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CP Violation in 5-Dimensional QED

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1. 5-Dimensional QED on a Circle and CP Violation.
2. One-loop Effective Potential for $\langle A_{4\,0} \rangle$.
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B.G, and Jóse Wudka, hep-ph/0401232

1. 5-Dimensional QED on a Circle and CP violation

- U(1) model in 5D, with coordinates x^M, $M = 0, \ldots, 4$ and $x^4 = y$ compactified on a circle of radius R.

- Two charged fermionic fields ($\psi_{1,2}$)

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{MN}^2 + \sum_{i=1,2} \bar{\psi}_i \left(i\gamma^M D_M - M_i \right) \psi_i + \mathcal{L}_{gf} \,,$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$, $D_M = \partial_M + ie_5 q_i A_M$, q_i denotes the charge of ψ_i, and $\mathcal{L}_{gf}$ stands for a gauge-fixing term.

Boundary conditions:

$$\begin{aligned} A_M(x^\mu, y+L) &= A_M(x^\mu, y), \\ \psi(x^\mu, y+L) &= e^{i\alpha} \psi(x^\mu, y) \,, \end{aligned}$$

where $L = 2\pi R$.

Symmetries:

- The Lagrangian and the BC are invariant under the gauge U(1) symmetry

$$\begin{aligned} \psi(x,y) &\xrightarrow{\mathrm{U}(1)} e^{-ie_5 q\Lambda(x,y)}\psi(x,y), \\ A_M(x,y) &\xrightarrow{\mathrm{U}(1)} A_M(x,y)+\partial_M\Lambda(x,y)\,. \end{aligned}$$

- The action is symmetric under the 5D CP transformations:

$$\begin{array}{llll} x^{0,4} \xrightarrow{\mathrm{CP}} +x^{0,4}\,, & & x^{1,2,3} \xrightarrow{\mathrm{CP}} -x^{1,2,3} \\ A^{0,4} \xrightarrow{\mathrm{CP}} -A^{0,4}\,, & & A^{1,2,3} \xrightarrow{\mathrm{CP}} +A^{1,2,3} \\ \psi \xrightarrow{\mathrm{CP}} \eta\gamma^0\gamma^2\psi^\star\,, & & |\eta|=1\,, \end{array}$$

KK expansion:

$$\begin{aligned} A_M(x,y) &= \frac{1}{\sqrt{L}}\sum_{n=-\infty}^{\infty} A_{M\,n}(x)\,e^{i\omega_n y} \qquad \text{for} \qquad \omega_n = 2\pi n/L \\ \psi(x,y) &= \frac{1}{\sqrt{L}}\sum_{n=-\infty}^{\infty} \psi_n(x)\,e^{i\bar{\omega}_n y} \qquad \text{for} \qquad \bar{\omega}_n = \omega_n + \alpha/L\,. \end{aligned}$$

- $A_{4\,n}$ become 4D scalars & $A_{4\,n} \xrightarrow{\text{CP}} -A_{4\,n}$.
- $\langle A_{4\,n}\rangle \neq 0 \quad \Longrightarrow \quad$ possibility of spontaneous violation of CP.
- The BCs are not symmetric under CP (unless $\alpha = 0, \pm\pi$), and this is an *additional* source of explicit CP violation, present even if $\langle A_4\rangle = 0$.

<u>Fermionic kinetic terms</u>

$$\mathcal{L}_\psi = \sum_n \bar{\psi}_n \left[i\gamma^\mu \partial_\mu - M + i\gamma_5 \mu_n\right] \psi_n \,,$$

where $\mu_n \equiv \left[2\pi n + (\alpha + eqLa)\right]/L$, with $e \equiv e_5/\sqrt{L}$ the 4D gauge coupling.

A chiral rotation to diagonalize the fermion mass:

$$\psi_n \to e^{i\gamma_5 \theta_n} \psi_n \,, \qquad \tan(2\theta_n) = \frac{\mu_n}{M}; \quad |\theta_n| \leq \pi/4$$

The physical fermion masses: $m_n = \sqrt{M^2 + \mu_n^2}$.

