Contributions to the 2-loop Bhabha process: new results

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* *Based on common work with Michał Czakon and Tord Riemann*

Why study 2-loop Bhabha scattering now?

- *Prominent luminosity-monitoring reaction for* e^+e^- *colliders*
- *Experimental accuracy points to* 10^{-4}
- *New Physics reach:*
  $e^+e^-e^+e^-$ *four fermion operators*

Plan

- *2-loop topologies for Bhabha scattering in QED*

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- *Cross checks of results*

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- Identification of Master Integrals (MIs)
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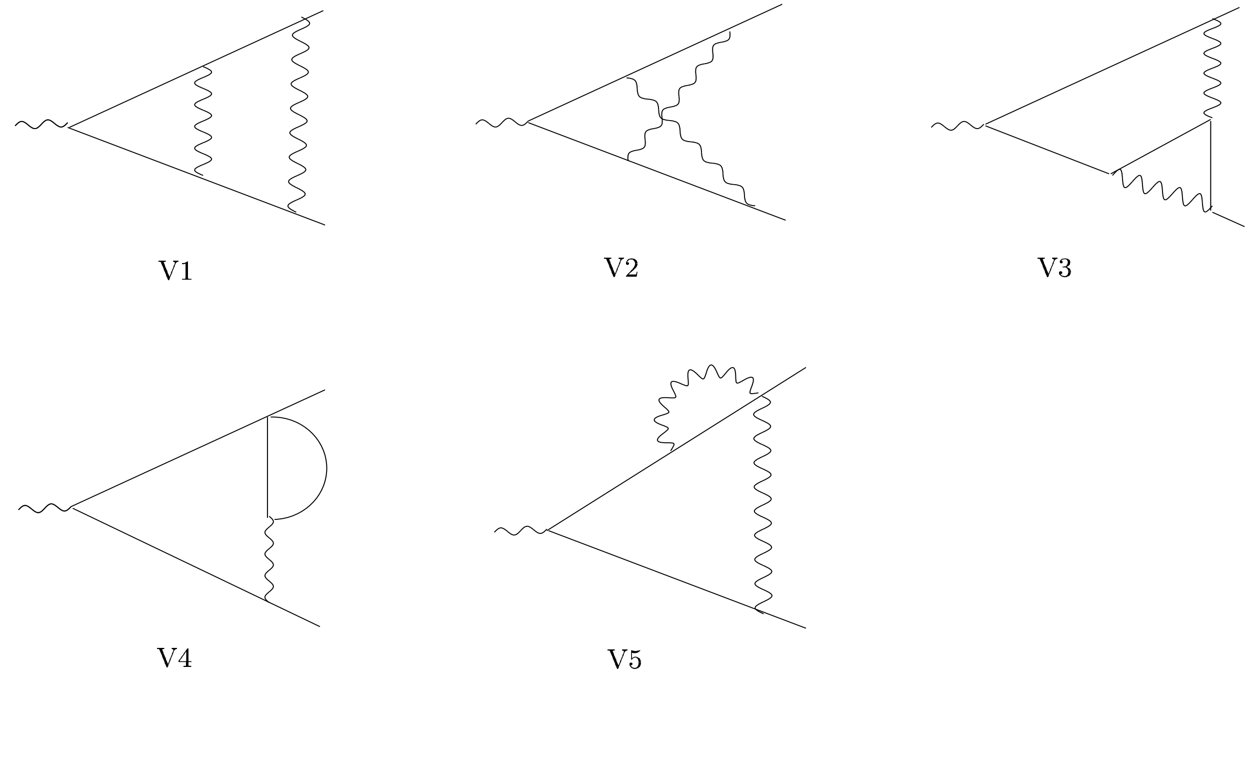
  last four 3-point functions and a new 2-loop box
- Cross checks of results

