Little Higgs models and precision EW data*

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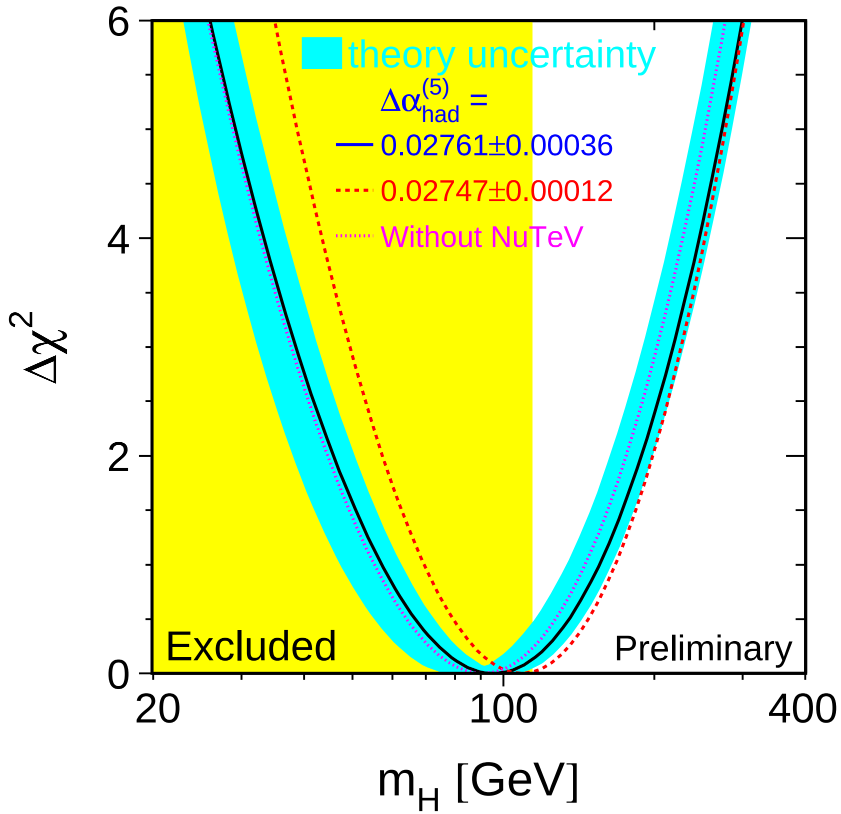
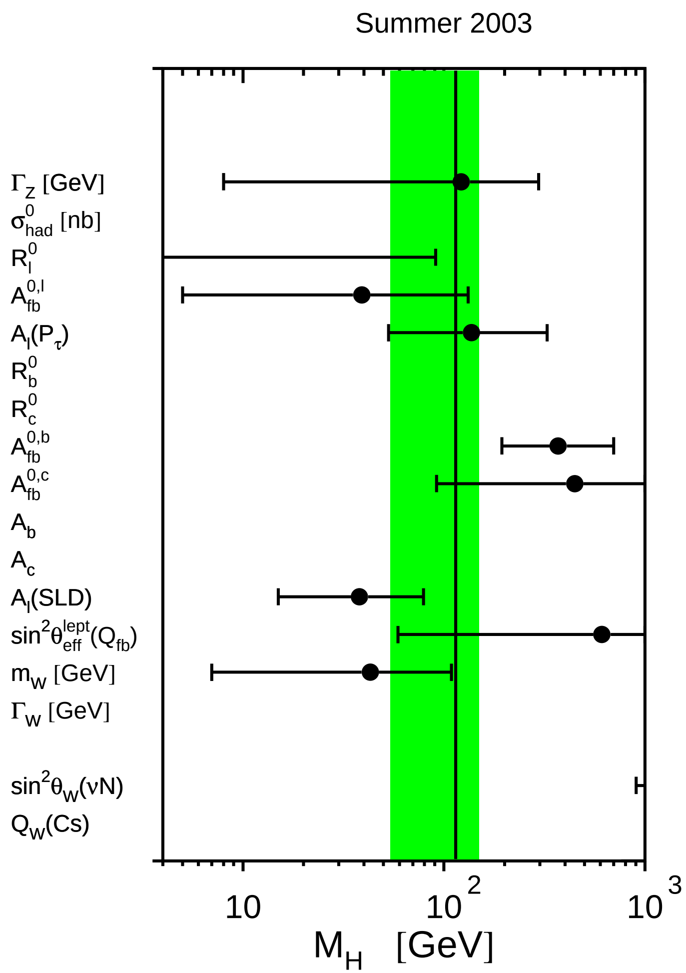
* hep-ph/0311038 (JHEP 0402:032 2004) with R. Casalbuoni and M. Oertel

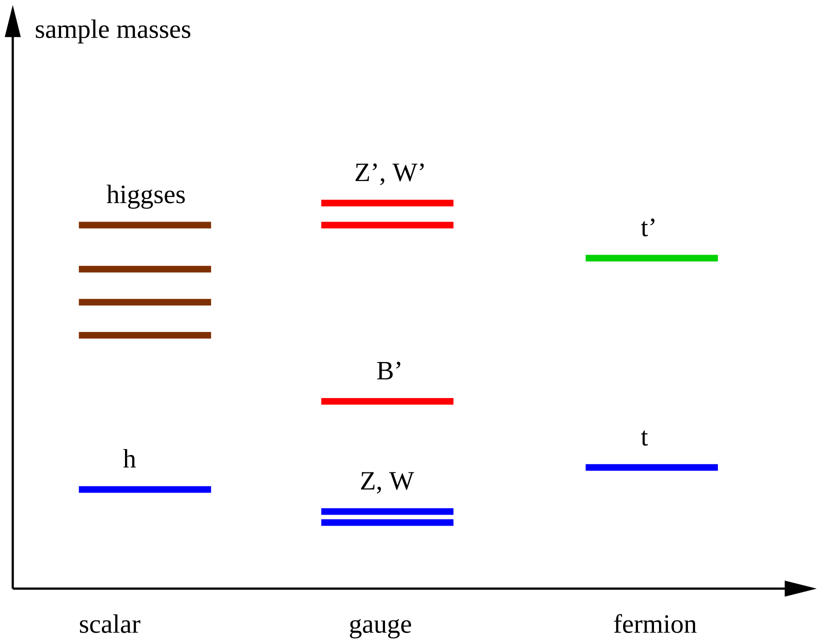
Introduction





Precision EW data indicate SM with a light Higgs $m_H < 219$ GeV at 95% CL (`lepewwg.web.cern.ch/LEPEWWG`)