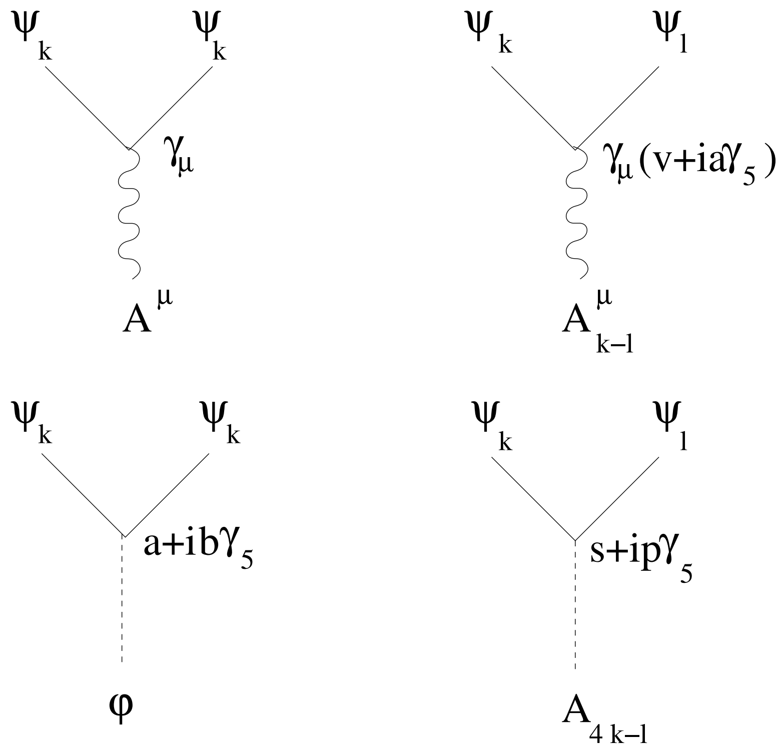
Interactions

$$\mathcal{L}_{A\psi} = -eq \left\{ A_\mu \sum_k \bar{\psi}_k \gamma^\mu \psi_k + \sum_{k\neq l} A_{\mu\ k-l} \bar{\psi}_k \Gamma^{(v)}_{kl} \gamma^\mu \psi_l \right\} ,$$

$$\mathcal{L}_{\varphi\psi} = -eq \left\{ \varphi \sum_k \bar{\psi}_k \Gamma^{(\varphi)}_k \psi_k + \sum_{k\neq l} A_{4\ k-l} \bar{\psi}_k \Gamma^{(s)}_{kl} \psi_l \right\} ,$$

where $\varphi \equiv A_{4\,0}$, $A_\mu \equiv A_{\mu\ 0}$ and

$$\begin{aligned}
\Gamma^{(\varphi)}_k &\equiv \sin(2\theta_k) - i\gamma_5 \cos(2\theta_k) \, , \\
\Gamma^{(s)}_{kl} &\equiv \sin(\theta_k + \theta_l) - i\gamma_5 \cos(\theta_k + \theta_l) \, , \\
\Gamma^{(v)}_{kl} &\equiv \cos(\theta_k - \theta_l) + i\gamma_5 \sin(\theta_k - \theta_l) \, .
\end{aligned}$$



$$
\begin{aligned}
\mathcal{L}_{A\psi} &= -eq\left\{A_\mu \sum_k \bar{\psi}_k \gamma^\mu \psi_k + \sum_{k\neq l} A_{\mu\ k-l}\bar{\psi}_k \Gamma^{(v)}_{kl}\gamma^\mu \psi_l\right\}\ , \\
\mathcal{L}_{\varphi\psi} &= -eq\left\{\varphi \sum_k \bar{\psi}_k \Gamma^{(\varphi)}_k \psi_k + \sum_{k\neq l} A_{4\ k-l}\bar{\psi}_k \Gamma^{(s)}_{kl}\psi_l\right\}\ ,
\end{aligned}
$$

