Impact of a Precise Top Mass Measurement

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Based on collaboration with S. Heinemeyer, S. Kraml, W. Porod

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2. Electroweak precision physics
3. Higgs physics in the MSSM
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1. Introduction

Top-quark mass is a fundamental parameter of the electroweak theory

By far the largest quark mass, largest mass of all known fundamental particles

Window to new physics?

Large coupling to the Higgs boson; physics of flavor; prediction of m_t from underlying theory?

Radiative corrections

$\Rightarrow$ non-decoupling effects proportional to powers of m_t

$\Rightarrow$ Need to know m_t very precisely in order to have sensitivity to effects of new physics

What is the top mass?

Particle masses are not observables; can only measure cross sections, decay rates, . . .

Additional problem for the top mass:

what is the mass of a colored object?

Top pole mass is not IR safe (affected by large long-distance contributions), cannot be determined to better than $\mathcal{O}(\Lambda_{\mathrm{QCD}})$

Measurement of m_{t}:

- At Tevatron, LHC: kinematic reconstruction, fit to invariant mass distribution
- At the LC: mainly from threshold behavior

Experimental accuracy of $m_{\rm t}$

Measurement $\Leftrightarrow$ comparison data – Monte Carlo

involves transition from actually measured quantity to suitably defined (short-distance) top mass

'Threshold mass' at the LC: accuracy $\lesssim 20$ MeV

[*M. Martinez, R. Miquel '03*]

Transition to $\overline{\rm MS}$ mass: [*A. Hoang et al. '00*]

$$\Rightarrow \quad \delta m_{\rm t} \lesssim 100 \text{ MeV} \quad \text{(LC)}$$

Are the uncertainties from unknown higher orders (QCD, electroweak, mixed) under control?

$$\text{LHC:} \quad \delta m_{\rm t} = 1\text{–}2 \text{ GeV} \quad \text{(LHC)}$$

2. Electroweak precision physics

Theoretical predictions for M_W, $\sin^2\theta_\mathrm{eff}$ in the SM:

Comparison of SM prediction for muon decay with experiment
(Fermi constant G_μ)

$$\Rightarrow \qquad M_\mathrm{W}^2\left(1-\frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_\mu}\left(1+\Delta r\right),$$

$\Updownarrow$

loop corrections

$\Rightarrow$ Theo. prediction for M_W in terms of M_Z, α, G_μ, $\Delta r(m_\mathrm{t}, M_\mathrm{H}, \ldots)$
Complete electroweak two-loop result known

Effective couplings at the Z resonance:

$$\Rightarrow \qquad \sin^2\theta_\mathrm{eff}=\frac{1}{4}\left(1-\frac{\mathrm{Re}\,g_V}{\mathrm{Re}\,g_A}\right)=\left(1-\frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right)\kappa(M_\mathrm{W}^2)$$

Leading m_t contributions

One-loop result for M_W in the SM:
[*A. Sirlin '80*], [*W. Marciano, A. Sirlin '80*]

$$\begin{array}{cccccccc} \Delta r_{1-\mathrm{loop}} = & \Delta\alpha & - & \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho & + & \Delta r_\mathrm{rem}(M_\mathrm{H}) \\ & \sim \log\frac{M_\mathrm{Z}}{m_f} & & \sim m_\mathrm{t}^2 & & \\ & \sim 6\% & & \sim 3.3\% & & \sim 1\% \end{array}$$

Leading m_t contribution to $\sin^2\theta_\mathrm{eff}$:

$$\Delta\sin^2\theta_\mathrm{eff} \approx -\frac{c_\mathrm{W}^2 s_\mathrm{W}^2}{c_\mathrm{W}^2 - s_\mathrm{W}^2}\Delta\rho \sim m_\mathrm{t}^2$$

Sources of theoretical uncertainties

- From unknown higher-order corrections ("intrinsic")
  [*M. Awramik, M. Czakon, A. Freitas, G.W. '03*]

$$\Delta M_{\mathrm{W}}^{\mathrm{intr}} \approx 4\ \mathrm{MeV}, \quad \Delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{intr}} \approx 6 \times 10^{-5}$$

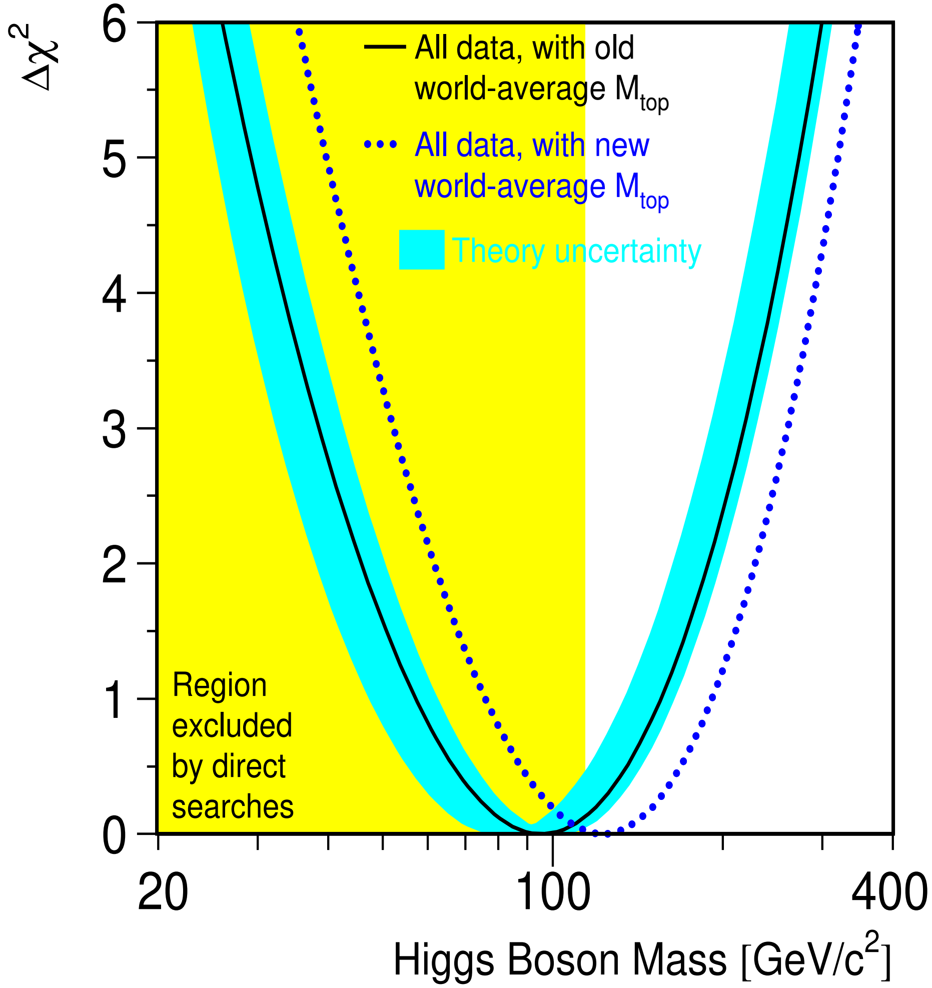
  $\sin^2\theta_{\mathrm{eff}}$: complete two-loop calculation in progress, will lead to reduction of $\Delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{intr}}$