Littlest Higgs*



Model parameters

$\tan\theta = s/c = g_2/g_1$, $\tan\theta' = s'/c' = g'_2/g'_1$ new $SU(2)$ and $U(1)$ mixing

f symmetry breaking scale $\mathcal{O}$ TeV $\quad$ v' scalar triplet vev

m_H Higgs mass $\quad$ m_T heavy vector top mass

* Arkani-Hamed et al. hep-ph/0206021

The model is based on $SU(5) \to SO(5)$ global symmetry breaking (24-10=14 Goldstone bosons) by a vev of the order f

$$\langle \Sigma \rangle = \begin{pmatrix} 0 & 0 & \mathbb{1}_2 \\ 0 & 1 & 0 \\ \mathbb{1}_2 & 0 & 0 \end{pmatrix} .$$

4 are eaten by the gauge bosons of the broken gauge group. The Goldstone boson matrix is

$$\Pi = \begin{pmatrix} 0 & h^\dagger/\sqrt{2} & \phi^\dagger \\ h/\sqrt{2} & 0 & h^*/\sqrt{2} \\ \phi & h^T/\sqrt{2} & 0 \end{pmatrix} .$$

h transforms as a doublet and ϕ as a triplet. The gauge symmetry breaking is $[SU(2) \times U(1)]^2 \to SU(2) \times U(1)$.

The scalar sigma model field can be written as

$$\Sigma = e^{i\Pi/f} \langle \Sigma \rangle e^{i\Pi^T/f} = e^{2i\Pi/f} \langle \Sigma \rangle$$

The kinetic term for the scalar fields is given by

$$\mathcal{L}_{kin} = \frac{1}{2} \frac{f^2}{4} \mathrm{Tr}[D_\mu \Sigma D^\mu \Sigma] \ ,$$

with the covariant derivative defined as

$$D_\mu \Sigma = \partial_\mu \Sigma - i(A_\mu \Sigma + \Sigma A_\mu^T) \ .$$

A_μ is the gauge boson matrix:

$$A_\mu = g_1 W_\mu^{1a} Q_1^a + g_2 W_\mu^{2a} Q_2^a + g_1' B_\mu^1 Y_1 + g_2' B_\mu^2 Y_2 \ ,$$

Q_i^a are the generators of the two $SU(2)$, Y_i those of the two $U(1)$ groups.

After symmetry breaking the gauge boson matrix can be diagonalized by the following transformations:

$$\begin{aligned} W &= sW_1 + cW_2 \qquad W' = -cW_1 + sW_2 \\ B &= s'B_1 + c'B_2 \qquad B' = -c'B_1 + s'B_2 \ . \end{aligned}$$

s, c, s', and c' denote the sines and cosines of two mixing angles, respectively. They can be expressed with the help of the coupling constants:

$$\begin{aligned} c' &= g'/g'_2 \qquad\qquad s' = g'/g'_1 \\ c &= g/g_2 \qquad\qquad s = g/g_1 \ , \end{aligned}$$

with the usual SM couplings g, g', related to g_1, g_2, g'_1 and g'_2 by

$$\frac{1}{g^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2}, \qquad \frac{1}{{g'}^2} = \frac{1}{{g'_1}^2} + \frac{1}{{g'_2}^2} \ .$$

Up to the order v^2/f^2 the equations of motion give :

$$
\begin{aligned}
W'^{\pm\mu} &= g\, x_W\, W^{\pm\mu} + \frac{x_W^F}{\sqrt{2}} \left(J^{\pm\mu} - (1 - c_L) J_3^{\pm\mu} \right) \\
W'^{3\mu} &= y_W (g\, W^{3\mu} + g'\, B^\mu) + x_W^F \left(J^{0\mu} - s_L^2 \bar{t}_L \gamma^\mu t_L \right) \\
B'^\mu &= x_B (g\, W^{3\mu} + g'\, B^\mu) + x_B^F \Bigg[(J_{em}^\mu + J^{0\mu}) - \frac{5c'^2}{2(3c'^2 - 2s'^2)} s_L^2 \bar{t}_L \gamma^\mu t_L \\
&- \frac{1}{3c'^2 - 2s'^2} s_R^2 \bar{t}_R \gamma^\mu t_R \Bigg] ,
\end{aligned}
$$

where :

$$
\begin{aligned}
x_W^F &= -\frac{4c^3 s}{g f^2} \qquad\qquad x_B^F = \frac{4c' s'}{g' f^2} (3c'^2 - 2s'^2) \\
x_W &= y_W = \frac{cs}{2g} (c^2 - s^2) \frac{v^2}{f^2} \\
x_B &= \frac{2c' s'}{g'} (c'^2 - s'^2) \frac{v^2}{f^2} .
\end{aligned}
$$

In terms of the model parameters we obtain:

$$\frac{G_F}{\sqrt{2}} = \frac{\alpha\pi(g^2+g'^2)}{2g^2g'^2m_Z^2}\left(1 - c^2(c^2-s^2)\frac{v^2}{f^2} + 2c^4\frac{v^2}{f^2} - \frac{5}{4}(c'^2-s'^2)^2\frac{v^2}{f^2}\right)$$

and defining the Weinberg angle as

$$\frac{G_F}{\sqrt{2}} = \frac{\alpha\pi}{2s_\theta^2c_\theta^2m_Z^2} \ .$$

we have

$$\begin{aligned} m_Z^2 &= (g^2+g'^2)\frac{v^2}{4}\left[1 - \frac{v^2}{f^2}\left(\frac{1}{6} + \frac{(c^2-s^2)^2}{4} + \frac{5}{4}(c'^2-s'^2)\right) + 8\frac{v'^2}{v^2}\right] \ , \\ m_W^2 &= \frac{g^2v^2}{4}\left[1 - \frac{v^2}{f^2}\left(\frac{1}{6} + \frac{(c^2-s^2)^2}{4}\right) + 4\frac{v'^2}{v^2}\right] \ . \end{aligned}$$

The ϵ_i parameters are calculated using the effective low energy theory by integrating out the heavy states. To the order v^2/f^2 we get:

$$\begin{aligned}
\epsilon_1 &= -\frac{v^2}{f^2}\left(\frac{5}{4}(c'^2 - s'^2)^2 + \frac{4}{5}(c'^2 - s'^2)(3c'^2 - 2s'^2) + 2c^4\right) + 4\frac{v'^2}{v^2} \\
\epsilon_2 &= -2c^4\frac{v^2}{f^2} \\
\epsilon_3 &= -\frac{v^2}{f^2}\left(\frac{1}{2}c^2(c^2 - s^2) + \frac{2}{5}(c'^2 - s'^2)(3c'^2 - 2s'^2)\frac{c_\theta^2}{s_\theta^2}\right)
\end{aligned}$$

where s, c, s', and c' denote the sines and cosines of two mixing angles.

