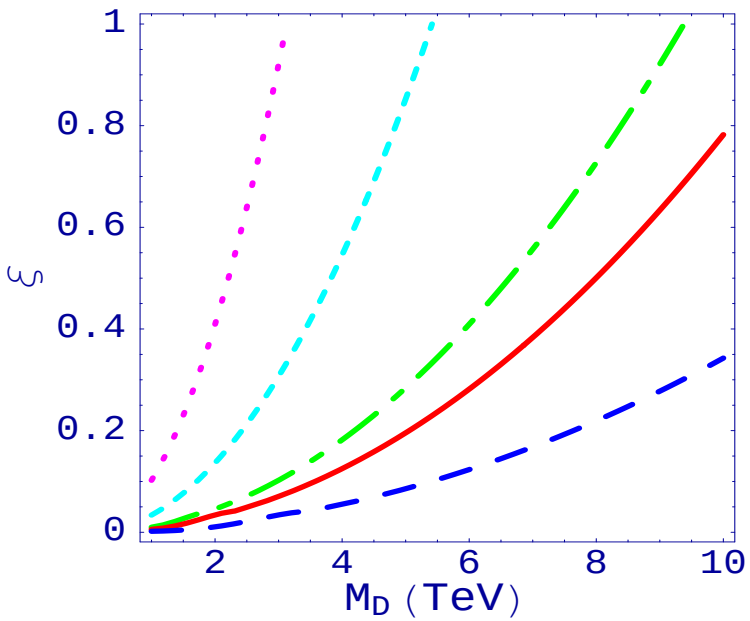
where $I$ is an integral coming from the sum over all the possible kinematically allowed $h' \to s_k s_l$ decays. The integral $I$ decreases rapidly as $\delta$ increases. The ratio of the two widths is given by:

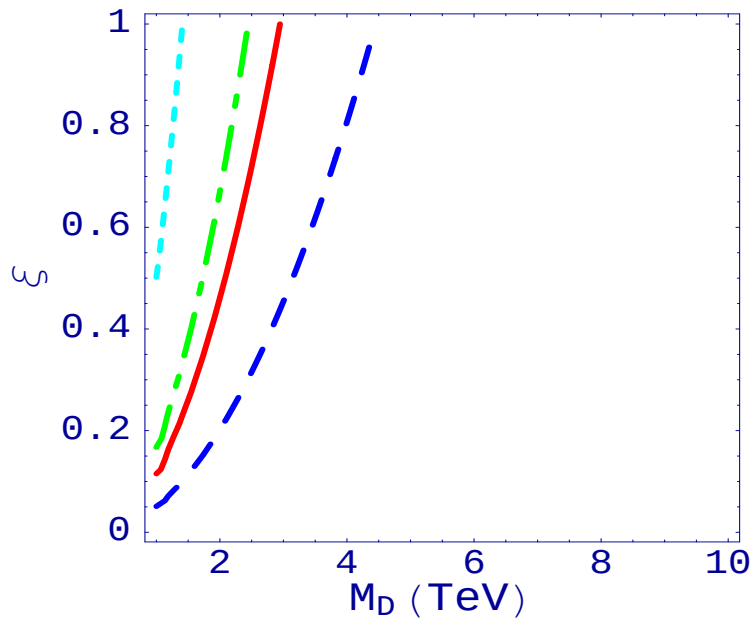
$$\frac{\Gamma(h' \to \text{graviscalar pairs})}{\Gamma(h' \to \text{graviscalar})} = \frac{3(\delta - 1)}{2\pi^2(\delta + 2)} \xi^2 \left(\frac{m_h}{M_D}\right)^{2+\delta} \frac{\pi^{\delta/2}}{\Gamma(\delta/2)} I \,.$$

**The ratio of the two-graviscalars decay width to the one-graviscalar decay width for a 1 TeV Higgs boson. ($\xi = 1$ solid, $\xi = 2$ dashed, $\xi = 3$ dotted), $\delta = 2$.**



Contours of fixed $Br(h \to \text{graviscalar}) = 0.01$ (blue dashes), $0.05$ (solid red line), $0.1$ (green long dash – short dash line), $0.5$ (cyan dashes), $.9$ (purple dots), in the $M_D$(TeV) – $\xi$ parameter space for $m_h = 120$ GeV (left) and $m_h = 400$ GeV (right), taking $\delta = 2$.