Summary and outlook

Basic 2-loop vertices

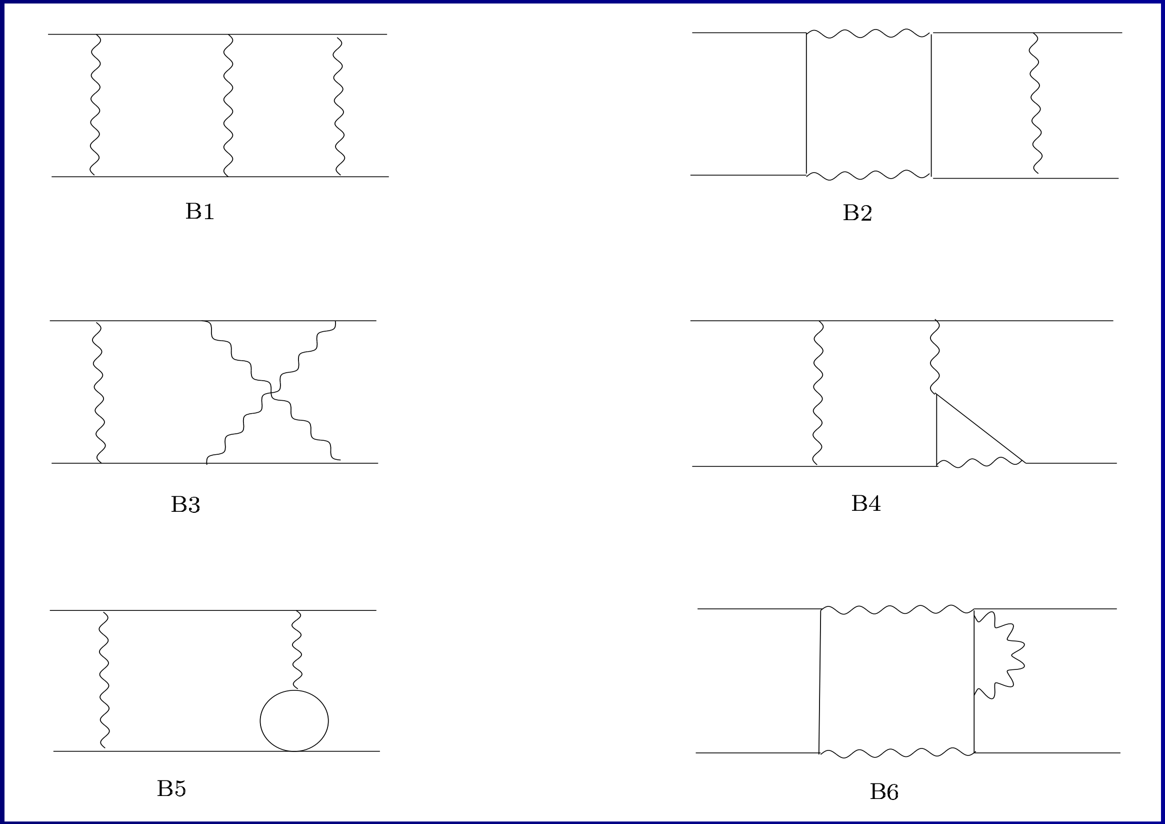


There are 16 Master Integrals for them (Tadpole, 5 SE, 10 V)

R. Bonciani, P. Mastrolia, E. Remiddi, Nucl. Phys. **B661***(2003)289*

Basic 2-loop boxes

Main prototype	*Number of MIs*
B1	*23*
B2	*42*
B3	*52*
B4	*35*
B5	*14*
B6	*15*



B5: Bonciani et al., Nucl. Phys. **B681***(2004)261*

B1: V. Smirnov, Phys. Lett. **B524***(2002)129*

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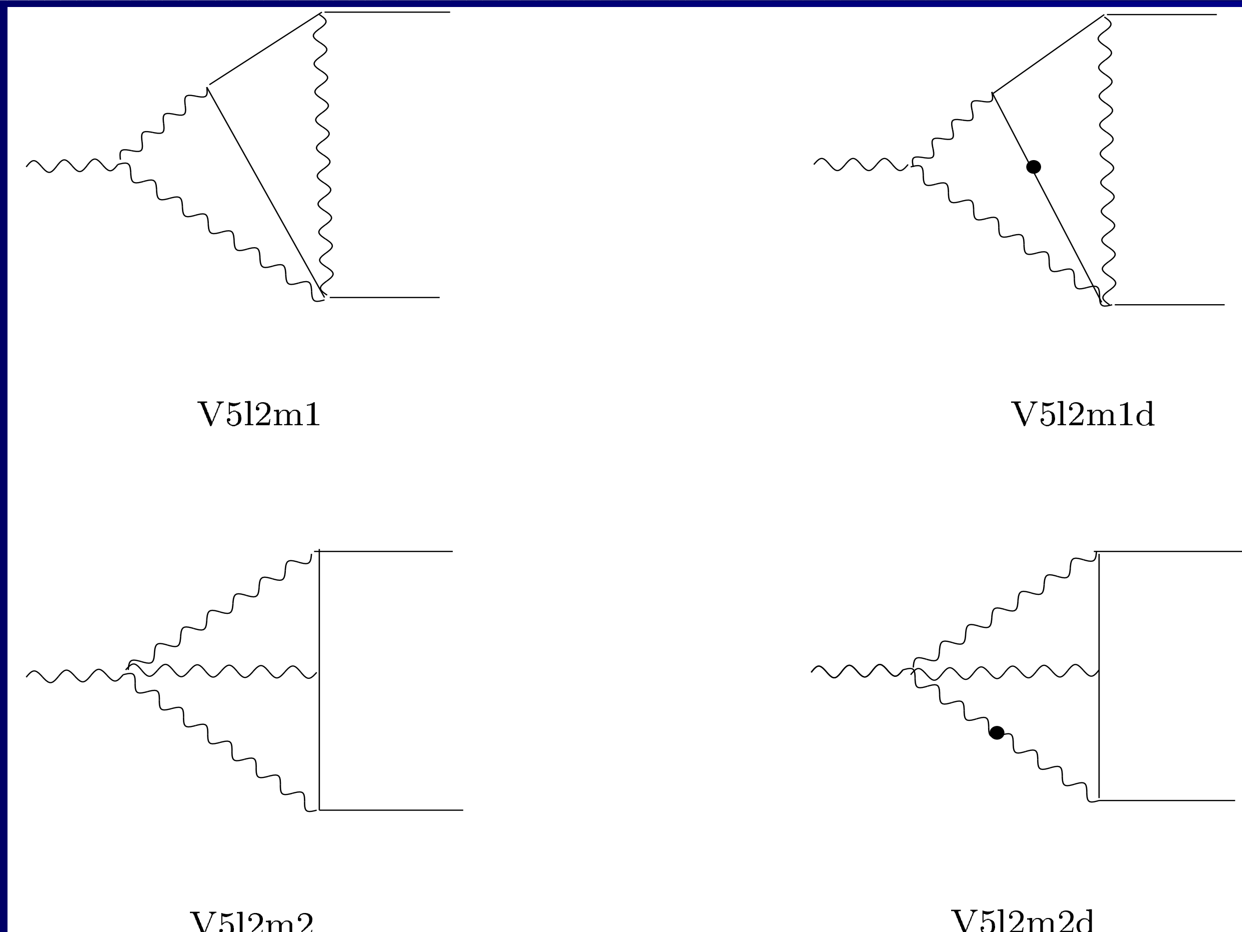
- *Identification of MIs for vertices takes a couple of minutes*
- *Boxes (high powers: 3 additional in denominator and 2 in numerator)*