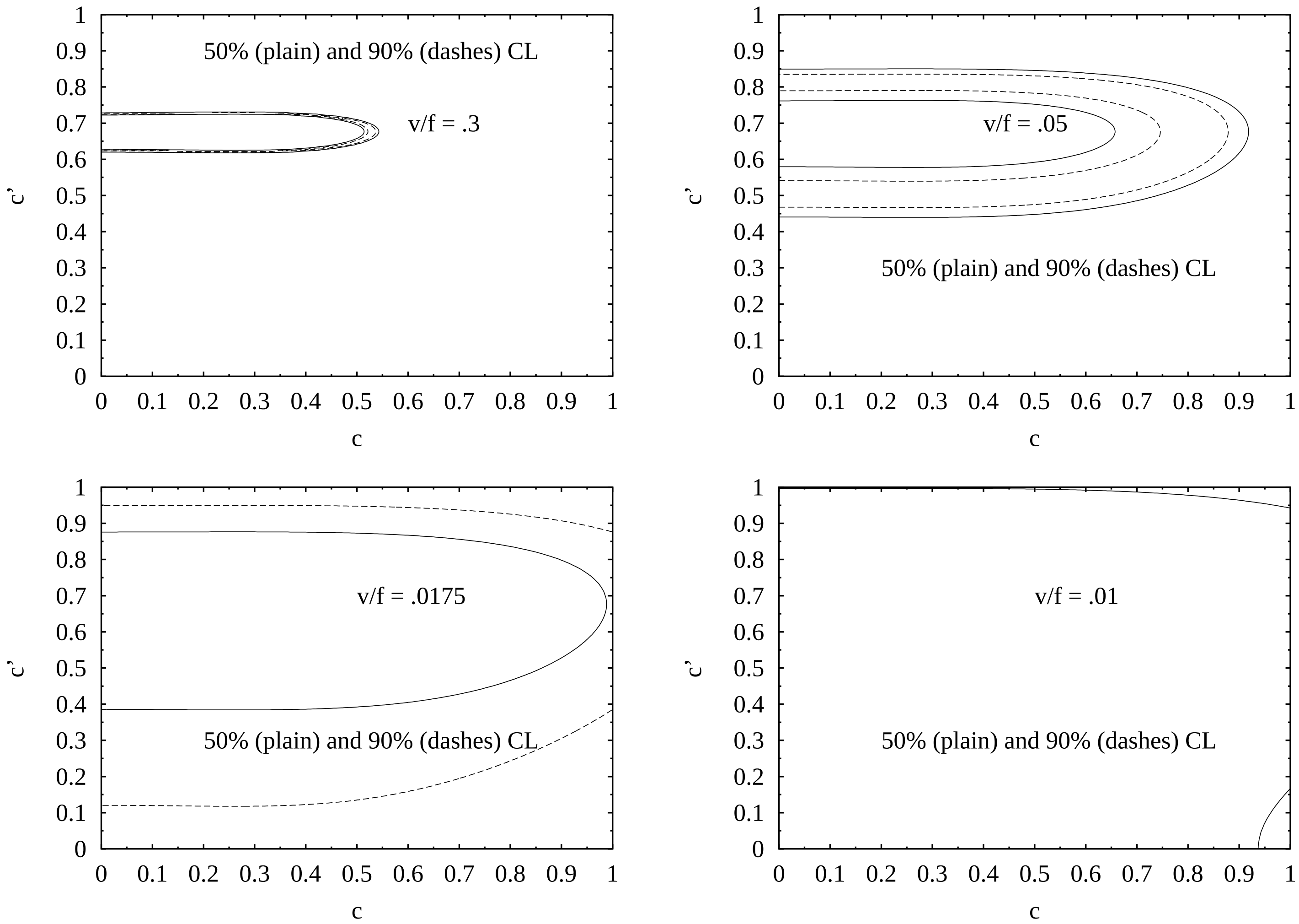
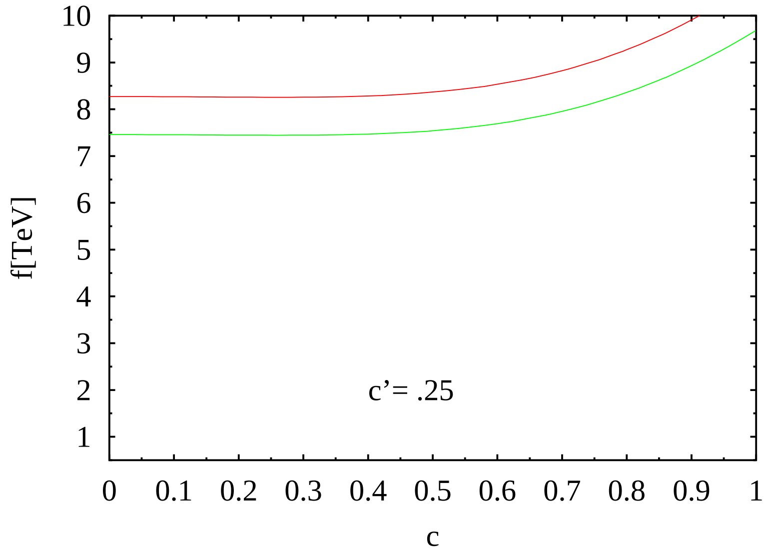