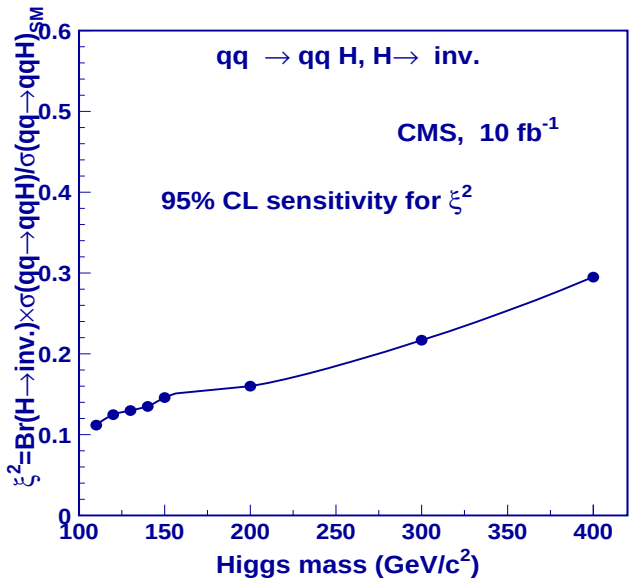


# Sensitivity to $\Gamma_{inv}$ at LHC

(Fusion channel: Eboli and Zeppenfeld, Di Girolamo et al, Abdullin et al, CMS note)

Higgs boson production in $qq \to qqVV \to qqh$ and subsequent $h$ invisible decay. Signal characterized by two very energetic forward jets well separated in pseudorapidity. With $B_{inv} = 1$ and 10 (100) fb$^{-1}$ it is possible discover Higgs up to 480 (770) GeV .