- $A_\mu(x) \equiv A_{\mu\,0}(x)$ is the 4D photon.
- $\varphi(x) \equiv A_{4\,0}(x)$ *is a new, physical,* <u>*tree-level massless*</u>*, 4D degree of freedom, Yukawa couplings of which appear to be CP-violating.*

$$A_M(x,y) \quad \xrightarrow{\mathrm{U}(1)} \quad A_M(x,y) + \partial_M \Lambda(x,y)$$

$$\Lambda(x,y) \quad = \quad \frac{1}{\sqrt{L}} \sum_{n=-\infty}^{+\infty} \Lambda_n(x) e^{i\omega_n y}$$

$$\Downarrow$$

$$A_{4\,k} \xrightarrow{\mathrm{U}(1)} A_{4\,k} + i\omega_k \Lambda_k \qquad \text{with} \qquad \omega_k = \frac{2\pi k}{L}$$

$$\Downarrow$$

$$\varphi \xrightarrow{\mathrm{U}(1)} \varphi$$

$$\Downarrow$$

The gauge symmetry does not protect $m_\varphi = 0$!!!

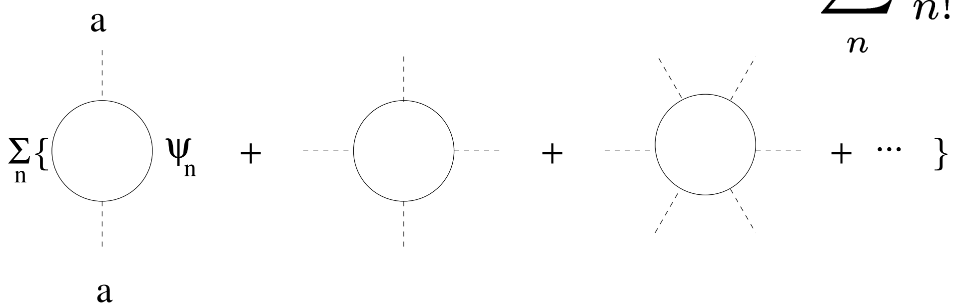
$$\Downarrow$$

φ it will receive calculable finite mass at higher orders.

2. One-loop Effective Potential for $a \equiv \langle A_{4\,0} \rangle$

SSB occurs if the quantum field φ develops a non-zero vacuum expectation value:

$$\frac{dV_{\rm eff}(a)}{da} = 0 \;\; \text{for} \;\; a \neq 0 \qquad\qquad \text{for} \qquad\qquad V_{\rm eff}(a) = -\sum_n \frac{1}{n!}\Gamma^{(n)}(0)a^n ,$$



- Dimensional regularization for the d^4p integral.
- Summation over the infinite tower of KK modes.

After dropping an irrelevant constant contribution:

$$V(M;E) = \frac{1}{2\pi^2 L^4}\left[x^2 Li_3(re^{-x}) + 3xLi_4(re^{-x}) + 3Li_5(re^{-x}) + \text{H.c.}\right] ,$$

$$x \equiv LM, \qquad E = (\alpha + eqLa)/L, \qquad r = \exp(iLE) \, .$$

$$V_{\rm eff}(a) = \sum_{i=1,2} V(M_i;E_i) \, .$$

One fermion field:

The minimum of V is at $\alpha + eqLa = \pi$.

- Eliminate α from the BC by the following field redefinition:

$$\psi'(x,y) = e^{-i\alpha y/L}\psi(x,y), \qquad e_5 q A'_M(x,y) = e_5 q A_M(x,y) + \alpha\delta_{M,4}\,,$$

  so that ψ' and A' are periodic in y with period L.

- Expand around the vacuum $e_5 q\langle A'_M\rangle = (\pi/L)\delta_{M4}$ by shifting the gauge field $e_5 q A'_M(x,y) = e_5 q A''_M(x,y) + (\pi/L)\delta_{M4}$.