Figure 1: *90% and 50% CL exclusion contours in the plane c-c'. The value of the triplet vev v' is fixed to $v'^2/v^2 = v^2/(17f^2)$. The allowed region lies inside the 90% and 50% bands, respectively. From hep-ph/0311038.*



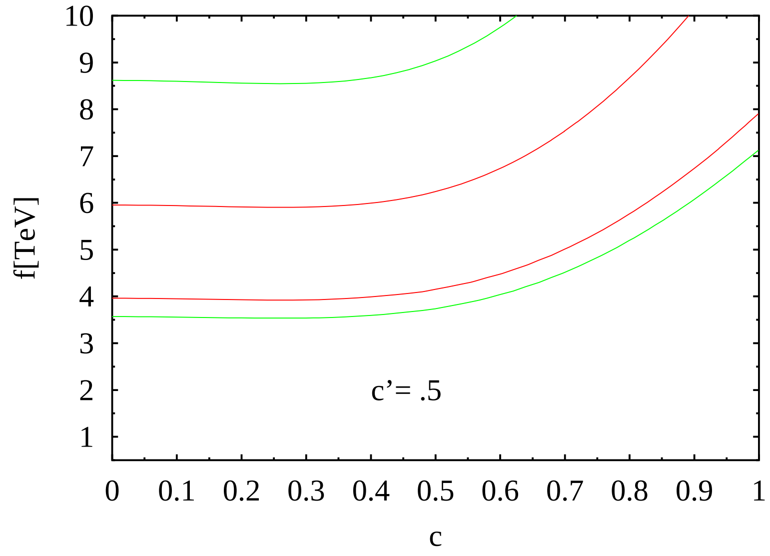


Figure 2: *90% and 50% CL exclusion contours in the plane f-c for two values of the cosine of the other mixing angle c' in the littlest Higgs model. The value of the triplet vev v' is chosen using $v'/v = v/(4f)$. Other choices of v' do not change much the above conclusions. The allowed region lies above the bands for the left figure and inside the bands for the right one (90% CL the narrowest (in red) and 50% CL the largest (in green)).*

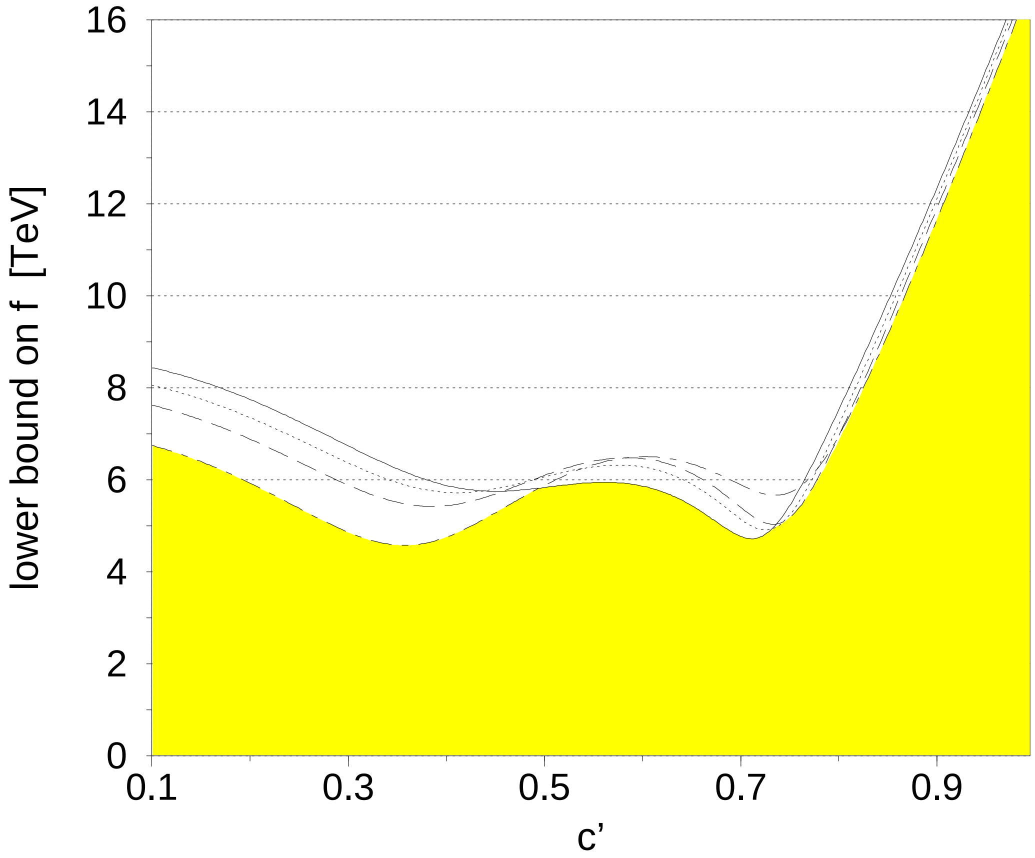


Figure 3: *The region below the contours is excluded to 95% C.L. for c equal to 0.1 (solid), 0.5 (dotted), 0.7 (dashed), 0.99 (dot-dashed). The yellow region is excluded for any choice of c. From hep-ph/0305157 based on hep-ph/0211124, hep-ph/0303236 by Csáki et al.*