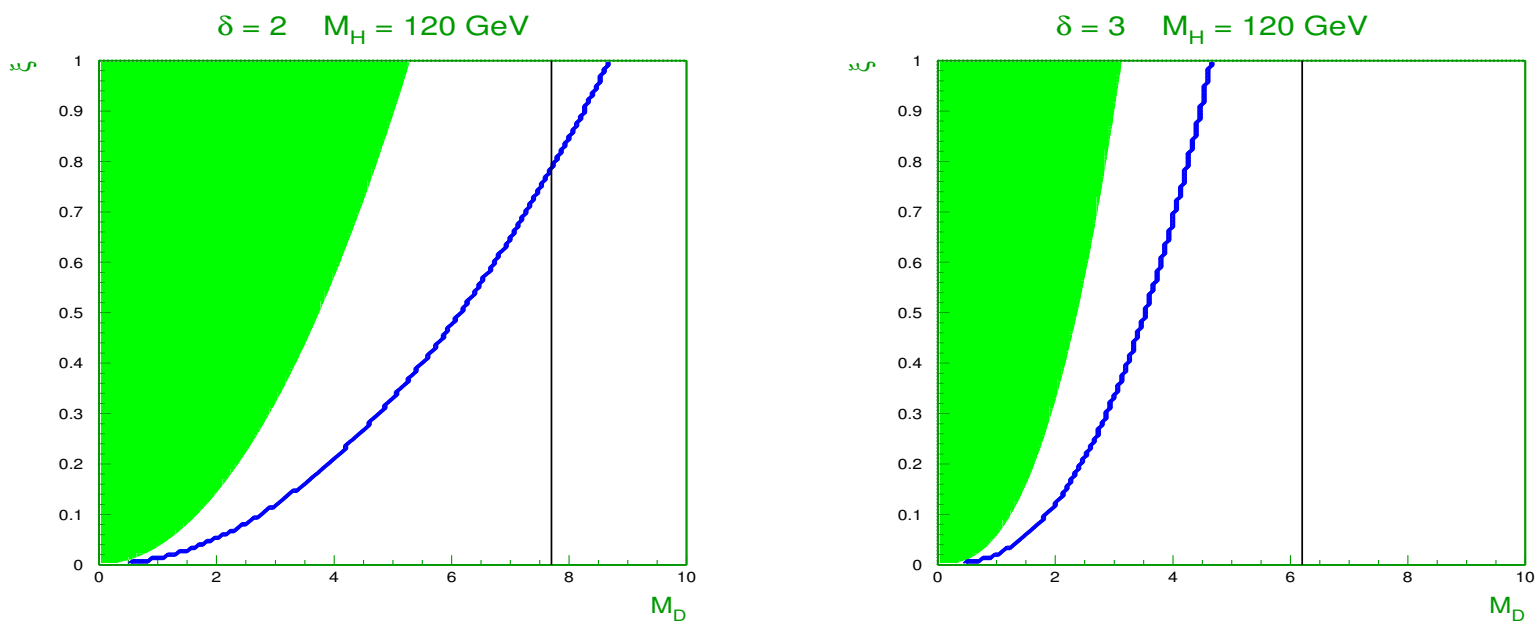



($ZH$ channel: Godbole, Guchait, Mazumdar, Moretti and Roy)

Dilepton +missing $\not{E}_T$ channel: $B_{inv} \sim 0.42(0.70)$ probed at 5$\sigma$ level for $m_H = 120(160)$ GeV with 100 fb$^{-1}$.

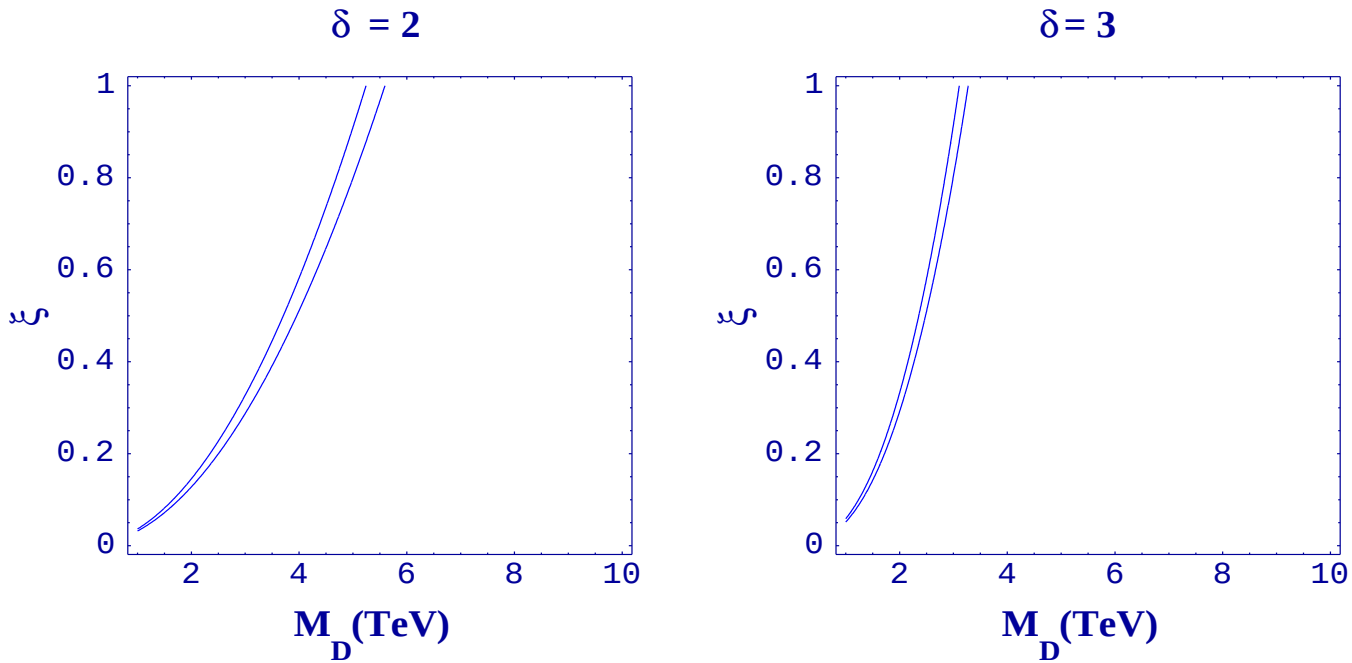
# Sensitivity to ADD $\Gamma_{inv}$ at LHC