- From experimental errors of input parameters ("parametric")

$$\delta m_{\mathrm{t}} = 4.3\ \mathrm{GeV} \;\Rightarrow\; \Delta M_{\mathrm{W}}^{\mathrm{para}} \approx 26\ \mathrm{MeV}, \quad \Delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{para}} \approx 14 \times 10^{-5}$$

Example: Effect of new m_{t} value on global fit to all data in the SM

[*LEPEWWG '04*]



Old m_{t} value:

$m_{\mathrm{t}} = 174.3 \pm 5.1$ GeV

New m_{t} value:

$m_{\mathrm{t}} = 178.0 \pm 4.3$ GeV

[*Tevatron EWWG '04*]

$\Rightarrow$ Best fit value (95% C.L. upper bound) of M_{H} shifted by more than 20 GeV (30 GeV)

Comparison of future parametric uncertainties

$$\delta m_{\rm t} = 1.5 \text{ GeV} \quad \Rightarrow \quad \Delta M_{\rm W}^{\rm para} \approx 9 \text{ MeV}, \quad \Delta \sin^2\theta_{\rm eff}^{\rm para} \approx 4.5 \times 10^{-5}$$

$$\delta m_{\rm t} = 0.1 \text{ GeV} \quad \Rightarrow \quad \Delta M_{\rm W}^{\rm para} \approx 1 \text{ MeV}, \quad \Delta \sin^2\theta_{\rm eff}^{\rm para} \approx 0.3 \times 10^{-5}$$

$$\delta(\Delta\alpha_{\rm had}) = 5 \times 10^{-5} \quad \Rightarrow \quad \Delta M_{\rm W}^{\rm para} \approx 1 \text{ MeV}, \quad \Delta \sin^2\theta_{\rm eff}^{\rm para} \approx 1.8 \times 10^{-5}$$

$$\delta M_{\rm Z} = 2.1 \text{ MeV} \quad \Rightarrow \quad \Delta M_{\rm W}^{\rm para} \approx 2.5 \text{ MeV}, \quad \Delta \sin^2\theta_{\rm eff}^{\rm para} \approx 1.4 \times 10^{-5}$$

$\Rightarrow$ Need LC accuracy on $m_{\rm t}$ in order to keep parametric error induced by $m_{\rm t}$ at / below level of other uncertainties

LC experimental errors: $\delta M_{\rm W}^{\rm exp} \approx 7$ MeV, $\delta \sin^2\theta_{\rm eff}^{\rm exp} \approx 1 \times 10^{-5}$

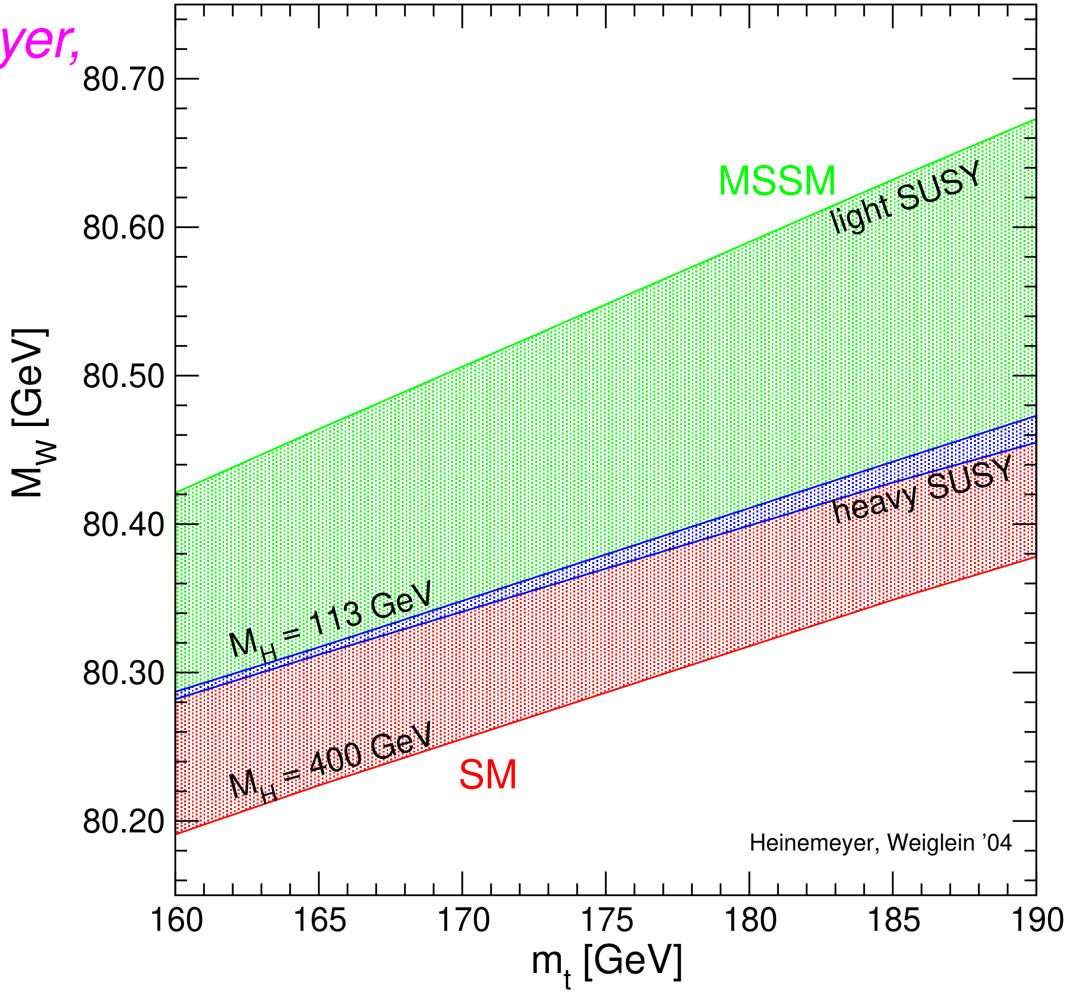
Effect of m_t on electroweak precision tests: SM vs. MSSM