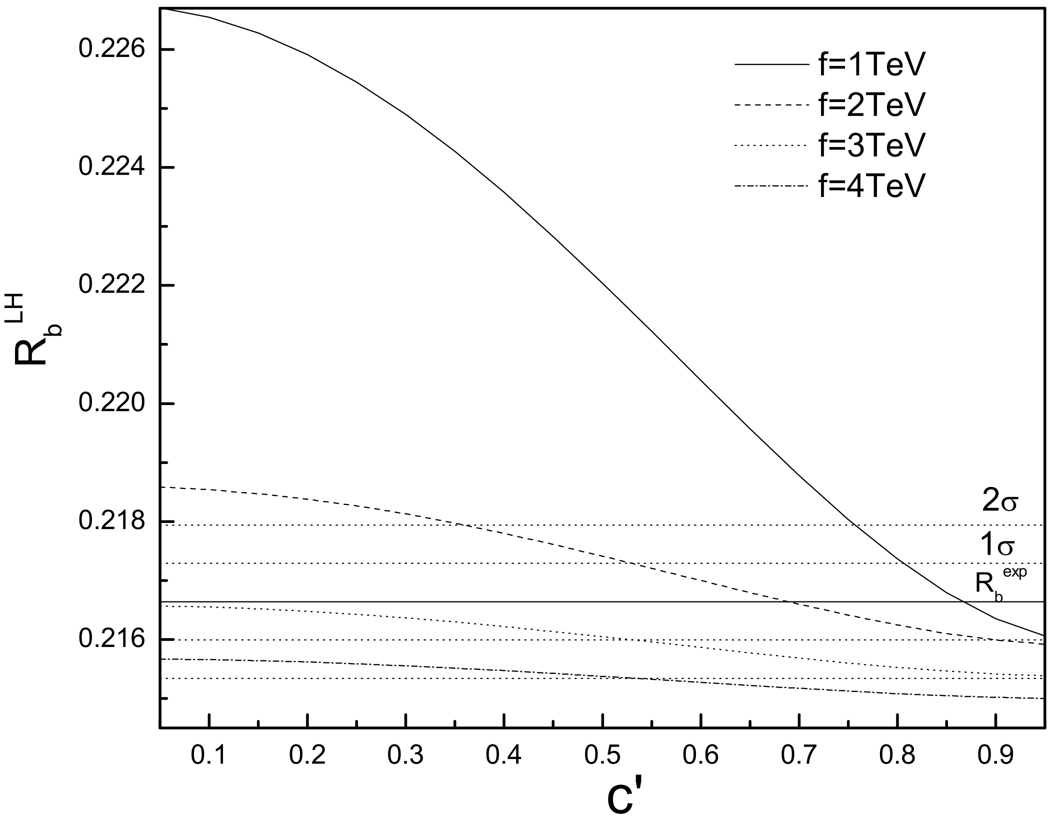


Figure 4: *The predicted value of* R_b^{LH} *in the LH model as a function of the mixing parameter* c' *for four values of the scale parameter* f*. From hep-ph/0401214 by Yue and Wang.*

Little Higgs with custodial $SU(2)^*$

The model is based on a $SO(9)/[SO(5) \times SO(4)]$ coset space, with $SU(2)_L \times SU(2)_R \times SU(2) \times U(1)$ subgroup of $SO(9)$ gauged. The vev is

$$\langle \Sigma \rangle = \begin{pmatrix} 0 & 0 & \mathbb{1}_4 \\ 0 & 1 & 0 \\ \mathbb{1}_4 & 0 & 0 \end{pmatrix}$$

breaking the $SO(9)$ global symmetry down to an $SO(5) \times SO(4)$ subgroup. This coset space has $20 = (36 - 10 - 6)$ light scalars. Of these 20 scalars, 6 will be eaten in the higgsing of the gauge groups down to $SU(2)_W \times U(1)_Y$. The remaining 14 scalars are : a single higgs doublet h, an electroweak singlet ϕ^0, and three triplets ϕ^{ab}.

* S.Chang hep-ph/0306034

The equations of motion up to the order v^2/f^2 are

$$
\begin{aligned}
W'^{\,1,2} &= -\frac{v^2\,cs}{2f^2}\left(c^2-s^2\right)\,W^{1,2} + \frac{s^3c}{f^2\,g}\,J^{1,2} \\
W'^{\,3} &= -\frac{v^2\,cs}{2f^2}\left(c^2-s^2\right)\,(W^3 - \frac{g'}{g}\,B) + \frac{s^3c}{f^2\,g}\,J^3 \\
B' &= \frac{v^2\,c's'}{2f^2}\left(c'^{\,2}-s'^{\,2}\right)(\frac{g}{g'}\,W^3 - B) + \frac{s'^{\,3}c'}{f^2\,g'}\,J^0 \\
W_R^{1,2} &= \frac{v^2}{2f^2}\,W^{1,2}\,.
\end{aligned}
$$

The expression for G_F in terms of the model parameters is

$$
\frac{G_F}{\sqrt{2}} = \frac{\alpha\pi(g^2+g'^2)^2}{2g^2g'^2}\left(1+\frac{v^2}{f^2}\frac{s^2(c^2-s^2)-s^4}{2}\right)\,,
$$

In this case the masses of Z- and W-bosons are given by

$$
m_Z^2 = (g^2+g'^2)\frac{v^2}{4}
$$

$$m_W^2 \quad = \quad \frac{g^2 v^2}{4}\left(1 + 2\frac{v'^2}{v^2}\right) \ .$$

The corrections to the ϵ parameters to the order v^2/f^2 are

$$\begin{aligned}
\epsilon_1 &= \frac{v^2}{4f^2}\left[4s'^{\,2}\left(c'^{\,2} - s'^{\,2}\right) + 2c^2 s^2 - s^4\right] + 2\frac{v'^2}{v^2} \\
\epsilon_2 &= \frac{v^2}{4c_{2\theta}\, f^2}\Big[4s'^{\,2}\left(c'^{\,2} - s'^{\,2}\right) c_\theta^2 c_{2\theta} + 2s^2\left(c^2 - s^2\right)\left(c_\theta^4 - 3c_\theta^2 s_\theta^2 + 2c_\theta^2 - s_\theta^2\right) \\
&+ s^4 (c_\theta^4 + s_\theta^4)\Big] \\
\epsilon_3 &= \frac{v^2}{2s_\theta^2\, f^2}\left[s^2\left(c^2 - s^2\right)\left(-c_{2\theta} + 2s_\theta^2 c_\theta^2\right) - s^4 c_\theta^2 s_\theta^2\right]
\end{aligned}$$