  B1: 4 h; B2: 9 h; B3,B4: 12 h; B5,B6: 1 h.

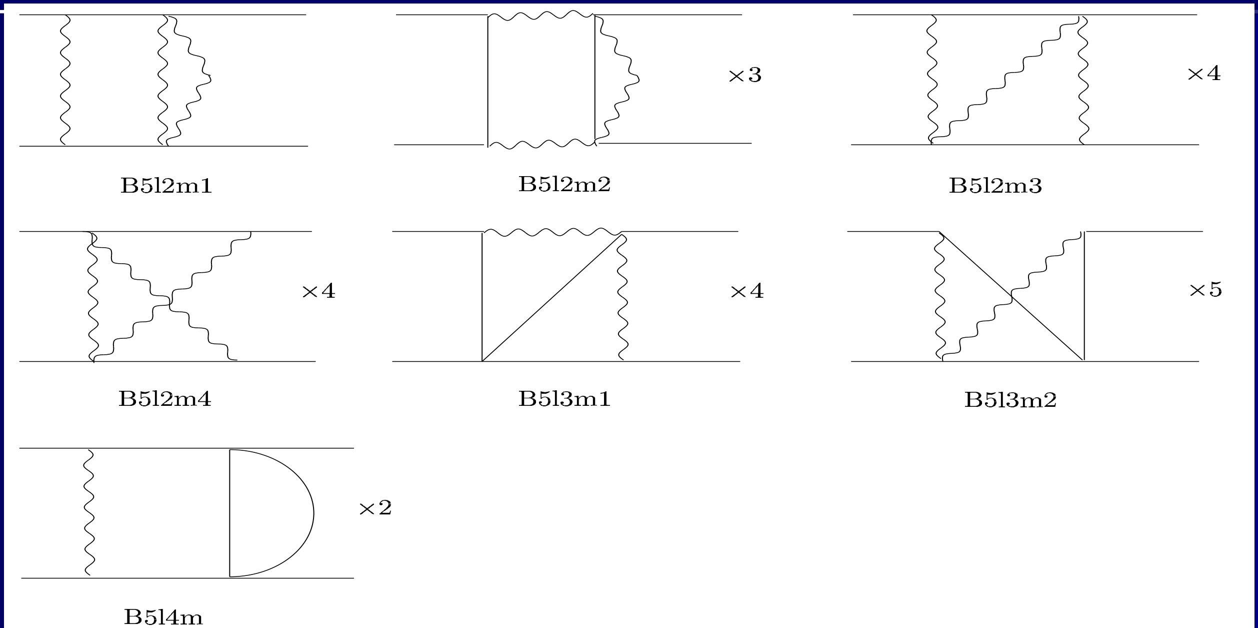
The complete set of MIs: additional vertices