- Again redefine the fermion field, so that the effect of this shift disappears from the Lagrangian density:

$$\chi(x,y) = e^{i\pi y/R}\psi'(x,y)\,,$$

  which is anti-periodic in y, $\chi(x, y+L) = -\chi(x,y)$.

$$\Downarrow$$

$$\langle A''_4\rangle = 0 \qquad \text{(no spontaneous CP violation)}$$

$$\chi(x, y+L) = -\chi(x,y) \qquad \text{(no explicit CP violation)}$$

A puzzle:

$$
\begin{aligned}
\mathcal{L}_{A\psi} &= -eq\left\{ A_\mu \sum_k \bar\psi_k \gamma^\mu \psi_k + \sum_{k\neq l} A_{\mu\ k-l} \bar\psi_k \Gamma^{(v)}_{kl} \gamma^\mu \psi_l \right\} , \\
\mathcal{L}_{\varphi\psi} &= -eq\left\{ \varphi \sum_k \bar\psi_k \Gamma^{(\varphi)}_k \psi_k + \sum_{k\neq l} A_{4\ k-l} \bar\psi_k \Gamma^{(s)}_{kl} \psi_l \right\} ,
\end{aligned}
$$

where $\varphi \equiv A_{4\,0}$, $A_\mu \equiv A_{\mu\ 0}$ and

$$
\begin{aligned}
\Gamma^{(\varphi)}_k &\equiv \sin(2\theta_k) - i\gamma_5 \cos(2\theta_k) , \\
\Gamma^{(s)}_{kl} &\equiv \sin(\theta_k + \theta_l) - i\gamma_5 \cos(\theta_k + \theta_l) , \\
\Gamma^{(v)}_{kl} &\equiv \cos(\theta_k - \theta_l) + i\gamma_5 \sin(\theta_k - \theta_l) .
\end{aligned}
$$

At the minimum of $V_{\rm eff}$,

$$
\tan 2\theta_n = \frac{(2n+1)\pi}{LM} .
$$

???

$$\Downarrow$$

$$m_n = m_{-n-1}$$

$$\Downarrow$$

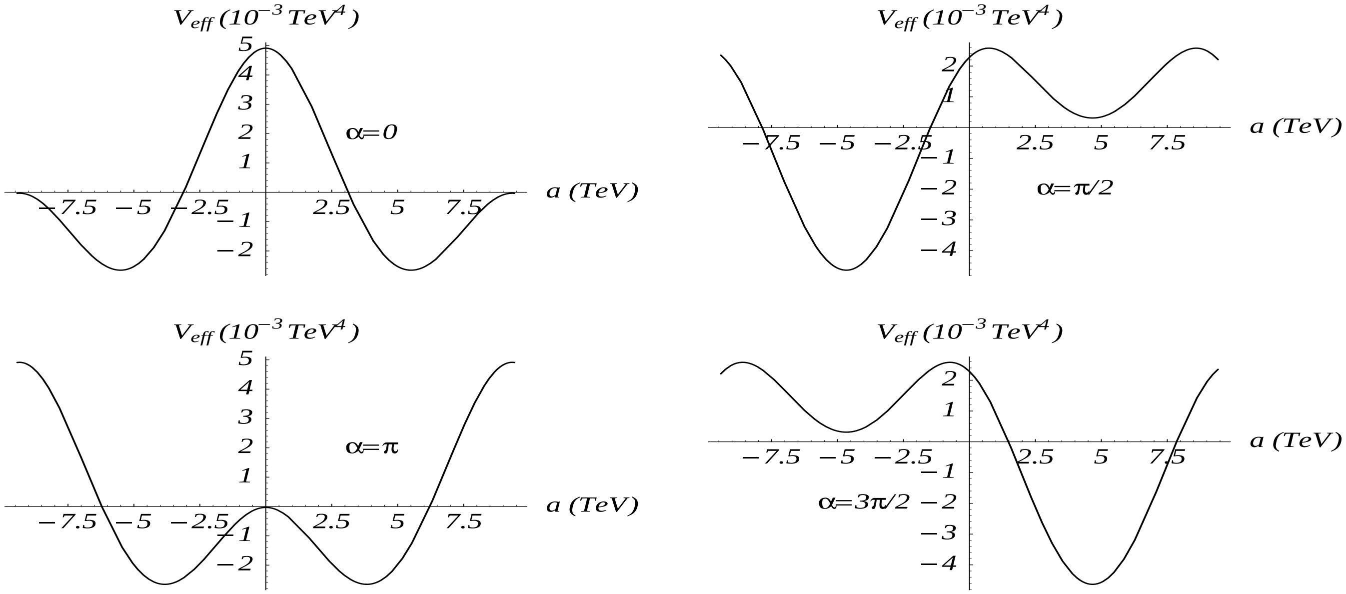
$$\psi_i \xrightarrow{\text{CP}} U_{ij} C \overline{(\gamma_0 \psi_j)}^T \qquad \text{for} \qquad i, j = n, -n-1\,,$$