Prediction for M_W in the SM and the MSSM:

[A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97]

[S. Heinemeyer, W. Hollik, G. W. '98]

[S. Heinemeyer, G. W. '02]



SM: M_H varied

MSSM: SUSY parameters varied

Effect of m_{t} on electroweak precision tests: SM vs. MSSM

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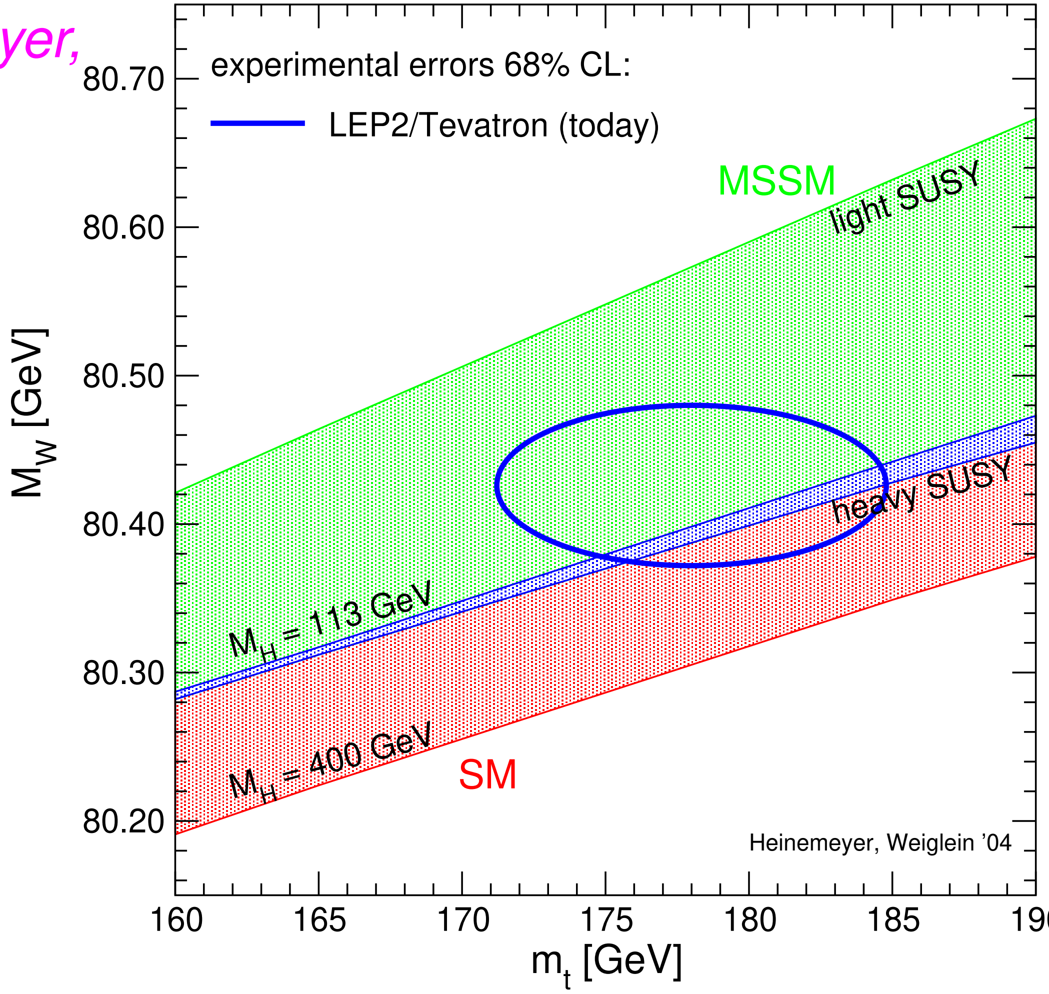