**Invisible decay width effects in the $\xi$ - $M_D$ plane for $m_h$ = 120 GeV. The green regions indicate where the Higgs signal in the canonical channels drops below the 5 $\sigma$ threshold for 30 $fb^{-1}$ of data. The regions above the blue line are where the invisible Higgs signal in the $WW$-fusion channel exceeds 5 $\sigma$ significance. The vertical lines show the upper limit on $M_D$ which can be probed by the analysis of jets/$\gamma$ with missing energy (Hinchliffe, Vacavant).**



# Sensitivity to $M_D$-$\xi$ at LC-500

Extracting the branching fraction into invisible final states from the Higgsstrahlung cross section and the sum of visible decay modes affords an accuracy of order 0.2-0.03% for values of the invisible branching fraction in the range 0.1-0.5. But the ultimate accuracy can be obtained with a dedicated analysis looking for an invisible system recoiling against a $Z$ boson in the $e^+e^- \to hZ$ process. $0.04 < \delta\mathrm{BR}/\mathrm{BR} < 0.025$ can be obtained for $0.1 < \mathrm{BR} < 0.5$ (Schumacher).



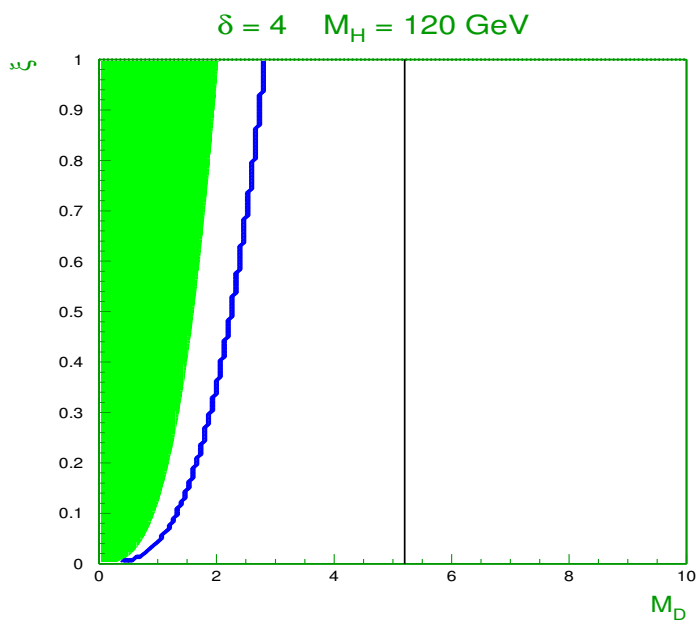

Limits from $e^+e^- \to \gamma + \text{missing energy}$ (Mirabelli, Perelstein, Peskin): $M_D \geq 6.5$ TeV ($\delta = 2$)