where $C\gamma_\mu C^{-1} = -\gamma_\mu^T$. Choosing

$$U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\Downarrow$$

the couplings of (ψ_n, ψ_{-n-1}) are invariant under CP.



For two fermions without losing any generality we can adopt the BCs

$$\psi_1(y+L) = \psi_1(y) \qquad \text{and} \qquad \psi_2(y+L) = e^{i\alpha}\psi_2(y)$$

The condition for an extremum

$$\frac{\partial V_{eff}}{\partial a} = e \sum_{i=1,2} q_i \frac{\partial V(M_i;E_i)}{\partial E_i} = 0$$

leads in general to a CP-conserving vacuum.

Summary:

- At least two fermions are necessary to observe CP violation in 5D QED compactified on a circle.
- Both fermions have CP-violating Yukawa couplings to φ, $A_{\mu\ n}$ and $A_{4\ n}$.
- For $\alpha = 0, \pi$ the Lagrangian and the BC are CP symmetric. Since $\varphi \xrightarrow{\mathrm{CP}} -\varphi$, therefore $a \neq 0$ implies *spontaneous CP violation.*
- For $\alpha \neq 0, \pi$ CP is violated explicitly by the BC.

3. The Georgi-Pais Theorem

H. Georgi and A. Pais, "CP violation as a quantum effect", Phys. Rev. D **10**, 1246 (1974).

<u>The theorem:</u>

If

- the symmetry U of the classical potential: $V^{\rm tree}(\varphi) = V^{\rm tree}(U\varphi)$,
- the tree-level vacuum λ consistent with the symmetry: $U\lambda = \lambda$,
- higher order corrections to the potential δV are perturbative:

$$\lim_{\delta V \to 0} \delta\lambda = 0\,,$$

- <u>there is no massless scalars except Goldstone bosons</u>,

then:

- the symmetry of the tree-level vacuum is preserved by the corrected vacuum:

$$U\delta\lambda = \delta\lambda.$$

5D QED:

- 5D gauge invariance leads to $m_{A_4} = 0$,
- "The massless scalar" is the zero mode $\varphi = A_{4\,0}$, it is massless as a consequence of the gauge invariance *and locality* of the 5D theory.
- Since $m_\varphi = 0$, there exists "flat directions" in the tree level-scalar potential.
- It is natural to expect that at higher orders
  - $\langle\varphi\rangle \neq 0$ will be generated, and consequently CP will be broken,
  - $m_\varphi \neq 0$ *and finite.*

4. EDM

$$\langle p'|j^{\mu}_{EM}|p\rangle = -(d/e)\bar{u}(p')\sigma^{\mu\nu}\gamma_5(p'-p)_\nu u(p)$$