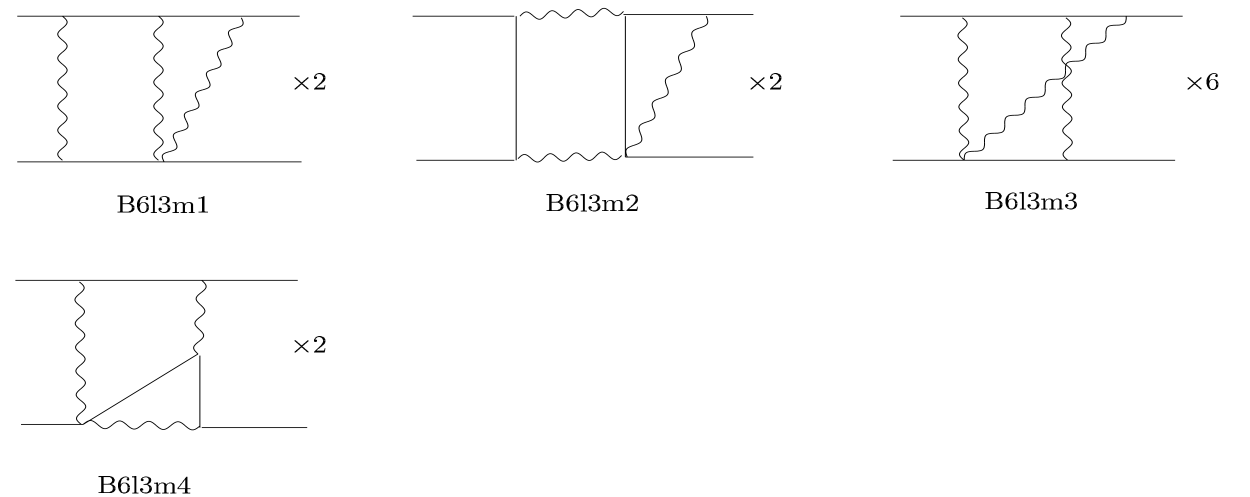


V5l2m1 $\longrightarrow$ $B2, B4$

V5l2m2 $\longrightarrow$ $B1, B3$

The complete set of MIs: boxes





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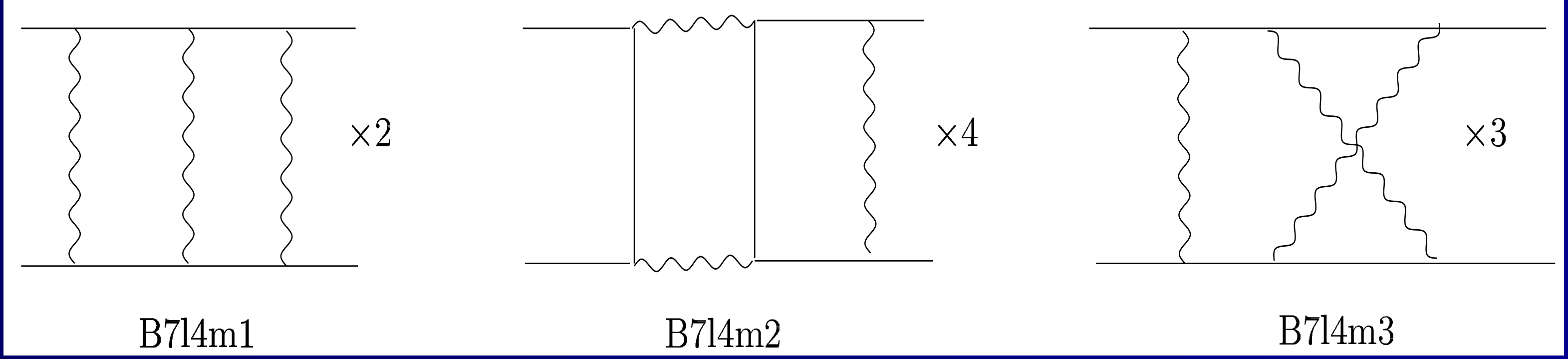
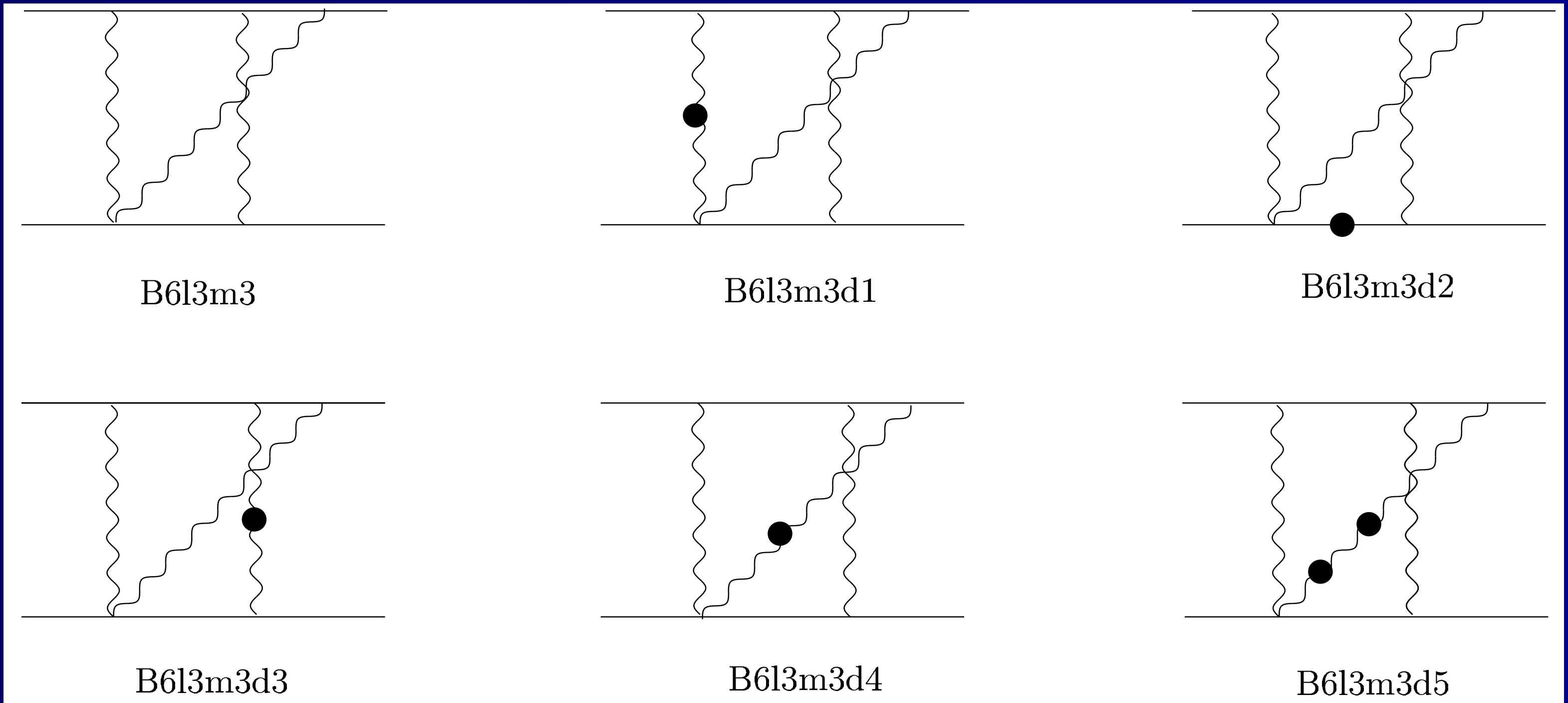