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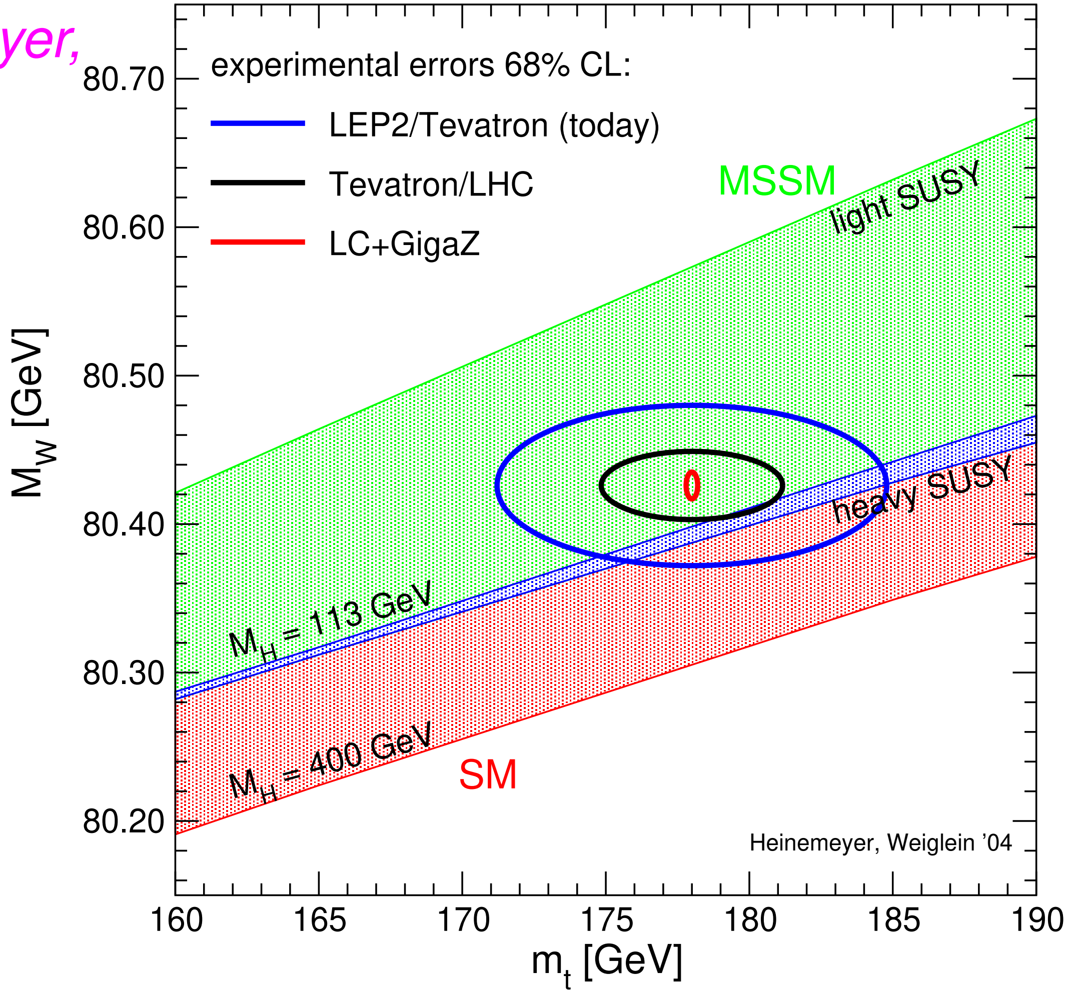

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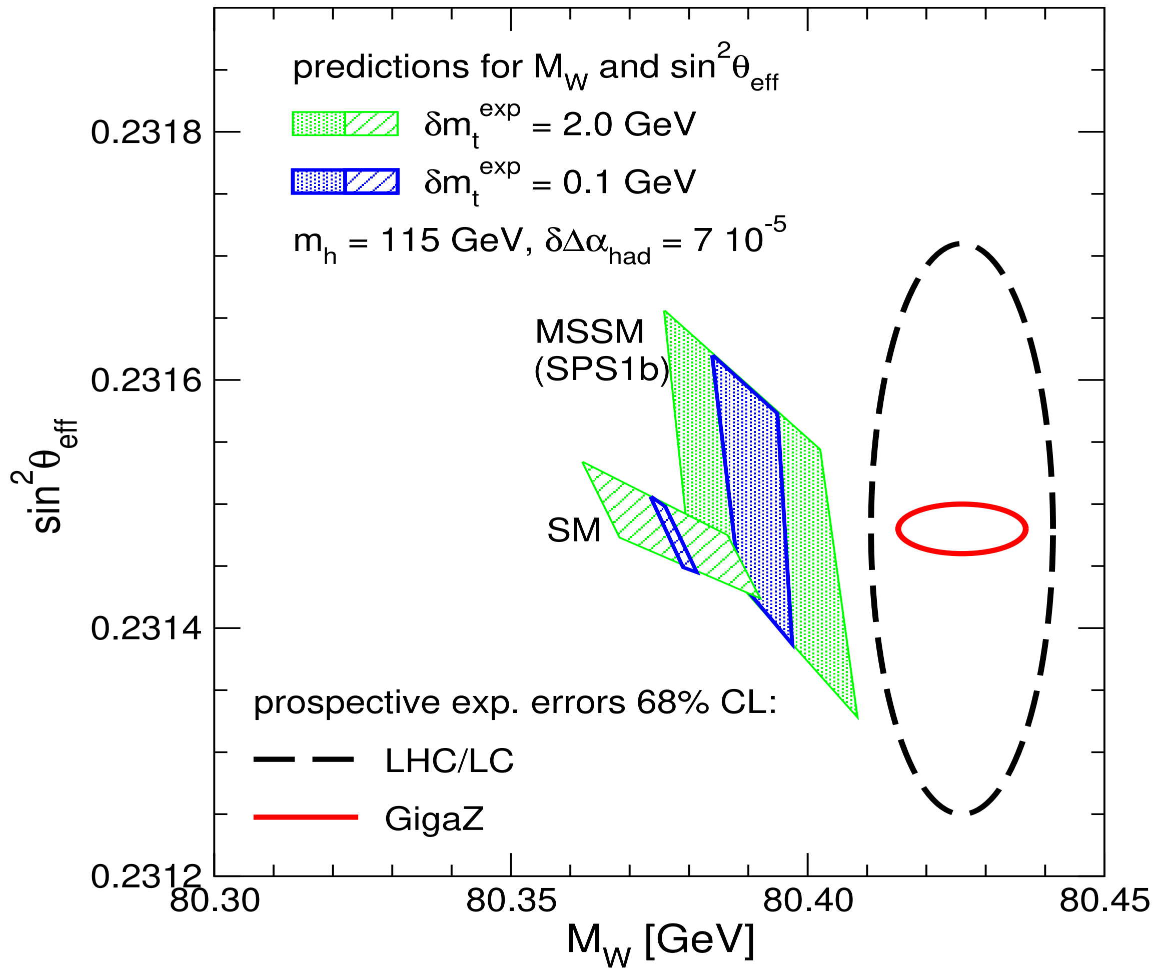


SM: $m_h = 115$ GeV

MSSM:

SPS 1b scenario,

all SUSY parameters varied within realistic errors

$\delta m_t = 0.1$ GeV vs. $\delta m_t = 2$ GeV:

$\Rightarrow$ SM: factor 10 improvement, MSSM: factor 2–3

3. Higgs physics in the MSSM

MSSM Higgs potential contains two Higgs doublets:

$$\begin{aligned} V &= m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\ &\quad + \underbrace{\frac{g'^2 + g^2}{8}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} \, |H_1 \bar{H}_2|^2 \end{aligned}$$

gauge couplings, in contrast to SM

Five physical states: $h^0, H^0, A^0, H^\pm$

Input parameters: $\tan\beta = \frac{v_2}{v_1}$, M_A

$\Rightarrow m_\mathrm{h}$, m_H, mixing angle α, $m_{\mathrm{H}^\pm}$: no free parameters

Higgs mass bound in the MSSM:

$\Rightarrow$ Prediction for $m_{\mathrm{h}}, m_{\mathrm{H}}, \ldots$

Tree-level result for $m_{\mathrm{h}}, m_{\mathrm{H}}$:

$$m^2_{\mathrm{H,h}} = \frac{1}{2}\left[M_{\mathrm{A}}^2 + M_{\mathrm{Z}}^2 \pm \sqrt{(M_{\mathrm{A}}^2 + M_{\mathrm{Z}}^2)^2 - 4M_{\mathrm{Z}}^2 M_{\mathrm{A}}^2 \cos^2 2\beta}\right]$$

$\Rightarrow m_{\mathrm{h}} \leq M_{\mathrm{Z}}$ at tree level

MSSM tree-level bound (gauge sector): excluded by LEP!