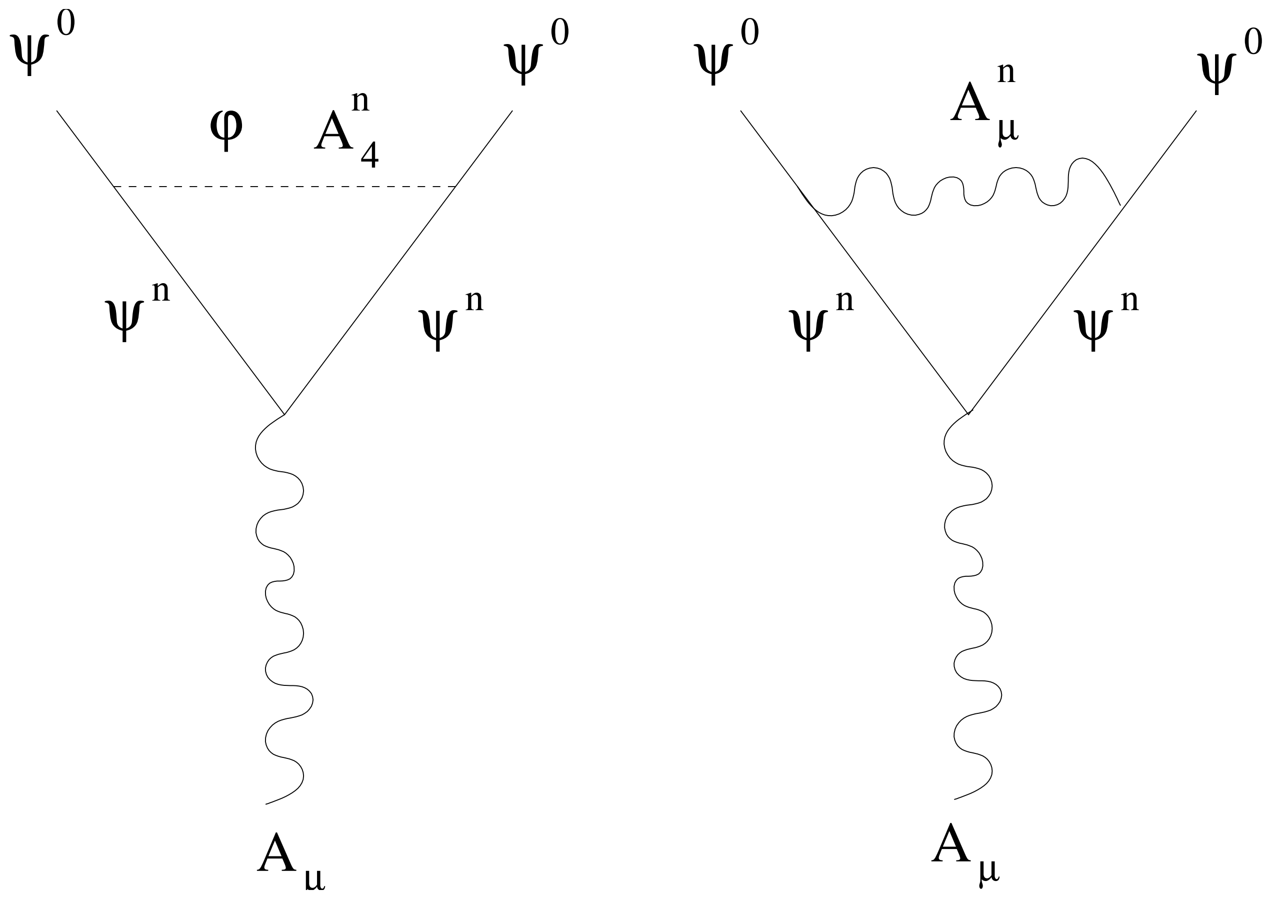
Non-zero EDM is generated already at the "one-loop" level:

$$\begin{aligned}
d_{i\,0} &= -\frac{(eq_i)^3 c^{(+)}_{i,\,0}}{16\pi^2 m_{i,\,0}} J^{(s)}(m^2_\varphi/m^2_{i,\,0},1) \quad \text{from } \varphi \text{ exchange}\,,\\
d^{(v)}_{i,\,n} &= \frac{(eq_i)^3 c^{(-)}_{i,\,n}}{4\pi^2}\frac{m_{i,\,n}}{m^2_{i,\,0}} J^{(v)}(x_{i,\,n},y_{i\,n}) \quad \text{from } A^\mu_n \text{ exchange}\,,\\
d^{(s)}_{i,\,n} &= -\frac{(eq_i)^3 c^{(+)}_{i,\,n}}{16\pi^2}\frac{m_{i,\,n}}{m^2_{i,\,0}} J^{(s)}(x_{i,\,n},y_{i\,n}) \quad \text{from } A^4_n \text{ exchange}\,,
\end{aligned}$$

where $c^{(\pm)}_{i,\,n} = \pm M_i(\mu_{i,\,n} \pm \mu_{i,\,0})/(m_{i,\,n}m_{i,\,0})$, $x_{i\,n} = (\omega_n/m_{i,\,0})^2$, $y_{i\,n} = (m_{i,\,n}/m_{i,\,0})^2$.

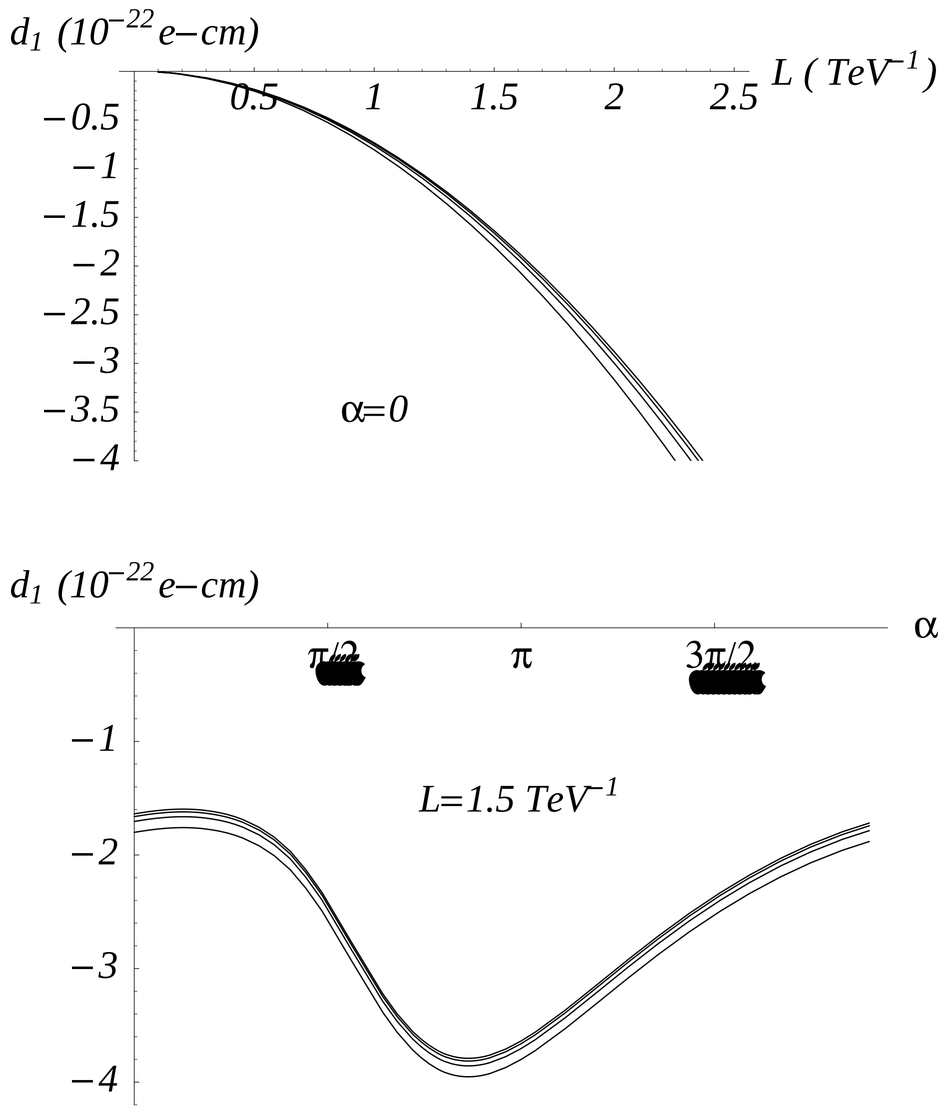
$$\begin{aligned}
J^{(s)}(x,y) &= 1 + \frac{x-y+1}{2}\ln\left(\frac{y}{x}\right) + \left(\frac{2x}{\rho} - \rho\right)\Theta\,,\\
J^{(v)}(x,y) &= -1 + \frac{y-x}{2}\ln\left(\frac{y}{x}\right) + (\rho - \cot\Theta)\Theta\,,
\end{aligned}$$