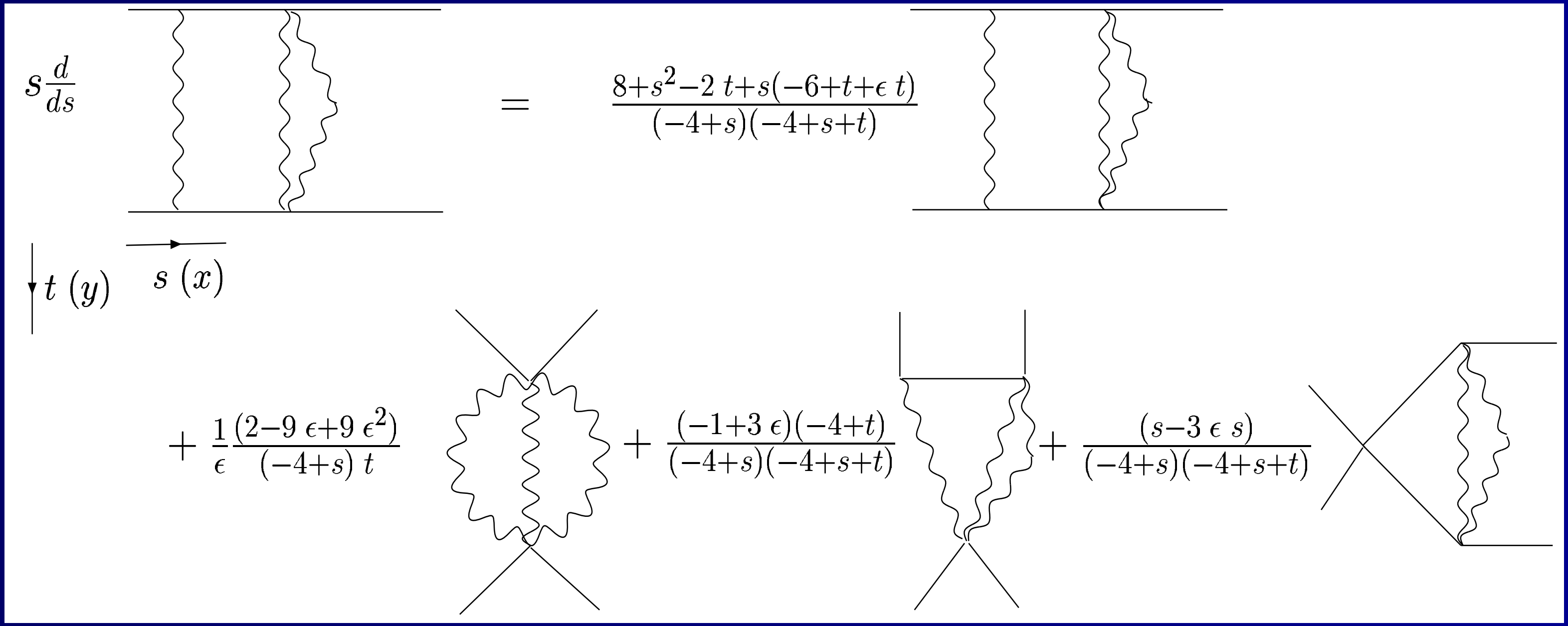
Altogether there are 42 MIs.

The complete set of MIs: B6l3m3 in more details



New MIs: 5 line box B5l2m1

- *Differential equation method (Kotikov, Remiddi)*

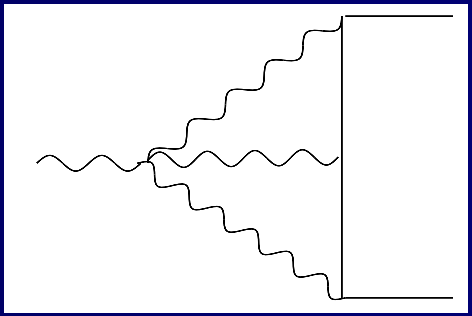


$$\frac{1}{\epsilon^2} \quad \rightarrow \quad \frac{1}{2}\log x\left(\frac{1}{1+x}+\frac{1}{-1+x}\right) \qquad\qquad x=\frac{\sqrt{-s+4}-\sqrt{-s}}{\sqrt{-s+4}+\sqrt{-s}}$$

$$\frac{1}{\epsilon} \quad \rightarrow \quad \frac{1}{4}\left(\frac{1}{1+x}+\frac{1}{1-x}\right)$$

$$[-2\zeta(2)+\log x(\log x-4[-1+\log(1+x)+\log(1-y)]+2\log y)-4Li_2(-x)]$$

New vertices: example



$$
\begin{aligned}
&= \quad \frac{2}{1-x^2}\left[8\zeta(4)+\zeta(2)\log^2(x)+\frac{1}{24}\log^4(x)-4\zeta(2)\ Li_2(x)+2\ Li_4(x)\right] \\
&+ \quad \mathcal{O}(\epsilon)
\end{aligned}
$$