Large radiative corrections (Yukawa sector, . . .):

Yukawa couplings: $\frac{\mathrm{e}\, m_{\mathrm{t}}}{2M_{\mathrm{W}} s_{\mathrm{W}}}, \frac{\mathrm{e}\, m_{\mathrm{t}}^2}{M_{\mathrm{W}} s_{\mathrm{W}}}, \ldots$

$\Rightarrow$ Dominant one-loop corrections: $G_\mu m_{\mathrm{t}}^4 \ln\left(\frac{m_{\tilde{\mathrm{t}}_1} m_{\tilde{\mathrm{t}}_2}}{m_{\mathrm{t}}^2}\right), \quad \mathcal{O}(100\%)$!

Current status of theoretical uncertainties in prediction for $m_{\rm h}$

[*G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. W. '02*]

– From unknown higher-order corrections: $\Rightarrow \Delta m_{\rm h} \approx 3$ GeV

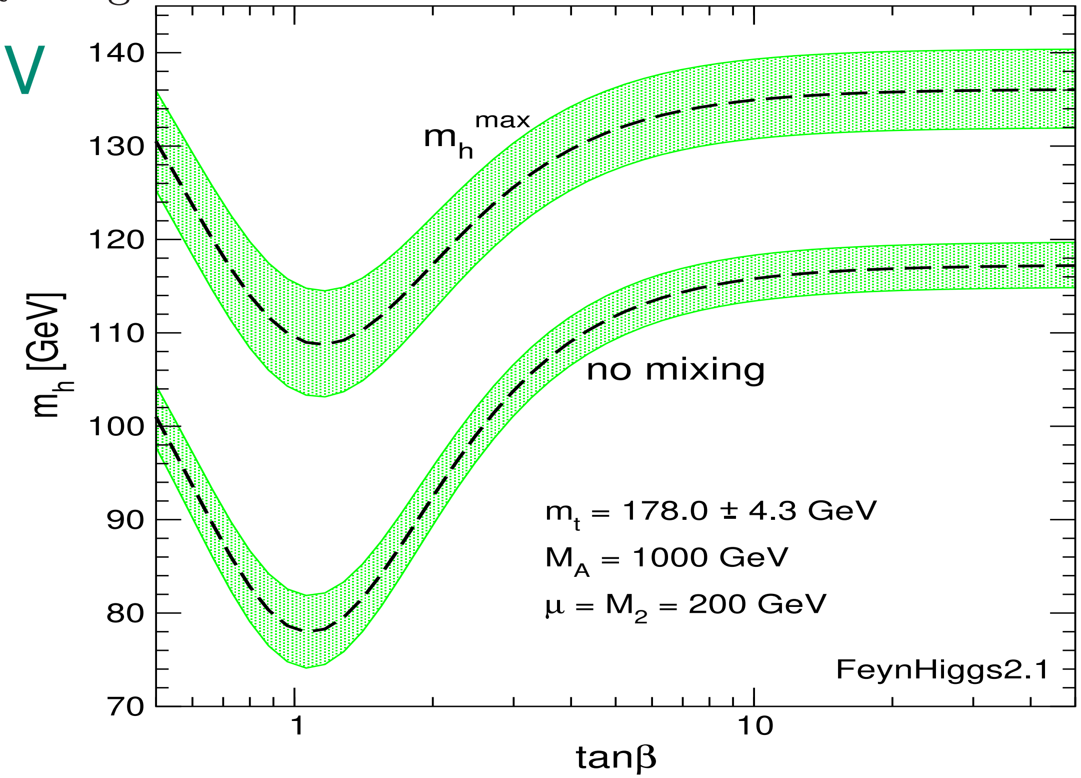
(in region where $\tilde{\rm b}$ sector corrections are under control)

– From uncertainties in input parameters

$m_{\rm t}, \ldots, M_{\rm A}, \tan\beta, m_{\tilde{\rm t}_1}, m_{\tilde{\rm t}_2}, \theta_{\tilde{\rm t}}, m_{\tilde{\rm g}}, \ldots$:

$\Delta m_{\rm t} \approx 4$ GeV $\Rightarrow \Delta m_{\rm h} \approx 4$ GeV

Upper bound on $m_{\rm h}$ saturated for large $M_{\rm A}$, large $\tan\beta$, significant mixing in $\tilde{\rm t}$ sector



Prospective experimental errors vs. parametric uncertainties

- LHC: $\delta m_{\rm h}^{\rm exp} \approx 0.2$ GeV (if $\gamma\gamma$ decay mode accessible)

  LC: $\delta m_{\rm h}^{\rm exp} \approx 0.05$ GeV

  Many more precision measurements in the Higgs sector

- Parametric uncertainty:

  $\Delta m_{\rm t} \approx 2$ GeV $\Rightarrow \Delta m_{\rm h} \approx 2$ GeV

  $\Delta m_{\rm t} \approx 0.1$ GeV $\Rightarrow \Delta m_{\rm h} \approx 0.1$ GeV