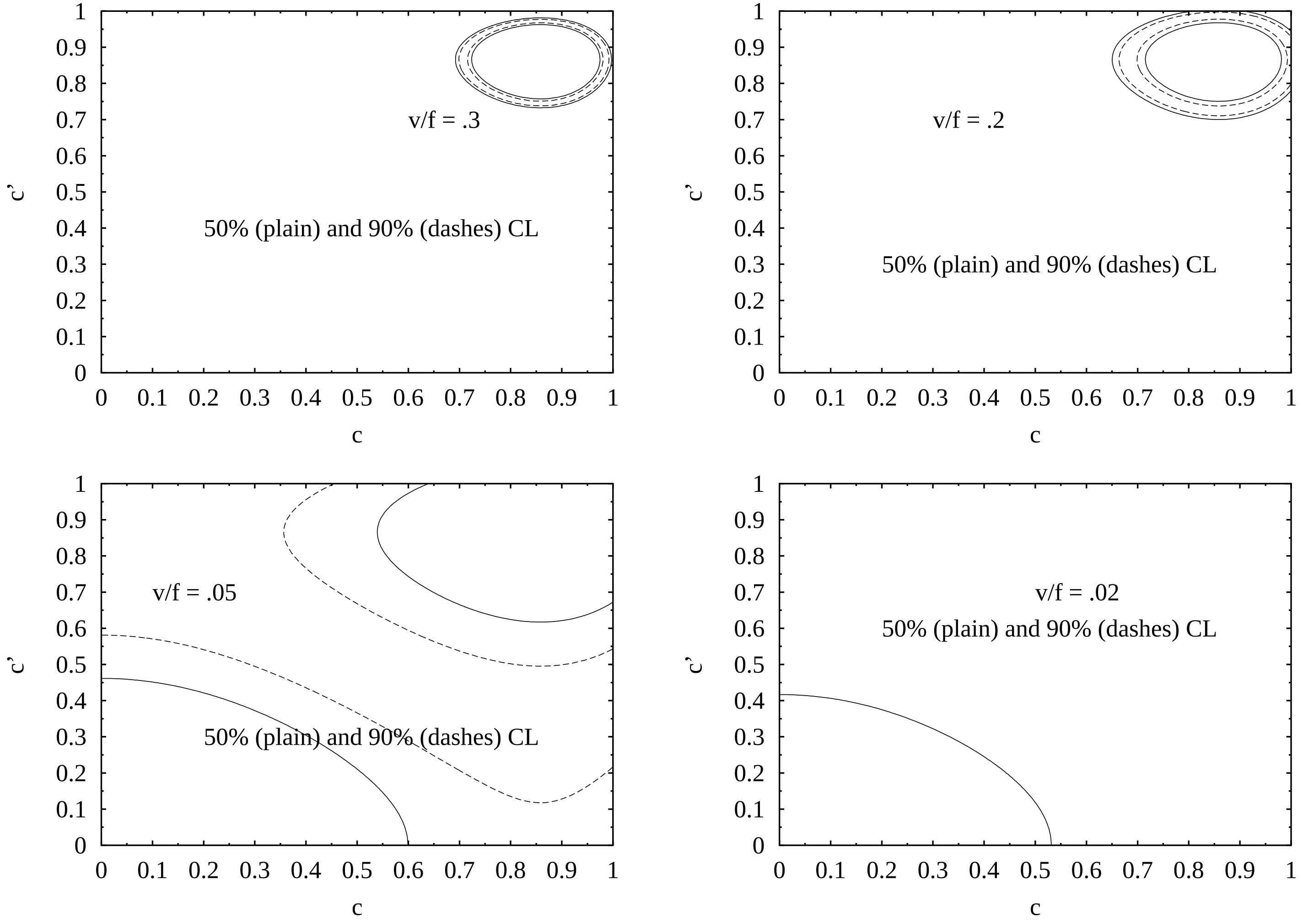


Figure 5: *90% and 50% CL exclusion contours in the plane c-c' of the $SO(9)/[SO(5) \times SO(4)]$ model. The value of the triplet vev v' is fixed to $v'^2/v^2 = v^2/(17f^2)$. The allowed region lies inside the 90% and 50% bands, respectively.*

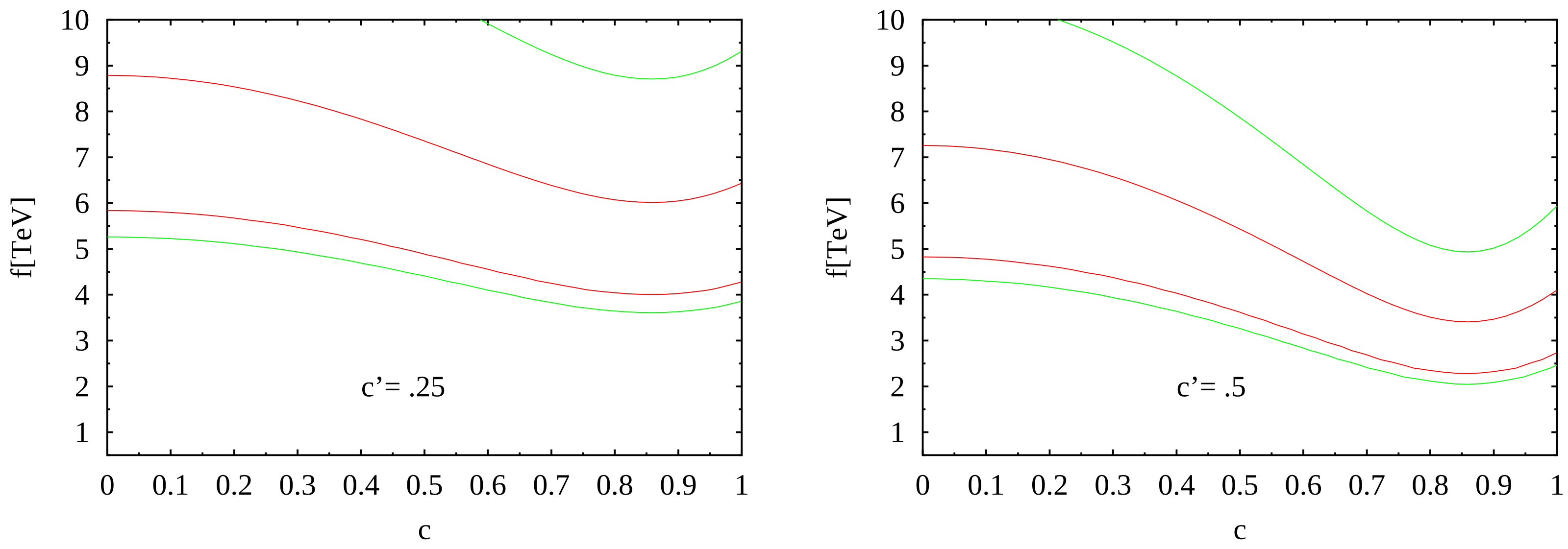


Figure 6: *90% and 50% CL exclusion contours in the plane f-c for two values of the cosine of the other mixing angle c' in the $SO(9)/[SO(5) \times SO(4)]$ model. The value of the triplet vev v' is chosen using $v'/v = v/(4f)$. Other choices of v' do not change much the above conclusions. The allowed region lies inside the bands (90% CL the narrowest (in red) and 50% CL the largest (in green)).*