Cross-checks

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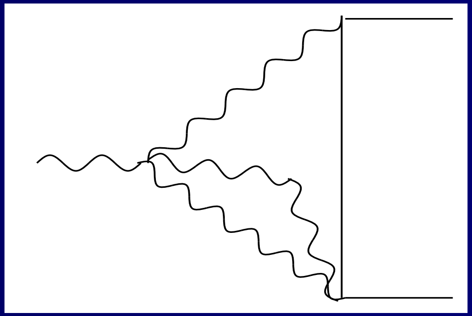
- For IR divergent integrals use IBP relations to the same integrals with irreducible numerators
- Divergent parts of IR safe integrals: propeties of the $\overline{MS}$ scheme exploited
- Subloop subtractions for numerical evaluation of finite parts

Subtraction in Feynman parameters for numerical evaluation



$$\sim \int \frac{d^d k_1 d^d k_2}{[k_2^2][(k_2+k_1-p_1)^2][k_1^2+m^2][(k_1+p_2)^2]}$$

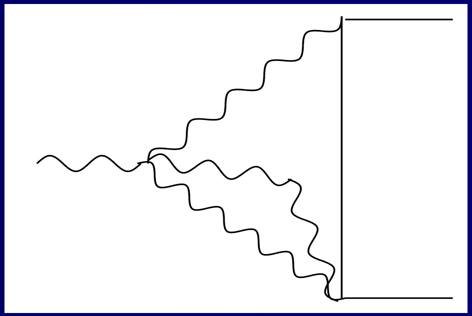
$$\rightarrow \quad \int \frac{d^d k_1}{[(k_1-p_1)^2]^\epsilon [k_1^2+m^2][(k_1+p_2)^2]}$$

$$= \quad \epsilon(1+\epsilon)\int_0^1 dxdydz x^{-1+\epsilon}\delta(1-x-y-z)\int \frac{d^d k_1}{(xD_1+yD_2+zD_3)^{2+\epsilon}}$$

$$\rightarrow \quad \frac{\Gamma(1-\epsilon)^2\Gamma(\epsilon)}{\Gamma(2-2\epsilon)}\epsilon(1+\epsilon)\frac{\Gamma(2\epsilon)}{\Gamma(2+\epsilon)}\int_0^1 dxdy\, x^{-1+\epsilon}(1-x)^{1-2\epsilon}F(x,y)$$

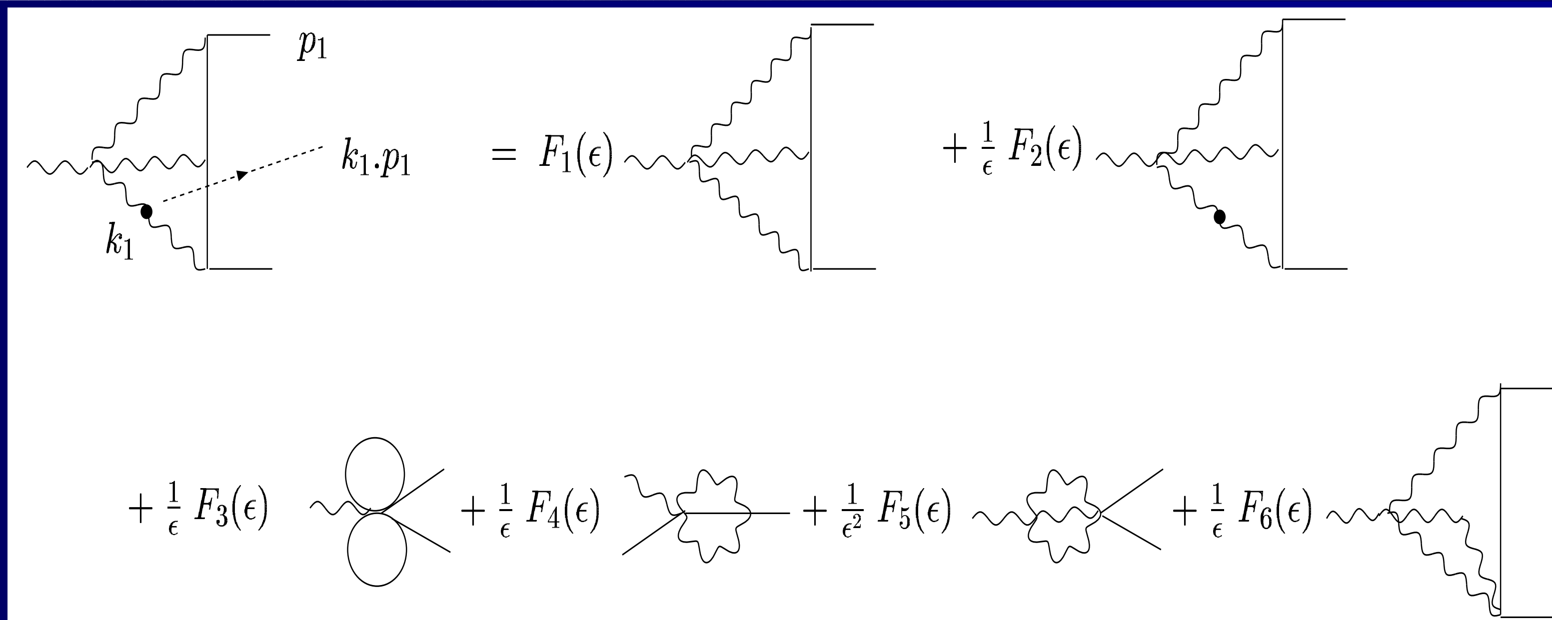
$$\text{Identity}: \quad \int_0^1 dy\,\{[F(x,y)-F(0,y)]+F(0,y)\}$$