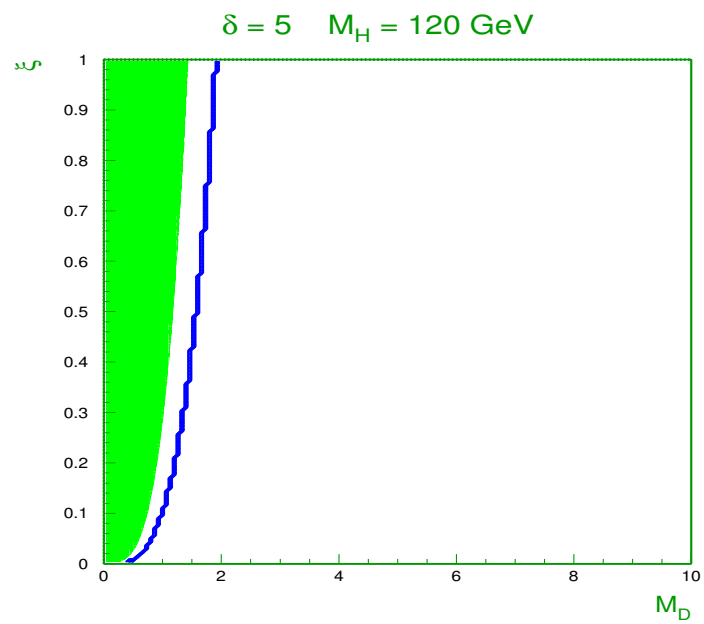
# Conclusions

- **For a light Higgs boson the process $pp \to W^*W^* + X \to \mathrm{Higgs, graviscalars} + \mathrm{X} \to \mathrm{invisible} + \mathrm{X}$ will be observable at the $5\ \sigma$ level at the LHC for the portion of the Higgs-graviscalar mixing ($\xi$) and $D$-dimensional Planck mass ($M_D$) parameter space where channels relying on visible Higgs decays fail to achieve a $5\ \sigma$ signal.**
- **Accuracy of $\Delta BR(H \to \mathrm{invisible})/BR(H \to \mathrm{invisible})$ at the $e^+e^-$ LC allows to constrain $M_D$ and $\xi$ parameters for known number of extra dimensions $\delta$.**

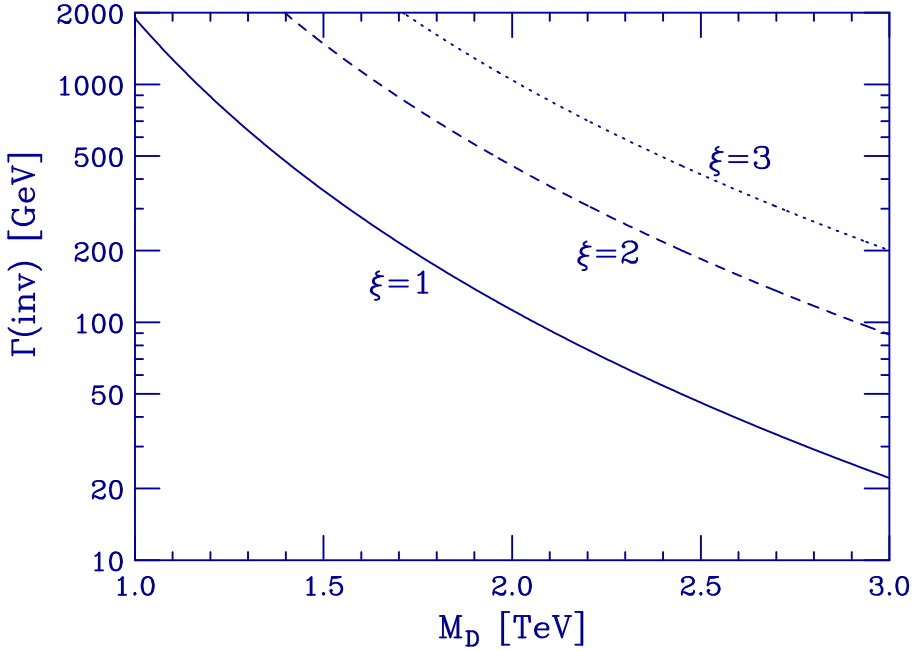
# Sensitivity to ADD $\Gamma_{inv}$ at LHC