$g-2$ of the muon

The relevant one-loop Feynman diagrams are

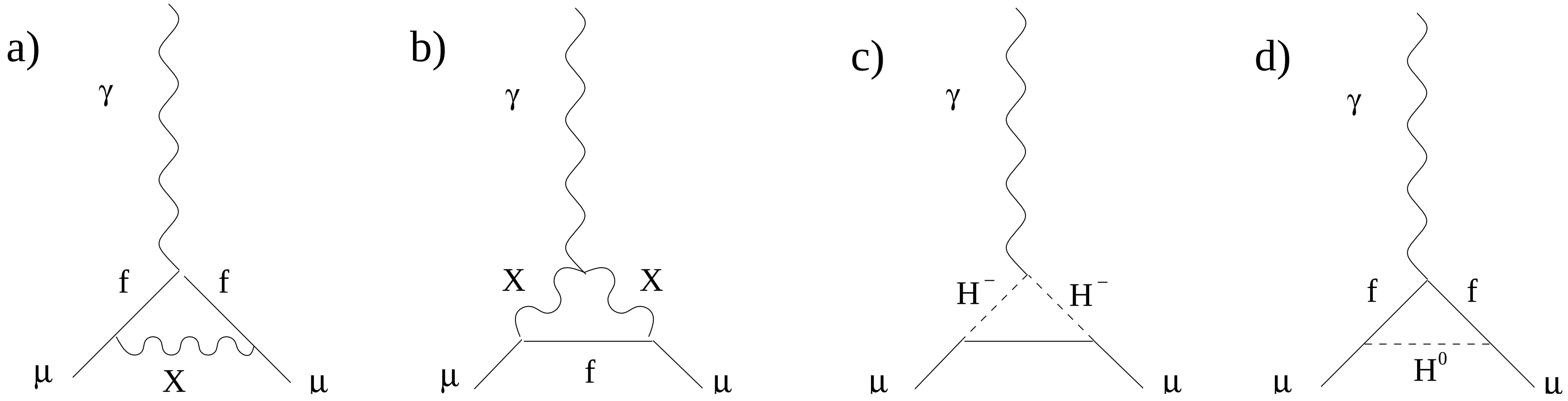


Figure 7: *Loop graphs contributing to the weak correction to Δg. a) and b) correspond to the exchange of a vector boson X while c) and d) are the Higgs sector contributions.*

The difference between experiment and the standard model prediction for a_μ is

$$\delta a_\mu = a_\mu^{exp} - a_\mu^{\mathrm{SM}} = 17(18) \times 10^{(-10)} \ .$$

The numerical results within the littlest Higgs model are relatively insensitive to the choice of parameter values of the model. We obtain a difference from the standard model value of at most $\delta a_\mu = a_\mu^{\mathrm{LH}} - a_\mu^{\mathrm{SM}}$ of the order of 1×10^{-10}. The contributions of the additional heavy particles are thereby completely negligible and the dominant contributions arise from the corrections to the light Z and W couplings. Similar results are obtained in the custodial model.

Weak charge of cesium atoms

At low energy, parity violation in atoms is due to the electron-quark effective Lagrangian

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}}(\bar{e}\gamma_\mu\gamma_5 e)(C_{1u}\bar{u}\gamma^\mu u + C_{1d}\bar{d}\gamma^\mu d) \ .$$

The experimentally measured quantity is the so-called “weak charge” defined as

$$Q_W = -2\left(C_{1u}(2Z+N) + C_{1d}(Z+2N)\right) \ ,$$

where Z, N are the number of protons and neutrons of the atom, respectively.