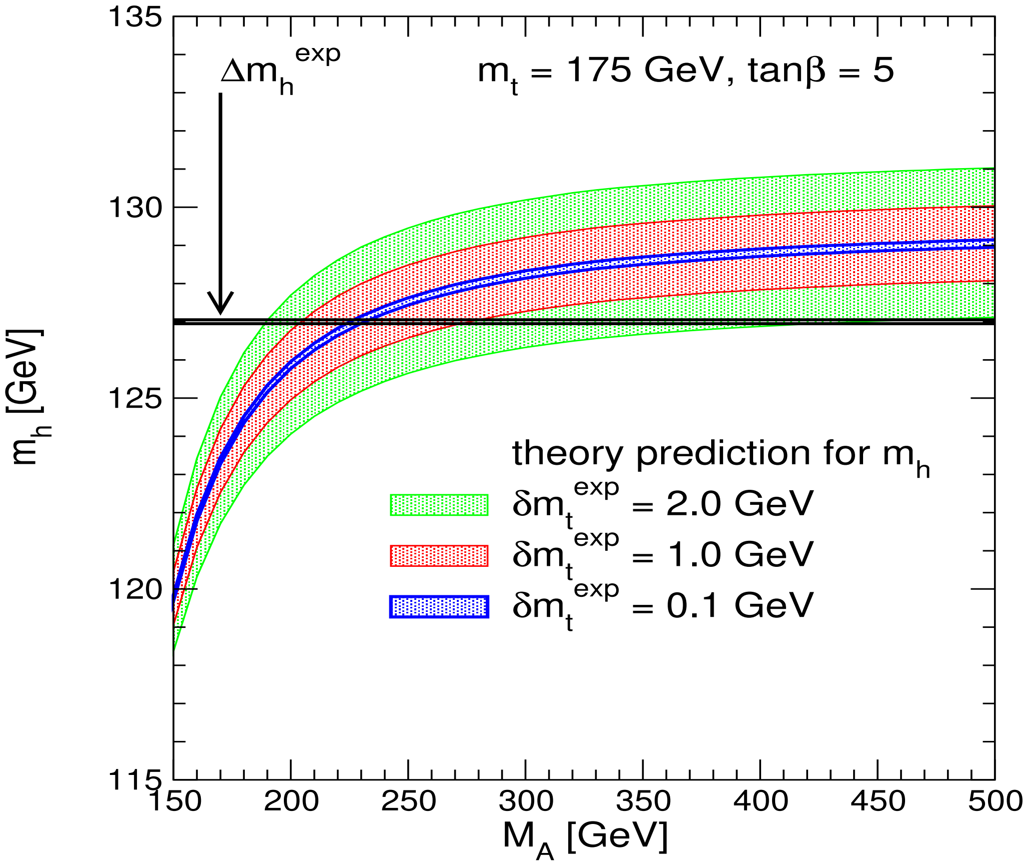
- All other future parametric uncertainties: $\Delta m_{\rm h} \lesssim 0.2$ GeV

$\Rightarrow$ Need LC precision on $m_{\rm t}$ in order to exploit LHC (and LC) precision on Higgs sector measurements

Note: need to improve uncertainty from unknown higher-order corrections by more than factor 10

Parametric uncertainty in m_h prediction induced by δm_t vs. experimental error of m_h

m_h^{max} benchmark scenario (no experimental errors of SUSY parameters):



$\Rightarrow$ LC precision on m_t needed to match experimental accuracy

Note: indirect determination of M_A or other SUSY parameters requires realistic estimate of other parametric $+$ intrinsic errors

Determination of parameters in stop sector: trilinear coupling A_t

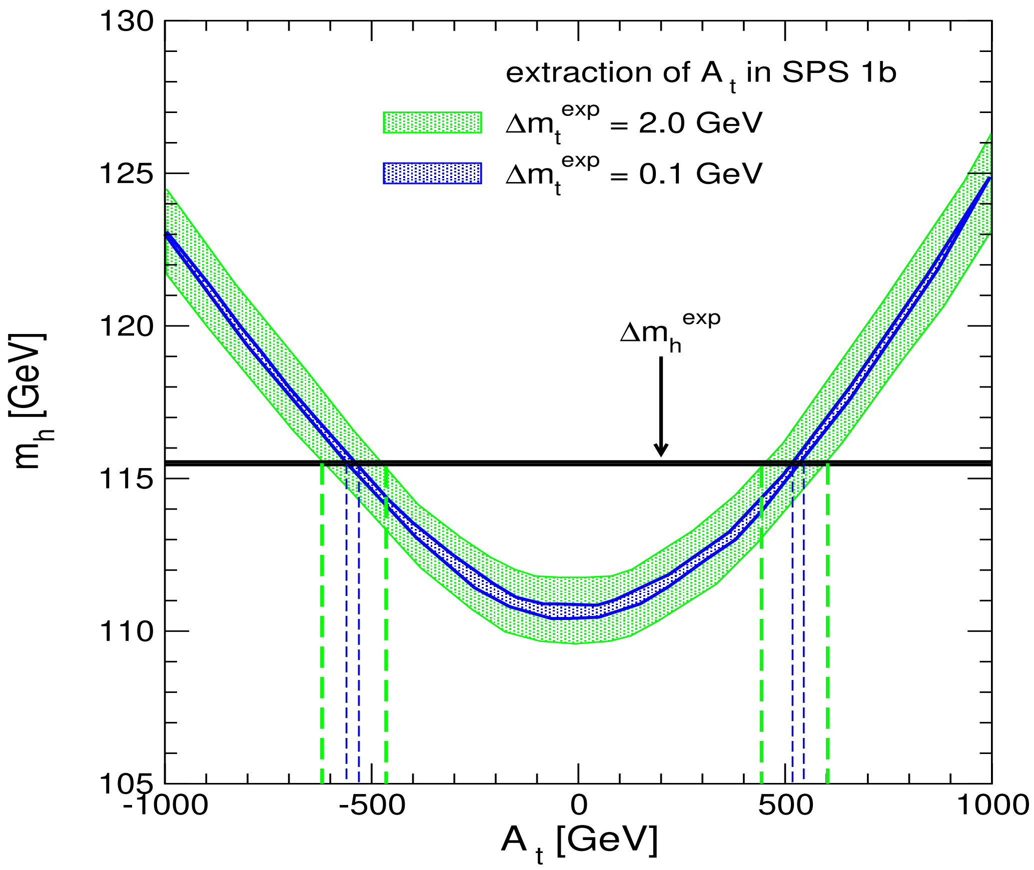
Example:

SPS 1b scenario

Assume:

$m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $m_{\tilde{b}_1}$, $m_{\tilde{b}_2}$, $\tan\beta$, M_A known

realistic estimate of parametric errors



$\Rightarrow$ Indirect determination of A_t from m_h measurement

LC accuracy on m_t $\Rightarrow$ improvement by factor 3–4

Determination of parameters in stop sector: heavier stop mass $m_{\tilde{t}_2}$

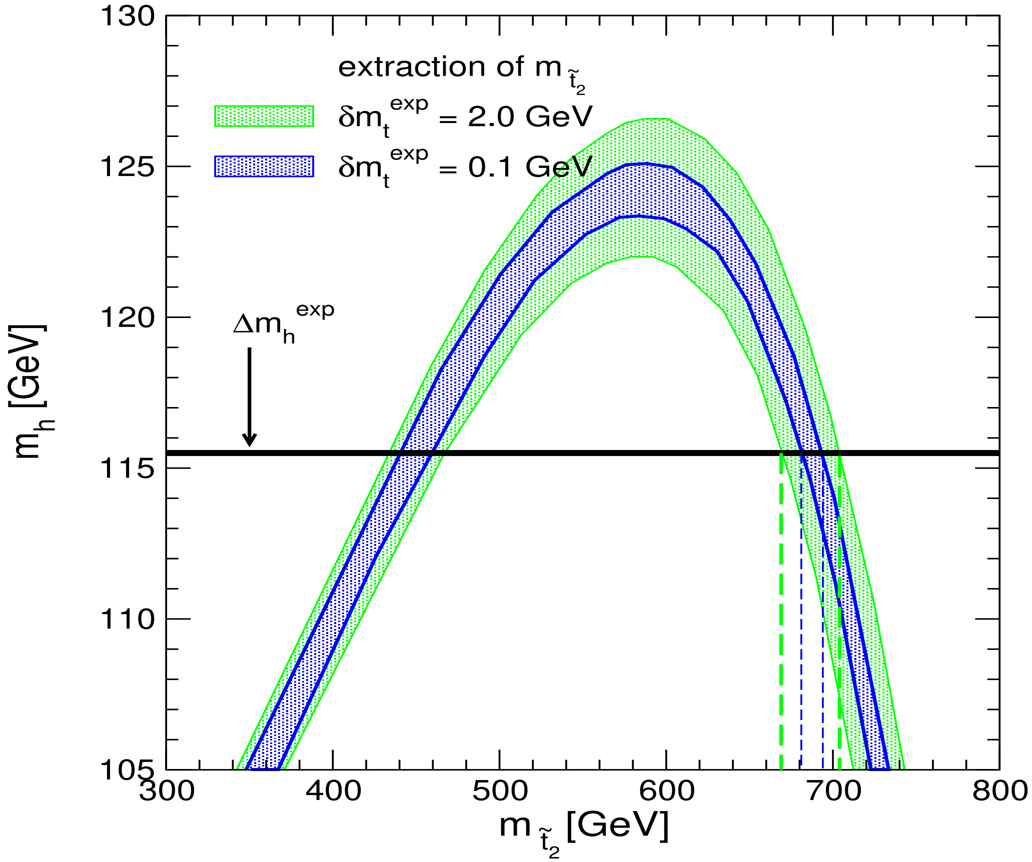
Example:

Scenario with light $m_{\tilde{t}_1}$, $m_{\tilde{t}_1} \approx 180$ GeV

Assume:

$m_{\tilde{t}_1}$, $\theta_{\tilde{t}}$, $m_{\tilde{b}_1}$, $m_{\tilde{b}_2}$, M_A known, $\tan\beta > 10$

realistic estimate of parametric errors



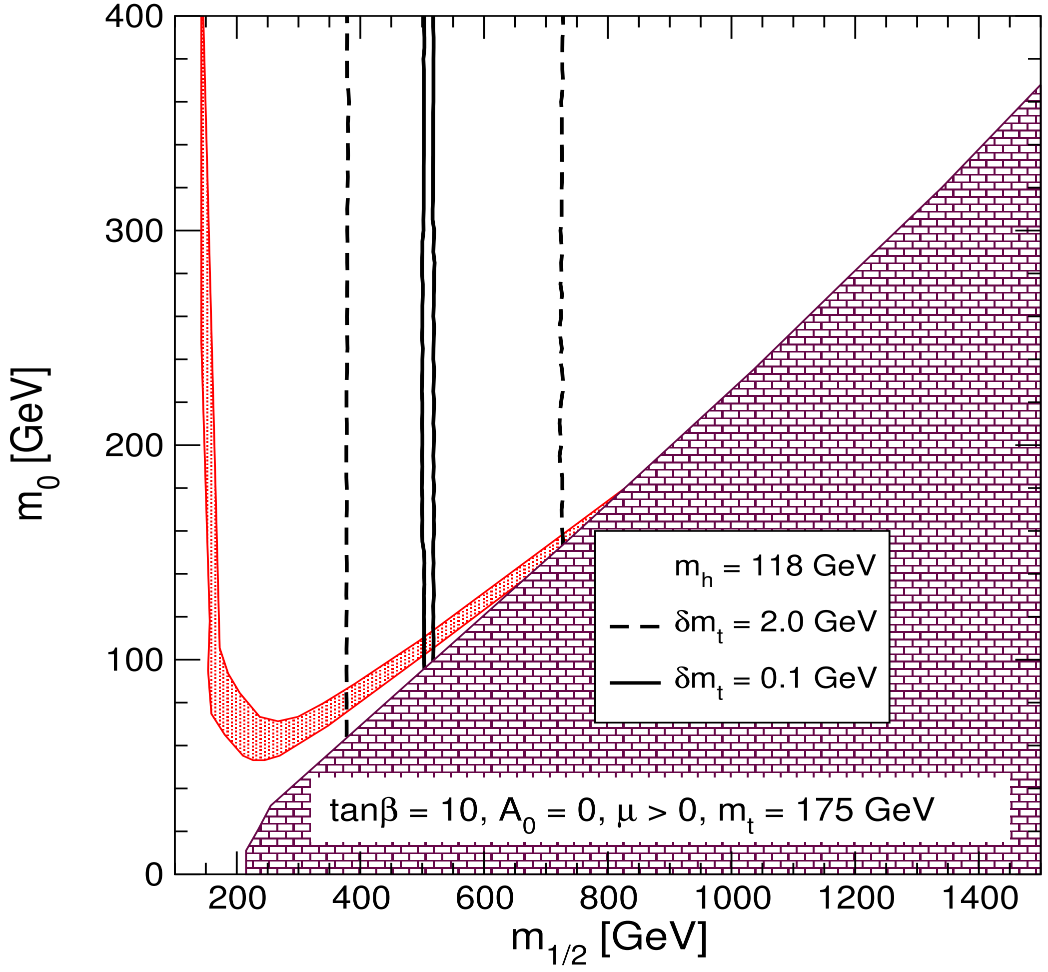
$\Rightarrow$ Indirect determination of $m_{\tilde{t}_2}$ from m_h measurement