Final result:



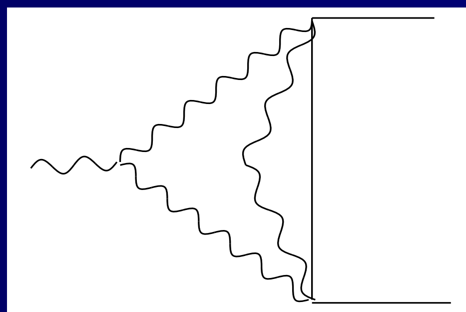
$$= \frac{1}{2}\frac{1}{\epsilon^2} + \frac{5}{2}\frac{1}{\epsilon} + \frac{1}{2}\left[19 + 4\zeta_2 + 2\left(\frac{x-1}{x+1}\right)\left[4\zeta_2 + \frac{1}{2}\ln^2 x + 2Li_2(x)\right]\right]$$

Using irreducible numerators

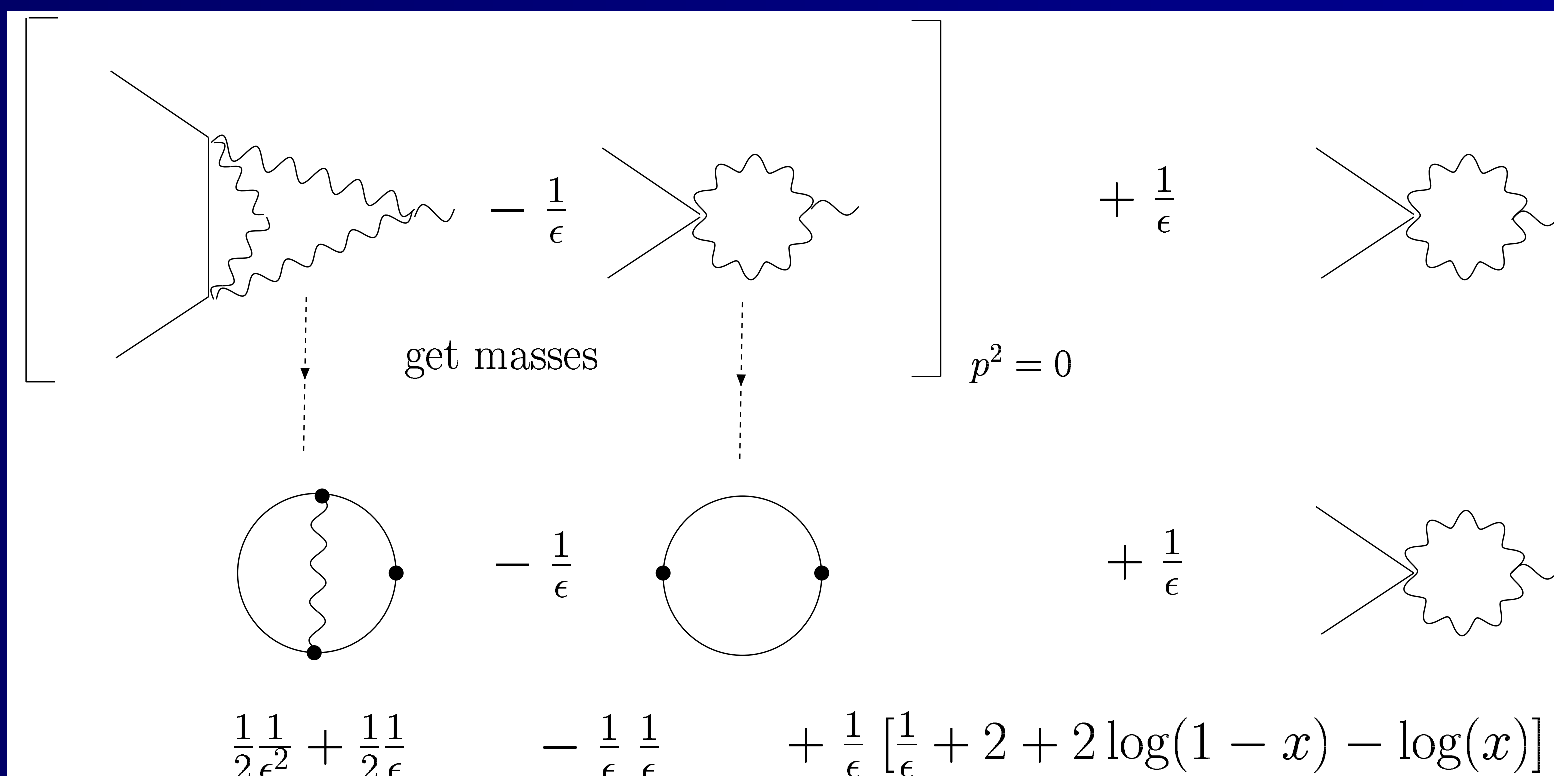


$$
\begin{aligned}
OBJd(s) &= A_{-2}(s)\frac{1}{\epsilon^2} + A_{-1}(s)\frac{1}{\epsilon} + A_0(s) + \cdots \\
OBJ(s) &= B_{-2}(s)\frac{1}{\epsilon^2} + B_{-1}(s)\frac{1}{\epsilon} + B_0(s) + \cdots \\
0 &= \frac{1}{\epsilon^3}\,[F_1(s)\ +\ F_2(s)A_{-2}(s)] \\
A_{-2}(s) &= \frac{1}{4\ s} \qquad \textit{also:}\quad A_{-1}(s)
\end{aligned}
$$