**Invisible decay width effects in the $\xi$ - $M_D$ plane for $M_H$ = 120 GeV. The green regions indicate where the Higgs signal drops below the 5 $\sigma$ threshold for 30 $fb^{-1}$ of data. The regions above the blue line are where the invisible Higgs signal in the $WW$-fusion channel exceeds 5 $\sigma$ significance. The vertical lines show the upper limit on $M_D$ which can be probed by the analysis of jets/$\gamma$ with missing energy.**



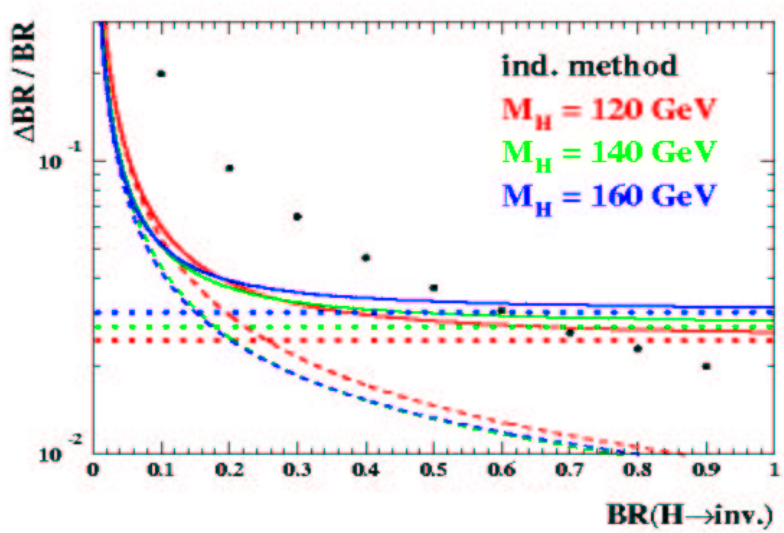



**The total invisible width of a 1 TeV Higgs boson into one and two graviscalars as a function of $M_D$ for various values of $\xi$ ($\xi = 1$ solid, $\xi = 2$ dashed, $\xi = 3$ dotted), $\delta = 2$.**



# Sensitivity to ADD $\Gamma_{inv}$ at the LC

**Relative accuracy of the measurement of the invisible branching as a function of the branching ratio, for $m_H = 120, 140, 160$ GeV for 500 fb$^{-1}$ at $\sqrt{s}$ =350 GeV. (Schumacher).**




**Signal process: $e^+e^- \to ZH \to twojetsE\!\!\!/_T$. Invisible Higgs discovered down to $B \sim 0.02$ for masses 120-160 GeV.**

# Sensitivity to $H_{inv}$ at the Tevatron

(Martin and Wells). $p\overline{p} \to ZH \to l^+l^- + \not{E}_T$ channel, assuming $BR(H \to inv) = 100\%$:

| $m_H$ [GeV] | 95% Exclusion Luminosity [$\mathrm{fb}^{-1}$] | $3\sigma$ Observation Luminosity [$\mathrm{fb}^{-1}$] | $5\sigma$ Discovery Luminosity [$\mathrm{fb}^{-1}$] |
|---|---|---|---|
| 90 | 3.1 | 7.3 | 20 |
| 100 | 5.0 | 11.6 | 32 |
| 110 | 7.5 | 17.5 | 49 |
| 120 | 10.9 | 26 | 71 |
| 130 | 15.7 | 37 | 103 |
| 140 | 23 | 53 | 146 |
| 150 | 32 | 74 | 206 |

Luminosity required to make a 95% confidence level exclusion, $3\sigma$ observation, and $5\sigma$ discovery of an invisibly decaying Higgs boson in the $ZH \to l^+l^- + \not{E}_T$ channel.

## Higgs signal significance as function of the Higgs boson mass. The curves show the signal significance for an integrated luminosity of 30 fb$^{-1}$

(Abdullin et al, CMS note)