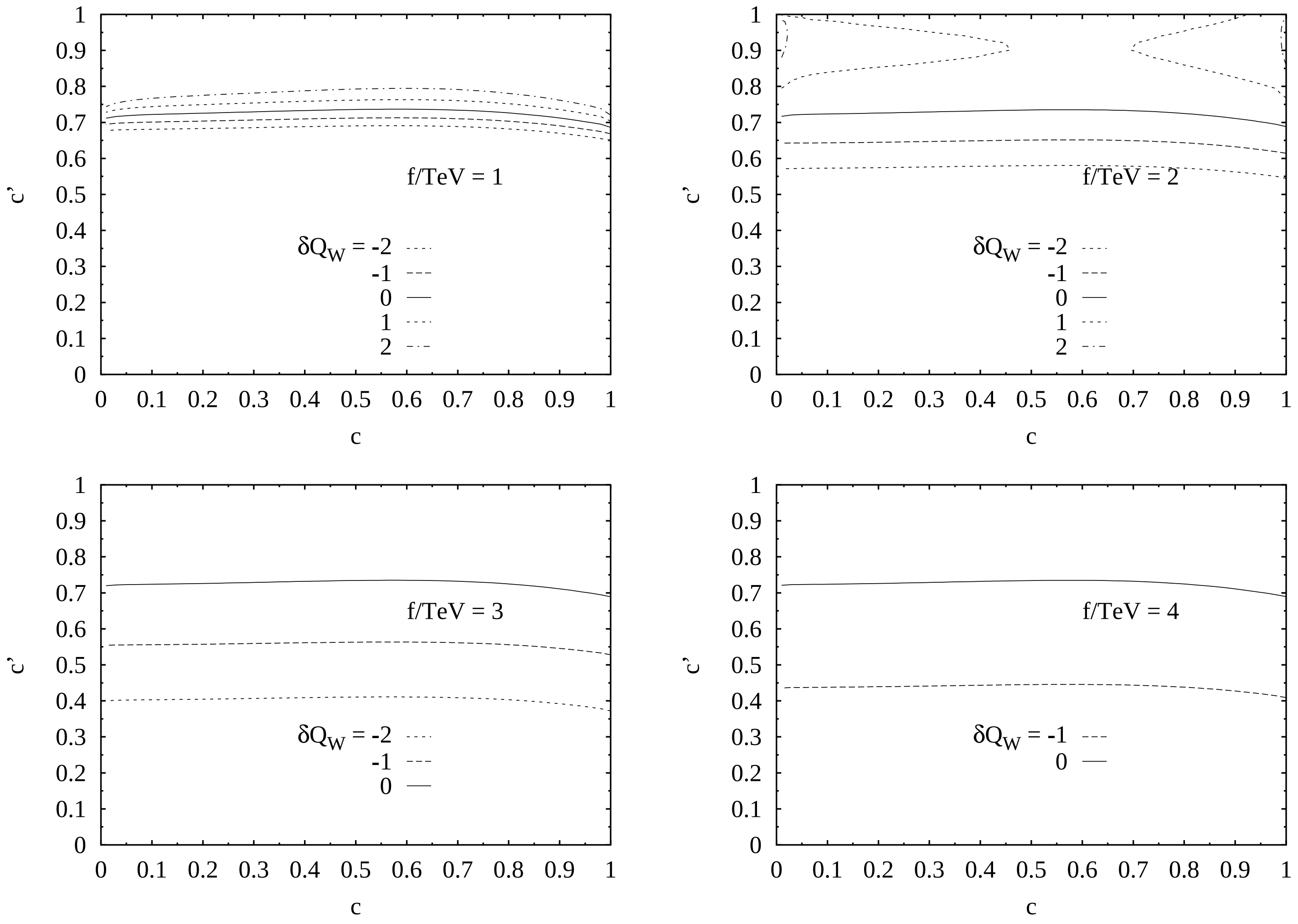


Figure 8: *Corrections to the weak charge of cesium atoms as a function of c and c' in the littlest Higgs model.*

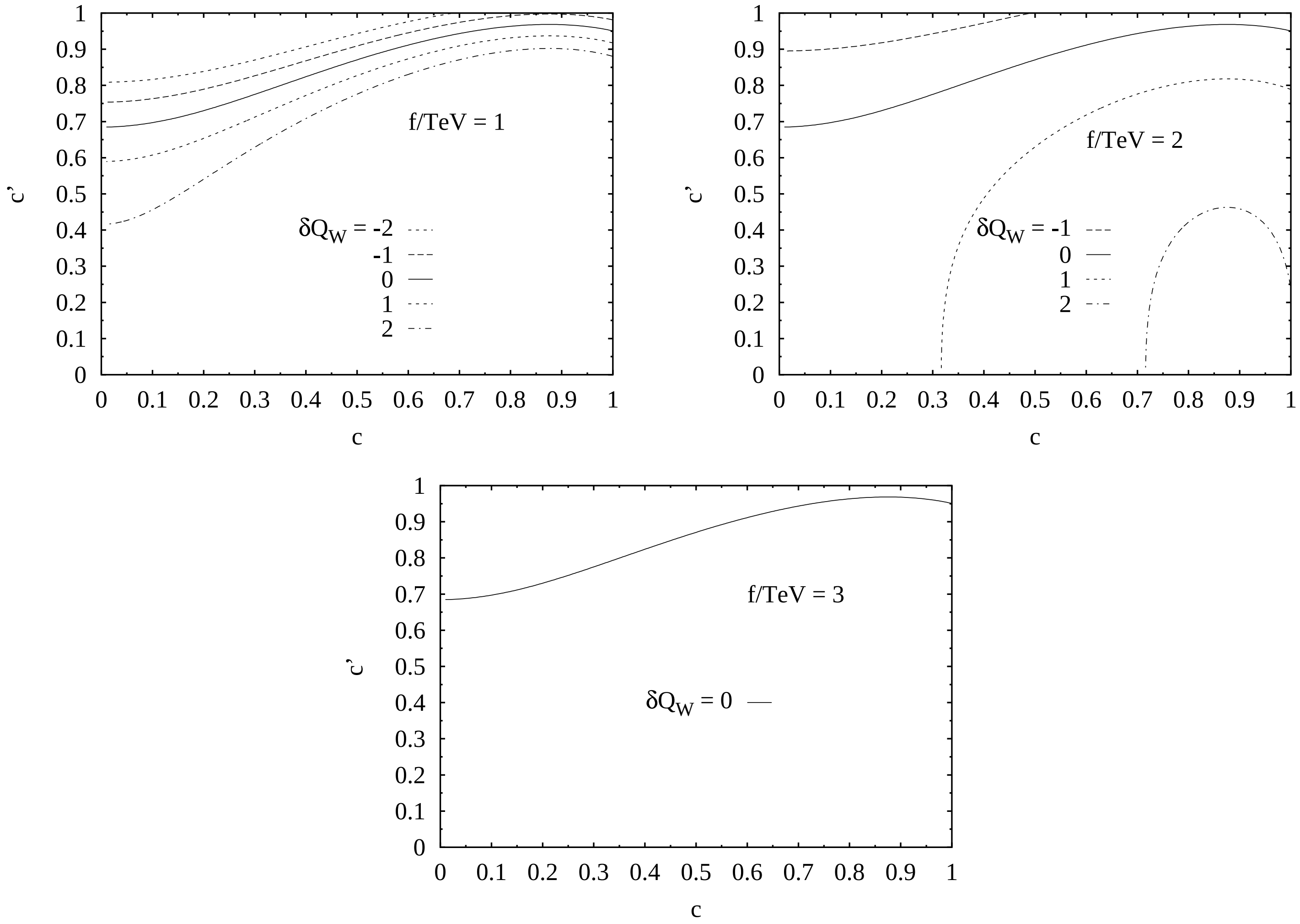


Figure 9: *Corrections to the weak charge of cesium atoms as a function of c and c' in the little Higgs model with approximate custodial symmetry.*

Conclusions

In the model without custodial symmetry a considerable fine tuning is necessary in order to satisfy the constraints imposed by LEP data. This problem is to a some extent avoided for the model with approximate custodial symmetry.

Low energy precision data does not change the above conclusions. For $g-2$ of the muon the corrections are too small. The weak charge does not allow for establishing new constraints either, even if the corrections are not negligible.