From renormalization theory:

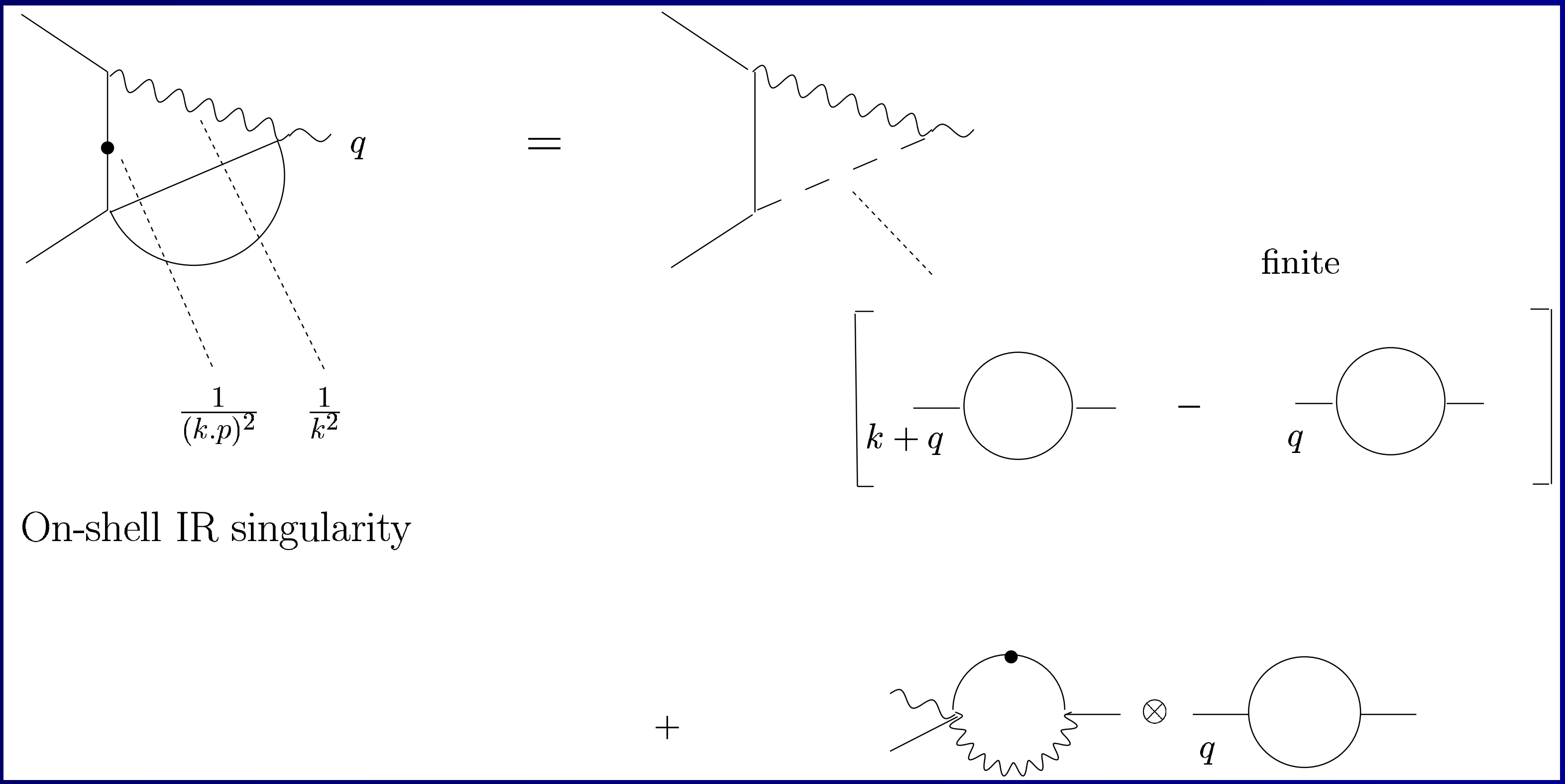


$$= \frac{1}{2}\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\frac{5}{2} + \log(x) - 2\log(1-x)\right) + \cdots$$



Subloop subtractions:

Cross check to all orders in ϵ



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- *The complete set of 3-point functions coming from vertex diagrams and from box diagrams after cancellations of some lines has been calculated. Apart from checking the known integrals this consisted in calculating 4 new master integrals.*
- *One new 5-line box integral coming from the planar double box diagram has been calculated.*
- *All master integrals have been identifi ed. Altogether, there are 42 two-loop box type MI's, 38 of them remain to be calculated (feasible within next few months)*