LC accuracy on $m_t \Rightarrow$ improvement by factor 2–3

Parameter region allowed by dark matter constraints in mSUGRA scenario: impact of m_h measurement

Allowed parameter region in m_0–$m_{1/2}$ plane from constraint that LSP yields correct CDM density

m_h constraint: $\delta m_\mathrm{t} = 2$ GeV vs. $\delta m_\mathrm{t} = 0.1$ GeV



$\Rightarrow$ CDM $+$ m_h constraints:

yield tight limits on m_0, $m_{1/2}$ for LC precision on m_t

4. Renormalization group running

Measurement of low-energy SUSY parameters

$\Rightarrow$ extrapolation to high scales via RGE

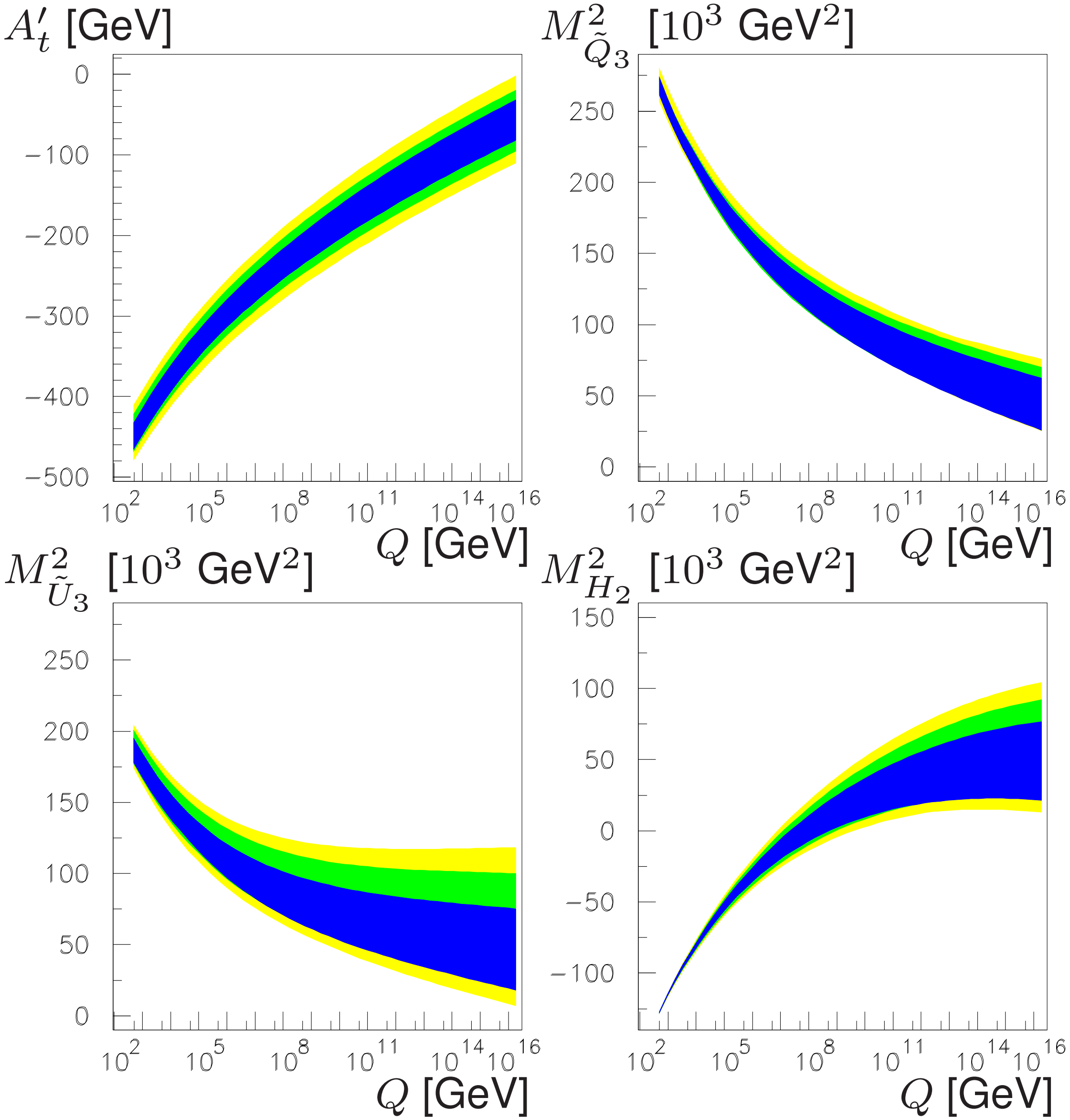
m_t enters via Yukawa coupling, stop masses, loop effects

Impact of

$\delta m_t = 0.1$ GeV,

$\delta m_t = 1$ GeV,

$\delta m_t = 2$ GeV



$\Rightarrow$ Large improvement in precision of GUT scale parameters

5. Conclusions

- Precise knowledge of $m_{\rm t}$ is crucial for electroweak physics:

  precision tests, constraints on the Higgs sector, sensitivity to new physics, . . .

- Comparison of $\delta m_{\rm t} = 0.1$ GeV (LC) with $\delta m_{\rm t} = 1$–2 GeV (LHC):

  LC precision mandatory for exploiting ew prec. observables, SUSY Higgs physics, RG running to high scales, . . . :

  $\delta m_{\rm t}^{\rm LHC}$ is dominant parametric error

  (examples discussed here are not an exclusive list)

  $\Rightarrow$ With LC precision: typical improvements by factors 2–10

  Even larger sensitivity to $m_{\rm t}$ in extreme parameter regions (e.g. SUSY focus point scenario)