with $\rho^2 \equiv 4xy - (x+y-1)^2$ and $\tan\Theta \equiv \rho/(x+y-1)$.



The total EDM of the i-th zero-mode fermion is then

$$d_i = d_{i,\,0} + \frac{(eq_i)^3}{16\pi^2 m_{i,\,0}^2} \sum_{n\neq 0} m_{i,\,n} \left[4c_{i,\,n}^{(-)} J^{(v)}(x_{i,\,n}, y_{i\ n}) - c_{i,\,n}^{(+)} J^{(s)}(x_{i,\,n}, y_{i\ n}) \right] .$$



Parameters: $M_1 = 0.2$ TeV, $M_2 = 0.005$ TeV, $q_1 = 2/3$, $q_2 = -1/3$

Comments:

- For large n, $m_{i,\,n} c^{\pm}_{i,\,n} J^{(s/v)}(x_{i,\,n}, y_{i\,n}) \sim 1/n + \mathcal{O}(1/n^2)$, so that d_i will be finite after symmetric summation over n. It follows that the EDM is finite and therefore insensitive to the cut off of the 5D theory.

- The parameters for the plot have been adjusted in such a way that the model has the mass scales and the coupling constants of the same order as those that are present in the SM.

- For parameters adopted here (with $\alpha = 0$, i.e. for the case of spontaneous CP), when L varies from $L = 0.1\ \ \mathrm{TeV}^{-1}$ to $L = 2.5\ \ \mathrm{TeV}^{-1}$:
  - The mass of the $\psi_{1\,0}$ depends on L and varies from $\sim 37\ \ \mathrm{TeV}$ to $\sim 1.5\ \ \mathrm{TeV}$.
  - The mass of φ ranges from $\sim 27\ \ \mathrm{GeV}$ to $\sim 1\ \ \mathrm{GeV}$.

- The leading contribution to d_i comes from the φ exchange; the contribution of the non-zero modes is of opposite sign and smaller by a factor $\mathcal{O}(5)$.

5. Conclusions

- It was shown that QED in $(1+4)$-dimensional space-time with at least two fermionic fields, when the fifth dimension is compactified on a circle, leads to CP violation (CPV). Depending on fermionic boundary conditions, CPV may be either
  - explicit, by the twisted boundary conditions,
  - spontaneous, via the Hosotani mechanism, through non-zero vacuum expectation value of $A_{4\,0}$.
- Low-energy effective theory contains the light scalar φ, which has CP-violating Yukawa couplings similar to those present in the CPV 2HDM model.
- The presence of CP-violating couplings leads to a non-zero EDM, which was calculated at the one-loop level for a zero-mode fermion.
- Any 5D model of electroweak interactions compactified on a circle is expected to lead to CP violation.